ELECTRON GREEN'S FUNCTION OF GRAPHENE IN THE AHARONOV–BOHM POTENTIAL

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The dynamics of electron excitations, which are described by the Dirac equation, in the Aharonov–Bohm field has been studied. The eigenfunctions and the spectrum of the Hamiltonian of a system have been used to construct the integral formula for the electron Green's function. Possible applications of the results obtained to numerically calculate the electronic properties of graphene have been discussed.

1. Introduction

The modern condensed matter physics has experienced a large impact in its development owing to the discovery of graphene, a new allotropic modification of carbon. Graphene is a two-dimensional crystal with a hexagonal (honey-comb) atomic arrangement. It was found to be a conductor with a spectrum of electron excitations that is unusual for condensed systems, the dynamics of which can be described on the basis of the Dirac equation rather than the Schrödinger one, as it happens as a rule. Such a spectrum together with a true two-dimensionality makes graphene an object interesting from the viewpoint of its electronic properties. It is also worth noting that, since the behavior of graphene can be described in the framework of field theory, this substance turned out useful for the sake of verification of some effects in quantum electrodynamics.

A considerable body of researches dealing with graphene aims at finding the dependence of its electronic properties on the external magnetic field. In the majority of works on this topic, the field is considered as uniform. At the same time, the study of the case of non-uniform field comprises the next stage of researches [1]. Among nontrivial models for such a field, the simplest example is a field created by an infinitesimally thin solenoid, the vector-potential of which is known as the Aharonov–Bohm potential (ABP). From the viewpoint of applied researches, the closest to the ABP is the vector-potential created by the Abrikosov vortex in type-II superconductors. Since most of electronic properties of a substance can be expressed in terms of its Green's function (GF), we tried to find GF for two-dimensional electron excitations in the AB field. It was the main purpose of this work.

2. Fundamentals

By the term "Green's function" we mean the solution of the equation

$$[E - \widehat{H}(\mathbf{r})]G(E, \mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}'), \tag{1}$$

where $\hat{H}(r)$ is the Hamiltonian of the system, r the radius-vector, and E the parameter of energy, which, generally speaking, can be complex-valued. It is known that GF can be constructed in the form

$$G(E, \mathbf{r}, \mathbf{r}') = \sum_n \frac{\Psi_n(\mathbf{r})\Psi_n^\dagger(\mathbf{r}')}{E - E_n + i0}, \tag{2}$$

using the eigenfunctions $\Psi_n(\mathbf{r})$ and eigenvalues E_n of the equation

$$[\widehat{H}(\mathbf{r}) - E_n]\Psi_n(\mathbf{r}) = 0, \tag{3}$$

where n runs through the complete set of quantum numbers that enumerate the characteristic wave functions. To eliminate the ambiguity, the quantity E is appended with an infinitesimally small imaginary term. The summation operator means the discrete summation, if the spectrum is discrete, and the integration, if it is continuous. If the spectrum is characterized by both continuous

and discrete quantum numbers, the summation operator means the integration and the summation, respectively.

In the case of the Dirac equation, the Hamiltonian and GF are matrices, whereas the solution of the Dirac equation is a spinor. The Hamiltonian, which describes electron excitations in graphene, can be presented as a sum [2]

$$\widehat{H}(\mathbf{r}) = \widehat{H}(\mathbf{r}, +1) \oplus \widehat{H}(\mathbf{r}, -1). \tag{4}$$

Both Hamiltonians $\hat{H}(\mathbf{r}, \zeta)$, where the parameter $\zeta = \pm 1$ corresponds to that or another Dirac cone, have identical structures,

$$\widehat{H}(\mathbf{r}, \zeta) = -i\hbar v_{\mathrm{F}} \gamma_\zeta^0 \gamma_\zeta^j D_j + \gamma_\zeta^0 \Delta, \tag{5}$$

where v_{F} is the Fermi velocity, Δ the quantity that determines a number of excitations (it was introduced for the sake of generality), and γ-matrices look like

$$\gamma_\zeta^0 = \sigma_3,\ \gamma_\zeta^1 = i\sigma_2,\ \gamma_\zeta^2 = -i\sigma_1\zeta. \tag{6}$$

It is worth noting that the γ-matrix representation (6) is only one of many representations that can be used to describe graphene. A transition from one representation to another is executed with the use of a certain unitary transformation. The extended derivative in the Dirac equation has the form

$$D_j = \frac{\partial}{\partial x_j} - i\frac{e}{\hbar c} A_j, \tag{7}$$

where $e < 0$ is the electron charge, c the light velocity, and A_j the j-th component of the vector-potential $\mathbf{A}$ in the Cartesian coordinates. Since Hamiltonians (5) have the same matrix structure, it is convenient to consider them simultaneously, using the parameter ζ to distinguish between them. The procedure of determination of eigenfunctions is reduced to the solution of a system of differential equations that are determined by the Hamiltonians $\hat{H}(\mathbf{r}, \zeta)$.

3. Aharonov–Bohm Potential

Let us also discuss what is meant by solutions of the Dirac equation with the ABP. The analysis of solutions obtained for the potential created by an infinitesimally thin solenoid contains a number of mathematical difficulties associated with the ABP singularity at the point, where the solenoid is located [3]. As was shown in work [4], regular potentials similar to the ABP do not contain the aforementioned difficulties (in particular, the solutions of the Dirac equation are unambiguous for such potentials).

Nevertheless, to avoid the necessity to analyze unphysical solutions, we will consider solutions of the Dirac equation in a regular potential. For this purpose, the potential of a solenoidal field will be characterized by the dimensional parameter R_s, a characteristic radius of the solenoid tube. In the limiting case $R_s \to 0$, this potential transforms into the ABP. By the solutions of the Dirac equation, which depend now on the parameter R_s, we mean the solutions of the regular problem in the same limit.

4. Solutions of the Dirac Equation with the Aharonov–Bohm Potential

Consider the equation of motion in polar coordinates $\mathbf{r} = (r, \varphi)$ for a two-dimensional electron in an axially symmetric magnetic field with the vector-potential

$$\mathbf{A} = \mathbf{e}_\varphi A_\varphi(r). \tag{8}$$

Expressing the spinor in the form

$$\Psi(\mathbf{r}) = \begin{pmatrix} \psi_1(\mathbf{r}) \\ i\psi_2(\mathbf{r}) \end{pmatrix}, \tag{9}$$

we obtain the system of equations

$$C_- e^{i\zeta\varphi}\psi_1(\mathbf{r}) - \left(\frac{\partial}{\partial r} - \frac{i\zeta}{r}\frac{\partial}{\partial\varphi} - \frac{e\zeta A_\varphi}{\hbar c}\right)\psi_2(\mathbf{r}) = 0, \tag{10}$$

$$\left(\frac{\partial}{\partial r} + \frac{i\zeta}{r}\frac{\partial}{\partial\varphi} + \frac{e\zeta A_\varphi}{\hbar c}\right)\psi_1(\mathbf{r}) + C_+ e^{-i\zeta\varphi}\psi_2(\mathbf{r}) = 0, \tag{11}$$

where $C_\pm = (E \pm \Delta)/\hbar v_{\mathrm{F}}$.

Let us consider the following regularized vector-potential:

$$A_\varphi^{\mathrm{reg}}(r) = \begin{cases} 0, & r < R; \\ \frac{\hbar c}{|e|}\frac{\eta}{r}, & r > R. \end{cases} \tag{12}$$

The dimensionless parameter η, where $\eta \in [0, 1)$, controls the magnetic field flux $\eta\Phi_0$, where $\Phi_0 = 2\pi\hbar c/|e|$, through the solenoid tube. The discontinuity of the potential determines the following matching conditions for the radial spinor components $\psi_1(r)$ and $\psi_2(r)$, the upper and lower ones, respectively:

$$\psi_1'(R_s + 0) - \psi_1'(R_s - 0) = \frac{\zeta\eta}{R_s}\psi_1(R_s),$$

$$\psi_1(R_s + 0) = \psi_1(R_s - 0), \tag{13}$$

$$\psi_2'(R_s+0)-\psi_2'(R_s-0)=-\frac{\zeta\eta}{R_s}\psi_2(R_s),$$

$$\psi_2(R_s+0)=\psi_2(R_s-0), \tag{14}$$

where the prime means the differentiation with respect to r. To find the functions $\psi_1(r)$ and $\psi_2(r)$, it is necessary to pass from the system of equations (10), (11) to the system of second-order differential equations: one equation for each spinor component separately. In this case, the new system of equations and the corresponding boundary conditions actual for $\zeta = 1$ transform into another system of equations and boundary conditions actual for $\zeta = -1$, provided that the permutation $\psi_1(r) \leftrightarrow \psi_2(r)$ is made. Therefore, without any loss of generality, it is enough to examine the problem in the case $\zeta = 1$. We introduce the notation $E(\mathbf{k}) = \sqrt{(\hbar v_{\rm F})^2 k^2 + \Delta^2}$, where $\mathbf{k}$ is a two-dimensional wave vector, and k is its absolute value. The normalized solutions of the Dirac equation with the normalizing coefficient

$$C_m = \sqrt{\frac{k}{4\pi E(\mathbf{k})}}\, e^{im\varphi} \tag{15}$$

have the following forms in the limit $R_s \to 0$: for positive energies ($E = E(\mathbf{k})$),

$$\Psi_m^{(+)}(\mathbf{r}) = C_m \begin{pmatrix} e^{-i\varphi}\sqrt{E(\mathbf{k})+\Delta}\, J_{|m+\eta-1|}(kr) \\ \pm i\sqrt{E(\mathbf{k})-\Delta}\, J_{|m+\eta|}(kr) \end{pmatrix}, \tag{16}$$

and, for negative energies ($E = -E(\mathbf{k})$),

$$\Psi_m^{(-)}(\mathbf{r}) = C_m \begin{pmatrix} e^{-i\varphi}\sqrt{E(\mathbf{k})-\Delta}\, J_{|m+\eta-1|}(kr) \\ \mp i\sqrt{E(\mathbf{k})+\Delta}\, J_{|m+\eta|}(kr) \end{pmatrix}. \tag{17}$$

The upper signs before the second terms in both formulas correspond to positive m-values, and the lower signs to negative ones. At $m = 0$, we obtain: for positive energies ($E = E(\mathbf{k})$),

$$\Psi_0^{(+)}(\mathbf{r}) = C_0 \begin{pmatrix} e^{-i\varphi}\sqrt{E(\mathbf{k})+\Delta}\, J_{1-\eta}(kr) \\ -i\sqrt{E(\mathbf{k})-\Delta}\, J_{-\eta}(kr) \end{pmatrix}, \tag{18}$$

and, for negative ones ($E = -E(\mathbf{k})$),

$$\Psi_0^{(-)}(\mathbf{r}) = C_0 \begin{pmatrix} e^{-i\varphi}\sqrt{E(\mathbf{k})-\Delta}\, J_{1-\eta}(kr) \\ i\sqrt{E(\mathbf{k})+\Delta}\, J_{-\eta}(kr) \end{pmatrix}. \tag{19}$$

Note that, at $m = 0$, the system of Dirac equations has two quadratically integrated, linearly independent solutions. This ambiguity is associated with the singular behavior of the ABP. The consideration of the regularized potential (12) does not meet those difficulties and gives rise to an unambiguous solution in this case.

5. Construction of Green's Function for the System

The electron GF for graphene in the ABP is determined by its diagonal elements. Really, from Eq. (1), the following equalities for non-diagonal matrix elements of GF are obtained (one should bear in mind that, owing to the aforesaid, the calculations given below correspond to the case $\zeta = 1$):

$$G_{12}(E,\mathbf{r},\mathbf{r}') = \frac{e^{-i\varphi}}{iC_-}\left(\frac{\partial}{\partial r} - i\frac{1}{r}\frac{\partial}{\partial\varphi} + \frac{\eta}{r}\right) G_{22}(E,\mathbf{r},\mathbf{r}'), \tag{20}$$

$$G_{21}(E,\mathbf{r},\mathbf{r}') = \frac{e^{i\varphi}}{iC_+}\left(\frac{\partial}{\partial r} + i\frac{1}{r}\frac{\partial}{\partial\varphi} - \frac{\eta}{r}\right) G_{11}(E,\mathbf{r},\mathbf{r}'). \tag{21}$$

Let us introduce the notation $q = \sqrt{E^2-\Delta^2}/\hbar v_{\rm F}$. Then, the diagonal elements of GF can be written down as follows:

$$G_{11}(E,\mathbf{r},\mathbf{r}') = \frac{(E+\Delta)}{2\pi(\hbar v_{\rm F})^2}\int\limits_0^\infty \frac{kdk\, G_{11}(\mathbf{r},\mathbf{r}')}{q^2-k^2+i0{\rm sgn}E}, \tag{22}$$

$$G_{22}(E,\mathbf{r},\mathbf{r}') = \frac{(E-\Delta)}{2\pi(\hbar v_{\rm F})^2}\int\limits_0^\infty \frac{kdk\, G_{22}(\mathbf{r},\mathbf{r}')}{q^2-k^2+i0{\rm sgn}E}, \tag{23}$$

where

$$G_{11}(\mathbf{r},\mathbf{r}') = \sum_{m=-\infty}^{\infty} e^{im(\varphi-\varphi')} J_{|m+\eta|}(kr) J_{|m+\eta|}(kr'), \tag{24}$$

$$G_{22}(\mathbf{r},\mathbf{r}') = G_{11}(\mathbf{r},\mathbf{r}') - \sum_{\alpha=\pm1} {\rm sgn}\alpha\; J_{\alpha\eta}(kr) J_{\alpha\eta}(kr'). \tag{25}$$

So, to find components (22) and (23), it is necessary to calculate the expressions

$$F_1(q,\mathbf{r},\mathbf{r}') = \int\limits_0^\infty \frac{kdk}{q^2-k^2+i0{\rm sgn}E}\times$$

$$\times \sum_{m=-\infty}^{\infty} e^{im(\varphi-\varphi')} J_{|m+\eta|}(kr) J_{|m+\eta|}(kr'), \tag{26}$$

and

$$F_2(q,\mathbf{r},\mathbf{r}') = \int\limits_0^{\infty} \frac{kdk}{q^2-k^2+i0\mathrm{sgn}E} \times$$

$$\times [J_{-\eta}(kr)J_{-\eta}(kr') - J_{\eta}(kr)J_{\eta}(kr')]. \tag{27}$$

Let us calculate quantity (26), by using the method proposed in work [5]. Namely, we consider the analytical continuation of the function, which is fixed by the rule $q+i0\mathrm{sgn}E \to z = iQ\mathrm{sgn}E$:

$$F_1(Q,\mathbf{r},\mathbf{r}') = -\int\limits_0^{\infty} \frac{kdk}{Q^2+k^2} \sum_{m=-\infty}^{\infty} e^{im(\varphi-\varphi')} \times$$

$$\times J_{|m+\eta|}(kr) J_{|m+\eta|}(kr'). \tag{28}$$

The values of the difference $\theta = \varphi - \varphi'$ fall within the interval $[-\pi, \pi]$. The absolute value of θ determines the smallest angles between the radius-vectors $\mathbf{r}$ and $\mathbf{r}'$, whereas its sign reflects their relative position. The angles $\theta = \pm\pi$ correspond to the same arrangement of radius-vectors. Let us calculate every integral in the sum, by using the equality [6]

$$\int\limits_0^{\infty} \frac{kdk}{Q^2+k^2} J_{|m+\eta|}(kr) J_{|m+\eta|}(kr') =$$

$$= I_{|m+\eta|}(Qr_-) K_{|m+\eta|}(Qr_+), \tag{29}$$

where r_- and r_+ are the smallest and largest, respectively, values of the radii r and r'. Let us consider the case where $r_- = r$ and $r_+ = r'$ (if $r_- = r'$ and $r_+ = r$, the corresponding final GF can be obtained, by making the permutation $r \leftrightarrow r'$ in the GF derived with regard for the first assignments). Let us apply the integral representations of cylindrical functions [7]

$$I_{\nu}(z) = \frac{1}{2\pi i} \int\limits_C e^{z\cosh\omega - \nu\omega} d\omega, \tag{30}$$

$$K_{\nu}(z) = \frac{1}{2} \int\limits_{-\infty}^{\infty} e^{-z\cosh v - \nu v} dv, \tag{31}$$

where the path of integration C starts and ends at the points $-i\pi+\infty$ and $i\pi+\infty$, respectively. Let us rewrite sum (28), by removing the absolute values in the subscripts of cylindrical functions:

$$F_1(Q,\mathbf{r},\mathbf{r}') = -\frac{1}{4\pi i} \int\limits_{-\infty}^{\infty} dv \int\limits_C d\omega S(Q,\mathbf{r},\mathbf{r}') \times$$

$$\times e^{Qr\,\mathrm{ch}\,\omega - Qr'\,\mathrm{ch}\,v}, \tag{32}$$

where

$$S(Q,\mathbf{r},\mathbf{r}') = \sum_{m=0}^{\infty} e^{im\theta} e^{-(m+\eta)(v+\omega)} +$$

$$+ \sum_{m=1}^{\infty} e^{-im\theta} e^{-(m-\eta)(v+\omega)}. \tag{33}$$

To calculate the function $F_1(Q,\mathbf{r},\mathbf{r}')$, the following trick is used. Consider the expression

$$F_1(Q,\mathbf{r},\mathbf{r}',a) = -\frac{1}{4\pi i} \int\limits_{-a}^{\infty} dv \int\limits_C d\omega S(Q,\mathbf{r},\mathbf{r}') \times$$

$$\times e^{Qr\,\mathrm{ch}\,\omega - Qr'\,\mathrm{ch}\,v}, \tag{34}$$

where a is a positive real number. The quantity $F_1(Q,\mathbf{r},\mathbf{r}')$ is sought as a limit of $F_1(Q,\mathbf{r},\mathbf{r}',a)$ as $a \to \infty$. Let us choose the path of integration C that satisfies the condition $\mathrm{Re}\,\omega > a$. The latter makes it possible to sum up the series in Eq. (33) taking the following properties of geometrical progressions:

$$S(Q,\mathbf{r},\mathbf{r}') = \frac{e^{-\eta(v+\omega)}}{1-e^{i\theta-(\omega+v)}} - \frac{e^{\eta(v+\omega)}}{1-e^{i\theta+(\omega+v)}}. \tag{35}$$

It is easy to verify that, at the substitutions $v \to -v$ in the first and $\omega \to -\omega$ in the second terms, they become identical to each other. Therefore, at such a change of variables, function (34) can be written down in the form

$$F_1(Q,\mathbf{r},\mathbf{r}',a) = \frac{1}{4\pi i} \left(\int\limits_{-\infty}^{a} dv \int\limits_C d\omega + \int\limits_{-a}^{\infty} dv \int\limits_{C'} d\omega \right) \times$$

$$\times \frac{e^{\eta(v-\omega)}}{e^{i\theta+v-\omega}-1} e^{Q(r\,\mathrm{ch}\,\omega - r'\,\mathrm{ch}\,v)}, \tag{36}$$

where C' is the path symmetric to the path C with respect to the coordinate origin. Let us consider the limit $a \to \infty$ and use the identity

$$\int\limits_{C} d\omega f(\omega) + \int\limits_{C'} d\omega f(\omega) =$$

$$= \int\limits_{i\pi-\infty}^{i\pi+\infty} d\omega f(\omega) + \int\limits_{-i\pi+\infty}^{-i\pi-\infty} d\omega f(\omega) + \oint_{C''} d\omega f(\omega), \tag{37}$$

where C'' is a rectangular path, the length of which exceeds $2a$ and the width is equal to $2\pi i$, and which is traced counter-clockwise, i.e.

$$F_1(Q, \mathbf{r}, \mathbf{r}') = I_1 + I_2, \tag{38}$$

where

$$I_1 = -\frac{1}{4\pi i} \int\limits_{-\infty}^{\infty} dv \oint d\omega \frac{e^{\eta(v-\omega)}}{1 - e^{i\theta+v-\omega}} e^{Q(r\cosh\omega - r'\cosh v)}, \tag{39}$$

$$I_2 = \frac{\sin\pi\eta}{2\pi} \int\limits_{-\infty}^{\infty} dv \int\limits_{-\infty}^{\infty} d\omega \frac{e^{\eta(v-\omega)}}{1 + e^{i\theta+v-\omega}} e^{-Q(r\cosh\omega + r'\cosh v)}. \tag{40}$$

The calculation of integral I_1 does not meet any difficulty in the case $\theta \neq \pi$, because there is only one integrand pole within the path C''. In the case $\theta = \pi$, one must take into account that two poles of the integrand are located on the path of integration. Calculating the path integral and using the integral representation of the McDonald function, we obtain

$$I_1 = \begin{cases} -e^{-i\eta\theta} K_0(QR), & \theta \neq \pm\pi; \\ -\cos\pi\eta K_0(QR), & \theta = \pm\pi, \end{cases} \tag{41}$$

where $R = |\mathbf{r} - \mathbf{r}'|$.

Proceeding from the integral representation of the McDonald function, integral I_2 is reduced to a one-fold one, so that

$$I_2 = \frac{\sin\pi\eta}{\pi} \int\limits_{-\infty}^{\infty} dx \frac{e^{\eta x}}{1 + e^{i\theta+x}} K_0(QR_x), \tag{42}$$

where $R_x = (r^2 + r'^2 + 2rr'\cosh x)^{1/2}$. Even in simple cases, this integral gives rise to rather cumbersome expressions, the application of which does not simplify calculations, where GF is required. Therefore, the explicit form of integral (42) or its asymptotic behavior has to be studied separately in every problem. Hence, let us preserve this term in its integral form.

The inverse change $Q \to -iq\,\mathrm{sgn}E$ allows the ultimate expression to be written down, namely:
if $\theta \neq \pi$,

$$F_1(q, \mathbf{r}, \mathbf{r}') = \mp\frac{i\pi}{2} e^{-i\eta\theta} H_0^{(j)}(qR) \pm$$

$$\pm\frac{i\sin\pi\eta}{2} \int\limits_{-\infty}^{\infty} dx \frac{e^{\eta x}}{1 + e^{i\theta+x}} H_0^{(j)}(qR_x), \tag{43}$$

where $j = 1, 2$ and the upper and lower signs correspond to $E > 0$ and $E < 0$, respectively;
if $\theta = \pm\pi$,

$$F_1(q, \mathbf{r}, \mathbf{r}') = \mp\frac{i\pi}{2} \cos\pi\eta H_0^{(j)}(qR) \pm$$

$$\pm\frac{i\sin\pi\eta}{2} \int\limits_{-\infty}^{\infty} dx \frac{e^{\eta x}}{1 - e^{x}} H_0^{(j)}(qR_x), \tag{44}$$

where the choice of signs and j-values is the same as that in formula (43), and the integral is taken in the principal value sense. The last equality was obtained with the use of the identities [8]

$$K_\nu(iz) = -\frac{i\pi}{2} e^{-i\pi\nu/2} H_\nu^{(2)}(z),$$

$$K_\nu(-iz) = \frac{i\pi}{2} e^{i\pi\nu/2} H_\nu^{(1)}(z). \tag{45}$$

Now, let us derive the expression for $F_2(q, \mathbf{r}, \mathbf{r}')$, by considering its analytical continuation

$$F_2(Q, \mathbf{r}, \mathbf{r}') = \int\limits_0^{\infty} \frac{kdk}{Q^2 + k^2} \sum_{\alpha=\pm1} \mathrm{sgn}\alpha\, J_{\alpha\eta}(kr) J_{\alpha\eta}(kr'). \tag{46}$$

Using formula (29) and the expression for the McDonald function $K_\nu(z)$ in terms of the modified Bessel function $I_\nu(z)$ [8],

$$K_\nu(z) = \frac{\pi}{2\sin\pi\nu} [I_{-\nu}(z) - I_\nu(z)], \tag{47}$$

we obtain the expression

$$F_2(Q, \mathbf{r}, \mathbf{r}') = -\frac{2\sin\pi\eta}{\pi} K_\eta(Qr) K_\eta(Qr'). \tag{48}$$

Analogously to the first expression, we find

$$F_2(q,\mathbf{r},\mathbf{r}') = \frac{\pi \sin \pi\eta}{2} e^{\pm i\pi\eta} H_\eta^{(j)}(qr) H_\eta^{(j)}(qr'), \qquad (49)$$

where $j = 1, 2$, and the upper and lower signs correspond to $E > 0$ and $E < 0$, respectively.

The formulas derived for the functions $F_1(q,\mathbf{r},\mathbf{r}')$ and $F_2(q,\mathbf{r},\mathbf{r}')$ are used to construct GF. Hence, this allows us to draw conclusion that the problem of electron GF derivation for graphene, which contains a solenoidal field–in other words, the ABP,–has been actually solved.

6. Conclusion

The obtained GF differs substantially from that for a free Dirac particle [9]. Such a difference is explained by the long-range character of the ABP. Consequently, provided that there are structures in graphene, which are characterized by a magnetic field flux, it is natural to use the GF obtained above as the main approximation, while calculating the electron properties of such systems.

Two examples of such systems could be mentioned. The first example includes systems with magnetic impurities (see, e.g., work [10], where the influence of magnetic impurities on the conductivity in a two-dimensional electron gas was studied). The second example includes graphene with defects of its crystal lattice. As was shown in works [11, 12], the equations that govern the dynamics of electron excitations in graphene with defects are equivalent to the equations that describe ideal graphene in the presence of additional ABPs, the centers of which are positioned at the defect locations. This fact allows one to determine the contribution of defects in graphene to graphene electron properties in the framework of the field-theory approach.

The fabrication of a two-dimensional electron system with a magnetic vortex is not difficult now [13, 14]. It makes possible to determine the efficiency of using the corresponding GF in the calculations of electron properties of graphene with the help of scanning tunnel microscopy [15].

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ЕЛЕКТРОННА ФУНКЦІЯ ГРІНА ГРАФЕНА В ПОТЕНЦІАЛІ ААРОНОВА–БОМА

А.О. Слободенюк

Р е з ю м е

Розглянуто динаміку електронних збуджень (які описуються рівнянням Дірака) у графені в полі Ааронова–Бома. Власні функції і спектр гамільтоніана системи використовуються для побудови електронної функції Гріна. Показано, що вона може бути представлена в інтегральній формі. Обговорено можливе застосування отриманих результатів для чисельних розрахунків електронних властивостей графена.