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## PHENOMENOLOGICAL THEORY OF BOUNDARY FRICTION IN THE STICK-SLIP MODE

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A deterministic theory describing the melting of an ultrathin lubricant film between two atomically smooth solid surfaces has been developed. The lubricant state is described by introducing a parameter of excess volume that arises owing to the solid structure chaotization at its melting. The thermodynamic and shear kinds of melting are described consistently. The dependences of the stationary friction force on the lubricant temperature and the shear velocity of rubbing surfaces that move with respect to each other with a constant velocity have been analyzed. In the framework of a simple tribological model, the stick-slip mode of friction, when the lubricant periodically melts and solidifies, has been described. The influence of velocity, temperature, and load on the stick-slip friction has been analyzed. A qualitative comparison between the results obtained and experimental data has been carried out.

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### 1. Introduction

If the thickness of a lubricant material does not exceed 10 atomic layers, there emerges a boundary friction mode [1]. Experiments show that such thin lubricant layer demonstrates abnormal properties in comparison with those in the case of bulk liquids or if the layer thickness is only a few times larger [2]. In particular, the intermittent (stick-slip) motion inherent to dry friction is observed [2, 3]. Such a mode is explained as a solidification induced by squeezing by rubbing surfaces, followed by an abrupt melting when shear stresses exceed the yield stress (“shear melting”).

There are a few phenomenological models that partially explain the results experimentally observed. These include, e.g., thermodynamic [4, 5], mechanistic [6], and

synergetic [7, 8] models. They are of either a deterministic [5–7] or a stochastic [9–11] origin. Methods of molecular dynamics are also applied [12–14]. It turns out that the lubricant can operate in a number of kinetic modes, and transitions between them take place in the course of friction, which is responsible for the intermittent motion [2]. Theoretical researches [9] gave rise to three modes of friction: a slip mode at low shear velocities, a regular stick-slip mode, and a slip mode at high shear velocities. Numerous experiments confirm the existence of those modes [1–3, 15].

In works [7, 8], in the framework of the Lorentz model approximating a viscoelastic medium, an approach was developed, according to which the transition of an ultrathin lubricant film from a solid-like into a liquid-like state occurs owing to the thermodynamic and shear melting. The processes invoked by the self-organization of shear stress and strain fields, as well as the lubricant temperature, were described with regard for the additive noises of those quantities [16, 17] and correlated temperature fluctuations [18]. It was shown that, in the case of additive noises, a self-similar mode of lubricant melting, in which the series expansion of the stress in time acquires multifractal properties, is established [19, 20]. The reasons for the jump-like melting and hysteresis phenomena, which were observed in experiments [21–23], were examined in works [24–26], and the conditions for the indicated features to be realized were also determined with regard for the deformation defect of the shear modulus. In the framework of this model, the periodic stick-slip friction mode was also described [11, 27]; however, it has

a stochastic component and, hence, can be realized only provided that there are fluctuations in the system. Another shortcoming of this model is the fact that it does not involve the load applied to the friction surfaces; in addition, a number of approximations were made, while deriving the basic equations [7].

A thermodynamic theory proposed in work [5] is based on the expansion of the free energy of the system into a power series in the parameter  $f$  which is an excess volume [28, 29] that emerges owing to the formation of a defect structure in the lubricant at its melting. The liquid-like state is interpreted as a section of plastic flow on the loading diagram and is characterized by the presence of defects in the lubricant [4]. In work [5], to describe strongly nonequilibrium processes that take place at the slipping of two solids that rub against each other, being separated by a lubricant layer, an approach based on the Landau theory of phase transitions [30–34] was used. However, work [5] was devoted to the study of the lubricant melting, when friction surfaces are moved relative to each other at a constant velocity, so it does not describe the stick-slip regime of motion observed in experiments [2]. The proposed work continues the consideration started in work [5]. It aims at studying the periodic mode of stick-slip friction in the framework of the model using a mechanical equivalent for the tribological system, because numerous experimental results evidence just the periodic character of stick-slip motion [2].

## 2. Basic Equations

If the thickness of a melting lubricant is less than 10 molecular layers, the stationary states, in which the lubricant operates, are not thermodynamic phases, but represent kinetic modes of friction – maybe, several ones. In this case, the terms solid-like and liquid-like phases are used rather than solid and liquid ones. Whether such lubricants melt or not is judged by the increase of their volume [12] and the diffusion coefficient [12, 13, 35, 36]. Since, it is the volume that is an experimentally observable quantity, let us introduce a parameter  $f$  to describe the lubricant state. The physical meaning of this parameter is an excess volume [28, 29] that appears owing to the stochastization of a solid structure in the course of its melting. As the parameter  $f$  grows, the defect concentration in the lubricant increases, and the lubricant, governed by the transport of those defects under the action of an applied tangential stress, goes into the kinetic mode of plastic flow (the liquid-like phase).

Let us write down the expansion for the free energy density in the form which includes the contributions

made by the elastic components of shear strains  $\varepsilon_{ij}^e$  and the entropy  $s$  [5]:

$$\Phi = \Phi_0^* + \frac{1}{2}\lambda (\varepsilon_{ii}^e)^2 + \mu (\varepsilon_{ij}^e)^2 - \alpha s^2 + \frac{c}{2} (\nabla f)^2 - \varphi_0 f + \frac{1}{2}\varphi_1 f^2 - \frac{1}{3}\varphi_2 f^3 + \frac{1}{4}\varphi_3 f^4, \quad (1)$$

where  $\Phi_0^*$ ,  $\lambda$ ,  $\mu$ ,  $\alpha$ ,  $c$ ,  $\varphi_0$ ,  $\varphi_1$ ,  $\varphi_2$ , and  $\varphi_3$  are expansion constants. In turn,

$$\varphi_0 = \varphi_0^* + \frac{1}{2}\bar{\lambda} (\varepsilon_{ii}^e)^2 + \bar{\mu} (\varepsilon_{ij}^e)^2 + \alpha_\varphi s. \quad (2)$$

Elastic stresses are taken into account to an accuracy of quadratic contributions, expressing them in terms of the strain tensor invariants  $\varepsilon_{ii}^e$ ,  $(\varepsilon_{ij}^e)^2 = \varepsilon_{ij}^e \varepsilon_{ji}^e$ , where the repeated indices mean summation. In this case, the first invariant is the trace of the strain tensor,  $\varepsilon_{ii}^e = \varepsilon_1^e + \varepsilon_2^e + \varepsilon_3^e$ , whereas the second one is determined by the expression [37]

$$(\varepsilon_{ij}^e)^2 \equiv (\varepsilon_{ii}^e)^2 - 2I_2 = (\varepsilon_1^e + \varepsilon_2^e + \varepsilon_3^e)^2 - 2(\varepsilon_1^e \varepsilon_2^e + \varepsilon_1^e \varepsilon_3^e + \varepsilon_2^e \varepsilon_3^e) = (\varepsilon_1^e)^2 + (\varepsilon_2^e)^2 + (\varepsilon_3^e)^2. \quad (3)$$

According to expression (1), elastic stresses

$$\sigma_{ij}^e = \frac{\partial \Phi}{\partial \varepsilon_{ij}^e} = \lambda \varepsilon_{ii}^e \delta_{ij} + 2\mu \varepsilon_{ij}^e - (\bar{\lambda} \varepsilon_{ii}^e \delta_{ij} + 2\bar{\mu} \varepsilon_{ij}^e) f \quad (4)$$

arise in the lubricant. Expression (4) can be written in the form of the effective Hooke law

$$\sigma_{ij}^e = 2\mu_{\text{eff}} \varepsilon_{ij}^e + \lambda_{\text{eff}} \varepsilon_{ii}^e \delta_{ij} \quad (5)$$

with effective elastic parameters<sup>1</sup>

$$\mu_{\text{eff}} = \mu - \bar{\mu} f, \quad (6)$$

$$\lambda_{\text{eff}} = \lambda - \bar{\lambda} f, \quad (7)$$

which decrease at the melting, if the parameter  $f$  grows.

It is not hard to demonstrate that the invariants are determined as follows:

$$\varepsilon_{ii}^e = \frac{n}{\lambda_{\text{eff}} + \mu_{\text{eff}}}, \quad (8)$$

<sup>1</sup> It is necessary to take  $\mu_{\text{eff}} = 0$ , if  $f > \mu/\bar{\mu}$ , and  $\lambda_{\text{eff}} = 0$ , if  $f > \lambda/\bar{\lambda}$ .

$$(\varepsilon_{ij}^e)^2 = \frac{1}{2} \left[ \left( \frac{\tau}{\mu_{\text{eff}}} \right)^2 + (\varepsilon_{ii}^e)^2 \right], \quad (9)$$

where  $n$  and  $\tau$  are the normal and tangential (shear), respectively, stress components which are induced in the lubricant by the rubbing surfaces.<sup>2</sup> Expressions (8) and (9) reflect relations between tensor components and their invariants in the linear theory of elasticity [37].

Let us write down an evolution equation for the nonequilibrium parameter  $f$  as the Landau–Khalatnikov equation,

$$\tau_f \dot{f} = -\frac{\partial \Phi}{\partial f}, \quad (10)$$

where the relaxation time  $\tau_f$  is introduced. In an explicit form, the equation reads

$$\tau_f \frac{\partial f}{\partial t} = -c \nabla^2 f + \varphi_0 - \varphi_1 f + \varphi_2 f^2 - \varphi_3 f^3 - \frac{n^2 (\bar{\lambda} + \bar{\mu})}{(\lambda_{\text{eff}} + \mu_{\text{eff}})^2}, \quad (11)$$

where the presence of the last term is associated with the fact that invariants (8) and (9) depend on the excess volume  $f$ . In work [5], this term was neglected. However, taking it into account is important, because, in this case, we can describe the influence of an external pressure on the melting.

The lubricant temperature is determined from the free energy of the system,

$$T = -\frac{\partial \Phi}{\partial s} = 2\alpha s + \alpha_\varphi f. \quad (12)$$

Therefore, the entropy is a function of the temperature and the excess volume. In this case, the free energy (1) is also a function of the temperature and the volume.

To describe the processes of heat exchange between the lubricant and its environment, let us introduce the temperature of friction surfaces,  $T_e$  [7]. If the medium is heated up non-uniformly, the heat equation is an ordinary continuity equation [38]

$$T \frac{\partial s}{\partial t} = \kappa \nabla^2 T, \quad (13)$$

where the heat conductivity coefficient  $\kappa$  is adopted to be constant. For the normal component  $\nabla_z^2$ , the approximation  $\kappa \nabla_z^2 T \approx (\kappa/h^2)(T_e - T)$ , where  $h$  is the lubricant

thickness, can be used with a sufficient accuracy. Then, Eq. (13) reads

$$\frac{\partial s}{\partial t} = \frac{\kappa}{h^2} \left( \frac{T_e}{T} - 1 \right) + \frac{\kappa}{T} \nabla_{xy}^2 T, \quad (14)$$

where the quantity  $h^2/\kappa$  plays the role of the relaxation time, during which the temperatures becomes uniform across the lubricant thickness due to the heat conductivity.

Let us take advantage of the Debye approximation, which couples elastic,  $\varepsilon_{ij}^e$ , and plastic,  $\varepsilon_{ij}^{\text{pl}}$ , strains [4],

$$\dot{\varepsilon}_{ij}^{\text{pl}} = \frac{\varepsilon_{ij}^e}{\tau_\varepsilon}, \quad (15)$$

where  $\tau_\varepsilon$  is the Maxwell time of internal stress relaxation. The total strain in the layer is determined as

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^{\text{pl}}. \quad (16)$$

This strain governs the motion velocity of the upper block,  $V_{ij}$ , according to the following relation [39]:

$$V_{ij} = h \dot{\varepsilon}_{ij} = h(\dot{\varepsilon}_{ij}^e + \dot{\varepsilon}_{ij}^{\text{pl}}). \quad (17)$$

Three last relations yield an expression for the elastic component of a shear strain [5],

$$\tau_\varepsilon \dot{\varepsilon}_{ij}^e = -\varepsilon_{ij}^e + \frac{V_{ij} \tau_\varepsilon}{h}. \quad (18)$$

Below, to simplify the analysis, we consider a uniform system, so that  $\nabla \equiv 0$  has to be put in relations (1), (11), and (14).

### 3. Friction Force

The system of kinetic equations (11), (14), and (18), taking definitions (2), (5)–(9), and (12) into account, is closed and can be used to study the melting kinetics. In this section, we consider the stationary modes of friction. According to Eqs. (14) and (18), the stationary values of lubricant temperature  $T_0$  and the elastic component of the shear strain  $\varepsilon_{ij0}^e$  are established in due course,

$$T_0 = T_e, \quad \varepsilon_{ij0}^e = \frac{V_{ij} \tau_\varepsilon}{h}. \quad (19)$$

To find the stationary states of all quantities, it is necessary to numerically solve the evolution equation (11), using formulas (2), (5)–(9), and determining the current entropy from Eq. (12), taken at  $T = T_e$ , and the strain from Eq. (19).

<sup>2</sup> The shear stresses  $\tau$  are determined by expression (5) taken at  $i \neq j$ , i.e., at  $\delta_{ij} = 0$ . In the case  $\mu_{\text{eff}} = 0$ , the term  $\tau/\mu_{\text{eff}}$  in Eq. (9) should be replaced, in accordance with Eq. (5), by  $2\varepsilon_{ij}^e$ .

In experimental works, the dependences of the friction force on the shear velocity, thickness of a lubricant layer, and normal pressure are often reported [1–3, 21–23]. In this section, we analyze the influence of the lubricant temperature and the shear velocity on the friction force.

In the lubricant, besides elastic stresses,  $\sigma_{ij}^e$ , the viscous ones,  $\sigma_{ij}^{\text{visc}}$ , also emerge. The total stress in the layer is a sum of those two contributions,

$$\sigma_{ij} = \sigma_{ij}^e + \sigma_{ij}^{\text{visc}}. \quad (20)$$

The full friction force is determined in a standard way,

$$F_{ij} = \sigma_{ij} A, \quad (21)$$

where  $A$  is the area of contacting surfaces. The viscous stresses in the layer are expressed by the formula [39]

$$\sigma_{ij}^{\text{visc}} = \frac{\eta_{\text{eff}} V_{ij}}{h}, \quad (22)$$

where  $\eta_{\text{eff}}$  is the effective viscosity which can be found only experimentally and in the boundary regime [39, 40],

$$\eta_{\text{eff}} \sim (\dot{\epsilon}_{ij})^\gamma, \quad (23)$$

where  $\gamma < 0$  for pseudo-plastic lubricants, and  $\gamma > 0$  for dilatant ones [40]. Taking Eqs. (17) and (23) into account, expression (22) for viscous stresses reads

$$\sigma_{ij}^{\text{visc}} = \left( \frac{V_{ij}}{h} \right)^{\gamma+1}. \quad (24)$$

After substituting Eqs. (20) and (24) into Eq. (21), we obtain the sought expression for the friction force [5]<sup>3</sup>

$$F_{ij} = \left[ \sigma_{ij}^e + \text{sgn}(V_{ij}) \left( \frac{|V_{ij}|}{h} \right)^{\gamma+1} \right] A, \quad (25)$$

where the quantity  $\sigma_{ij}^e$  is given by formula (5) at  $i \neq j$ .

As experimental friction surfaces, atomically smooth mica surfaces are usually used, whereas quasispherical molecules of octamethylcyclotetrasiloxane (OMCTS) and linear chain molecules of either tetradecane or hexadecane play the role of a lubricant [2, 23]. The indicated experiments are carried out under the following conditions: the thickness of a lubricant layer  $h \approx 10^{-9}$  m, the contact area  $A \approx 3 \times 10^{-9}$  m<sup>2</sup>, and the load on the upper friction surface lies within the interval  $L = (2 \div 60) \times 10^{-3}$  N, which corresponds to normal stress

$n = -L/A = -(6.67 \div 200) \times 10^5$  Pa. The friction force is  $F \approx (2 \div 40) \times 10^{-3}$  N. In the course of the cited experimental works, it was revealed that the lubricant melts, if either the temperature exceeds the critical value,  $T_e > T_{c0} \approx 300$  K, or the shear velocity  $V$  is larger than  $V_c \approx 400$  nm/s. Those thresholds can substantially vary, depending on the lubricant type and the experimental geometry.

In the model concerned, according to experimental data, the following values of theoretical constants were selected [5]:  $\Phi_0^* = 20$  J/m<sup>3</sup>,  $\lambda = 2 \times 10^{11}$  Pa,  $\bar{\lambda} = 10^8$  Pa,  $\mu = 4.1 \times 10^{11}$  Pa,  $\bar{\mu} = 4 \times 10^{11}$  Pa,  $\varphi_0^* = 5$  J/m<sup>3</sup>,  $\varphi_1 = 1100$  J/m<sup>3</sup>,  $\varphi_2 = 2700$  J/m<sup>3</sup>,  $\varphi_3 = 2070$  J/m<sup>3</sup>,  $\alpha = 0.055$  K<sup>2</sup>·m<sup>3</sup>/J,  $\alpha_\varphi = 0.05$  K,  $h = 10^{-9}$  m,  $\tau_f = 1$  Pa·s, and  $\tau_\varepsilon = 10^{-8}$  s. Note that the time of the excess volume relaxation  $\tau_f$  has the dimension of viscosity. Actually, this means that the time of the establishment of a stationary friction mode increases with the effective viscosity of a a lubricant.

Dependence (25) is shown in Fig. 1. Figure 1,*a* illustrates the fact that the friction force decreases, as the temperature grows. Let us consider curve 2 in more details. At first, the excess volume monotonously grows together with the temperature. On the contrary, the effective shear modulus  $2\mu_{\text{eff}}$  (see Eq. (6)) decreases, which results in a decrease of the elastic component of shear stresses (5) and, accordingly, in a reduction of the friction force (25). When the temperature exceeds the critical value,  $T_e > T_{c0}$ , the magnitude of excess volume  $f$  increases in a jump-like manner, and the lubricant melts, which causes a drastic decrease of the total friction force. If the lubricant temperature decreases further, the lubricant solidifies already at a lower value  $T_e = T_c^0$ . The temperature dependence has a hysteretic character, which corresponds to the first-order phase transitions. According to Fig. 1,*a*, if the shear velocity increases, the lubricant melts at a lower temperature. At a velocity higher than a certain critical value, the lubricant is always liquid-like, irrespective of the temperature (curve 4), and the friction force decreases together with the temperature owing to a reduction of the shear modulus (lubricant fluidization).

Hence, at low temperatures ( $T_e < T_c^0$ ), the potential  $\Phi(f)$  has one minimum which corresponds to a stationary state with small  $f$  (a solid-like lubricant). In the temperature interval  $T_c^0 < T_e < T_{c0}$ , two minima of  $\Phi(f)$  coexist. However, the system cannot go into the state that corresponds to the second minimum, because those minima are separated by a potential barrier. If the temperature increases further,  $T_e > T_{c0}$ , the separating maximum disappears, and the lubricant, following the

<sup>3</sup> Here, we use both the sign function,  $\text{sgn}(x)$ , and the absolute value of shear velocity,  $|V_{ij}|$ , because the velocity can accept negative values as well.

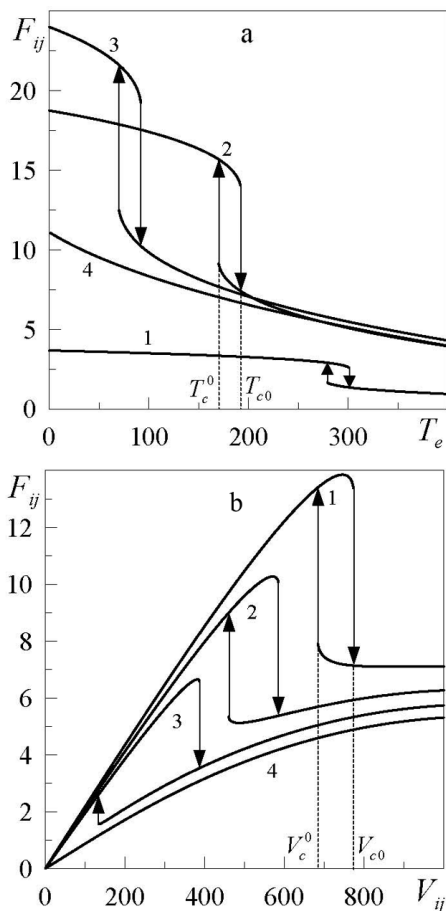


Fig. 1. Dependences of the stationary value of total friction force  $F_{ij}$  [mN] (see formula (25)) on the friction surface temperature  $T_e$  [K] (a) and on the shear velocity  $V_{ij}$  [nm/s] (b) at  $\gamma = 2/3$ ,  $A = 3 \times 10^{-9}$  m<sup>2</sup>, and  $n = -7 \times 10^5$  Pa. Curves 1 to 4 in panel a correspond to the constant shear velocity  $V_{ij} = 150, 800, 1100,$  and  $1400$  nm/s, respectively; curves 1 to 4 in panel b correspond to the fixed temperatures  $T_e = 200, 245, 279,$  and  $310$  K, respectively

mechanism of the first-order phase transition, goes into a state that corresponds to a unique minimum at large  $f$ , i.e., it melts. If then the temperature is reduced, the appearance of the first minimum cannot force the system to transit back into the corresponding state because of the separating maximum. When the latter disappears at  $T_e = T_c^0$ , the lubricant undergoes a jump-like solidification.

Figure 1,b illustrates a somewhat different behavior. Here, in accordance with Eq. (25), the lubricant is solid-like, if the shear velocities are low, and the corresponding  $\sigma_{ij}^e$ -values are large. The velocity growth in such a mode gives rise to an increase of both friction force components (25); therefore, the friction force increases rapidly.

If the velocity grows further, the lubricant melts, and the elastic shear stress (5) decreases considerably, which results, in turn, in a drastic reduction of the total friction force. If the velocity increases even more, the value of  $F_{ij}$  grows again, driven by both friction force components which increase together with the shear velocity. In the liquid-like state, according to curve 4, the friction force (25) also grows owing to an increase in the velocity. That is, we have a situation which is similar to the behavior of the system, when the temperature is elevated (Fig. 1,a); the difference is that, in the case of the shear melting, the growth of the friction surface temperature is accompanied by an increase of the area confined by the hysteresis loop. As the temperature increases, the lubricant melts at lower shear velocities. We emphasize that the results depicted in Fig. 1,b qualitatively coincide with a new friction map for the boundary lubrication proposed in work [39] as a generalization of experimental data. No dependences of the friction force on the temperature – of the type exhibited in Fig. 1,a – are measured experimentally now.

#### 4. Stick-Slip Mode

The dependences depicted in Fig. 1 were obtained at a fixed shear velocity of the upper friction surface. However, the dynamic characteristics of the tribological system are governed not only by the friction force presented in this figure, but also by the properties of the system in whole. In particular, according to experiments, in the hysteresis section of the dependence shown in Fig. 1, an intermittent (stick-slip) mode of friction is possible [2, 6, 9, 10, 12, 23, 36, 39], and this work is devoted to the elucidation of its features. A typical scheme of tribological system is shown in Fig. 2. A spring, the rigidity of which is  $k$ , is connected with a block of mass  $M$ , to which an additional loading  $L$  is applied. The block is located on a smooth surface, being separated from it by a lubricant layer of the thickness  $h$ . The free end of the spring is forced to move with a constant velocity  $V_0$ . When the block moves, there emerges a friction force  $F$  (25) that interferes with the block motion. In the case of ultrathin lubricant layers operating under the boundary friction conditions, the block,  $V$ , and spring,  $V_0$ , velocities can differ from each other because of the oscillating character of the force  $F$ , which results in the stick-slip motion of the block. This mode is similar to dry friction (with no lubricant).

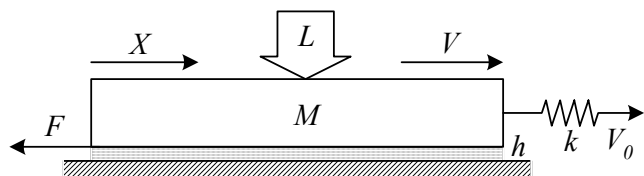


Fig. 2. Scheme of a tribological system

The equation of motion for the upper block looks like [2, 4, 6]<sup>4</sup>

$$M\ddot{X} = k \left( \int_0^t V_0 dt' - X \right) - F, \quad (26)$$

where  $t = t'$  is the time of motion. If the shear velocity  $V_0$  is constant, the integral in expression (26) can be substituted by  $V_0 t$ , of course. In order to calculate the time evolution of the friction force, this equation has to be solved together with Eqs. (11), (14), and (18); then, the friction force is determined by expression (25). However, since the lubricant layer is thin, the relaxation times of strain,  $\tau_\varepsilon$ , and entropy,  $\tau_s = \hbar^2/\kappa$ , can be considered short in comparison with the relaxation time of excess volume  $\tau_f$ . Therefore, within the approximation  $\tau_f \gg \tau_\varepsilon, \tau_s$ , we have to solve two equations, (26) and (11), consistently; then, the temperature and strain can be determined from Eq. (19), and the entropy from Eq. (12).

The solution of the specified equations is depicted in Fig. 3. The figure demonstrates that the friction force monotonously grows at first, because the lubricant is solid-like, and the shear velocity increases. When the latter exceeds the critical value  $V_{c0}$ , the lubricant melts. As a result, the friction force decreases, the motion velocity of the upper block  $V$  grows, and the block shifts over a large distance. The spring tension and, accordingly, the shear velocity decrease at that. When the latter becomes less than it is necessary to keep the lubricant in the liquid-like state, the lubricant solidifies, and the friction force starts to grow. The described process is periodically repeated. Note that the velocity, at which the lubricant solidifies, does not coincide with the analogous velocity presented in Fig. 1. This fact is associated with a drastic growth of the shear velocity  $V$  at the melting and the corresponding increase of the parameter  $f$ . According to Eq. (6), the shear modulus becomes negative, so it is necessary to consider it as zero-valued, which changes the form of potential (1). Then, in the

<sup>4</sup> Since a unidirectional shear is considered, tensor notations are omitted hereafter for convenience.

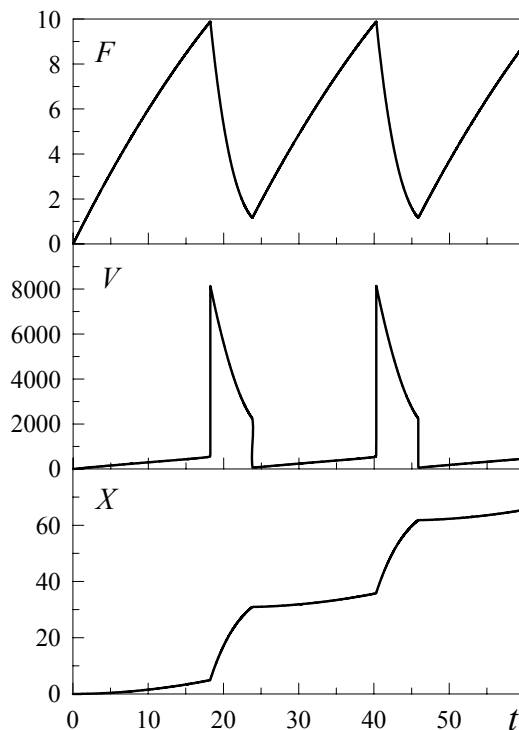


Fig. 3. Dependences of the friction force  $F$  [mN] (a), shear velocity  $V$  of the friction surface [nm/s] (b), and coordinate  $X$  of the friction surface [ $\mu\text{m}$ ] on the time  $t$  [s] at  $n = -7 \times 10^5$  Pa,  $M = 0.4$  kg,  $k = 480$  N/m,  $T_e = 250$  K, and  $V = 1400$  nm/s

presence of elastic strains (18), the elastic stresses in the lubricant, according to formula (5), vanish, which is responsible for the friction force reduction, so that the lubricant can flow.

In Fig. 4, the time dependences of the total friction force  $F$  (25), excess volume  $f$ , and elastic component of shear stresses  $\sigma_{ij}^e$  (5) at various-increasing-values of shear velocity  $V_0$  are shown. First, the motion of the upper movable block ( $V_0 = V_{01}$ ) results in the growth of the excess volume  $f$ . When, the  $f$ -value reaches the critical magnitude, the lubricant melts following the mechanism of the first-order phase transitions, so that the parameter  $f$  increases stepwise. Afterwards, the lubricant starts to solidify again, because the relative shear velocity of friction surfaces decreases (see Fig. 3). After the lubricant has totally solidified, an elastic stress appears in it. The further growth of the stress leads to a new increase of the parameter  $f$ , until the latter reaches the critical value necessary for melting, and the process repeats again. As a result, a periodic intermittent (stick-slip) regime melting/solidification is established.

When the velocity is increased to the value  $V_0 = V_{02}$ , the frequency of the stick-slip peak appearance also in-

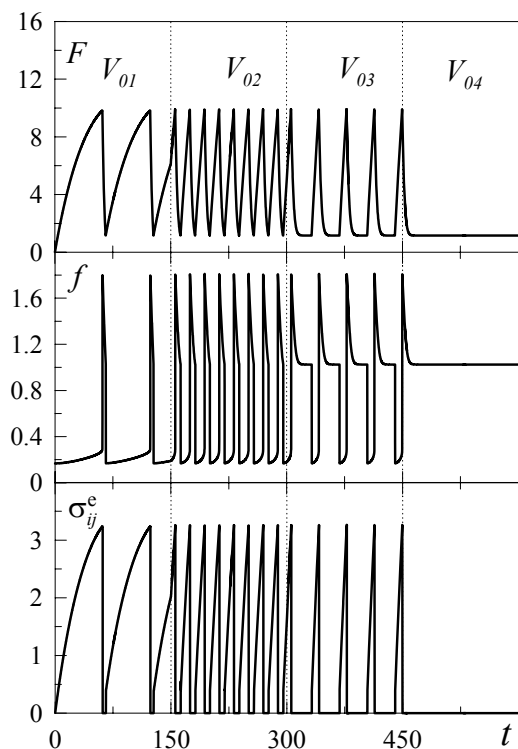


Fig. 4. Dependences of the friction force  $F$  [mN] (a), excess volume  $f$  (b), and elastic stress component  $\sigma_{ij}^e$  [MPa] (c) on the time  $t$  [s] for various shear velocities  $V_{01} = 650$  nm/s,  $V_{02} = 1800$  nm/s,  $V_{03} = 2246.7$  nm/s, and  $V_{04} = 2247$  nm/s. The other parameters are the same as in Fig. 3

creases, because the critical  $f$ -value is attained more rapidly at this velocity. Respectively, the lubricant melts more quickly; therefore, the system undergoes a larger number of melting/solidification transitions within the same time interval.

A further velocity growth to  $V_0 = V_{03}$  results in a reduction of the stick-slip peak frequency. This occurs, because there appear long kinetic sections in the dependence  $F(t)$ , where  $F = \text{const}$ . It should be noted that, in this mode, the parameter  $f$  drastically grows first at the melting owing to a rapid increase of the shear velocity  $V$  of the upper block. The stationary kinetic section is associated with a smaller excess volume  $f$ , which is established after the sharp initial shift of the upper rubbing surface. This shift is induced by a partial release of the mechanical potential energy stored by the stretched spring.

As the shear velocity increases to  $V_0 = V_{04}$ , the stick-slip mode disappears, and a kinetic friction regime for the liquid-like lubricant is established, which is characterized by a larger value of excess volume  $f$  and zero

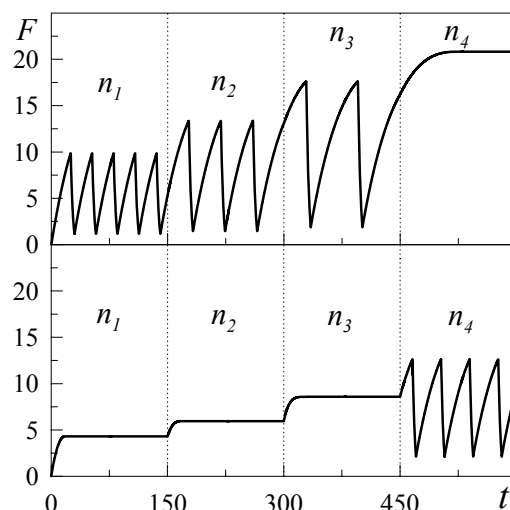


Fig. 5. Dependences of the friction force  $F$  [mN] on the time  $t$  [s] at the same parameters as in Fig. 3 and for various external normal loads  $n_1 = -7 \times 10^5$  Pa,  $n_2 = -50 \times 10^5$  Pa,  $n_3 = -80 \times 10^5$  Pa, and  $n_4 = -100 \times 10^5$  Pa and various temperatures  $T_e = 250$  (upper panel) and 400 K (lower panel)

elastic stresses  $\sigma_{ij}^e$ . Note that the liquid-like state is not always characterized by zero stresses  $\sigma_{ij}^e$  [5]. In our case, this fact stems from the equality of the effective shear modulus of a lubricant (6) to zero in the liquid-like state [4].

Hence, when the shear velocity grows, the frequency of stick-slip peaks increases firstly; then, it decreases owing to the appearance of long kinetic sections. If the critical velocity  $V_0$  is exceeded, the stick-slip mode disappears. Such a behavior agrees well with experimental data [2].

In experiments, the influence of an external pressure applied normally to the friction surfaces on the melting of a lubricant is also often studied [2, 36]. Such experiments testify that the pressure affects the parameters of a tribological system in a nontrivial manner. For example, the critical shear velocity decreases as the pressure grows for lubricants containing hexadecane chain molecules and, on the contrary, increases for quasispherical OMCTS ones [2]. The pressure also influences the frequency and the amplitude of stick-slip transitions [2]. In the framework of our model, according to Eq. (11), the growth of a loading on the friction surface results in a reduction of the excess volume, which should favor the lubricant solidification.

Figure 5 illustrates the time dependences of the friction force at various values of normal pressure that squeezes the friction surfaces. At a temperature lower than the critical value (the upper panel in the figure), the stick-slip mode of friction is realized. In this case,

the pressure growth is accompanied by an increase of the amplitude of stick-slip transitions and the magnitudes of kinetic and static friction forces, as well as by a reduction of the transition frequency. At a pressure that corresponds to the normal stress  $n = n_4$ , the stick-slip regime does not take place. However, the kinetic mode, which corresponds to the liquid-like lubricant, is not established. Instead, the lubricant solidifies owing to its squeezing by walls. As a result, the lubricant cannot melt any more, and a large friction force  $F$  is established, which corresponds to the case of a solid-like lubricant and a small value of excess volume  $f$ , because the squeezing of the lubricant by walls promotes the emergence of an atomic long-range order in it.

The lower panel of Fig. 5 exhibits the time dependence of the friction force that is observed at an elevated temperature of friction surfaces,  $T_e$ . One can see that the kinetic mode of friction is established, which corresponds to low values of friction force and large values of excess volume  $f$ . However, at  $n = n_4$ , the stick-slip mode starts, because, according to Eq. (11), the lubricant cannot always be liquid-like at such values of normal stress  $n$ . If the pressure grows further, one has to expect a complete solidification of a lubricant, as it takes place in the upper panel of the figure at  $n = n_4$ . Therefore, we revealed three friction modes: 1) a kinetic mode, in which the lubricant is always liquid-like, 2) a stick-slip mode, which corresponds to periodic melting/solidification cycles, and 3) a dry friction mode, which is characterized by a large value of friction force and a solid-like structure of the lubricant. These modes were also found in work [9] in the framework of a stochastic model.

In Fig. 6, the time dependences of the friction force at various temperatures of friction surfaces, which coincide in this consideration with the lubricant temperatures, are shown. One can see that the temperature elevation results in a decrease of the friction force oscillation amplitude and in a frequency growth for the phase transitions “liquid-like lubricant–solid-like lubricant”. At  $T_e = T_{e4}$ , the slip mode starts, which is characterized by a constant value of the kinetic friction force and a constant shear velocity of the upper block. Hence, the temperature growth promotes the lubricant melting. This dependence is a prediction, because we do not know about the experiments of this type aimed at studying the temperature effects.

## 5. Conclusions

The proposed theory allows one to describe the effects that are observed at the melting of an ultrathin lubricant

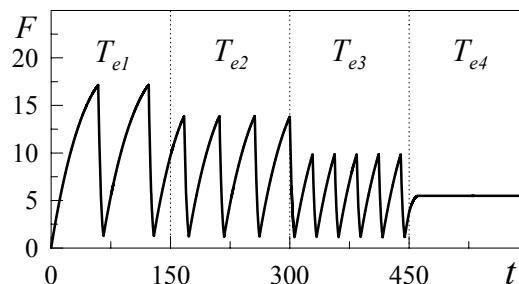


Fig. 6. Dependences of the friction force  $F$  [mN] on the time  $t$  [s] at the same parameters as in Fig. 3 and for various temperatures of friction surfaces:  $T_{e1} = 150$  K,  $T_{e2} = 200$  K,  $T_{e3} = 250$  K, and  $T_{e4} = 300$  K

film in the boundary friction mode. A consistent consideration of the thermodynamic and shear melting has been carried out. The dependences of the friction force on the shear velocity and the temperature have been examined. At high temperatures of friction surfaces, the shear melting begins at lower values of shear velocities (shear stresses), and, if the temperature grows further, the lubricant melts even at the zero shear velocity. In the model proposed, the influences of the temperature, the shear melting, and the external pressure are taken into account. Those parameters are the main factors, which are studied experimentally.

In the framework of the proposed theory, a simple tribological system is studied, and the time dependences of the friction force are obtained for increasing the shear velocity, pressure, and temperature. It is shown that, in a wide range of parameters of the system, the intermittent friction mode is realized, which is observed experimentally. The pressure is found to affect the system in a nontrivial way. The results obtained qualitatively coincide with known experimental data. Since the model is quantitative, its modifications can be used for the description of specific experiments.

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#### ФЕНОМЕНОЛОГІЧНА ТЕОРІЯ ПЕРЕРИВЧАСТОГО РЕЖИМУ МЕЖОВОГО ТЕРТЯ

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#### Резюме

Побудовано детерміністичну теорію плавлення ультратонкої плівки мастила, яку затиснуто між двома атомарно-гладкими твердими поверхнями. Для опису стану мастила введено параметр надлишкового об'єму, що виникає за рахунок хаотизації структури твердого тіла у процесі плавлення. Узгоджено описано термодинамічне і зсувне плавлення. Проаналізовано залежності стаціонарної сили тертя від температури мастила і швидкості зсуву поверхонь, що труться, при їх рівномірному зсуві зі сталою швидкістю. У межах простої трибологічної моделі описано переривчастий режим тертя, при якому мастило періодично плавиться і твердне. Проаналізовано вплив швидкості, температури і навантаження на переривчасте тертя. Проведено якісне порівняння отриманих результатів із експериментальними даними.