FREQUENCY MODULATION OF RECOMBINATION RADIATION EMITTED BY AN InAs/GaAs HETEROSTRUCTURE WITH InAs QUANTUM DOTS UNDER THE INFLUENCE OF AN ACOUSTIC WAVE

R.M. PELESHCHAK, O.O. DAN'KIV, O.V. KUZYK

Ivan Franko Drohobych State Pedagogical University
(24, Ivan Franko Str., Drohobych 82100, Ukraine; e-mail: `peleshchak@rambler.ru`*)*

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We have developed a theoretical model that describes the process of frequency modulation of radiation emitted at the recombination transition between the ground states of an electron and a hole in the InAs/GaAs heterostructure with InAs/GaAs quantum dots, the modulation being induced by an acoustic wave. The character of the dependence of the frequency modulation amplitude on the acoustic wave frequency is determined.

1. Introduction

Sources of infra-red radiation, which are capable of quickly changing their lasing frequency, are important elements in high-resolution laser spectroscopy and optical communication systems. In experimental works [1,2], effects of the influence of an ultrasonic wave on the process of laser generation in the InGaAs/InP heterostructure, which manifested themselves in a rapid variation of the radiation wavelength, were detected. In recent years, the interest of researchers in InAs/GaAs semiconducting heterostructures with zero-dimensional strained InAs nanoobjects (quantum dots (QDs)) [3–7] has considerably grown up. Such nanoobjects are characterized by a high quantum yield of photoluminescence, being promising objects for the creation of lasers in the near infra-red spectral range [8, 9]. One of the important research directions is a possibility to control the lasing frequency of QD-based heterolasers.

An important factor that affects the spectral characteristics of radiation emitted by InAs/GaAs heterostructures with InAs quantum dots is the elastic deformation. For instance, in work [10], the influence of the field of internal elastic deformations, which is a result of a discrepancy between the lattice parameters, as well as of different coefficients of thermal expansion, in the QD and matrix materials, was studied. However, the external stresses can also affect the electron subsystem in semiconducting heterostructures [11]; in particular, they can change the energy gap width, the energy spectrum of charge carriers, and, respectively, the frequency of emitted radiation. In works [12, 13], the thermal deformation mechanism of sound generation was considered, and the opto-acoustic effects emerged in a dielectric matrix with a metal nanocluster under the influence of laser irradiation were studied.

In this work, a theoretical model for the modulation of the radiation frequency of InAs/GaAs heterostructures with InAs quantum dots by an acoustic wave has been developed, and the influence of a dynamic deformation on the energy spectrum of charge carriers in such structures has been examined.

2. Model

Consider an InAs/GaAs nanoheterosystem with strained spherical InAs quantum dots, which undergoes an acoustically induced deformation. The lattice constant $a_1 = 0.608$ nm in the grown material, InAs, is larger than that in the GaAs matrix ($a_2 = 0.565$ nm). Therefore, when InAs is grown heteroepitaxially on a GaAs layer under the pseudomorphic growth, the InAs substance undergoes the compressive deformation, and the GaAs material does the tensile one. Hence, a spherical quantum dot of radius R_0 can be regarded as an elastic dilatational microinclusion, which is the elastic sphere (the solid thin curve in Fig. 1) located in a spherical cavity (the dashed curve in Fig. 1) in the GaAs matrix. The cavity volume is smaller than the microinclusion one by ΔV. The radius of the GaAs matrix is R_1. For such a spherical microinclusion to be inserted, it has to be compressed and the surrounding GaAs matrix to be stretched in radial directions. The result of the simultaneous action of deformations in the contacting nanomaterials is presented by the solid bold curve in Fig. 1.

An acoustic wave induces a periodic deformation field $U_{\text{out}}(t)$ in the materials of the nanoheterosystem. A

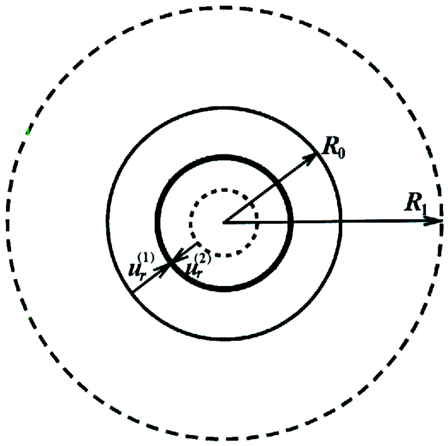

Fig. 1. Model of a strained spherical quantum dot. $u_r^{(i)}$ are the radial components of shift vectors in the quantum dot ($i = 1$) and matrix ($i = 2$) materials

mismatch between the edges of the conduction (valence) bands at the quantum dot–matrix heterointerface, which is associated with the difference between the energy gap widths in the QD and matrix substances, governs the potential energies of electrons and holes in the nanoheterosystem. In the case of heterosystems with strained QDs, the depth of the quantizing potential is determined by both the internal deformation in the contacting matrix and QD materials, which arises owing to the mismatch between the lattice parameters in them, and the deformation induced by the acoustic wave.

In Fig. 2, the geometry of the InAs/GaAs heterosystem with spherical InAs quantum dots is illustrated, and the coordinate dependences for the potential energies of an electron and a hole without (the solid curve) and with regard for (the dotted and dashed curves, respectively) the deformation of the QD material caused by the action of an ultrasonic wave are shown. The dotted (dashed) curve corresponds to the time moment, when the tensile (compressive) deformation induced by the external field is maximal. The energy shifts of the edges of the allowed bands under the action of elastic strains are

$$\Delta E_c^{(i)}(t) = a_c^{(i)}\varepsilon^{(i)}(t) = \Delta E_{c1}^{(i)} + \Delta E_{c2}^{(i)}(t);$$

$$\Delta E_v^{(i)}(t) = a_v^{(i)}\varepsilon^{(i)}(t) = \Delta E_{v1}^{(i)} + \Delta E_{v2}^{(i)}(t);$$

where $\Delta E_{c1}^{(i)}$ and $\Delta E_{v1}^{(i)}$ are the energy shifts of the edges of the conduction band in the QD and the valence band in the matrix, respectively, caused by a mismatch between the lattice parameters in the contacting materials; $\Delta E_{c2}^{(i)}(t)$ and $\Delta E_{v2}^{(i)}(t)$ are their counterparts emerging owing to the action of an ultrasonic wave; $\varepsilon^{(i)} = \mathrm{Sp}\,\varepsilon^{(i)}$ is the overall deformation stemming from both a mismatch between the lattice parameters of contacting materials and the action of an ultrasonic wave; $a_c^{(i)}$ and $a_v^{(i)}$ are the constants of the hydrostatic deformation potential in the conduction and valence bands, respectively; and $i = 1$ for InAs and 2 for GaAs.

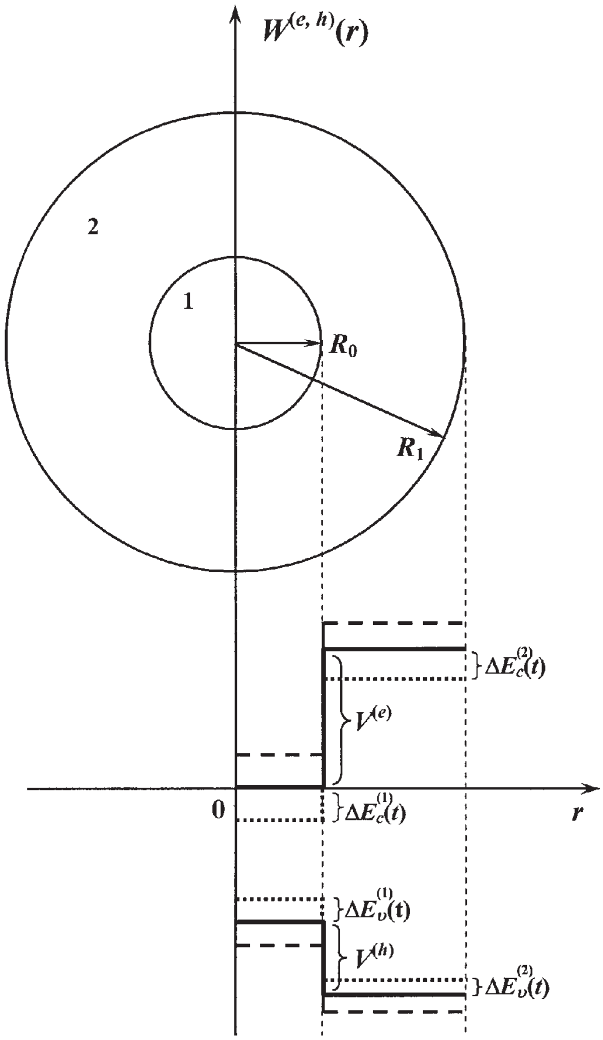

Fig. 2. Coordinate dependences of the potential energies of an electron and a hole in the InAs/GaAs heterostructure with spherical InAs quantum dots without the influence of an ultrasonic wave (solid curve), at the time moment when the ultrasonic wave creates the maximal tensile deformation (dotted curve), and at the time moment when the ultrasonic wave creates the maximal compressive deformation (dashed curve). $V^{(e)}$ and $V^{(h)}$ are the depths of potential wells for an electron and a hole, respectively, without an acoustic wave

Hence, the potential energies of an electron and a hole, $W^{(e,h)}(r)$, in the strained InAs/GaAs heterosystem with spherical InAs quantum dots are

$$W^{(e,h)}(r,t) = \begin{cases} 0, & 0 \le r \le R_0, \\ W^{(e,h)}(r,t), & R_0 \le r \le R_1, \end{cases} \quad (1)$$

where

$$W^{(e)} = \Delta E_c(0) - a_c^{(1)}\varepsilon^{(1)} + a_c^{(2)}\varepsilon^{(2)},$$
$$W^{(h)} = \Delta E_v(0) + a_v^{(1)}\varepsilon^{(1)} - a_v^{(2)}\varepsilon^{(2)},$$

$\Delta E_{c,v}(0)$ is the depths of potential wells for an electron and a hole, respectively, in an unstrained quantum dot,

$$\Delta E_c(0) = \chi_1 - \chi_2,$$

$$\Delta E_v(0) = E_g^{(2)}(0) + \chi_2 - E_g^{(1)}(0) - \chi_1, \quad (2)$$

and χ_i and $E_g^{(i)}(0)$ are the electron affinity and the energy gap width, respectively, in the i-th bulk unstrained material. The energy is reckoned from the edge of the corresponding allowed band in InAs.

3. Calculation of Strain Tensor Components in a Nanoheterosystem with Quantum Dots with regard for Ultrasonic Wave Effects

In order to determine the strain tensor components, it is necessary to find the shift vectors $\mathbf{u}^{(i)}(t,\mathbf{r})$ in the QD and matrix materials which satisfy the equations ($i = 1, 2$)

$$\rho^{(i)}\frac{\partial^2 u_i^{(i)}}{\partial t^2} = \sum_j \frac{\partial \sigma_{ij}^{(i)}}{\partial x_j}, \quad (3)$$

where $\rho^{(i)}$ and $\sigma_{ij}^{(i)}$ are the density and the stress tensor components, respectively, for the QD and matrix materials;

$$\sigma_{ij}^{(i)} = K^{(i)}\sum_k \varepsilon_{kk}^{(i)}\delta_{ij} + 2\mu^{(i)}\left(\varepsilon_{ij}^{(i)} - \delta_{ij}\frac{1}{3}\sum_k \varepsilon_{kk}^{(i)}\right), \quad (4)$$

where $K^{(i)}$ and $\mu^{(i)}$ are the modulus of uniform compression and the shear modulus, respectively; and

$$\varepsilon_{ij}^{(i)} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \quad (5)$$

are the strain tensor components.

Let the displacement vectors $\mathbf{u}^{(i)}(t,\mathbf{r})$ be represented as a sum of two terms, $\mathbf{u}^{(i)}(t,\mathbf{r}) = \mathbf{u}_l^{(i)}(t,\mathbf{r}) + \mathbf{u}_T^{(i)}(t,\mathbf{r})$ which satisfy the conditions

$$\operatorname{rot}\mathbf{u}_l^{(i)}(t,\mathbf{r}) = 0, \quad \operatorname{div}\mathbf{u}_T^{(i)}(t,\mathbf{r}) = 0.$$

As a result, we obtain

$$\Delta\mathbf{u}_l^{(i)} = \frac{1}{c_l^{(i)2}}\frac{\partial^2\mathbf{u}_l^{(i)}}{\partial t^2}, \quad \Delta\mathbf{u}_T^{(i)} = \frac{1}{c_T^{(i)2}}\frac{\partial^2\mathbf{u}_T^{(i)}}{\partial t^2}, \quad (6)$$

where $c_l^{(i)} = \sqrt{\frac{3K^{(i)}+4\mu^{(i)}}{3\rho^{(i)}}}$ and $c_T^{(i)} = \sqrt{\frac{\mu^{(i)}}{\rho^{(i)}}}$ are the longitudinal and transverse, respectively, velocities of acoustic vibrations in either the QD or matrix material. The transverse acoustic wave – $\mathbf{u}_T^{(i)}$ in Eq. (6) – does not change the volume [14], because div $\mathbf{u}_T^{(i)}(t,\mathbf{r}) = 0$. On the contrary, the propagation of a longitudinal wave is accompanied by the volumetric expansion and compression.

Elastic vibrations in the heterosystem with QDs are considered to take place against the background of static stresses that arise owing to a mismatch between the lattice parameters in contacting materials. Let the displacement be represented in the form

$$\mathbf{u}_l^{(i)}(\mathbf{r},t) = \mathbf{u}_0^{(i)}(\mathbf{r}) + \mathbf{u}_{1l}^{(i)}(\mathbf{r},t), \quad (7)$$

where $\mathbf{u}_0^{(i)}(\mathbf{r})$ are static displacements in the QD or matrix material which satisfy the equilibrium equation [14]

$$\boldsymbol{\nabla}\operatorname{div}\mathbf{u}_0^{(i)}(r) = 0 \quad (8)$$

and the boundary conditions

$$\begin{cases} 4\pi R_0^2\left(u_{0r}^{(2)}\big|_{r=R_0} - u_{0r}^{(1)}\big|_{r=R_0}\right) = \Delta V, \\ \sigma_{0rr}^{(1)}\big|_{r=R_0} = \sigma_{0rr}^{(2)}\big|_{r=R_0} - P_L, \\ \sigma_{0rr}^{(2)}\big|_{r=R_1} = 0. \end{cases} \quad (9)$$

In formula (9), $\sigma_{0rr}^{(i)}$ are the radial components of static stresses in the QD and matrix materials emerging owing to a mismatch between their lattice parameters. The left-hand side of the first equation in system (9) amounts to the geometrical difference ΔV between the volumes of the microinclusion and the cavity in the GaAs matrix; $P_L = \frac{2\alpha}{R_0}$ is the Laplace pressure; α is the specific QD (InAs) surface energy which is determined by the condition

$$\int_0^{R_1} \rho^{(i)} c_l^{(i)2}\varepsilon^{(i)2}(r) r^2 dr = \alpha\Delta S,$$

where $\Delta S = 2\pi R_0 u^{(1)}(R_0)$ is a change of the QD surface area, and $\varepsilon^{(i)}$ is the overall deformation of the QD or matrix material. The choice of signs in the second equation of system (9) is dictated by the relationship between the directions of forces that induce the compressive deformation in the QD material, the tensile deformation in the matrix material, and the surface tension force in the QD, which makes it additionally squeezed.

An ordered arrangement of strained QDs in the crystalline matrix stems from the elastic interaction between them. To reduce the problem with a large number of QDs to a problem with a single QD, the following approximation was made in work [10]: the energy of the pairwise elastic interaction between QDs was replaced by the energy of interaction between every QD and an averaged field of elastic deformations σ_{ef} of all other QDs. In this work, when solving Eq. (8) with boundary conditions (9), the elastic interaction between the QDs was not taken into account, which is justified, provided that the distance between the QDs is much larger than their dimensions.

The second term in Eq. (7), $\mathbf{u}_{1l}^{(i)}(\mathbf{r},t)$, describes dynamic displacements in the QD or matrix materials induced by the action of an acoustic wave.

Since a spherically symmetric system is considered, the displacement vector has only the radial component u_r, and the radial stress looks like [12]

$$\sigma_{rr}^{(i)} = \left(K^{(i)} + \frac{4}{3}\mu^{(i)}\right)\frac{\partial u_r^{(i)}}{\partial r} + \left(K^{(i)} - \frac{2}{3}\mu^{(i)}\right)\frac{2u_r^{(i)}}{r}. \tag{10}$$

The solution of Eq. (8), taking conditions (9) into account and provided that a displacement at the point $r = 0$ is finite in the case of spherical QDs, looks like

$$u_{0r}^{(1)} = C_1 r, \quad 0 \le r \le R_0, \tag{11}$$

$$u_{0r}^{(2)} = C_2 r + C_3\frac{1}{r^2}, \quad R_0 \le r \le R_1, \tag{12}$$

where the integration constants C_1, C_2, and C_3 are determined from conditions (9).

Changing to the scalar potential, $\mathbf{u}_{1l}^{(i)} = \boldsymbol{\nabla}\varphi^{(i)}$, and making allowance for Eq. (7), Eq. (6) can be written down in the form

$$\Delta\varphi^{(i)} = \frac{1}{c_l^{(i)2}}\frac{\partial^2\varphi^{(i)}}{\partial t^2}. \tag{13}$$

The solutions of this equation in each heterostructure region must satisfy the boundary conditions

$$\begin{cases} \sigma_{1rr}^{(1)}(t)\big|_{r=R_0} = \sigma_{1rr}^{(2)}(t)\big|_{r=R_0}; \\ \sigma_{1rr}^{(2)}(t)\big|_{r=R_1} = -\sigma_{us}\sin\omega t. \end{cases} \tag{14}$$

Here, $\sigma_{1rr}^{(i)}(t)$ are the radial components of dynamical stresses in the QD or matrix material caused by the action of an acoustic wave.

The last boundary condition in system (14) determines the influence of an acoustic wave on the strained state of the nanosystem as the action of a periodic driving force with frequency ω, and the quantity σ_{us} in it is the amplitude of a mechanical stress created by an acoustic wave on the matrix surface. This external periodic force and the elastic force that arises in the nanoheterosystem under its action are oppositely directed at any time moment, and this fact dictates the choice of the sign in the second equation of system (14).

Therefore, taking Eqs. (7), (10)–(12), and (14) into account, as well as the fact that $u_r^{(i)} = \frac{\partial\varphi^{(i)}}{\partial r}$, we obtain the following expressions for the radial components of displacement vectors in the QD and the matrix:

$$u_r^{(1)}(r,t) = C_1 r - A_1\times$$

$$\times\left(\frac{\sin\left(\omega t - \frac{\omega r}{c_l^{(1)}} + \alpha_1\right)}{r^2} + \frac{\omega\cos\left(\omega t - \frac{\omega r}{c_l^{(1)}} + \alpha_1\right)}{r c_l^{(1)}}\right), \tag{15}$$

$$u_r^{(2)}(r,t) = C_2 r + C_3\frac{1}{r^2} - A_2\times$$

$$\times\left(\frac{\sin\left(\omega t - \frac{\omega r}{c_l^{(2)}} + \alpha_2\right)}{r^2} + \frac{\omega\cos\left(\omega t - \frac{\omega r}{c_l^{(2)}} + \alpha_2\right)}{r c_l^{(2)}}\right), \tag{16}$$

where

$$A_1 = -\frac{R_1}{\rho^{(1)}}\frac{\sigma_{us}}{\sqrt{\left(\omega_{01}^2 - \omega^2\right)^2 + 4\gamma_1^2\omega^2}},$$

$$\alpha_1 = \frac{\omega R_0}{c_l^{(1)}} - \frac{\omega R_0}{c_l^{(2)}} + \frac{\omega R_1}{c_l^{(2)}} + \arctan\frac{2\gamma_1\omega}{\omega^2 - \omega_{01}^2},$$

$$A_2 = -\frac{R_1}{\rho^{(2)}} \frac{\sigma_{us}}{\sqrt{\left(\omega_{02}^2 - \omega^2\right)^2 + 4\gamma_2^2\omega^2}},$$

$$\alpha_2 = \frac{\omega R_1}{c_l^{(2)}} + \arctan \frac{2\gamma_2 \omega}{\omega^2 - \omega_{02}^2},$$

$$\omega_{01} = \frac{2c_T^{(1)}}{R_0}, \quad \omega_{02} = \frac{2c_T^{(2)}}{R_1}, \quad \gamma_1 = \frac{2c_T^{(1)2}}{R_0 c_l^{(1)}}, \quad \gamma_2 = \frac{2c_T^{(2)2}}{R_1 c_l^{(2)}}.$$

The components of the strain tensor for the QD and matrix materials are

$$\varepsilon_{rr}^{(i)} = \frac{\partial u_r^{(i)}}{\partial r^{(i)}}, \ \varepsilon_{\theta\theta}^{(i)} = \varepsilon_{\varphi\varphi}^{(i)} = \frac{u_r^{(i)}}{r^{(i)}},$$

$$\mathrm{Sp}\ \varepsilon^{(i)} = \varepsilon_{rr}^{(i)} + \varepsilon_{\theta\theta}^{(i)} + \varepsilon_{\varphi\varphi}^{(i)}. \qquad (17)$$

We are interested in the influence of an acoustic wave on a change of the frequency of radiation emitted by the heterostructure with QDs. Therefore, in what follows, only the dynamic component of a deformation will be paid attention to in calculations. The influence of internal static stresses on the energy spectrum of charge carriers in the heterostructure with QDs was studied in work [10].

4. Modulation of the Frequency of Radiation Emitted by a Heterostructure with QDs

The energy spectra and the wave functions for an electron and a hole in the system under consideration are found by solving the non-stationary Schrödinger equation

$$\hat{H}_{e,h}(r,\theta,\varphi,t)\,\Psi^{(e,h)}(r,\theta,\varphi,t) = -\frac{\hbar}{i}\frac{d\Psi^{(e,h)}}{dt}(r,\theta,\varphi,t) \qquad (18)$$

with the Hamiltonian

$$\hat{H}(r,\theta,\varphi,t)_{e,h} = -\frac{\hbar^2}{2}\nabla\frac{1}{m^{(e,h)}(r)}\nabla + W^{(e,h)}(r,t),$$

$$m^{(e,h)}(r) = \begin{cases} m_1^{(e,h)}, & r \in KT, \\ m_2^{(e,h)}, & r \notin KT, \end{cases}$$

where the potential energies $W^{(e,h)}(r,t)$ of an electron and a hole in the strained InAs/GaAs heterosystem with spherical InAs quantum dots are defined by formula (1). Provided that the Hamiltonian changes in time slowly enough, one may expect that the stationary characteristic functions of the energy operator calculated for the given time moment will approximate the solutions of the Schrödinger equation. Therefore, any characteristic function found for a definite time moment continuously transforms into the corresponding characteristic function for a later time moment (the adiabatic approximation) [15]. This approximation is correct, if the following condition [16] is satisfied:

$$\frac{1}{\hbar}\left|\frac{\partial H}{\partial t}\frac{1}{\omega_{kn}^2}\right| \ll 1, \qquad (19)$$

where ω_{kn} is the radiation frequency of a transition between the corresponding energy levels.

The solution of the stationary equation (18) and the energies of the stationary ground, $E_0^{(e,h)}$, and excited, $E_1^{(e,h)}$, electron and hole states in a spherical QD, which take into account static stresses caused by a mismatch between the lattice parameters of contacting materials (matrix and QD), were given in work [10]. Whether condition (19) is satisfied or not depends on the relation between the acoustic wave frequency and the frequency ω_{kn}, as well as on the relation between the deformation potential and the energy distance between the ground, $E_0^{(e,h)}$, and the excited, $E_1^{(e,h)}$, state of an electron or a hole. As the further calculations demonstrate, $\omega \ll \omega_{kn}$ and $a_{c,v}^{(i)}\varepsilon^{(i)} \ll E_1^{(e,h)} - E_0^{(e,h)}$, i.e. condition (19) is obeyed.

Let us seek the potential energies of an electron and a hole in the form

$$W^{(e,h)}(r,t) = V^{(e,h)} + \Delta V^{(e,h)}(r,t),$$

where $\Delta V^{(e,h)}$ are the components associated with a dynamic deformation. In the first approximation, the corrections to the energies of an electron and a hole, which arise owing to a perturbation induced by the action of an acoustic wave, are calculated by the formula [15]

$$E_{1n}^{(e,h)} = \int\limits_V \psi_n{*}^{(e,h)}(r)\Delta V^{(e,h)}(r,t)\psi_n^{(e,h)}(r)dV, \qquad (20)$$

where $\psi^{(e,h)}(r)$ are the eigenfunctions of an electron and a hole in the nonperturbed (stationary) state. The components of the potential energies of an electron and a hole associated with a dynamic deformation, taking Eqs. (1)

and (15)–(17) into account, look like

$$\Delta V^{(e)}(r,t) = a_c^{(1)} A_1 \left(\frac{\omega}{c_l^{(1)}}\right)^2 \frac{\sin\left(\omega t - \frac{\omega r}{c_l^{(1)}} + \alpha_1\right)}{r} -$$

$$-a_c^{(2)} A_2 \left(\frac{\omega}{c_l^{(2)}}\right)^2 \frac{\sin\left(\omega t - \frac{\omega r}{c_l^{(2)}} + \alpha_2\right)}{r}, \quad (21)$$

$$\Delta V^{(h)}(r,t) = -a_v^{(1)} A_1 \left(\frac{\omega}{c_l^{(1)}}\right)^2 \frac{\sin\left(\omega t - \frac{\omega r}{c_l^{(1)}} + \alpha_1\right)}{r} +$$

$$+a_v^{(2)} A_2 \left(\frac{\omega}{c_l^{(2)}}\right)^2 \frac{\sin\left(\omega t - \frac{\omega r}{c_l^{(2)}} + \alpha_2\right)}{r}. \quad (22)$$

Then, the periodic variation of the radiation frequency, which corresponds to the recombination transition between the ground states of an electron and a hole in a strained InAs quantum dot, can be determined as follows:

$$\Delta\nu(t) = \frac{1}{2\pi\hbar}\left(E_{10}^{(e)}(t) + E_{10}^{(h)}(t) + \Delta E_g(t)\right), \quad (23)$$

where $\Delta E_g = a_c^{(1)}\varepsilon^{(1)} - a_v^{1}\varepsilon^{(1)}$ is a change of the energy gap width in the QD material under the influence of an acoustic wave. Formula (23) takes into account that the acoustic deformation results in a variation ΔE_g of the energy gap width in the QD material and a variation of the electron and hole energies $E_{10}^{(e,h)}$. The change in the charge carrier energies is connected with the variation of the depth in the corresponding potential wells (see formula (1)), which is associated with different shifts of the conduction band bottom in the QD material and the valence band top in the matrix. While calculating the changes in the electron and hole energies $E_{10}^{(e,h)}$ by formula (20), we used the results of work [10], where the wave functions of an electron and a hole in the nonperturbed state were determined.

Hence, owing to the interaction between the electron subsystem and the periodic deformation field induced by an acoustic wave, the process of frequency modulation of radiation emitted by the InAs/GaAs heterostructure with InAs quantum dots takes place. Specifically, the emission frequency which corresponds to the recombination transition between the ground states of an electron and a hole in a strained InAs quantum dot varies with the period of an acoustic wave. Taking Eqs. (20)–(23) into account, the variation of the recombination radiation frequency can be determined as follows:

$$\Delta\nu(t) = \nu_0 \sin(\omega t + \theta), \quad (24)$$

where

$$\nu_0 = \frac{1}{2\pi\hbar}\left(\left(E_{10\,\max}^{(e)} + E_{10\,\max}^{(h)}\right)^2 + \Delta E_{g\,\max}^2 +\right.$$

$$\left. +2\Delta E_{g\,\max}\left(E_{10\,\max}^{(e)} + E_{10\,\max}^{(h)}\right)\cos(\alpha_1 - \varphi)\right)^{1/2},$$

$$E_{10\,\max}^{(e)} = \left(\left(B_c^{(1)}\right)^2 + \left(B_c^{(2)}\right)^2 -\right.$$

$$\left. -2B_c^{(1)}B_c^{(2)}\cos(\alpha_1 - \alpha_2)\right)^{1/2} \int\limits_V \psi_n *^{(e)}(r)\frac{1}{r}\psi_n^{(e)}(r)dV,$$

$$E_{10\,\max}^{(h)} = \left(\left(B_v^{(1)}\right)^2 + \left(B_v^{(2)}\right)^2 -\right.$$

$$\left. -2B_v^{(1)}B_v^{(2)}\cos(\alpha_1 - \alpha_2)\right)^{1/2} \int\limits_V \psi_n *^{(h)}(r)\frac{1}{r}\psi_n^{(h)}(r)dV,$$

$$\Delta E_{g\,\max} = \frac{1}{R_0}\left(-B_c^{(1)} + B_v^{(1)}\right),$$

$$B_{c,v}^{(i)} = a_{c,v}^{(i)} A_i \left(\frac{\omega}{c_l^{(i)}}\right)^2,$$

$$\tan\varphi = \frac{B_c^{(1)}\sin\alpha_1 - B_c^{(2)}\sin\alpha_2}{B_c^{(1)}\cos\alpha_1 - B_c^{(2)}\cos\alpha_2},$$

$$\tan\theta = \frac{\left(E_{10\,\max}^{(e)} + E_{10\,\max}^{(h)}\right)\sin\varphi + \Delta E_{g\,\max}\sin\alpha_1}{\left(E_{10\,\max}^{(e)} + E_{10\,\max}^{(h)}\right)\cos\varphi + \Delta E_{g\,\max}\cos\alpha_1}.$$

In Fig. 3, the results of numerical calculations of the dependence of the modulation amplitude ν_0 for the radiation frequency that corresponds to the recombination transition between the electron and hole ground states in the InAs/GaAs nanoheterosystem with strained InAs

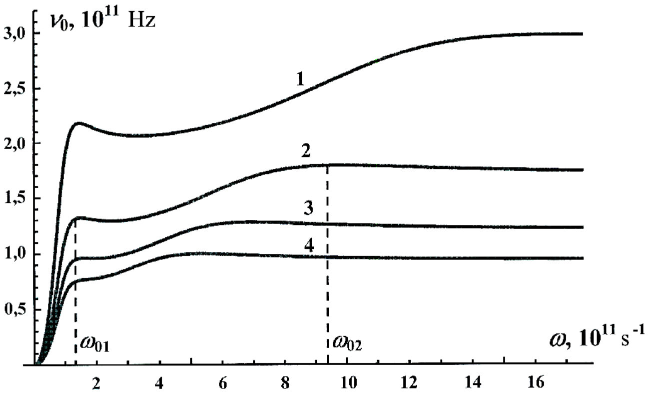

Fig. 3. Dependences of the modulation amplitude for the recombination radiation frequency which corresponds to the transition between the ground states of an electron and a hole in the InAs/GaAs heterostructure with InAs quantum dots on the acoustic wave frequency for various quantum dot dimensions $R_0 = 3$ (*1*), 5 (*2*), 7 (*3*), and 9 nm (*4*)

quantum dots on the acoustic wave frequency are depicted. The calculations were carried out using the following parameters [17–20]: $\chi_1 = 4.9$ eV, $\chi_2 = 4.07$ eV, $a_c^{(1)} = -5.08$ eV, $a_c^{(2)} = -7.17$ eV, $a_v^{(1)} = 1$ eV, $a_v^{(2)} = 1.16$ eV, $E_g^{(1)}(0) = 0.36$ eV, $E_g^{(2)}(0) = 1.45$ eV, $m_1^{(e)} = 0.057m_0$, $m_2^{(e)} = 0.065m_0$, $m_1^{(h)} = 0.41m_0$, $m_2^{(h)} = 0.45m_0$, $R_1 = 50$ nm, $K^{(1)} = 0.58$ Mbar, $\mu^{(1)} = 0.19$ Mbar, $K^{(2)} = 0.79$ Mbar, $\mu^{(2)} = 0.33$ Mbar, $\rho^{(1)} = 5680$ kg/m^3, $\rho^{(2)} = 5320$ kg/m^3, and $\sigma_{us} = 10$ bar. Figure 3 demonstrates that the dependence of the modulation amplitude for the frequency of the recombination radiation emitted by the InAs/GaAs heterostructure with InAs quantum dots on the acoustic wave frequency has a nonmonotonous profile which is characterized by two maxima in a vicinity of the points $\omega = \omega_{01}$ and $\omega = \omega_{02}$. Such a dependence can be explained as follows. A change of the recombination transition energy is governed by two factors, which the action of an acoustic wave is responsible for: (i) the variation of the electron and hole energies in the heterostructure with QDs and (ii) the change of the energy gap width in the QD material. At the frequency $\omega = \omega_{01}$ of an acoustic wave, the deformation in the matrix material reaches the maximal value, whereas the deformation in the QDs can be neglected ($\varepsilon^{(1)} \ll \varepsilon^{(2)}$). Therefore, only the shifts of the conduction band bottom and the valence band top in the matrix material can be considered. In turn, this induces a variation of the electron (hole) potential well depth and, respectively, the shifts of charge carrier energy levels. A further increase in the acoustic wave frequency results in a reduction of the deformation in the matrix material and, respectively, in a reduction of the influence of the first factor on the recombination radiation energy.

At the frequency $\omega = \omega_{02}$ of acoustic wave, it is the deformation in the QD material that achieves the maximal value ($\varepsilon^{(1)} \gg \varepsilon^{(2)}$). In this case, the second factor, i.e. the variation of the energy gap width in the QD material, plays the dominant role in the change of the recombination radiation energy.

Moreover, the amplitude of the frequency modulation for radiation that corresponds to the recombination transition between the ground states of an electron and a hole in the InAs/GaAs heterostructure with InAs quantum dots depends considerably on the QD size. In particular, if the QD radius is reduced from 9 to 3 nm, the maximal increase of the frequency modulation amplitude changes from 100 to 300 GHz (Fig. 3). This can be explained by the fact that QDs with smaller dimensions are more sensitive to deformation. The reduction of QD dimensions also results in that the maxima of the frequency modulation amplitude are shifted toward higher frequencies, which is explained by the growth of the frequency of characteristic vibrations in a spherical nanoinclusion.

The theoretical calculations, which were carried out in the framework of the presented model, qualitatively coincide with the experimental data obtained in works [1, 2]. Namely, in those works, on the basis of the spectral analysis data, it was established that laser 2D-heterostructures created on the basis of InGaAsP/InP can modulate their lasing frequency under the influence of an acoustic wave. In particular, the action of a bulk ultrasonic wave with an acoustic power of 1 W (an intensity of 100 W/cm^2) gave rise to a variation of the radiation wavelength by 0.7–0.8 nm [1], which corresponded to a frequency change of about 110 GHz.

5. Conclusions

A theoretical model describing the process of radiation frequency modulation induced by an ultrasonic wave at the recombination transition between the ground states of an electron and a hole in the InAs/GaAs heterostructure with InAs quantum dots is developed. The dependence of the frequency modulation amplitude on the acoustic wave frequency was found to have a nonmonotonous character with two maxima, the positions of which are governed by the geometrical dimensions of the heterostructure and the elastic constants of its components. The reduction of the QD size is shown to result in an increase in the frequency modulation amplitude, which is explained by the growth of deformations in the QD material.

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ЧАСТОТНА МОДУЛЯЦІЯ РЕКОМБІНАЦІЙНОГО ВИПРОМІНЮВАННЯ ГЕТЕРОСТРУКТУРИ InAs/GaAs З КВАНТОВИМИ ТОЧКАМИ InAs ПІД ВПЛИВОМ АКУСТИЧНОЇ ХВИЛІ

Р.М. Пелещак, О.О. Даньків, О.В. Кузик

Р е з ю м е

Побудовано теоретичну модель процесу частотної модуляції випромінювання при рекомбінаційному переході між основними станами електрона та дірки в гетероструктурі InAs/GaAs з квантовими точками InAs за допомогою акустичної хвилі. Встановлено характер залежності амплітуди частотної модуляції від частоти акустичної хвилі.