FORMATION OF THE EXCITON CONDENSED PHASES IN DOUBLE QUANTUM WELLS IN THE PRESENCE OF EXTERNAL HARMONIC POTENTIAL

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PACS 71.35.Lk, 78.67.De, 74.20.De

The properties of structures of the spatial distribution of indirect excitons in semiconductor double quantum wells in the presence of an external harmonic potential for excitons are investigated. The calculations indicate that if the radiation density exceeds some threshold value, the structures of the exciton density distribution appear. The appearance of islands is explained by the existence of the exciton condensed phase and the non-equilibrium state of the system due to the finite lifetime of excitons and the presence of a pumping. As the pumping increases, the distribution of excitons acquires the form of concentric rings which are divided into separate islands of the exciton condensed phase, and then the structure transforms into continuous rings. At the further increase in the intensity of the external excitation, the condensed phase with inclusions of the gaseous phase islands (antiislands) emerges. It is shown that enlarging the depth of the potential trap allows one to observe the exciton condensation at lower intensities of the pumping. The dependence of the structures on the depth and the radius of the potential and on the pumping intensity is found.

1. Introduction

Semiconductor double quantum wells (QWs) is a perspective system for the investigation of exciton collective effects. In such a system, the lifetime of excitons is very long in the presence of an electric field which separates electrons and holes over different wells [1]. A special interest is stimulated by a possibility to find the Bose–Einstein condensation of excitons [2] which has still not been revealed in bulk semiconductors. At applying the electric field, the recombination of indirect excitons is slowed down, and the excitons lifetime prolongs. Therefore, a large concentration of excitons can be gathered at lower pumping intensities. A series of new experimental results in semiconductor systems with double QWs was obtained. V.B. Timofeev with co-authors revealed a very thin luminescence line in the excitation spectra of indirect excitons [3] which depends unusually on the temperature and the pumping. Non-trivial peculiarities were detected in the spatial structure of the exciton luminescence emission: the periodic breaking of the emission ring which is concentric to the laser excitation spot and localized at large distances from the laser spot [4]; a structure in the form of islands of the emission under the perimeter of a round window in the electrode, through which excitons were created in the QW [5]; and the division of the exciton density into separate islands in the direction which was perpendicular to the periodic modulation of a potential [6]. Meanwhile, the shape of formed structures was not caused by a symmetry of the system or by external factors. Therefore, this circumstance gave rise to a considerable interest of theorists. There are theoretical models, in which the appearance of structures is connected with the Bose statistics [7–9], the description of a system by the non-linear Schrödinger equation [10], or the Mott transition [11]. A possibility of a periodic distribution of the exciton density is shown in some models in [7–11]. But these papers give no explanation of the peculiarities of a spatial location of the emission islands and the structure behavior with changing the pumping, temperature, and external conditions.

In a series of our papers [12–15], the aforementioned and other peculiarities of a periodic location of the emission from the QWs were successfully explained. The main idea for the explanation was the fact that there is the exciton condensed phase, and the system is not equilibrium due to the finite exciton lifetime. Two models of phase transition theory, which were generalized for a finite lifetime of particles, were applied. The statistical theory of nucleation-growth (the Lifshits–Slyozov model) [12, 13] is one of these models. By means of this theory, the phase diagram in V.B. Timofeev's works [3] was explained, as well as the dependence of the emission intensity on the pumping and the temperature. For the visualization, the theory of spinodal decomposition [14, 15] (the Cahn–Hillert model) turned out to be very useful. Applying this theory, the transition from the fragmented ring to a continuous ring of the luminescence emission with changing the parameters of the system [14], periodic location of emission islands under the round window of a metallic electrode [15], and the behavior of the structure with changing the pumping and the temperature were satisfactorily described, as well as other effects which were hardly attainable in the framework of the statistical model. It should be emphasized that the consideration of a non-equilibrium state of the system caused by a finite value of the lifetime is the principal condition for the explanation of experiments within both models.

Last years, some authors considered the possibility to increase the exciton density at the same pumping, by creating the macroscopic traps for excitons [16,17]. Such a potential can have a harmonic profile [18,19].

In the present paper, the phase conversion in the exciton system at the laser pumping in an external field will be analyzed, namely in the potential of an axisymmetric harmonic profile.

2. Model of the System

Let us investigate the exciton density distribution in the QW plane in case of the laser pumping and an external potential (Fig. 1,a). For solving this task, we utilize the approach which was employed at studying the spinodal decomposition, by generalizing it for unstable particles with regard for the pumping. The conservation law for the density n of excitons, which have a finite lifetime τ, is as follows:

$$\frac{\partial n}{\partial t} = -\mathrm{div}\mathbf{j} + G - \frac{n}{\tau}, \qquad (1)$$

where G is the pumping (the number of excitons which are created by the laser radiation in a unit area for a unit time), $\mathbf{j} = -M\nabla\mu$ is the current density, and μ is the chemical potential. We use the Einstein formula $M = nD/k_\mathrm{B}T$ for the exciton mobility. The chemical potential is expressed in terms of the free energy as $\mu = \delta F/\delta n$. The free energy is chosen in the Landau model:

$$F[n] = \int d\mathbf{r}\left[\frac{K}{2}(\nabla n)^2 + f(n) + nV\right]. \qquad (2)$$

The term $\frac{K}{2}(\nabla n)^2$ characterizes the energy of the inhomogeneity, and the additional energy of an exciton in the external field is described by the term nV. The free energy density f is approximated in the form

$$f(n) = kTn\,(\ln n - 1) + \frac{a}{2}n^2 + \frac{b}{3}n^3 + \frac{c}{4}n^4, \qquad (3)$$

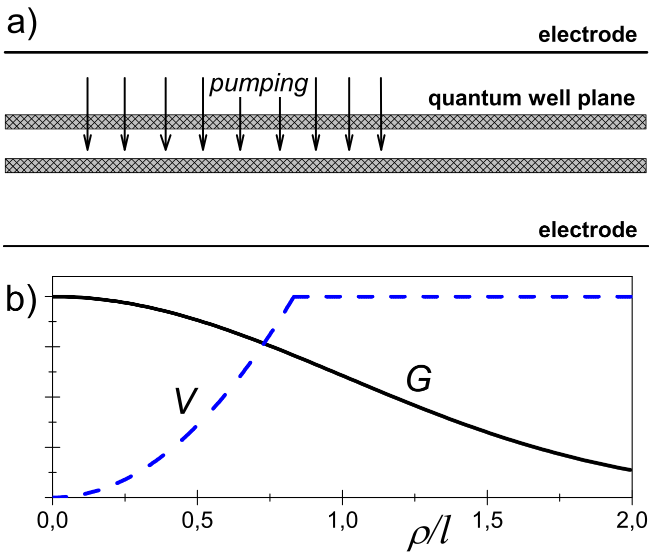

Fig. 1. Scheme of the system (a). Radial profiles of pumping (6) and the harmonic potential (7) (b)

where the term $kTn\,(\ln n - 1)$ dominates at small densities of excitons and describes their diffusive movement outside of the laser spot, and the terms of the power expansion of the exciton concentration play the main role at large values of n. The parameters a, b, and c in Eq. (3) are phenomenological. They are chosen in such a way that the free energy has a minimum corresponding to the condensed phase, and the shift of the spectrum to the region of high frequencies with increase in n was reproduced (see the detailed analysis in [15]). To this end, we suppose $a > 0$, $b < 0$, $c > 0$.

For numerical simulations, we choose the following units of length, concentration, and time:

$$l_\mathrm{u} = \sqrt{\frac{K}{a}}, \quad n_\mathrm{u} = \sqrt{\frac{a}{c}}, \quad V_\mathrm{u} = an_\mathrm{u}, \quad t_\mathrm{u} = \frac{d_1 l_\mathrm{u}^2}{D}, \qquad (4)$$

where $d_1 = kT/V_\mathrm{u}$. Introducing the dimensionless units and using the formulas for the free energy (2) and its density (3), we can reduce the equation for the exciton density (1) to the form

$$\frac{\partial n}{\partial t} = d_1\Delta n + \nabla\left[n\nabla\left(-\Delta n + n + b_1 n^2 + n^3 + V\right)\right] +$$

$$+G - \frac{n}{\tau}, \qquad (5)$$

where $b_1{=}b/\sqrt{ac}$. Equation (5) is a non-linear 2D phenomenological equation which describes the high-density exciton distribution with regard for the pumping, external potential, and finite lifetime of particles.

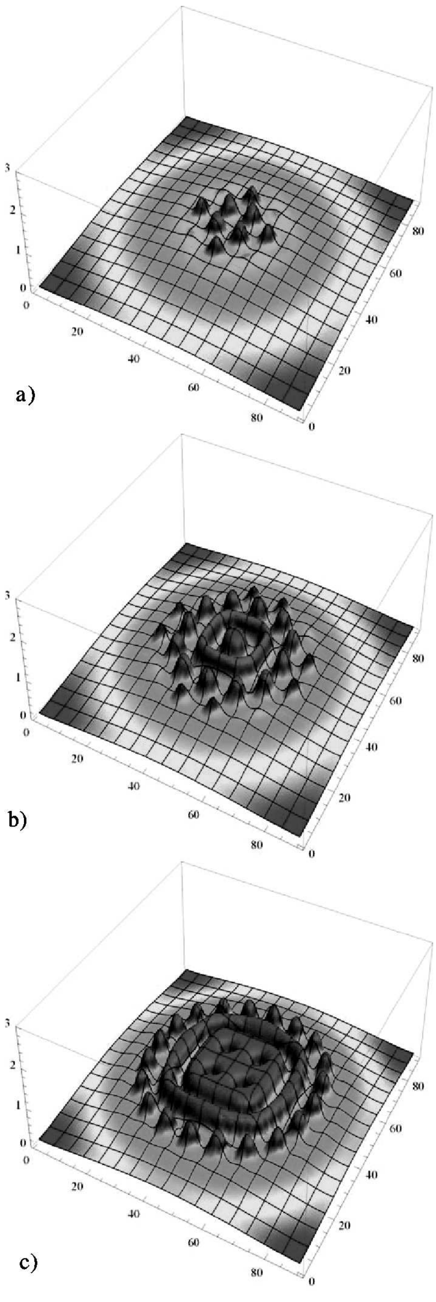

Fig. 2. Exciton density distribution $n(x, y)$ in the QW plane in the system without potential at different values of the pumping intensity P: a – 150; b – 180; c – 230. Other parameters of the system are $\tau = 100$, $b_1 = -2.1$, $d_1 = 0.1$, $l = 60$

To study the exciton density distribution in the QW plane, we suppose that the pumping is determined by the Gauss dependence on coordinates:

$$G(\rho) = \frac{P}{2\pi l^2} \exp\left(-\frac{\rho^2}{2l^2}\right), \qquad (6)$$

where ρ is the distance from the laser spot center (the radial coordinate in the QW plane), and l denotes the Gauss curve half-width.

The integral pumping P is measured in units $t_{\rm u}^{-1}$.

Equation (5) was solved numerically in a 2D domain, whose sizes substantially exceeded the half-width of the pumping distribution (6), so that the density of excitons was negligible at the boundaries. Below, the results of calculations will be presented namely for the region of a laser spot.

3. Numerical Calculation of the Exciton Density in the Case of the Pumping without External Potential

Let us consider the structures of the density of indirect excitons at the pumping with a laser spot of shape (6). The results of the solution of Eq. (5) for the exciton density are shown in Fig. 2 for various intensities of the pumping. At small intensities of the irradiation, the maximal density is concentrated in the laser spot region, and the structure is monotonous in the form of a disk, i.e., the structure is of axial symmetry. When the pumping exceeds some threshold value, the homogeneous distribution of the density becomes unstable with respect to the formation of islands of the condensed phase (Fig. 2,a). Under some conditions, the structure gets form of the condensed phase islands. We assume that their appearance in the plane of the gaseous phase is a consequence of both the formation of the condensed phase of excitons and the manifestation of the instability of the homogeneous solution of Eq. (5). At higher intensities of the radiation, some islands merge into concentric rings of the condensed phase (Fig. 2,b).

As the pumping intensity increases, the structure of the exciton density changes: from the form of the condensed phase islands in the gaseous phase to "antiislands" of the gaseous phase in the dominant condensed phase (Fig. 2,c). The gaseous phase islands (antiislands) are located on the spot of the high-density condensed phase. With increase in the pumping, the antiislands are transformed into a continuous circle, and then a structure in the form of a disk with nonuniform edges emerges. At high pumping intensities, a large-size domain of the condensed phase is formed near the laser spot, where the density of excitons is high.

4. Numerical Calculation of the Exciton Density in the Case of a Harmonic Potential

In this section, we investigate the exciton density in the presence of an external harmonic potential. Let the shape of the potential be as follows:

$$V = \begin{cases} V_0 \left(\frac{\rho^2}{R^2} - 1 \right), & \rho \le R, \\ 0, & \rho > R. \end{cases} \quad (7)$$

Here, R is the parameter which characterizes the radius of a potential parabolic trap, and V_0 determines the trap depth (see Fig. 1,*b*). One can expect that the presence of such a potential leads to the exciton drift toward the region, where the potential has a minimum. Just there, the formation of the condensed phase takes place. Such a potential was realized experimentally [18].

The results of the solution of Eq. (5) for the exciton density with a potential energy in the form (7) are presented in Fig. 3 for various intensities of the pumping. At low intensities of the excitation, the maximum of the exciton density is observed in the minimum of the harmonic potential. When the pumping reaches some threshold, the homogeneous distribution becomes unstable with respect to the formation of spatially inhomogeneous islands of the condensed phase. At else higher intensities of the pumping, the formed fragments merge into a continuous ring of the condensed phase. We note that the potential trap depth V_0 is $17.5\,k_{\mathrm{B}}T$ at the chosen parameters.

The fragments of the condensed phase merge into a continuous ring also with increase in the depth of the harmonic potential (Fig. 4). Such behavior is caused by the fact that, as the potential depth increases, the exciton density increases in the minimum of the potential trap, which is equivalent to the pumping growth.

Figures 2–4 are drawn for dimensionless values of the parameters. For the quantitative description, we chose the following values of the parameters: $\tau = 10\,\mathrm{ns}$, $T = 4\,\mathrm{K}$, $n_{\mathrm{u}} = 3.57 \times 10^{10}\,\mathrm{cm}^{-2}$, $D = 4\,\mathrm{cm}^2\mathrm{c}^{-1}$, $Kn_{\mathrm{u}}^2 = 492\,\mathrm{meV}$, $an_{\mathrm{u}} = 3.45\,\mathrm{meV}$. In this case, the potential depth V_0 in Fig. 3 is equal to 6.04 meV, the parameter l of the Gauss pumping half-width in Fig. 2 is 37.9 μm, and the condensed phase is observed at the densities of the order of $5 \times 10^{10}\,\mathrm{cm}^{-2}$.

The system with the external potential, whose shape is similar to a harmonic one, was realized in the experiment [20].

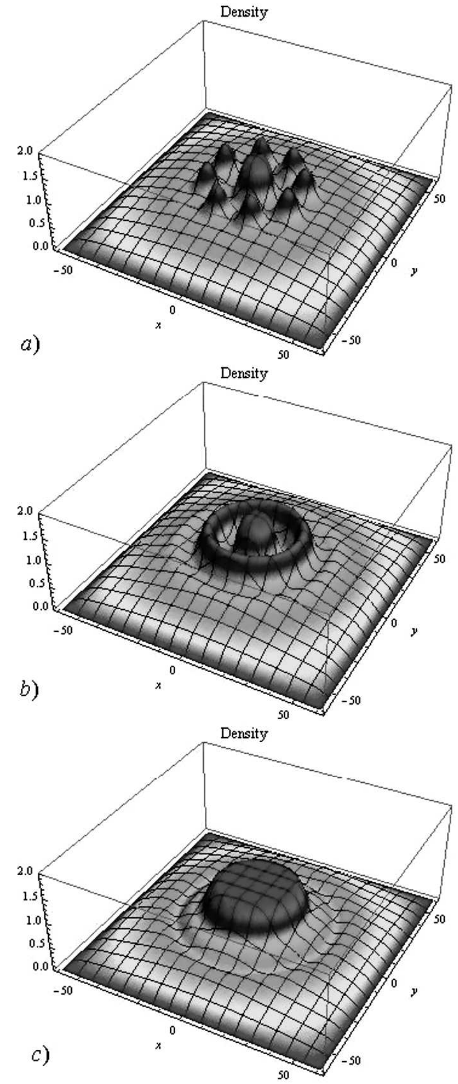

Fig. 3. Exciton density distribution in case of the harmonic potential versus the pumping intensity P: a – 0.007; b – 0.008; c – 0.01. The parameters of the system are $\tau = 100$, $b_1 = -2.1$, $d_1 = 0.1$, $l = 80$, $V_0 = 1.75$, $R = 50$

5. Conclusions

In the present paper, we have studied the structures which emerge during the exciton condensation in the system of indirect excitons in semiconductor double QWs, taking the finite exciton lifetime, pumping, and external harmonic potential into account. The following results are obtained:

1. The exciton condensed phase appears at the pumping which exceeds a certain threshold value.
2. At the pumping growth, the structure of the condensed phase gets different forms: the gaseous phase,

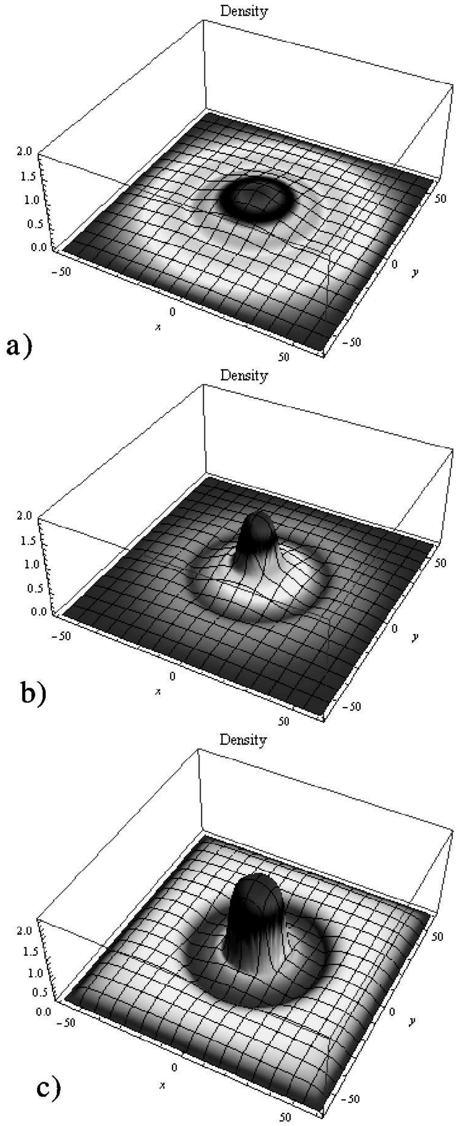

Fig. 4. Exciton density distribution in case of the harmonic potential versus the depth of the potential V_0: a – 0.9; b – 2.7; c – 8.1. $P = 0.003$, $R = 30$, $l = 20$. Other parameters of the system are as those in Fig. 3

the system of islands of the condensed phase against the background of the gaseous phase, the system of islands of the gaseous phase (antiislands) against the background of the condensed phase, and the continuous condensed phase.

3. Increasing the depth of the potential harmonic trap causes a decrease in the pumping threshold for the exciton condensation appearance.

The emerging structures are an example of the self-organizing processes under non-equilibrium conditions.

In the present work, the condensed phase was not concretized, but it is described by several parameters of the free energy. The investigation of the processes of condensation and formation of structures at phase transitions in the systems under consideration can be important for the development of a microscopic theory of the exciton condensed phase and optoelectronics [21].

1. T. Fukuzawa, E.E. Mendez, and J.M. Hong, Phys. Rev. Lett. **64**, 3066 (1990).
2. S.A. Moskalenko and D.W. Snoke, *Bose–Einstein Condensation of Excitons and Biexcitons and Coherent Nonlinear Optics with Excitons* (Cambridge University Press, Cambridge, 2000).
3. A.V. Larionov and V.B. Timofeev, Pis'ma Zh. Eksp. Teor. Fiz. **73**, 342 (2001); V.B. Timofeev, Usp. Fiz. Nauk **175**, 315 (2005).
4. L.V. Butov, A.C. Gossard, and D.S. Chemla, Nature **418**, 751 (2002).
5. A.V. Gorbunov and V.B. Timofeev, Pis'ma Zh. Eksp. Teor. Fiz. **83**, 178 (2006); Usp. Fiz. Nauk **176**, 652 (2006).
6. M. Remeika, J.C. Graves, A.T. Hammark *et al.*, Phys. Rev. Lett. **102**, 186803 (2009).
7. L.S. Levitov, B.D. Simons, and L.V. Butov, Phys. Rev. Lett. **94**, 176404 (2005).
8. A.V. Paraskevov and T.V. Khabarova, Phys. Lett. A **151**, 368 (2007).
9. R.B. Saptsov, Pis'ma Zh. Eksp. Teor. Fiz. **86**, 779 (2007).
10. C.S. Liu, H.G. Luo, and W.C. Wu, J. Phys.: Condens. Matter. **18**, 9659 (2006); Phys. Rev. B **80**, 125317 (2009).
11. V.S. Babichenko, arXiv/cond-mat:0706.0994 (2007).
12. V.I. Sugakov, Ukr. J. Phys. **49**, 1117 (2004); Solid State Commun. **134**, 63 (2005).
13. V.I. Sugakov, Fiz. Tverd. Tela **48**, 1868 (2006); V.I. Sugakov, Fiz. Nizk. Temper. **32**, 1449 (2006); Phys. Rev. B **76**, 115303 (2007).
14. A.A. Chernyuk and V.I. Sugakov, Phys. Rev. B **74**, 085303 (2006).
15. V.I. Sugakov and A.A. Chernyuk, Pis'ma Zh. Eksp. Teor. Fiz. **85**, 699 (2007).
16. A.T. Hammack, N.A. Gippius, S. Yang *et al*, J. of Appl. Phys. **99**, 066104 (2006).
17. A. Gartner, L. Prechtel, D. Schuh *et al.*, Phys. Rev. Lett. **76**, 085304 (2007).
18. Z. Voros, D.W. Snoke, L. Pfeiffer *et al.*, Phys. Rev. Lett. **97**, 016803 (2006).
19. V. Negoita, D.W. Snoke, and K. Eberl, Appl. Phys. Lett. **75**, 2059 (1999).

20. G. Chen, R. Rapaport, L.N. Pffeifer *et al.*, Phys. Rev. B **74**, 045309 (2006).

21. G. Grosso, J. Graves, A.T. Hammack *et al.*, Nature Photonics **3**, 5770 (2009).

Received 14.02.11

ФОРМУВАННЯ КОНДЕНСОВАНИХ ФАЗ ЕКСИТОНІВ У ПОДВІЙНИХ КВАНТОВИХ ЯМАХ ЗА НАЯВНОСТІ ЗОВНІШНЬОГО ГАРМОНІЙНОГО ПОТЕНЦІАЛУ

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Р е з ю м е

Вивчено властивості структур у просторовому розподілі густини непрямих екситонів у подвійних квантових ямах у напівпровідниках за наявності зовнішнього гармонійного потенціалу для екситонів. Розрахунки показали, що при накачках, більших за порогові, виникають структури густини екситонів. Поява острівців пояснюється існуванням конденсованої фази екситонів та нерівноважністю системи внаслідок скінченного часу життя екситонів і наявності накачки. Зі зростанням накачки розподіл екситонів має вигляд концентричних кілець, розбитих на окремі острівці конденсованих екситонів, потім структура набуває вигляду суцільних кілець. За подальшого збільшення інтенсивності зовнішнього збудження утворюється конденсована фаза із вкрапленнями острівців газової фази (антиострівців). Показано, що збільшення глибини потенціальної ями дозволяє спостерігати конденсацію екситонів при менших накачках. Знайдено залежність виникаючих структур від глибини і радіуса потенціалу та інтенсивності накачки.