FORMATION OF DARK DISSIPATIVE SOLITONS IN MEDIA WITH NONLOCAL RESPONSE

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For the problem of two-wave self-diffraction in a nonlocal nonlinear medium considered in the reflection geometry, the steady state solutions in terms of the tanh function have been found for the distribution of dynamical grating amplitudes and for the distribution of intensity maxima in the interference pattern. The solutions for the mixed-wave intensities turned out to depend on the area under the curve describing the grating-amplitude distribution function. The distribution in the form of the tanh function shifts along the direction of wave propagation, when the ratio of the intensities for the input waves changes. The dynamical problem is solved numerically for the case of two interacting Gaussian beams. It has been demonstrated that the shape of output beams can be controlled by varying the time delay between the input pulses, hence creating various dissipative solitons, including grating-amplitude distributions, in the medium bulk.

1. Introduction

A lot of phenomena based on a nonlinear interaction between waves are known. The scope of their application includes the formation of beams with different wavelengths or frequencies, optical parametrical amplifiers and oscillators, the wave front inversion, the creation of holographic images, the treatment of optical signals and images, and so forth [1].

As has been shown in recent years, besides the generation of components with different wavelengths or frequencies, amplification, and oscillations, the wave interaction can result in the formation of stable space-time localized states [2–6]. This effect is observed at the wave diffraction at dynamic gratings, if the exciting "light grating" of the interference pattern and the dynamical grating of the refractive index are shifted in space with respect to each other. Such a situation is realized in the cases where a nonlinear medium is characterized by either a nonlocal or an inertial response [7]. The shift between the light and dynamical gratings brings about a phase delay between the mixing waves, which reveals itself as the well-known effect of energy exchange between those waves. As a result, the ratio between wave intensities in the nonlinear medium changes, with localized stable structures being formed at that for both the interference pattern and the dynamical-grating amplitude. Nowadays, many media characterized by a nonlocal and nonlinear response have been studied, including optically nonlinear media, plasma, and the Bose-Einstein condensate [8, 9].

As a rule, a nonlocal response arises, when some transport process invokes a nonlinearity. For instance, it can be a nonlinearity associated with the heat conduction in media with a thermal response [10], diffusion of molecules or atoms that accompanies the propagation of light beams in atomic vapor [11], or charge transfer in photorefractive crystals [12]. The nonlocal response in nematic liquid crystals, which arises owing to a reorientation of anisotropic molecules under the light beam propagation, has been studied as well [13, 14]. Recently, a nonlocal nonlinearity and the formation of dissipative optical solitons have been revealed in wide-aperture lasers with saturable absorption [15, 16].

Two basic geometries of nonlinear wave mixing, the transmission and reflection ones, are usually considered in the self-diffraction problems [7]. In the former case, the space-time localized structure behaves itself as a bright dissipative soliton [3–5], and its profile is described in terms of the sinh function. The corresponding complex-valued Ginzburg–Landau equation, which describes the space-time dynamics of a dissipative soliton, was derived in [5]. The localization degree of a bright dissipative soliton and the position of its maximum were found to be controllable, by varying the ratio of the wave intensities at the input into the medium. The output intensities turned out to be determined by the integral area under the soliton profile calculated within the medium boundaries. They were shown to depend very strongly on the intensity ratio for input waves, because the space-time localization of a dissipative soliton changes at that.

In the reflection geometry, the profile of the grating-amplitude distribution has the shape of a dark dissipative soliton, and it is described by the tanh function. The motion of such a soliton in the course of the grating erasure process owing to the two-beam wave mixing was

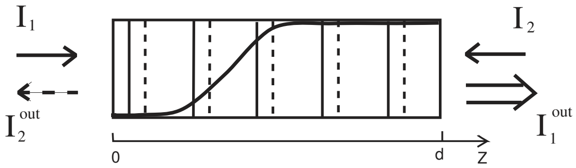

Fig. 1. Diagram of the two-beam interaction in a medium with a nonlocal nonlinear response in the reflection geometry of the problem. Straight lines mark the interference pattern maxima, and dashed ones the maxima of the dynamical refractive-index grating. The curve illustrates the grating amplitude distribution

studied theoretically in work [6]. The authors of work [17] theoretically showed that the intensities of output waves in the reflection geometry substantially depend on the ratio of the intensities of input waves, but not equivalently. However, no physical interpretation was given for the results of those calculations.

In this work, we obtained the stationary solutions for the problem of two-beam wave interaction in the reflection geometry in a medium with a nonlocal and nonlinear response. We obtained tanh-like profiles for the curves describing the dynamical-grating amplitude distribution and the intensity distribution at the interference pattern maxima. We showed how one can control the position of a dark dissipative soliton in the medium and how it affects the intensities of output waves. We also demonstrates a capability to govern the pulse profiles, if the light pulses at the input of the system are Gaussian-like.

The complex-valued Ginzburg–Landau equation is widely known in many branches of nonlinear physics, chemistry, and biology, while describing various time- or space-localized structures [18]. It is regarded as the simplest mathematical model, which allows solutions in the form of dissipative solitons to be obtained. Dissipative solitons are known to manifest a lot of unusual properties, such as stable periodic pulsations, coupled soliton waves, bistability, coupled pairs composed of holes and fronts, periodic "explosions", collapses, the formation of helical waves in two-dimensional systems and stable threads in three-dimensional ones, and many others [19]. All those unique properties can find application, e.g., when using a nonlinear wave interaction, in particular, in dynamical holography in nonlocal media.

2. Formation of a Non-uniform Distribution of Intensity Maxima in the Interference Pattern at the Self-diffraction of Waves in a Nonlinear Medium

In this section, we consider stationary solutions for the problem of two-beam wave interaction (TBCI) in a medium with a nonlocal response. The consideration is carried out for the reflection geometry of the problem, and the corresponding TBWI schematic diagram is depicted in Fig. 1. In contrast to the previous publications, stationary solutions were obtained for the dynamical-grating amplitude and the intensity at the maxima of the interference pattern. Now, we shall demonstrate that the non-uniform distributions of those quantities are formed in the nonlinear medium bulk along the coordinate of wave propagation z. The solutions for the amplitudes (or, respectively, the intensities) of output waves are determined by calculating the area under the distribution curve for the dynamical-grating amplitude within the medium boundaries.

Consider the TBWI diagram depicted in Fig. 1. Two input waves, I_1 and I_2, propagate toward each other in a nonlinear medium, where they form an interference pattern I_m. Under the action of the modulated I_m-intensity distribution and by means of nonlinear optical mechanisms, the refractive index $\Delta n(I)$ changes. The variation is also modulated, i.e. a dynamical diffraction grating is recorded. If the nonlinear medium is characterized by a purely nonlocal response, the amplitude maxima of the recorded grating become shifted with respect to the intensity maxima by one-quarter of the period of the interference pattern. The self-diffraction process consists in that the mixing waves create a dynamical diffraction grating and, simultaneously, diffract at this very grating. The output intensities $I_1^{\rm out}$ and $I_2^{\rm out}$ are obtained as a result of the interference between two waves: one of them propagates in the given direction, and the other diffracts at the dynamical grating. Since the refractive-index grating is shifted with respect to the light one, the well-known effect of energy exchange between mixing waves is observed [7]. If the dynamical grating is shifted in the z-axis direction with respect to the interference pattern (this case is depicted in Fig. 1), wave 1 that propagates in the same direction becomes amplified, because it and the diffracting wave interfere in phase. At the same time, the intensity of wave 2 diminishes at the output, because it and the wave diffracting in this direction interfere in anti-phase.

The maximum amplitude of the recorded grating is determined by the light intensity in the interference pattern maxima. However, it is known that this intensity depends on the ratio between the intensities of mixing waves: $I_m(z) \propto \sqrt{I_1(z)I_2(z)}$. Those intensities change over the bulk of the nonlinear medium owing to the energy transfer effect. Therefore, the intensity distribution over the interference pattern maxima $I_m(z)$ is not uniform along the z-axis. We intend to demonstrate that the distributions for both the grating amplitude maximum, $E(z)$, and the normalized intensity at the interference pattern maxima, $\frac{I_m(z)}{I_0(z)}$, are described by the tanh function. Formally, this function coincides with the known solutions for dark solitons [20]. However, in our case, this function does not describes unstable waves, solitons, but stable spatially localized structures, which are referred to as dissipative solitons in the modern literature [18]. Therefore, we shall use below the term "dark dissipative soliton" to designate the structure that is formed in our case. The example of such a structure is shown in Fig. 2. We obtained that the position of the inflection point in the tanh functions along the z-axis depends on the intensity ratio for input waves, I_1/I_2

The dynamical process of wave self-diffraction in the reflection geometry is described by a system of equations that includes [6, 7] an equation for coupled waves,

$$\frac{\partial A_1}{\partial z} = -i\frac{E}{I_0}A_2; \quad \frac{\partial A_2^*}{\partial z} = -i\frac{E}{I_0}A_1^* \tag{1}$$

and a dynamical equation for grating recording,

$$\frac{\partial E}{\partial t} = \gamma\frac{A_1 A_2^*}{I_0} - \frac{1}{\tau}E, \tag{2}$$

where $A_1(t,z)$ and $A_2(t,z)$ are the amplitudes of waves 1 and 2, respectively, which change slowly; the asterisk means the complex conjugation; $E(t,z)$ is the grating amplitude (in the approximation of small variations of the refractive index, $E \equiv \Delta\varepsilon \cong 2n_0\Delta n$, where $\Delta\varepsilon$ and Δn are variations of the dielectric permittivity and the refractive index, respectively, of the medium induced by the laser radiation, and n_0 is the average refractive index in the medium); $I_0(t,z) = |A_1|^2 + |A_2|^2$ is the total intensity; $I_m(t,z)$ are the interference pattern maxima; $\gamma = \gamma_L + i\gamma_N$ is the nonlinear gain coefficient of the medium: γ_L and γ_N describe, respectively, the local and nonlocal responses of the medium (according to work [7], in nonlocal media, $\gamma = 2\pi\Delta n_{\max}(\cos\Phi_g + i\sin\Phi_g)$, where Φ_g describes a shift of the dynamical grating with respect to the interference pattern maxima, and $\Delta n_{\max}$ is the maximally possible grating amplitude in the given medium); and τ is the relaxation time for the refractive-index grating. The first integral of system (1), (2) is $I_d = |A_1|^2 - |A_2|^2 = \text{const}$.

Let us consider a case of purely nonlocal response, when the shift between the interference pattern and the dynamical grating equals one-quarter of the grating period, i.e. $\gamma = i\gamma_N$. Then, system (1)–(2) can be substantially simplified, because it transforms into a system of real-valued equations, for which the stationary solutions

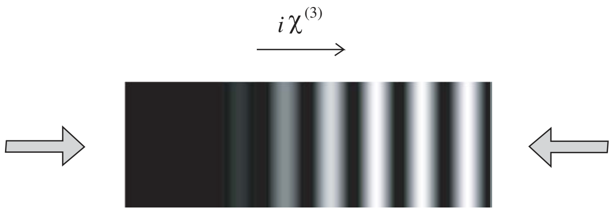

Fig. 2. Localized structure of a dark dissipative soliton formed at TBWI

can be found easily. For the stationary system, we have

$$E = \gamma_N\tau\frac{I_m}{I_0} \tag{3}$$

and another first integral is $I_0^2 - 4I_m^2 = I_d^2$. The solutions of the stationary system are as follows.

The distribution of grating amplitudes is

$$E(z) = \frac{1}{\sqrt{2}}\sqrt{1 + \tanh\left(\gamma\tau\cdot z - p + \frac{1}{2}\ln\left(\frac{4}{I_d^2}\right)\right)}. \tag{4}$$

Taking Eq. (3) into account, we see that the normalized intensity at the interference pattern maxima, I_m/I_0, has the same distribution, but the distributions $E(z)$ and I_m/I_0 are shifted in space with respect to each other by one-quarter of the period along the z-axis (or, equivalently, by a phase of $\pi/2$).

The amplitudes of mixing waves look like

$$A_1(z) = C_1 e^{U(z)} + C_2 e^{-U(z)},$$

$$A_2(z) = C_1 e^{U(z)} - C_2 e^{-U(z)}, \tag{5}$$

where U is the integral area under the distribution curve for the grating amplitude,

$$U(z) = \int_0^z E(z)dz = \frac{1}{4}\ln\left[\frac{1}{2} + e^w + \sqrt{(e^w)^2 + e^w}\right], \tag{6}$$

$w = 2\gamma\tau z - 2p + \ln\left(4/I_d^2\right)$, $U_0 = U(z=0)$, $U_d = U(z=d)$, and d is the medium thickness. The constants C_1 and C_2 are determined from the expressions

$$C_1 = (A_{2d}e^{-U_0} + A_{10}e^{-U_d})/(2\cosh(U_d - U_0)),$$

$$C_2 = (A_{10}e^{U_d} - A_{2d}e^{U_0})/(2\cosh(U_d - U_0)), \tag{7}$$

where $A_{10} = A_1(z=0)$ and $A_{2d} = A_2(z=d)$ are the amplitudes of input waves.

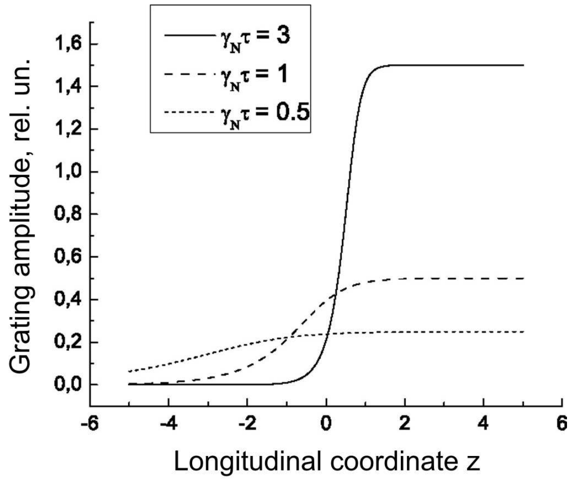

Fig. 3. Grating-amplitude profiles $E(z)$ for various gain coefficients in the nonlinear medium. The output waves have identical intensities: $I_1 = I_2 = 0.5$

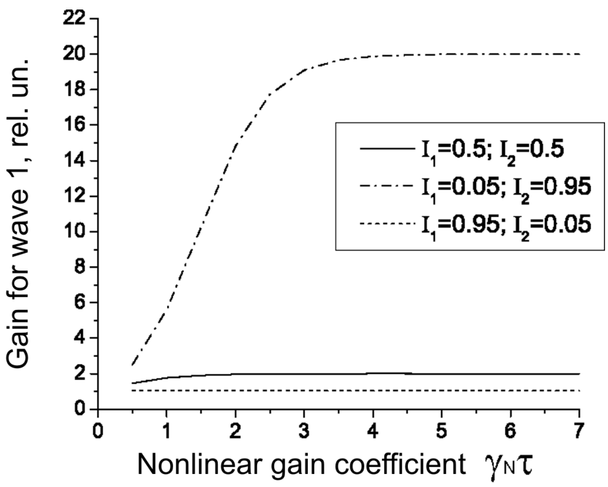

Fig. 4. Gain coefficient for wave 1, I_1^{out}/I_1, at various ratios I_1/I_2 between input-wave intensities

The integration constant p can be found from the conditions at the medium boundaries,

$$E(0) = \gamma\tau \frac{A_{10}A_2(0)}{A_{10}^2 + A_2^2(0)} =$$

$$= \frac{1}{\sqrt{2}}\sqrt{1 + \tanh\left(-p + \frac{1}{2}\ln\left(\frac{4}{I_d^2}\right)\right)},$$

$$E(d) = \gamma\tau \frac{A_1(d)A_{2d}}{A_1^2(d) + A_{2d}^2} =$$

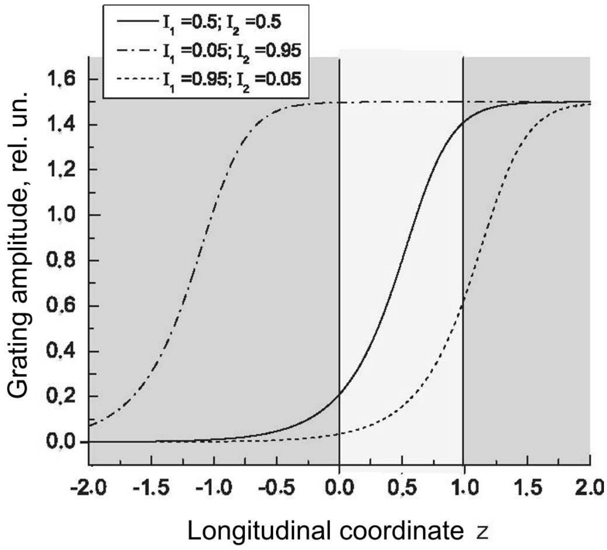

Fig. 5. Stationary distribution of the grating amplitude for input wave intensities presented in Fig. 4. The nonlinear gain coefficient of the medium $\gamma_N\tau = 3$. The nonfilled interval $z = [0,1]$ corresponds to the nonlinear medium bulk ($d = 1$)

$$= \frac{1}{\sqrt{2}}\sqrt{1 + \tanh\left(\gamma\tau\, d - p + \frac{1}{2}\ln\left(\frac{4}{I_d^2}\right)\right)}, \quad (8)$$

into which the solutions $A_1(d)$ and $A_2(0)$ for the output waves (see Eq. (5)) should be substituted.

We calculated the grating amplitude profile for various values of nonlinear response coefficient. The results of calculations are shown in Fig. 3. Note that the amplification of the output signal in a medium with a nonlocal response depends not only on the medium gain coefficient, but also on the ratio between the input wave intensities, as is illustrated in Fig. 4. It is evident that the lower the input intensity of wave 1, the higher is the gain coefficient. This occurs, because various stationary profiles of the grating-amplitude distribution are formed in the medium. In Fig. 5, the plots for the grating profile are depicted, which correspond to various ratios between input intensities taken from Fig. 4. One can see that the inflection point in the function $E(z)$ is located beyond the nonlinear medium, if the input intensity I_1 is low in comparison with I_2 (we adopted that the nonlinear medium occupies the interval $[0, 1]$). In this case, the grating amplitude in the medium is constant. However, if the ratio I_1/I_2 increases, the grating amplitude profile "moves" toward the medium, and a non-uniform distribution $E(z)$ in the medium is observed.

Therefore, our calculations testify that the coefficient of energy exchange between mixing waves can be controlled not only by varying the nonlocal gain coefficient

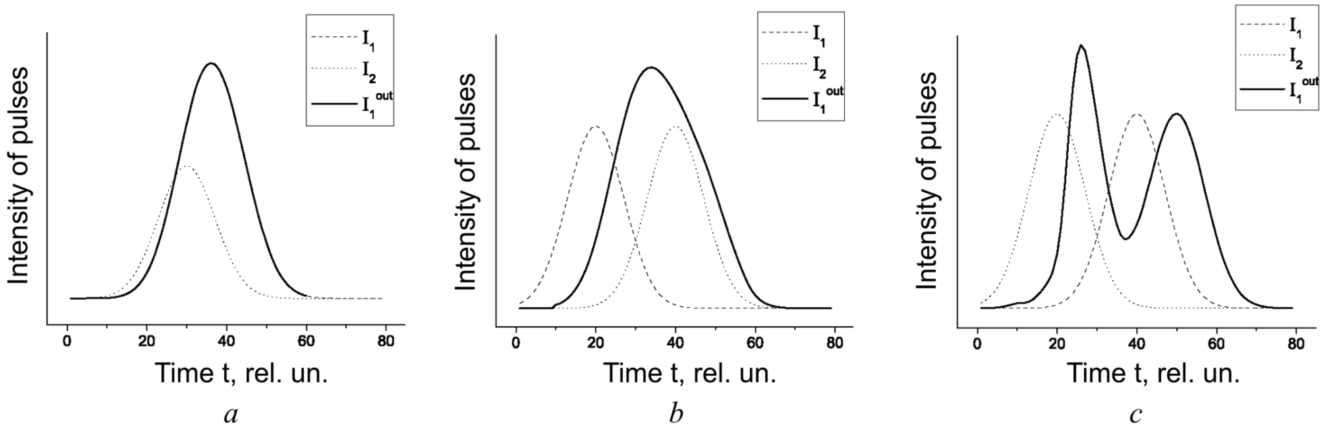

Fig. 6. Transformation of the pulse shape at the medium output, I_1^{out}, induced by the interaction between two Gaussian input beams (I_1, I_2) in a nonlinear medium with a nonlocal response in the reflection geometry. The time delays between the input pulses: (*a*) input beams are synchronized ($\Delta_1 = \Delta_2$), (*b*) $\Delta_1 < \Delta_2$, and (*c*) $\Delta_1 > \Delta_2$

of a nonlinear medium, but also by changing the ratio between the intensities of input waves. In this case, the distribution of grating amplitudes shifts: the lower the input intensity of the amplified beam, the more uniform is the grating amplitude in the medium bulk.

3. Control over Beam Parameters at TBWI

In the previous section, we showed that the position of a grating amplitude profile changes, by depending on the ratio between the input intensities of mixing waves. At the same time, this position – to be more specific, the coordinate of the inflection point in the tanh-function profile – governs the magnitude of energy exchange between the interacting beams and the amplitudes of output beams. Let us apply this idea in order to discover new effects in the control of beam parameters, when the beams interact in a nonlinear medium with a nonlocal response. For this purpose, consider the case where two Gaussian beams are applied to the input,

$$I_1 = I_{01} \exp\left[-\frac{(t-\Delta_1)^2}{\tau_1^2}\right],$$

$$I_2 = I_{02} \exp\left[-\frac{(t-\Delta_2)^2}{\tau_2^2}\right],$$

where I_{01} and I_{02} are the maximum intensities, τ_1 and τ_2 the times that correspond to the half-widths of Gaussian pulses, and Δ_1 and Δ_2 the time delays for beams 1 and 2, respectively; and τ is the time. The problem of interaction between laser beams providing TBWI is reduced to the dynamical equations (1), (2) in the case where the widths of Gaussian beams are much larger than the relaxation time of the dynamical grating. Therefore, we choose $\tau_1 = \tau_2 = 10$ and $\tau = 1$. Again, for the sake of simplicity, let us consider the case where the maximum input intensities are identical, $I_{01} = I_{01} = 1$.

Figure 6 demonstrates how the profile of Gaussian beam 1 changes at the medium output, depending on the time delay between input pulses. In Fig. 6,*a*, the input pulses are synchronized, and the output beam has the same Gaussian profile, but it is amplified owing to the energy exchange and delayed in time. If the input pulses are not synchronized, but their relative shift is not large, so that the pulses overlap, the output beam profile changes. This effect depends on which of the input pulses is applied first. If it is the pulse in beam 1, the output beam profile broadens, and it has one maximum. In the opposite situation, when beam 2 comes earlier to the input, the Gaussian beam changes its shape cardinally: it decays to form two maxima. The origin of such inadequate behavior lies in the fact that different dark dissipative solitons are generated in the medium, depending on different input conditions. This effect of a beam shape transformation can be used, e.g., in laser spectroscopy.

4. Conclusions

To summarize, the formation of a non-uniform distribution profile of the dynamical-grating amplitude at the two-beam wave interaction in a nonlinear media with a nonlocal response has been considered in the reflection TBWI geometry of the problem. This distribution was found to be described by the tanh function. The same distribution, to within a constant, is also inherent to the intensity at the interference pattern maxima. At the two-beam interaction, the steady state of this distribution is attained, which is stable in time. If the intensity ratio for the input waves varies, the distribution curve does not change its shape, but moves as a whole in the direction of wave propagation (the z-axis). Therefore, both the intensity distribution over the maxima of the interference pattern and the grating-amplitude profile reveal the properties of a dark dissipative soliton. The degree of energy exchange between the interacting beams and the intensities of the output waves are determined by the integral area under the curve of the grating-amplitude distribution. In the stationary case of TBWI scheme, the corresponding solutions were obtained.

The dynamical TBWI problem has been considered in the case of the interaction between two identical Gaussian input pulses, which are applied to the input of the system with a certain time delay. Numerical calculations demonstrate that the input conditions at the nonlinear medium boundaries are not equivalent, if the medium has a nonlocal response. The shape of the output beam changes, by transforming into a non-Gaussian one, if the input beams partially overlap in time. The resulting beam shape also depends on which of the beams is applied firstly. For instance, if input beam 2 has a time delay with respect to input beam 1, we obtained that the former broadens and shifts, but a single maximum remains preserved. In the opposite case, i.e. when input beam 1 has a time delay with respect to input beam 2, the shape of output beam 1 is quite different, namely, it has one minimum and two maxima. The origin of such a behavior is the formation of various dark dissipative solitons in the nonlinear medium.

The results obtained could be of interest for various applications, which are based on the wave interaction in nonlinear media with a nonlocal response, such as a signal transformation, optical switches, holographic interferometers, laser spectroscopy, and so forth. From this viewpoint, the subsequent progress in the development of the proposed approach consists in studying the dynamics of grating recording and the formation of dissipative solitons for various specific nonlinear media with a nonlocal response (liquid crystals, photorefractive crystals, atomic vapor, various gas media, and others), as well as in the development of corresponding calculation techniques for specific applications. Experimental researches on the basis of the proposed theory would doubtless be of interest.

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УТВОРЕННЯ ТЕМНИХ ДИСИПАТИВНИХ СОЛІТОНІВ У СЕРЕДОВИЩАХ З НЕЛОКАЛЬНИМ ВІДГУКОМ

С.А. Бугайчук

Р е з ю м е

Знайдено стаціонарні розв'язки у формі *tanh*-функції для розподілу амплітуди динамічної ґратки та для розподілу максимумів інтенсивності картини інтерференції при самодифракції двох хвиль у відбиваючій геометрії в середовищах з нелокальним нелінійним відгуком. Розв'язки для інтенсивностей взаємодіючих хвиль залежать від інтеграла під кривою розподілу амплітуди ґратки. Розподіл за формою *tanh*-функції зсувається вздовж координати поширення хвиль при зміні співвідношення інтенсивностей хвиль на вході у середовище. Динамічну задачу розв'язано чисельно для взаємодії двох гаусівських імпульсів. Показано, що залежно від часової затримки між вхідними імпульсами можна управляти формою вихідних імпульсів, створюючи у середовищі різні дисипативні солітони – розподіли амплітуди ґратки.