OPERATOR OF PHOTON DENSITY IN THE PHASE SPACE

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The possibility to describe the evolution of an electromagnetic field by means of the photon distribution function in the phase space ($\mathbf{r}, \mathbf{q}$-space) is studied. This function defined by analogy with the coarse-grained Mandel operator of photon density in the configuration space is used to characterize the local density of photons with a given momentum. Approximate eigenfunctions and eigenvalues of the distribution function, corresponding to one-photon localized states of the electromagnetic field, are obtained. It is shown that the photon transport is governed by the Newton mechanics if the "external force" acting on photons is a slowly varying function of spatial variables. It is shown that the distribution function at any time can be expressed via the initial distribution and photon's trajectories.

1. Introduction

Many important applications of quantum theory of light require a representation of radiation fields as ensembles of local excitations. For example, when the localized space-time interactions (like the spontaneous emission from an atom) are considered, the plane-wave representation can be inefficient. This circumstance has motivated the development of the photon-wave-function formalism (see, e.g., Refs. [1–3]), in which the single-photon states are treated as the states of individual particles (photons).

A somewhat different situation is in the case of photodetection. The electromagnetic field density averaged over the volumes with sizes of the order of several or even many wavelengths can be quite sufficient for the description of a measuring process, if the aperture of a measuring device is much greater than the wavelengths. The volume of averaging should be smaller than the volume involved in the course of measurement. In this case, the theoretical data obtained within this coarse-grained description, are quite suitable for comparing with the experimental ones. It seems that the formalism of the Mandel density operator (see Refs. [4] and [5]) can be useful for theoretical purposes. The definition of the Mandel operator does really includes the procedure of coarse-graining of the photon density in the manner explained above (see also Section 2). At the same time, it is worth pointing out that the Mandel operator depends on a complete set of the system variables (a set of all modes). This circumstance makes the Mandel operator to be inconvenient in theoretical studies. Moreover, the so-detailed description does not correspond to real experimental setups: the actual measuring devices are sensitive to very restricted sets of field modes.

There is another operator [6] of photon density, $f(\mathbf{r}, \mathbf{q}, t)$ describing the coarse-grained photon distribution in the $\mathbf{r}, \mathbf{q}$ space, i.e., in the phase space (see Section 3). In our opinion, this operator is preferable in practice. It is defined as a quadratic form of the creation and annihilation operators acting on the number of excitations of the actual field modes. These modes really contribute to the detected signals in individual experimental setups. The restriction of the plane-wave basis to the subspace of only important modes allows us to derive a linear kinetic equation for $f(\mathbf{r}, \mathbf{q}, t)$ (Section 5). This equation governs the evolution of the photon density. The solution of the kinetic equation relates an arbitrary-time photon distribution with the initial distribution. In contrast, the approach based on the Mandel's operator does not provide such useful opportunity.

The definition of photon density in the phase space, $f(\mathbf{r}, \mathbf{q}, t)$ is similar to the definition of distribution functions commonly used in practice for electrons, holes, phonons, *etc.* For example, the electron and hole distribution functions are the quantum analogs of the classical microscopic distribution functions commonly used for the description of kinetic phenomena in gas plasmas and semiconductors. With account for the mentioned similarity, the term "distribution function" seems to be quite adequate to designate the operator of photon density in the phase space.

In what follows, we will obtain approximate eigenfunctions and eigenvalues of the operator $f(\mathbf{r}, \mathbf{q}, t)$. These quantities describe the localized states (particle-like states) of the radiation field. The localization occurs in

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small (but not infinitely small) volumes of the $\mathbf{r},\mathbf{q}$-space. The localization corresponds to the coarse-graining procedure and agrees with the uncertainty principle. The evolution of the localized states of individual photons is such as if they represent a set of classical particles with a specific (linear) momentum-energy relation, whose motion is governed by the Newton mechanics. The conditions, under which the photon ensemble behaves like a classical system of noninteracting particles, are discussed.

The averaged distribution function, $\langle f(\mathbf{r},\mathbf{q},t)\rangle$, can be used for obtaining such important characteristics of radiation fields as the local densities of energy, flux, momentum, angular momentum, *etc.* The value $\langle f(\mathbf{r},\mathbf{q},t)\rangle$ depends on statistical properties of both a source of the radiation field and propagation paths.

In what follows, the temporal dependence of $\langle f(\mathbf{r},\mathbf{q},t)\rangle$ will be obtained for the simplest cases. Besides that, the pair correlation function,

$$\langle f(\mathbf{r},\mathbf{q},t)f(\mathbf{r}',\mathbf{q}',t')\rangle,$$

describing fluctuations of the intensity of electromagnetic fields will be derived.

2. The Mandel Operator

The Mandel operator describes the coarse-grained photon density in the configuration space ($\mathbf{r}$-space):

$$\hat{n}^{\mathrm{M}}(\mathbf{r},t)=\frac{1}{V}\sum_{\mathbf{q},\mathbf{k},s}\int_{\Omega}d\mathbf{r}'e^{-i\mathbf{k}(\mathbf{r}+\mathbf{r}')}a^{+}_{\mathbf{q}+\frac{\mathbf{k}}{2},s}a_{\mathbf{q}-\frac{\mathbf{k}}{2},s},\qquad(1)$$

where V is the volume of the system; $\Omega=l^3$ is the volume of averaging ($V\gg\Omega$); s denotes two polarizations of plane-wave modes with the same wave-vectors. The value of l is considered to be much greater than the characteristic wavelength of light, λ. All operators are defined in the Heisenberg representation.

Due to the integration, the terms with $k>\pi/l$ do not contribute significantly to the value of $\hat{n}^{\mathrm{M}}(\mathbf{r},t)$. This observation shows that a similar effect of cause-graining can be realized using the alternative procedure, i.e., by eliminating the harmonics with large k from the sum which defines the photon density. Then the modified photon density is given by

$$\hat{n}(\mathbf{r},t)=\frac{1}{V}\sum_{\mathbf{q},s,k<k_0}e^{-i\mathbf{k}\mathbf{r}}a^{+}_{\mathbf{q}+\frac{\mathbf{k}}{2},s}a_{\mathbf{q}-\frac{\mathbf{k}}{2},s},\qquad(2)$$

where the inequality, $k<k_0$, means here $|k_{x,y,z}|<k_0=\pi/l$. There is the qualitative correspondence between the Mandel operator and the operator given by Eq. (2):

$$\hat{n}(\mathbf{r},t)\rightarrow\frac{1}{\Omega}\hat{n}^{\mathrm{M}}(\mathbf{r},t).\qquad(3)$$

Both of them provide an adequate description of the smoothly varying photon density.

3. Photon Density in the Phase Space

The photon distribution function is defined by analogy with the distribution functions for electrons, phonons, *etc.*, as

$$f_s(\mathbf{r},\mathbf{q},t)=\frac{1}{V}\sum_{k<k_0}e^{-i\mathbf{k}\mathbf{r}}a^{+}_{\mathbf{q}+\frac{\mathbf{k}}{2},s}a_{\mathbf{q}-\frac{\mathbf{k}}{2},s}.\qquad(4)$$

It can be easily seen from Eqs. (2) and (4) that the operator of photon density in the configuration space can be expressed in terms of f_s as

$$\hat{n}(\mathbf{r},t)=\sum_{\mathbf{q},s}f_s(\mathbf{r},\mathbf{q},t).\qquad(5)$$

Equation (5) indicates that $f_s(\mathbf{r},\mathbf{q},t)$ represents the photon density in the $(\mathbf{r},\mathbf{q})$ space. An additional support of this statement follows from the observation that the integral of $f_s(\mathbf{r},\mathbf{q},t)$ over $\mathbf{r}$ is the operator of total photon numbers, $\hat{n}_{\mathbf{q},s}$, in the mode $\{\mathbf{q},s\}$:

$$\int_V d\mathbf{r}f_s(\mathbf{r},\mathbf{q},t)=a^{+}_{\mathbf{q},s}a_{\mathbf{q},s}\equiv\hat{n}_{\mathbf{q},s}.\qquad(6)$$

In the next section, we consider the properties of the operator $f_s(\mathbf{r},\mathbf{q},t)$.

4. Eigenfunctions and Eigenvalues of the Operator $f_s(\mathbf{r},\mathbf{q},t)$

Let us define the operator

$$\hat{\mathbf{V}}^{+}(\mathbf{r},\mathbf{q},t)=C\sum_{q'<k_0,s}e^{-i(\mathbf{q}+\mathbf{q}')\mathbf{r}}\mathbf{e}_{\mathbf{q}+\mathbf{q}',s}a^{+}_{\mathbf{q}+\mathbf{q}',s},\qquad(7)$$

where $C=V^{-1/2}(\pi/k_0)^{3/2}$, $\mathbf{e}_{\mathbf{q}+\mathbf{q}',s}$ is the unit vector showing the polarization of the mode denoted by a set $(\mathbf{q}+\mathbf{q}',s)$. With account for the inequality, $q\gg q'$, we can approximate $\mathbf{e}_{\mathbf{q}+\mathbf{q}',s}$ by the value $\mathbf{e}_{\mathbf{q},s}$. Then the vector $\hat{\mathbf{V}}^{+}(\mathbf{r},\mathbf{q},t)$ can be represented as a sum

$$\hat{\mathbf{V}}^{+}(\mathbf{r},\mathbf{q},t)=\mathbf{e}_{\mathbf{q},1}\hat{v}^{+}(\mathbf{r},\mathbf{q},t,1)+\mathbf{e}_{\mathbf{q},2}\hat{v}^{+}(\mathbf{r},\mathbf{q},t,2),$$

where the numbers $1,2$ denote the polarizations of two modes with a given wave-vector $\mathbf{q}$. To shorten notations, we will use a single letter $\mathbf{q}$ to denote a set of variables $\{\mathbf{q}, s\}$.

The action of $\hat{v}^+(\mathbf{r},\mathbf{q},t)$ on the vacuum state, $|\mathrm{vac}\rangle$, generates a normalized excited state, $|\mathbf{r},\mathbf{q},t\rangle$:

$$|\mathbf{r},\mathbf{q},t\rangle = \hat{v}^+(\mathbf{r},\mathbf{q},t)|\mathrm{vac}\rangle, \quad \langle\mathbf{r},\mathbf{q},t|\mathbf{r},\mathbf{q},t\rangle = 1. \tag{8}$$

It can be seen that the function $|\mathbf{r},\mathbf{q},t\rangle$ is a superposition of the single-photon Fock states of modes with wave-vectors close to $\mathbf{q}$. Using the evident relation

$$a_{\mathbf{q}'}a^+_{\mathbf{q}}|\mathrm{vac}\rangle = \delta_{\mathbf{q},\mathbf{q}'}|\mathrm{vac}\rangle, \tag{9}$$

we can show that $|\mathbf{r},\mathbf{q},t\rangle$ is the exact eigenfunction of the operator of total number of photons, $\hat{N} = \sum_{\mathbf{q}} \hat{n}_{\mathbf{q}}$, with the eigenvalue equal to 1:

$$\hat{N}|\mathbf{r},\mathbf{q},t\rangle = 1 * |\mathbf{r},\mathbf{q},t\rangle. \tag{10}$$

Equation (10) shows that $|\mathbf{r},\mathbf{q},t\rangle$ is the state representing an elementary portion of the electromagnetic field (a one-photon state) which is smeared out over a set of photon modes around the mode with the wave-vector $\mathbf{q}$. The spatial structure of this field can be "probed" by the action of the density operator on the state $|\mathbf{r},\mathbf{q},t\rangle$. Using the inequality $k_0 \ll q$ and Eq. (9), we can easily obtain

$$\hat{n}(\mathbf{r},t)|\mathbf{r}',\mathbf{q},t\rangle \approx \tilde{\delta}(\mathbf{r}-\mathbf{r}')|\mathbf{r}',\mathbf{q},t\rangle, \tag{11}$$

where

$$\tilde{\delta}(\mathbf{r}) = \frac{1}{V}\sum_{k<k_0} e^{-i\mathbf{k}\mathbf{r}} = \frac{1}{\pi^3}\frac{\sin(k_0x)}{x}\frac{\sin(k_0y)}{y}\frac{\sin(k_0z)}{z}.$$

When $k_0 \to \infty$, $\tilde{\delta}(\mathbf{r})$ approaches the Dirac δ-function. For finite values of k_0, this function is localized in the region with size of the order of π/k_0.

It follows from Eq. (11) that the function, $|\mathbf{r}',\mathbf{q},t\rangle$, is the approximate eigenfunction of the operator of photon density in the configuration space. The corresponding approximate eigenvalue is $\tilde{\delta}(\mathbf{r}-\mathbf{r}')$. Using a similar consideration, we can show that $|\mathbf{r},\mathbf{q},t\rangle$ is also the eigenfunction of the operator, $f(\mathbf{r},\mathbf{q},t)$, i.e.,

$$f(\mathbf{r},\mathbf{q},t)|\mathbf{r}',\mathbf{q}',t\rangle = \tilde{\delta}(\mathbf{r}-\mathbf{r}')\tilde{\delta}_{\mathbf{q},\mathbf{q}'}|\mathbf{r}',\mathbf{q}',t\rangle, \tag{12}$$

where the tilde over the Kronecker delta means that $\mathbf{q} \approx \mathbf{q}'$ with accuracy up to k_0/q. The wave function $|\mathbf{r}',\mathbf{q}',t\rangle$ represents a one-particle state (see Eq. 10) localized in the phase-space volume of the order of $\Delta V k_0^3 \sim 1$. [The estimate of $\Delta V \sim k_0^{-3}$ can be seen from the explicit form of $\tilde{\delta}(\mathbf{r})$.] Taking into account that the uncertainty of q is of the order of k_0, we obtain that $\Delta r \Delta q \sim 1$. This agrees with the Heisenberg uncertainty principle.

Our analysis can be generalized for a set of particles localized in different regions of the phase space. The wave function of N particles, $|N\{\mathbf{r}_i,\mathbf{q}_i,t\}\rangle$, is given by

$$|N\{\mathbf{r}_i,\mathbf{q}_i,t\}\rangle = \prod_{i=1}^{N}\hat{v}^+(\mathbf{r}_i,\mathbf{q}_i,t)|\mathrm{vac}\rangle. \tag{13}$$

Repeating the previous argumentation, we derive

$$f(\mathbf{r},\mathbf{q},t)|N\{\mathbf{r}_i,\mathbf{q}_i,t\}\rangle \approx \sum_{i=1}^{N}\tilde{\delta}(\mathbf{r}-\mathbf{r}_i)\tilde{\delta}_{\mathbf{q},\mathbf{q_i}}|N\{\mathbf{r}_i,\mathbf{q}_i,t\}\rangle. \tag{14}$$

The eigenvalue,

$$\rho(\mathbf{r},\mathbf{q}) = \sum_{i=1}^{N}\tilde{\delta}(\mathbf{r}-\mathbf{r}_i)\tilde{\delta}_{\mathbf{q},\mathbf{q_i}}, \tag{15}$$

can be interpreted as the photon density in the phase space.

Equation (14) can be easily generalized to the case of different times of the wave functions and the density operators. For this purpose, the following relation for the Heisenberg operators $a^+_{\mathbf{q+k/2}}, a_{\mathbf{q-k/2}}$ is useful:

$$a^+_{\mathbf{q+k/2}}(t)a_{\mathbf{q-k/2}}(t) =$$

$$= e^{i(\omega_{\mathbf{q+k/2}}-\omega_{\mathbf{q-k/2}})t}a^+_{\mathbf{q+k/2}}(0)a_{\mathbf{q-k/2}}(0) \approx$$

$$\approx e^{i\mathbf{c_q}\mathbf{k}t}a^+_{\mathbf{q+k/2}}(0)a_{\mathbf{q-k/2}}(0), \tag{16}$$

where $\mathbf{c_q} = \partial\omega_{\mathbf{q}}/\partial\mathbf{q}$ is the photon velocity, and only the term linear in $\mathbf{k}$ is retained in the exponent. Using Eq. (16), we obtain

$$f(\mathbf{r},\mathbf{q},t)|N\{\mathbf{r}_i,\mathbf{q}_i,0\}\rangle \approx$$

$$\approx \sum_{i=1}^{N}\tilde{\delta}(\mathbf{r}-\mathbf{r}_i-\mathbf{c_{q_i}}t)\tilde{\delta}_{\mathbf{q},\mathbf{q_i}}|N\{\mathbf{r}_i,\mathbf{q}_i,0\}\rangle. \tag{17}$$

It follows from Eq. (17) that, independently of the time, t, the function $|N\{\mathbf{r}_i,\mathbf{q}_i,0\}\rangle$ is an eigenfunction of the operator $f(\mathbf{r},\mathbf{q},t)$. The corresponding eigenvalue $\rho(\mathbf{r},\mathbf{q},t)$ given by

$$\rho(\mathbf{r},\mathbf{q},t) = \sum_{i=1}^{N}\tilde{\delta}(\mathbf{r}-\mathbf{r}_i-\mathbf{c_{q_i}}t)\tilde{\delta}_{\mathbf{q},\mathbf{q_i}} \tag{18}$$

depends essentially on the time and differs from (15). The difference arises from the displacements of wave-packets over the distances $\mathbf{c}_{\mathbf{q_i}} t$. At the same time, it should be noted that the volume of localization of each photon state does not depend on t. The initial localization volume is displaced as a whole to the distance $\mathbf{c}_{\mathbf{q_i}} t$. This very simple physical picture occurs for a not too long propagation distance (when the linear-in-k approximation of the difference $\omega_{\mathbf{q+k/2}} - \omega_{\mathbf{q-k/2}}$ used in Eq. (16) is sufficient for the description of the wave packet evolution). For very long propagation paths, the terms nonlinear in k and potentially responsible for distorting (varying) the volumes of localizations should be also accounted for. This peculiarity does not exist for non-relativistic finite-mass ($m \neq 0$) particles. In the case of the usual energy-momentum relation ($\varepsilon(p) = p^2/2m$), we obtain

$$\varepsilon_{\mathbf{p}+\hbar\mathbf{k}/2} - \varepsilon_{\mathbf{p}-\hbar\mathbf{k}/2} = \hbar\mathbf{k}\mathbf{v}_{\mathbf{p}}, \quad \mathbf{v}_{\mathbf{p}} = \frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{p}}. \tag{19}$$

As we see, only the terms linear in $\mathbf{k}$ describe the wave-packet in course of its propagation. As the velocity $v = p/m$ increases, the relativistic effects become important, and, in the limit $v \to c$ ($p \to \infty$), the energy-momentum relation resembles that in the photon case: $\varepsilon(\mathbf{p}) \to cp$. This means that the particles with finite masses become delocalized after long propagation paths.

The photon density $\rho(\mathbf{r}, \mathbf{q}, t)$ resembles the microscopic distribution function used for the classical description of noninteracting point particles. Hence, Eq. (18) illustrates the particle-like properties of photon fields.

5. Kinetic Equation for the Photon Distribution Function in the Phase Space

Using the definition of the distribution function (6) and considering it as a Heisenberg operator, we can easily obtain the kinetic equation

$$\left\{ \frac{\partial}{\partial t} + \mathbf{c}_{\mathbf{q}} \frac{\partial}{\partial \mathbf{r}} \right\} f(\mathbf{r}, \mathbf{q}, t) = 0, \tag{20}$$

governing the evolution of a photon distribution in the vacuum. When photons propagate in a medium with slowly varying refractive index $n_{\mathrm{ref}}(\mathbf{r})$ (e.g., in the Earth's atmosphere), the effect of the medium can be accounted for by introducing an "external force" (see more details in Ref. [7])

$$\mathbf{F}(\mathbf{r}) \sim \frac{\partial n_{\mathrm{ref}}(\mathbf{r})}{\partial \mathbf{r}},$$

which modifies the photon momentum. In this case, the additional term describing the particle drift in the momentum space appears in the kinetic equation:

$$\left\{ \frac{\partial}{\partial t} + \mathbf{c}_{\mathbf{q}} \frac{\partial}{\partial \mathbf{r}} + \mathbf{F}(\mathbf{r}) \frac{\partial}{\partial \mathbf{q}} \right\} f(\mathbf{r}, \mathbf{q}, t) = 0. \tag{21}$$

Its solution can be expressed in terms of the initial distribution $f(\mathbf{r}, \mathbf{q}, t = 0)$ as

$$f(\mathbf{r}, \mathbf{q}, t) = f\left(\mathbf{r} - \int_0^t dt' \frac{\partial \mathbf{r}(t')}{\partial t'}, \mathbf{q} - \int_0^t dt' \frac{\partial \mathbf{q}(t')}{\partial t'}, 0 \right), \tag{22}$$

where the derivatives should obey the equations

$$\frac{\partial \mathbf{r}(t')}{\partial t'} = \mathbf{c}[\mathbf{q}(t')],$$

$$\frac{\partial \mathbf{q}(t')}{\partial t'} = \mathbf{F}[\mathbf{r}(t')] \tag{23}$$

and the initial conditions $\mathbf{r}(t' = t) = \mathbf{r}, \quad \mathbf{q}(t' = t) = \mathbf{q}$ (see [7–9]).

It seems to be reasonable to represent a state of the photon ensemble using the set of approximate eigenfunctions (13) as a basis and to describe the photon kinetics by means of Eqs. (21) and (23).

Equations (23) can be interpreted as the Newton equations of motion for the zero-size particles which have the energy $\hbar\omega_{\mathbf{q}}$ and the momentum $\hbar\mathbf{q}$. In the above, we have used the term "photons" for them. In our opinion, this is a quite adequate name for the next reason. Historically, the term "photon" was used to define the smallest (elementary) "portion" of light. In the case of a monochromatic electromagnetic field, this portion can be considered as a single quantum of the plane-wave mode. By analogy, it seems reasonable to use the term "photon" for a portion of the electromagnetic field described by the superposition state $|\mathbf{r}, \mathbf{q}, t\rangle$. This state, being the eigenfunction of the operator $\hat{N}$ with the eigenvalue equal to 1 (one-particle state), does really represent the elementary "portion of light".

Concluding this paragraph, Eq. (21) completed by Eqs. (23) coincides with the corresponding equations for classical particles. Nevertheless, the wave nature of light is not missed here, because it is accounted for by the initial distribution $f(\mathbf{r}, \mathbf{q}, 0)$ formed by the source of radiation. The "memory" about an initial state of f is present, for example, in average values and correlations of distribution functions. (The detailed description of one-photon and two-photon interferences can be found in [10].)

6. Average Values and Correlations of Distribution Functions

For practical purposes, the average values of the distribution functions and their correlation functions are required. The simplest situation is realized when the radiation field is in the state $|N\{\mathbf{r}_i, \mathbf{q}_i, t\}\rangle$. Using the property given by Eq. (17), we can easily obtain

$$\langle f(\mathbf{r}, \mathbf{q}, t)\rangle = \langle N\{\mathbf{r}_i, \mathbf{q}_i, 0\}|f(\mathbf{r}, \mathbf{q}, t)|N\{\mathbf{r}_i, \mathbf{q}_i, 0\}\rangle \approx$$

$$\approx \sum_{i=1}^{N} \tilde{\delta}(\mathbf{r} - \mathbf{r}_i - \mathbf{c}_{\mathbf{q_i}} t)\tilde{\delta}_{\mathbf{q},\mathbf{q_i}}. \quad (24)$$

It can be seen from Eq. (24) that the photon created at $t = 0$ moves with its individual velocity $\mathbf{c}_{\mathbf{q_i}}$. The correlation function in this case is given by

$$\langle f(\mathbf{r}, \mathbf{q}, t) f(\mathbf{r}', \mathbf{q}', t')\rangle = \langle f(\mathbf{r}, \mathbf{q}, t)\rangle\langle f(\mathbf{r}', \mathbf{q}', t')\rangle. \quad (25)$$

For the thermal radiation, the average value of the distribution function is equal to

$$\langle f(\mathbf{r}, \mathbf{q}, t)\rangle = \frac{1}{V}\bar{n}_q, \quad (26)$$

where $\bar{n}_q = \left[\exp\left(\frac{\hbar\omega_q}{k_\mathrm{B}T}\right) - 1\right]^{-1}$ is the Planck distribution. There is no spatial dependence of $\langle f\rangle$ because of the spatial homogeneity of equilibrium radiation.

The correlation function is given by

$$\langle f(\mathbf{r}, \mathbf{q}, t) f(\mathbf{r}', \mathbf{q}', t')\rangle \approx$$

$$\approx \frac{1}{V^2}\bar{n}_q\bar{n}_{q'} + \frac{1}{V}\tilde{\delta}[\mathbf{r} - \mathbf{r}' - \mathbf{c}_\mathbf{q}(t - t')]\tilde{\delta}_{\mathbf{q},\mathbf{q}'}\bar{n}_q(1 + \bar{n}_q). \quad (27)$$

Integrating Eq. (27) over $\mathbf{r}$ and $\mathbf{r}'$, we obtain

$$\langle \hat{n}_\mathbf{q}\hat{n}_{\mathbf{q}'}\rangle = \bar{n}_q\bar{n}_{q'} + \delta_{\mathbf{q},\mathbf{q}'}\bar{n}_q(1 + \bar{n}_q).$$

This is the result known for photon number correlations in the black-body radiation.

As is seen, the photon's trajectories in both cases, as well as the initial statistical properties of the radiation, determine the average of distribution functions and their correlations.

7. Conclusion

We have analyzed the possibility to describe the evolution of optical fields by means of the operator of photon density (the photon distribution function) in the phase space. Ideologically, our approach is in the course of the Mandel operator procedure. Approximate eigenfunctions and eigenvalues of the coarse-grained distribution function are obtained. It is shown that the evolution of the photon density in the phase space can be described in terms of photon trajectories. These trajectories can be obtained by solving the Newton equations of motion, in which an "external force" may originate from spatial variations of the refractive index. We believe that this method can be useful for the description of various optical phenomena, including the problem of quasi-one-dimensional light propagation in fibers.

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ОПЕРАТОР ФОТОННОЇ ГУСТИНИ У ФАЗОВОМУ ПРОСТОРІ

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Р е з ю м е

Вивчається можливість опису еволюції електромагнітного поля за допомогою функції розподілу фотонів у фазовому просторі (в **r**,**q**-просторі). Локальна густина фотонів із заданим значенням імпульса виражається через цю функцію, побудовану за аналогією із "загрубленим" оператором Манделя фотонної густини в конфігураційному просторі. Знайдено наближені власні функції та власні значення функції розподілу, що відповідають однофотонним локалізованим станам електромагнітного поля. Показано, що рух фотонів описується механікою Ньютона у тому випадку, коли "зовнішня сила", що діє на фотони, є плавною функцією просторових змінних. Показано також, що функцію розподілу у довільний момент часу можна виразити через початковий розподіл та фотонні траєкторії.