THE GEOMETRODYNAMIC NATURE OF THE QUANTUM POTENTIAL

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PACS 03.65.-w, 03.65.Ud, 03.65.Ta

The de Broglie–Bohm theory allows us to have got a satisfactory geometrodynamic interpretation of quantum mechanics. The fundamental element, which creates a geometrodynamic picture of the quantum world in the non-relativistic domain, a relativistic curved space-time background, and the quantum gravity domain, is the quantum potential. It is shown that, in the non-relativistic domain, the geometrodynamic nature of the quantum potential follows from the fact that it is an information potential containing a space-like active information on the environment; the geometric properties of the space expressed by the quantum potential determine non-local correlations between subatomic particles. Moreover, in the de Broglie–Bohm theory in a curved space-time, it is shown that the quantum, as well as the gravitational, effects of matter have geometric nature and are highly related: the quantum potential can be interpreted as the conformal degree of freedom of the space-time metric, and its presence is equivalent to the curved space-time. It is shown on the basis of some recent research that, in quantum gravity, we have a generalized geometric unification of gravitational and quantum effects of matter; Bohm's interpretation shows that the form of a quantum potential and its relation to the conformal degree of freedom of the space-time metric can be derived from the equations of motion.

1. Introduction

Understanding the quantum theory in terms of a geometrodynamic interpretation is certainly an interesting theme in the physical research. Many attempts toward a geometric picture have been made recently, which base themselves on the standard version of quantum mechanics. In this regard, we can mention, for example, that J.T. Wheeler suggested a new quantum theory characterized by a purely geometric picture on the basis of the Weyl picture [1]. In this approach, the observables are introduced as zero Weyl weight quantities. Moreover, any weightful field has a Weyl conjugate such as the complex conjugate of a state vector in quantum mechanics. By these dual fields, the probability can be defined. These are the elements of a consistent quantum theory, which is equivalent to the standard quantum mechanics. Moreover, the quantum measurement and the related uncertainty emerge from the Weyl geometry naturally. One more approach to geometrize quantum mechanics was suggested by W.R. Wood and G. Papini [2]. In this approach, a modified Weyl–Dirac theory is used to join the particle aspects of matter and the Weyl symmetry breaking; the result is just a geometrization of quantum mechanics. Moreover, B.G. Sidharth developed a geometric interpretation of quantum mechanics from the point of view of a non-commutative non-integrable geometry [3].

According to the author, all the attempts toward a geometric picture of quantum theory, which base themselves on the standard interpretation, cannot be considered completely satisfactory. In fact, it is important to underline that the standard version of quantum mechanics presents several conceptual problems concerning the interpretation of atomic and subatomic processes and the measurement processes, namely the objectivation of macroscopic properties. As is known, we must ascribe a special role to the observer in the description of atomic processes, according to the standard version of quantum theory, and it is not possible to provide a causal explanation of atomic phenomena. Since the atomic processes cannot be explained theoretically as events happening in space-time, namely the dogma of formulation of physics in terms of a motion in space-time (motion dogma) must be abandoned, the standard version of quantum theory cannot be considered an intrinsic geometrodynamic picture of microphysics. In virtue of its peculiar features, the standard version cannot be considered satisfactory if we want to develop a coherent geometrodynamic picture of the quantum world.

Today, we have got, however, an important consistent version of quantum mechanics, which is able to explain the quantum behavior of matter remaining faithful to the principle of causality and the motion dogma: it is the de Broglie–Bohm pilot wave theory. The de Broglie–Bohm version of quantum mechanics reproduces all the empirical results of quantum theory and, at the same time, has the merit to describe atomic and subatomic processes without ascribing a crucial role to the observer and to recover some causality also in the microscopic world [4–9]. Therefore, this theory seems better than the standard one in order to give a geometrodynamic picture of the quantum world. In this article, we want to show that Bohm's theory can be considered intrinsically as a satisfactory geometrodynamic interpretation of quantum mechanics in virtue of its most important element, the quantum potential: it is possible to provide a geometrodynamic interpretation to the key point of the de Broglie–Bohm theory, namely the quantum potential.

2. Non-relativistic Bohmian Mechanics

The Bohmian mechanics, known also as the de Broglie–Bohm pilot wave theory, is the most significant and satisfactory theory with hidden variables, which is predictably equivalent to quantum mechanics and able to give a causal completion to quantum mechanics. It can be inserted inside that important research stream directed to the complete standard quantum theory in a deterministic sense.

This theory is based on two fundamental starting hypotheses. Before all, we mean the idea of that quantum mechanics is not complete and must be completed by adding supplementary parameters to the formalism, the so-called hidden variables. The hidden variables of the model are the positions of all the particles constituting the physical system under study. The first starting hypothesis of Bohm's pilot wave theory is just this: the physical system is prepared in such a way that, at the initial time $t = 0$, it is associated with a specific wave function $\psi(\mathbf{x}, 0)$, which is assumed to be known perfectly, and, moreover, it is at a point $\mathbf{x}$ (among those compatible with the wave function under examination) that instead we ignore (it is in this sense that the position is a hidden variable of this theory).

The second starting hypothesis of Bohm's pilot wave theory is de Broglie's objective wave-corpuscle dualism. On the ground of this idea proposed by de Broglie in 1927 at Solvay Conference, each fundamental particle of physics is assumed to be constituted by a corpuscle and by a wave which surrounds it and accompanies it during its motion. As regards the non-relativistic problem, de Broglie suggested that the wave function of such an object was associated with a set of identical particles which have different positions and are distributed in space according to the usual quantum formula, given by $|\psi(\mathbf{x})|^2$. But he recognized a dual role for the wave function: on one hand, it determines the probable position of the particle (just like in the standard interpretation); on the other hand, it influences the position by exerting a force on the orbit. According to de Broglie, the wave function would act like a pilot wave, which guides the particles in regions where such wave function is more intense [10].

Bohm's version of quantum mechanics is practically the de Broglie pilot-wave theory carried to its logical conclusion. In his classic works [4, 5, 11], Bohm succeeded in developing a mathematical treatment of de Broglie's objective wave-particle dualism. He showed that if we interpret each individual physical system as composed by a corpuscle and a wave guiding it, the movement of the corpuscle under the guide of the wave happens in agreement with a law of motion which assumes the following form

$$\frac{\partial S}{\partial t} + \frac{|\nabla S|^2}{2m} - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} + V = 0 \tag{1}$$

(where R is the amplitude and S is the phase of the wave function, $\hbar$ is Planck's reduced constant, m is the mass of the particle, and V is the classical potential). This equation is equal to the classical Hamilton–Jacobi equation except for the appearance of the additional term

$$Q = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} \tag{2}$$

having the dimension of energy and containing the Planck constant and, therefore, appropriately defined quantum potential. The equation of motion of the particle can be expressed also in the form

$$m\frac{d^2\mathbf{x}}{dt^2} = -\nabla(V + Q), \tag{3}$$

where $\mathbf{x} = \mathbf{x}(t)$ is the trajectory of the particle associated with its wave function. Equation (3) is equal to Newton's second law of classical mechanics, always with the additional term Q of the quantum potential. The movement of an elementary particle, according to Bohm's pilot wave theory, is thus tied to a total force which is given by the sum of two terms: a classical force (derived from the classical potential) and a quantum force (derived just from the quantum potential) [6, 8].

To summarize, we can say that, according to the de Broglie–Bohm theory, each subatomic particle is completely described by its wave function (which evolves according to the usual Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi \tag{4}$$

and its configuration and follows a precise trajectory $\mathbf{x} = \mathbf{x}(t)$ in space-time that is originated by the action of a classical potential and a quantum potential (and that evolves according to Eq. (1) or to the equivalent equation (3)). An ensemble of particles (distinguished by their initial locations) is associated with each wave. In analogy with classical statistical mechanics, the quantity $\rho(\mathbf{x},t) \equiv R^2(\mathbf{x},t) = |\psi(\mathbf{x},t)|^2$ describes the space-temporal distribution of an ensemble of particles (namely the density of particles in the element of volume d^3x around a point $\mathbf{x}$ at time t) associated with the same wave function. Just like in classical statistical mechanics, the density of particles in the Bohmian mechanics satisfies a continuity equation, which has the form

$$-\frac{\partial\rho}{\partial t} = \nabla\left(\rho\frac{\nabla S}{m}\right). \tag{5}$$

If we consider an ensemble of particles distributed initially according to a given R_0^2, these determine a quantum potential in the surrounding space. This potential is subjected, for non-stationary problems, to the temporal evolution, as a consequence of the motion of the packet of particles; the quantum potential retroacts on each particle, by determining its trajectory, with the classical potential.

It is important to remark that the equations of Bohm's approach to quantum mechanics are nonlinear in nature, via the dependence of the quantum potential on the wave function given by Eq. (2). In other words, different initial conditions yield different quantum potentials. Moreover, the total effective potential $(Q+V)$ always acts as to preserve the properties of single-valuedness and non-nodalness of the trajectories, which follow directly from the properties of the phase. At each point in space and at each instant of time, only one trajectory passes through that point, for each time t. The Bohmian trajectories of the particles (derived from the combined action of the classical and quantum potentials) cannot cross or even touch. But there is an important exception to this rule: particles cannot pass at all through nodal regions (namely where $\psi(\mathbf{x},t) = 0$), for there ∇S (which represents the momentum of a particle) is undefined and does not define the tangent to a curve. This property is consistent with the continuity equation for density (5) which maps non-nodal regions into themselves along trajectories.

Because of the requirements of boundedness and continuity satisfied by the wave function, the nodes are the points, where the quantum potential becomes singular. The singularities of the quantum potential corresponding with the nodes of the wave function result in a large quantum force, rapid changes in the momentum, and "jumps" in the phase of the Bohmian trajectories, as they move around a node. In the nodes, the time propagation becomes very expensive even if the quantum potential is provided and nearly impossible with approximate methods. The trajectory momenta in the nodal regions grow rapidly and reverse their direction after the minimum of the wave function density reaches zero. The ranges of the momenta for trajectories in the nodal regions can differ by orders of magnitude, depending on how close a trajectory is to the node. In contrast, the wave function in its complex form remains smooth and simply changes sign at the node. The only exception to this picture are the nodes of excited eigenstates, for which singularities in the quantum potential cancel exactly (because here the quantum force is equilibrated by the classical force), the density is stationary and the trajectories have zero momenta (and, thus, the corresponding particle is at rest). Stationary states with real-valued spatial wave function part constitute therefore particular cases of singularities of the quantum potential. In fact, in stationary states, the wave function is a real function for many bound states problems of interest. For these problems, the particle is always at rest, where one would classically expect it to move, since the quantum force cancels the classical force (and thus even at the nodes of the wave function).

The nodes of the wave function lead to quantum potentials complicated and rapidly varying in time and space and to quantum forces that are very difficult to compute accurately. The accuracy of the quantum potential and, consequently, the stability and the accuracy of a dynamics were found to deteriorate with time, especially in the presence of the density nodes, which motivated the development of the representation transformation, adaptive moving grids, artificial viscosity techniques, covering functions, and wave function decomposition [12, 14]. In this regard, it is also important to mention that recently Garashchuk and Rassolov proposed a mixed coordinate space/polar representation of the wave function [15]. The modified trajectory dynamics resulting from this mixed representation has the advantage to avoid the problems associated with the instability of the quantum trajectories and with the singularities of the quantum potential at the nodes of the wave function. At the same time, this mixed description incorporates the feature of the quantum trajectories, being the "ultimate" moving grid for the wave function.

3. Quantum Potential in a Non-relativistic Domain and Its Geometrodynamic Nature

In quantum mechanics, the quantum potential must not be considered a term which is introduced *ad hoc*, contrary to the opinion of the supporters of the Copenhagen interpretation. In the formal plant of Bohm's non-relativistic theory, it emerges directly from the Schrödinger equation, and the energy should not be conserved without it. In fact, taking into account that the quantity $-\frac{\partial S}{\partial t}$ is the total energy of the particle and that $\frac{|\nabla S|^2}{2m}$ is its kinetic energy, Eq. (1) can be also written in the equivalent form as

$$\frac{|\nabla S|^2}{2m} - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} + V = -\frac{\partial S}{\partial t}, \tag{6}$$

which can be seen as a real energy conservation law in quantum mechanics. Here, one can easily see that the energy could not be conserved without the quantum potential (2), and this means that the quantum potential plays an essential role in the quantum formalism. It must be observed, as it was showed recently by Hiley, that the quantum potential can be derived also within Heisenberg's formalism by choosing a particular representation for operators, and such a term must be present to assure the conservation of the total energy of the system [16].

The basic equations (1) and (3) of the non-relativistic de Broglie–Bohm theory could give the impression that we have a return to a classical account of quantum processes. However, this is not the case just because of the features of the quantum potential. If we examine its form, we may note that the quantum potential does not have the usual properties expected from a classic potential. Relation (2) tells us clearly that the quantum potential depends on how the amplitude of the wave function varies in space. The presence of the Laplace operator indicates that the action of this potential is like-space, namely it renders a non-local instantaneous action on a particle. The appearance of the amplitude of the wave function in the denominator also explains why the quantum potential can produce strong long-range effects that do not necessarily fall off with distance and so the typical properties of entangled wave functions. Even though the wave function spreads out, the effects of the quantum potential need not necessarily decrease. This is just the type of behavior required to explain the EPR paradox.

If we examine the expression of the quantum potential in the two-slit experiment, we find that it depends on the width of the slits, their distance apart, and the momentum of the particle. In other words, it has a contextual nature, namely it brings a global information on the process and its environment. It contains an instantaneous information about the overall experimental arrangement, the environment. Moreover, this information can be regarded as being active in the sense that it modifies the behavior of the particle. In a double-slit experiment, for example, if one of the two slits is closed, the quantum potential changes, and this information arrives instantaneously to the particle, which behaves as a consequence.

Now the fact that the quantum potential produces an active information, a global information on the environment means that it cannot be seen as an external entity in space but as an entity which contains a spatial information, as an entity which represents space. It is thus possible to provide a geometrodynamic picture to the quantum potential (2). In virtue of its features, the quantum potential can be considered a geometrodynamic entity: it has a geometric nature, just because it has a contextual nature and contains a global information on the environment, in which the experiment is performed; at the same time, it is a dynamical entity just because its information about the process and the environment is active, determining the behavior of the particle. The geometrodynamic picture of the quantum potential derives therefore just from the interpretation of the quantum potential as an "information potential" (proposed by Bohm and Hiley in 1984).

In this geometrodynamic picture, we can say that the quantum potential indicates and represents the geometric properties of space, from which the quantum force and thus the behavior of quantum particles are derived. Considering the double-slit experiment, the fact that the quantum potential is linked with the width of the slits, their distance apart, and the momentum of the particle, namely that brings a global information on the environment means just that it describes the geometric properties of the experimental arrangement (and therefore of space), which determine the quantum force and the behavior of the particle. Moreover, the presence of the Laplace operator indicates that the geometric properties contained in the quantum potential determine a non-local instantaneous action on the particle. We can say therefore that Bohm's theory manages to make manifest this essential feature of quantum mechanics, just by means of the geometric properties of space described and expressed by the quantum potential. In virtue of the features of the quantum potential, namely in virtue of the geometric properties of space, which determine the quantum forces, Bohm's theory turns out to be intrinsically holistic, in which "the whole is more than the sum of the parts". It is a merit of the pilot wave theory (and, in particular, of the geometrodynamic nature of the quantum potential) to show, in such a direct way, the non-locality that, according to Bohm, "... is the newest and most fundamental ontological characteristic implied by quantum theory" [17].

The appearance of non-separability and non-locality in the Bohm approach led Bell to his famous inequalities [18]. Of course, the non-locality is not a feature that fits comfortably with the mechanical paradigm, but it was this feature that led Bohm to the conclusion that his approach was not mechanical. In this regard, more details can be found in [19].

Detailed investigations into these questions in the Bohm approach and in the review of other approaches to quantum mechanics led to the idea of that the Cartesian order could no longer be used to explain quantum processes, in particular the quantum non-locality. What is needed is a radically new order in which to understand quantum phenomena.

In this regard, G. Chew [20] pointed out that there is no necessity to explain quantum processes on the basis of the space-time manifold. This consideration of G. Chew appears legitimate if it is applied to the interpretation of EPR-type experiments. We encounter problems in explaining the instantaneous communication between subatomic particles, if we assume that space-time is a fundamental entity. If space-time is assumed as primary, then, *ipso facto*, the locality should be absolute. Instead, the quantum particles show non-local correlations.

In 1980, Bohm suggested that the new order, in which to understand quantum phenomena, would be based on a process and called this new order the implicate order: the quantum potential must be considered an active information source linked to a quantum background, namely just the implicate order. Taking its geometrodynamic nature into account, now we can also say that the quantum potential expresses the geometric properties of space, which determine the behavior of the particles and follow just from the implicate order.

The intention behind the introduction of the implicate order was simply to develop new physical theories together with the appropriate mathematical formalism, which will lead to new insights into the behavior of matter and ultimately to new experimental tests. In this way, Bohm in his last years departed from de Broglie's pilot wave: he suggested the necessity to consider the non-locality as a primary fundamental characteristic of space-time and to introduce an intrinsic non-locality of the quantum world. The idea of the implicate order can be collocated just in this context. Bohm's work was practically directed toward overcoming the traditional role of space-time (which instead was present in de Broglie's original view) and developing a theory of space-time, where the quantum concepts appear as structural elements of the world, which can be expressed through opportune topological constructions. In Bohm's view, the non-locality is a characteristic subtended of space-time, and the particles are seen as vibration modes of the global field, which is the dynamical expression of the fundamental level, i.e., of the deep geometric structure. Bohm's project was to develop a top-down approach: to introduce a global ontological structure and to try to obtain the form of the objects which emerge from this as manifestations of the undivided totality. As regards his research on the implicate order, conducted mainly with Hiley, Bohm used very refined mathematical instruments. In particular, he directed his attention toward the non-commutative geometries, the non-linearity, and the discreteness.

As regards this research line, Hiley recently suggested that the quantum processes evolve not in space-time but in a more general space called pre-space, which is not subjected to the Cartesian division between *res extensa* and *res cogitans*. In this view, the space-time of the classical world would be some statistical approximation, and not all quantum processes can be projected into this space without producing the familiar paradoxes, including non-separability and non-locality [21]. According to Hiley's approach, a quantum domain is to be regarded as a structure or order evolving in space-time, but space-time is to be regarded as a higher order abstraction arising from this process involving events and abstracted notions of space or space-like points [22]. These points are active in the sense that each point is a process that preserves its identity and its incidence relations with neighboring points. Thus, the points themselves are not static concepts, but a part of the underlying process. Hiley and Monk showed that this could be realized in a very simple algebraic structure, namely the discrete Weyl algebra [23]. According to Hiley's view, the process must be taken as fundamental, while space-time, fields, and matter can be derived from this basic process on the basis of the idea of that process is describable by elements of an algebra, and the relevant structure process is defined by the algebra itself. In particular, Hiley used a symplectic Clifford algebra which can be constructed from the boson annihilation and creation operators. This algebra contains the Heisenberg algebra, suggesting thus it will strongly feature in a process-oriented approach to quantum theory. It was these possibilities that lead Hiley and Monk to explore a simpler finite structure, the discrete Weyl algebra.

In synthesis, the basic underlying assumption of Hiley's general approach is that the ontology is based on a process that cannot be described explicitly. It can only be described implicitly with the terminology "implicate order". This implicate order is a structure of relationships and is not some woolly metaphysical construction; it is a precise description of the underlying process mathematically expressed in terms of a non-commuting algebra. This process allows partial views, because the nature is basically participatory.

The considerations of Chew and the research of Bohm and Hiley clearly show the legitimacy to understand and to explain quantum processes on the basis of approaches different from the space-time manifold. The space-time manifold characteristic of special relativity cannot be considered as basic and fundamental, because it does not seem compatible with non-locality, with the instantaneous communication between subatomic particles. In particular, by virtue of the peculiar characteristics of the quantum potential and by virtue of its geometrodynamic nature, according to the author, it seems legitimate to suggest the idea that the Bohmian implicate order (or, analogously, Hiley's pre-space and notion of underlying process of quantum phenomena) can be assimilated to a physical space as an immediate information medium.

The features of the quantum potential imply that the geometric properties of space have clearly an important role in determining the motion of a subatomic particle. On the basis of relation (2), one can say that it is the geometric properties of space expressed by the quantum potential, the medium responsible for the behavior of quantum particles and, thus, for the instantaneous connection between them. One can say that the quantum potential (2) contains the idea of space as an immediate information medium in an implicit way.

In other words, when one considers an atomic or subatomic process (such as, e.g., the case of an EPR-type experiment with two subatomic particles, before joined and then separated and carried away at big distances one from the other), the physical space assumes a special "state" represented by the quantum potential, and this state is characterized by geometric properties which allow an instantaneous communication between the particles into consideration [24]. It is the geometric properties of space expressed by the quantum potential that produce an instantaneous connection between two particles A and B: by disturbing system A, system B may indeed be instantaneously influenced despite the big distance separating the two systems thanks to space which puts them in an immediate contact.

Space expressed by the quantum potential allows us to explain why and in which sense, in an EPR experiment, two particles coming from the same source and going away remain joined by a mysterious link, why and in which sense, if we intervene on one of two particles A and B, the other feels the effects instantaneously despite the relevant distances separating them. By virtue of the features of the quantum potential, the instantaneous connection between two quantum particles, when they are at a big distance, can be seen as an effect of space. One can say also that information does not travel between particle A and particle B, information between particle A and particle B has no speed: by means of the quantum potential, space itself is informing particle A about the behavior of particle B and conversely [25].

4. Geometrodynamic Nature of the Quantum Potential of the de Broglie–Bohm Theory in a Curved Space-Time

The next important step of the geometrodynamic nature of the quantum potential concerns the de Broglie–Bohm theory in a curved space-time. The treatment of the behavior of a spin-zero particle moving in a curved background is not very difficult. Before all, by writing the wave function in its polar form $\psi = |\psi| \exp\left(\frac{iS}{\hbar}\right)$ and decomposing the real and imaginary parts of the Klein–Gordon equation, one obtains the quantum Hamilton–Jacobi equation

$$\partial_\mu S \partial^\mu S = m^2 c^2 (1 + Q), \tag{7}$$

with the quantum potential defined as

$$Q = \frac{\hbar^2}{m^2 c^2} \frac{\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)|\psi|}{|\psi|} \tag{8}$$

and the continuity equation

$$\partial_\mu (\rho \partial^\mu S) = 0, \tag{9}$$

where ρ is the ensemble of particles. The above Hamilton–Jacobi equation shows that, in the relativistic case, the quantum potential is essentially the mass square. Thus, one can define the quantum mass M of a particle on the basis of the relation

$$M^2 = m^2 (1 + Q) . \tag{10}$$

Since the quantum potential can be a negative number, tachyonic solutions would emerge, in general. In this regard, it must be remarked that Eq. (7) is not the correct equation of motion [26]. A correct relativistic equation of motion should not only be Poincarè-invariant but also give the correct non-relativistic limit. In [26], it was shown that, on the basis of these requirements, one obtains the correct equation of motion as

$$\partial_\mu S \partial^\mu S = m^2 c^2 \exp Q, \tag{11}$$

and, thus, the quantum mass must be defined in the following manner:

$$M^2 = m^2 \exp Q, \tag{12}$$

which is clearly free from the mentioned problem.

Now, as it has been underlined by F. Shojai and A. Shojai [27], by starting from Bohm's version of the Klein–Gordon equation, it is possible to combine the de Broglie–Bohm quantum theory of motion and gravity and to show that the key point of the de Broglie–Bohm theory, the quantum potential, can be interpreted as the conformal degree of freedom of the space-time metric. This means that the effects of gravity on the geometry and the quantum effects on the geometry of space-time are highly coupled.

In this regard, one must write the equations of motion for a particle (of spin 0) in a curved background and simply utilize the de Broglie remark [28] that the quantum theory of motion for relativistic spinless particles is very similar to the classical theory of motion in a conformally flat space-time, in which the conformal factor is related to Bohm's quantum potential.

Starting from Bohm's version of the Klein–Gordon equation, the extension to the case of a particle moving in a curved background can be done by changing the ordinary differentiation ∂_μ with the covariant derivative ∇_μ and by changing the Lorentz metric with the curved metric $g_{\mu\nu}$. In this way, we obtain the equations of motion for a particle (of spin 0) in a curved background:

$$\nabla_\mu (\rho \nabla^\mu S) = 0, \tag{13}$$

$$g^{\mu\nu} \nabla_\mu S \nabla_\nu S = m^2 c^2 \exp Q, \tag{14}$$

where

$$Q = \frac{\hbar^2}{m^2 c^2} \frac{\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)_g |\psi|}{|\psi|} \tag{15}$$

is the quantum potential. Utilizing now the above-mentioned fruitful observation of de Broglie [28], the quantum Hamilton–Jacobi equation can be written as

$$\frac{m^2}{M^2} g^{\mu\nu} \nabla_\mu S \nabla_\nu S = m^2 c^2. \tag{16}$$

From this relation, it can be concluded that the quantum effects are equivalent to a change of the space-time metric from $g_{\mu\nu}$ to

$$\tilde{g}_{\mu\nu} = \frac{M^2}{m^2} g_{\mu\nu}, \tag{17}$$

which is a conformal transformation. In this way, Eq. (16) can be written as

$$\tilde{g}^{\mu\nu} \tilde{\nabla}_\mu S \tilde{\nabla}_\nu S = m^2 c^2, \tag{18}$$

where $\tilde{\nabla}_\mu$ represents the covariant differentiation with respect to the metric $\tilde{g}_{\mu\nu}$. Moreover, in this new curved space-time, the continuity equation takes the form

$$\tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \left(\rho \tilde{\nabla}^\mu S\right) = 0. \tag{19}$$

The important conclusion we can draw from this treatment is that the presence of the quantum potential is equivalent to a curved space-time with its metric being given by (17).

In this way, we have achieved the geometrization of the quantum aspects of matter. It seems that there is a dual aspect to the role of geometry in physics. The space-time geometry sometimes looks like what we call gravity and sometimes looks like what we understand as the quantum behavior.

F. Shojai's and A. Shojai's treatment of the motion of a particle of spin zero in a curved background can be considered a very relevant result: it provides a further development to the geometrodynamic nature of the quantum potential, which concerns the non-relativistic de Broglie–Bohm theory. In fact, on the ground of relation (17), we can say that the geometric properties which are expressed by the quantum potential and which determine the behavior of a particle of spin zero are linked to the curved space-time. In other words, we can say that the particles determine the curvature of space-time, and, at the same time, the space-time metric is linked to the quantum potential, which influences the behavior of the particles. The quantum potential creates itself a curvature which may have a large influence on the classical contribution to the curvature of the space-time.

It can be also interesting to observe that the particle trajectory can be derived from the guidance formula and by differentiating Eq. (11), which leads to Newton's equation of motion

$$M\frac{d^2x^\mu}{d\tau^2} + M\Gamma^\mu_{\nu\kappa}u^\nu u^\kappa = \left(c^2 g^{\mu\nu} - u^\mu u^\nu\right)\nabla_\nu M, \qquad (20)$$

by using the above conformal transformation. Equation (20) reduces to the standard geodesic equation via the above conformal transformation.

F. Shojai's and A. Shojai's research suggests to us that there are practically two equivalent pictures for investigating the quantum effects of matter in a curved space-time background. According to the first – standard - picture, the space-time metric contains only the gravitational effects of matter. The quantum effects affect the path of the particles via the quantum force. In the second picture, that we have analyzed now in F. Shojai's and A. Shojai's treatment of the de Broglie–Bohm theory in a curved space-time, the space-time metric is related to the quantum force by a conformal factor and contains the gravitational and quantum effects of matter. This second picture is the real geometrodynamic picture of the world, because it shows just that the quantum, as well as the gravitational, effects of matter have geometric nature and are strictly related. This second picture provides a unified geometric framework for understanding the gravitational and quantum forces. Accordingly, we can call the conformal metric (containing both gravity and quantum forces) as the physical metric while the other metric (including only gravity) is the simple background metric.

5. Geometrodynamic View in Bohmian Quantum Gravity

The next important step in the interpretation of the quantum potential as a geometrodynamic entity is represented by the results regarding the quantum gravity domain (in particular, in the context of a scalar-tensor Bohmian model). In this regard, before all, it is important to underline that, despite some problems and weak points (for example, the still open question among the so-called Bohmian community, which sense can be given – if any – to the wave function of the Universe), some recent researches indicate that the Bohmian interpretation of canonical quantum gravity turns out to have several useful aspects and merits [6, 29, 30, 31, 32].

Some of them are:

– It leads to the time evolution of dynamical variables whether the wave function depends on time or not. Therefore, in the Bohmian quantum gravity, we have no time problem.

– Bohm's theory describes a single system, unlike the standard interpretation of quantum theory, which does not tell anything about a single system. About an ensemble of systems both interpretations are equivalent. This is because of the specific form of Bohm's equations of motion. They are the Bohmian version of the Hamilton–Jacobi equation and the conservation equation of probability density. These equations can be transformed to the Schrödinger equation by some canonical transformation. This aspect is useful in quantum cosmology, where the system is the Universe, and no ensemble of systems exists. Therefore, from the Bohmian point of view, we have not the conceptual problem of the meaning of the Universe's wave function in quantum cosmology.

– Normalization of the wave function is needed only for the probabilistic description. Here, there is no need to normalize the wave function for a single system.

– The classical limit has a well-defined meaning. When the quantum potential is less than the classical potential, and the quantum force is less than the classical force, we are in the classical domain.

– There is no need to separate the classical observer and the quantum system in the measurement problem. In the Bohmian picture of the measurement process, we have two interacting systems, the system and the observer. After the interaction takes place, the wave function of the system is reduced in a causal way. It must be noted that the same statistical results for the standard and Bohmian interpretations do not mean that the two theories are equivalent. They are different in physical concepts. The most important difference is that, in the Bohmian interpretation, one deals with trajectories. This can lead to new concepts. For example, one can evaluate the tunneling time of a particle through a potential barrier in the non–relativistic quantum mechanics. This is a concept that has no clear meaning in the standard interpretation [6, 33].

– Till now, the Bohmian interpretation of the Wheeler–De Witt quantum gravity and cosmology has given some physical results that could be found in the literature:

– In the Bohmian quantum cosmology, the quantum force can remove the Big-Bang singularity, because it can behave as a repulsive force [34, 35].

– The quantum force may be present on large scales, because the quantum effects of the quantum potential are independent of the scale [36].

– One can find the graceful exit behavior in the super-inflation model in a super string cosmology. The evolution begins with inflation and smoothly changes to the decelerating expansion, without any singularity in the transition [37]. For a more detailed discussion of the de Broglie–Bohm interpretation of quantum super string cosmology, pre–Big-Bang inflation, and graceful exit problem considering various classes of wavepackets, see [38].

– Real time tunneling can be occurred in the classically forbidden regions, through the quantum potential. For this effect in a closed de-Sitter Universe in 2+1 dimensions, see [39].

– Finally, and this is the point toward which now it is important to focus our attention, in a generalized geometric picture of Bohm's interpretation, one can unify the quantum effects and gravity [35, 40–45].

The latest point represents a very important result that the Bohmian version of quantum gravity can achieve. In this regard, F. Shojai and A. Shojai developed a toy model of quantum gravity (providing a scalar-tensor picture of the ideas developed in Section 4), in which the form of the quantum potential and its relation to the conformal degree of freedom of the space-time metric can be derived using the equations of motion. This model can be considered another interesting development in the geometrodynamic nature of the quantum potential. By showing that it is just the quantum gravity equations of motion which make the quantum potential the entity expressing the geometric properties, which influence the behavior of the particles, and relating to the space-time metric, F. Shojai's and A. Shojai's model suggests a sort of unification of the gravitational and quantum aspects of matter at the fundamental level of physical reality.

Starting from the most general scalar-tensor action

$$A=\int d^4x\left\{\phi R-\frac{\omega}{\phi}\nabla^\mu\phi\nabla_\mu\phi+2\Lambda\phi+L_m\right\}, \qquad (21)$$

in which ω is a constant independent of the scalar field ϕ, Λ is the cosmological constant, and L_m is the matter Lagrangian (which is assumed to be in the form

$$L_m=\frac{\rho}{m}\phi^a\nabla^\mu S\nabla_\mu S-m\rho\phi^b-\Lambda\left(1+Q\right)^c, \qquad (22)$$

in which a, b, and c are constants), using a perturbative expansion for the scalar field and the matter distribution density as

$$\phi=\phi_0+\alpha\phi_1+...,$$

$$\sqrt{\rho}=\sqrt{\rho_0}+\alpha\sqrt{\rho_1}+...$$

(and imposing the opportune physical constraints in order to determine the parameters a, b, and c), F. Shojai and A. Shojai have found the following quantum gravity equations:

$$\phi=1+Q-\frac{\alpha}{2}\left(\nabla^2-\frac{\partial^2}{\partial t^2}\right)Q, \qquad (23)$$

$$\nabla^\mu S\nabla_\mu S=m^2\phi-\frac{2\Lambda m}{\rho}\left(1+Q\right)\left(Q-\tilde{Q}\right)+$$

$$+\frac{\alpha\Lambda m}{\rho}\left[\left(\nabla^2-\frac{\partial^2}{\partial t^2}\right)Q-2\nabla_\mu Q\frac{\nabla^\mu\sqrt{\rho}}{\sqrt{\rho}}\right], \qquad (24)$$

$$\nabla_\mu\left(\rho\nabla^\mu S\right)=0, \qquad (25)$$

$$G^{\mu\nu}-\Lambda g^{\mu\nu}=-\frac{1}{\phi}T^{\mu\nu}-\frac{1}{\phi}\left[\nabla^\mu\nabla^\nu-g^{\mu\nu}\left(\nabla^2-\frac{\partial^2}{\partial t^2}\right)\right]\times$$

$$\times\phi+\frac{\omega}{\phi^2}\nabla^\mu\phi\nabla^\nu\phi-\frac{1}{2}\frac{\omega}{\phi^2}g^{\mu\nu}\nabla^\alpha\phi\nabla_\alpha\phi, \qquad (26)$$

where $\tilde{Q}=\alpha\frac{\nabla_\mu\sqrt{\rho}\nabla^\mu\sqrt{\rho}}{\sqrt{\rho}}$ and $T^{\mu\nu}=-\frac{1}{\sqrt{-g}}\frac{\delta}{\delta g_{\mu\nu}}\int d^4x\sqrt{-g}L_m$ is the energy-momentum tensor.

This geometrodynamic quantum gravity model suggested by F. Shojai and A. Shojai (and synthesized in Eqs. (23) and (26) allows us to draw some important conclusions:

– In this model, Eq. (26) shows that the causal structure of the space-time $g^{\mu\nu}$ is determined by the gravitational effects of matter. On the basis of Eq. (23), the quantum effects determine directly the scale factor of space-time.

– The mass field given by the right-hand side of Eq. (24) consists of two parts. The first part, which is proportional to α, is a purely quantum effect, while the second part, which is proportional to $\alpha\Lambda$, is a mixture of the quantum effects and the large-scale structure introduced via the cosmological constant.

– In this model, the scalar field produces the quantum force, which appears on the right-hand side and violates the equivalence principle (just like, in the Kaluza–Klein theory, the scalar field – dilaton – produces a fifth force leading to the violation of the equivalence principle [46]).

In conclusion, it is also important to underline that the geometrodynamic quantum gravity model suggested by F. Shojai's and A. Shojai's equations (23), (26) (according to which the geometry of space-time and the trajectories of quantum particles are simultaneously defined by a closed set of equations of evolution) intends, in some way, to compete general relativity. Therefore, it would be interesting to check whether it also leads to the same experimental confirmations (gravitational redshift, deflection of light grazing the Sun, shift of Mercury's perihelion, and so on). It would also be interesting to check whether the standard effects of gravity on quantum systems (for instance, the analogous effect of the Bohm–Aharonov phase shift in the presence of gravitational forces in an interference experiment) are also predicted by this model, and how this model could lead to new experimental predictions in similar set-ups. The further research about these topics will give you more information.

6. Geometrization of the Quantum Effects and the Generalized Equivalence Principle

One of the new points of the approach treated in Sections 4 and 5 (regarding the Bohmian theory in a curved space-time and the Bohmian scalar-tensor quantum gravity model) on the geometrization of quantum effects is the dual role of the geometry in physics. The gravitational effects determine the causal structure of space-time, as long as quantum effects give its conformal structure. This does not mean that the quantum effects have nothing to do with the causal structure; they can act on the causal structure through back-reaction terms appearing in the metric field equations [43, 42, 40]. We only mean that a dominant role in the causal structure belongs to the gravitational effects. The same is true for the conformal factor. The conformal factor of the metric is a function of the quantum potential, and the mass of a relativistic particle is a field produced by quantum corrections to the classical mass. In Section 4, we have shown that the presence of the quantum potential is equivalent to a conformal mapping of the metric. Thus, in conformally related frames, we measure different quantum masses and different curvatures. It is possible to consider two specific frames. One of these frames contains the quantum mass field (appearing in the quantum Hamilton–Jacobi equation) and the classical metric, while the other contains the classical mass (appearing in the classical Hamilton–Jacobi equation) and the quantum metric. In other frames, both the space-time metric and the mass field have quantum properties. By virtue of this argument, one can say that different conformal frames are equivalent pictures of the gravitational and quantum phenomena.

Considering the quantum force, the conformally related frames are not distinguishable. This is just what happens when we consider gravity: different coordinate systems are equivalent. Since the conformal transformation changes the length scale locally, we measure different quantum forces in different conformal frames. This is analogous to what happens in general relativity, in which a general coordinate transformation changes the gravitational force at any arbitrary point. Then, the following basic question becomes natural. Does applying the above correspondence between quantum and gravitational forces and between the conformal and general coordinate transformations mean that the geometrization of quantum effects implies the conformal invariance just as gravitational effects imply the general coordinate invariance?

In order to discuss this question, we need to recall what has happened in the development of general relativity. The general covariance principle leads to the identification of gravitational effects of matter with the geometry of the space-time. In general relativity, the important fact which supports this identification is the equivalence principle. According to it, one can always remove the gravitational field at some point by a suitable coordinate transformation. Similarly, according to the new approach to quantum gravity in the context of the Bohmian theory illustrated in Sections 4 and 5, the quantum effects of matter can be removed at any point (or even globally) by a suitable conformal transformation. Thus, in that point(s), matter behaves classically. In this way, we can introduce a new quantum equivalence principle, similar to the standard equivalence principle, and we can call it the conformal equivalence principle. According to this quantum equivalence principle, the gravitational effects can be removed by going to a freely falling frame, while the quantum effects can be eliminated by choosing an appropriate scale. The latter interconnects gravity and general covariance, while the former has the same role about quantum and conformal covariance. Both these principles state that there is no preferred frame, either coordinate or conformal.

Moreover, according to the geodesic equation (23), the appearance of quantum mass justifies Mach's principle, which leads to the existence of an interrelation between the global properties of the Universe (space-time structure, the large-scale structure of the Universe) and its local properties (local curvature, motion in a local frame, *etc.*). In the approach analyzed in Sections 4 and 5, it can be easily seen that the space-time geometry is determined by the distribution of matter. A local variation of the matter field distribution changes the quantum potential acting on the geometry. Thus, the geometry is altered globally (in conformity with Mach's principle). In this sense, the Bohmian approach to quantum gravity is highly non-local, as it is forced by the nature of the quantum potential. What we call geometry is only the gravitational and quantum effects of matter. Without matter, the geometry would be meaningless.

7. Conclusions

The de Broglie–Bohm theory allows us to portray a satisfactory geometrodynamic description of quantum mechanics. Under this point of view, it presents some important advantages with respect to the standard version of quantum mechanics. On one hand, the inherent conceptual problems of standard quantum mechanics concerning the interpretation of quantum processes, the meaning of the wave function and the measurement processes are not present. On the other hand, and this is the fundamental result, the most important element of the de Broglie–Bohm theory, namely the quantum potential, has got an intrinsic geometrodynamic nature. The quantum potential can be considered the fundamental element which creates a geometrodynamic picture of the quantum world. It is a geometrodynamic entity in the non-relativistic domain, the relativistic curved space-time background, and the quantum gravity domain described by a scalar-tensor model.

The first important step concerning the geometrodynamic nature of the quantum potential lies in the fact that, in the non-relativistic domain, the quantum potential is an information potential, a potential which contains a space-like active information on the environment. As a consequence of this feature, the quantum potential can be seen as the entity indicating the geometric properties of space, from which the quantum force and, thus, the behavior of quantum particles follow. In particular, the geometric properties of space expressed by the quantum potential (linked to the presence of the Laplace operator) determine a non-local instantaneous action onto a particle. We can say therefore that it is a merit of the geometrodynamic nature of the quantum potential to reveal the non-local correlations between subatomic particles, i.e., to create the quantum non-locality. As a consequence, the Bohmian implicate order (or equivalently, Hiley's pre-space) can be interpreted as a deep level of physical reality, which follows from the geometric properties of space expressed by the quantum potential and determines the quantum force acting on the particles.

The next important step concerning the geometrodynamic nature of the quantum potential can be found in the de Broglie–Bohm theory in a curved space-time. In this regard by investigating the coupling of purely gravitational effects and purely quantum effects of a particle in a general background space-time metric, we have achieved a very important result: the equivalence of quantum effects of matter and a curved space-time. By analyzing the quantum effects of matter in the framework of the Bohmian mechanics, we have shown that the motion of a particle (of spin zero) with quantum effects is equivalent to its motion in a curved space-time. The quantum effects of matter, as well as the gravitational effects of matter, have geometric nature and are highly related: the quantum potential can be interpreted as the conformal degree of freedom of the space-time metric, and its presence is equivalent to the curved space-time. In fact, the presence of the quantum force is just like having a curved space-time, which is conformally flat, and the conformal factor is expressed in terms of the quantum potential.

Finally, the last important step able to close the circle lies in the fact that, in a scalar-tensor version of quantum gravity, Bohm's interpretation achieves a generalized geometric unification of gravitational and quantum effects of matter, showing that the form of the quantum potential and its relation to the conformal degree of freedom of the space-time metric can be derived from the equations of motion. This scalar–tensor model shows clearly that it is the equations of motion that lead to the correct form of the quantum potential. We can therefore say that not only quantum effects are geometric in nature, but also that it is possible to derive the form of the quantum potential from the Bohmian approach to quantum gravity (precisely in the form of a scalar-tensor model). Thus, it is just the Bohmian quantum gravity equations of motion that make the quantum potential the entity expressing the geometric properties, which influence the behavior of the particles and determine the quantum force.

1. J.T. Wheeler, Phys. Rev. D **41**, 431 (1990).
2. W.R. Wood and G. Papini, in *The Present Status of Quantum Theory of Light, Proceedings of a Symposium in Honour of Jean-Pierre Vigier*, edited by S. Jeffers, S. Roy, J.-P. Vigier, G. Hunter (Springer, Berlin, 1996), p. 247.
3. B.G. Sidharth, *Geometry and Quantum Mechanics*, e-print arXiv:physics/0211012 (2002).
4. D. Bohm, Phys. Rev. **85**, 166 (1952).
5. D. Bohm, Phys. Rev. **85**, 180 (1952).
6. P.R. Holland, *The Quantum Theory of Motion* (Cambridge Univ. Press, Cambridge, 1993).
7. D. Fiscaletti, *I Fondamenti della Meccanica Quantistica. Un'analisi Critica dell'Interpretazione Ortodossa, della*

Teoria di Bohm e della Teoria GRW (CLEUP, Padova, 2003).
8. D. Fiscaletti, *I Gatti di Schrödinger. Meccanica Quantistica e Visione del Mondo* (Muzzio Editore, Roma, 2007).
9. D. Fiscaletti, Quantum Biosystems **2**, 93 (2007); www.quantumbionet.org.
10. L. de Broglie, in *Solvay Congress (1927), Electrons and photons: rapports et discussions du Cinquime Conseil de Physique tenu Bruxelles du 24 au Octobre 1927 sous les auspices de l'Istitut International de Physique Solvay* (Gauthier-Villars, Paris, 1928).
11. D. Bohm, Phys. Rev. **89**, 458 (1953).
12. E.R. Bittner and R.E. Wyatt, J. Chem. Phys. **113**, 8898 (2000).
13. B.K. Kendrick, J. Chem. Phys. **119**, 5805 (2003).
14. R.E. Wyatt, *CNLS Workshop: Quantum and Semiclassical Molecular Dynamics of Nanostructures* (2004).
15. S. Garashchuk and V.A. Rassolov, J. Chem. Phys. **121**, 8711 (2004).
16. B.J. Hiley, in *Proc. Conf. Theory: Reconsiderations of Foundations* (Växjö Univ. Press, Växjö, Sweden, 2002), p. 141.
17. D. Bohm, in *Symposium on the foundation of modern physics – 1987*, edited by P. Lahti and P. Mittelstaedt (World Scientific, Singapore, 1988).
18. J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge Univ. Press, Cambridge, 1987).
19. D. Bohm and B.J. Hiley, *The Undivided Universe: an Ontological Interpretation of Quantum Theory* (Routledge, London, 1993).
20. G. Chew, Sci. Progr. **51**, 529 (1960).
21. B.J. Hiley, in *Proc. CASYS'2000, Liege, Belgium, Aug. 7* (2000).
22. B.J. Hiley and M. Fernandes, in *Time, Temporality, Now*, edited by H. Atmanspacher and E. Ruhnau (Springer, Berlin, 1997), p. 365.
23. B.J. Hiley and N. Monk, Mod. Phys. Lett. A **8**, 3225 (1993; Frontier Persp. **14**, 2 (2005/2006).
24. D. Fiscaletti, Electronic J. of Theor. Phys. **2**, 6 (2005), www.ejtp.com.
25. A. Shojai and F. Shojai, Phys. Scr. **64**, 413 (2001).
26. F. Shojai and A. Shojai, *Understanding Quantum Theory in terms of Geometry*, e-print arXiv:gr-qc/0404102 v1 (2004).
27. L. de Broglie, *Non-Linear Wave Mechanics* (Elsevier, Amsterdam, 1960).
28. T. Horiguchi, Mod. Phys. Lett. A **9**, 1429 (1994).
29. A. Blaut and J.K. Glikman, Class. Quant. Grav. **13**, 39 (1996).
30. S.P. Kim, Phys. Lett. A **236**, 11 (1997).
31. S.P. Kim, Phys. Rev. D **55**, 7511 (1997).
32. *Bohmiam mechanics and quantum theory: An Appraisal*, edited by J.T. Cushing, A. Fine, and S. Goldstein (Kluwer, Boston, 1996).
33. J.A. de Barros, N. Pinto-Neto, and M.A. Sagioro-Leal, Phys. Lett. A **241**, 229 (1998).
34. F. Shojai and M. Golshani, Int. J. Mod. Phys. A. **13**, 4, 677 (1998).
35. R. Colistete (jr.), J.C. Fabris, and N. Pinto-Neto, Phys. Rev. D **57**, 4707 (1998).
36. P. Pinto-Neto and R. Colistete, Phys. Lett. A. **290**, 219 (2001).
37. J. Marto and P.V. Moniz, Phys. Rev. D **65**, 023516 (2001).
38. M. Kenmoku, R. Sato, and S. Uchida, Class. Quantum Grav. **19**, 799 (2002).
39. F. Shojai, A. Shojai, and M. Golshani, Mod. Phys. Lett. A **13**, 2725 (1998).
40. F. Shojai, A. Shojai, and M. Golshani, Mod. Phys. Lett. A., **13**, 2915 (1998).
41. A. Shojai, F. Shojai, and M. Golshani, Mod. Phys. Lett. A **13**, 2965 (1998).
42. F. Shojai and A. Shojai, Int. J. Mod. Phys. A **15**, 1859 (2000).
43. A. Shojai, Int. J. Mod. Phys. A **15**, 1757 (2000), e-print arXiv:gr-qc/0010013.
44. F. Shojai and A. Shojai, *Weyl Geometry and Quantum Gravity*, e-print arXiv: gr-qc/0306099 (2003).
45. Y.M. Cho and D.H. Park, Nuovo Cimento B **105**, 817 (1990).

Received 23.06.11

ГЕОМЕТРОДИНАМІЧНА ПРИРОДА КВАНТОВОГО ПОТЕНЦІАЛУ

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Р е з ю м е

Теорія де Бройля–Бома дозволяє отримати задовільну геометродинамічну інтерпретацію квантової механіки. Фундаментальним елементом, який створює геометродинамічну картину квантового світу в нерелятивістській області, в релятивістському викривленому просторі-часі і в квантової гравітації, є квантовий потенціал. Показано, що в нерелятивістській області геометродинамічна природа квантового потенціалу випливає з того факту, що він є інформаційним потенціалом, що містить просторово-подібну активну інформацію про середовище; геометричні властивості простору, виражені квантовим потенціалом, визначають нелокальні кореляції між субатомними частинками. В рамках теорії де Бройля–Бома у викривленому просторі-часі показано, що як квантові, так і гравітаційні ефек-

ти матерії мають геометричну природу і сильно пов'язані: квантовий потенціал може бути інтерпретований як конформаційний ступінь вільності просторово-часової метрики, і його наявність еквівалентна викривленому простору-часу. Ґрунтуючись на недавніх дослідженнях, показано, що в квантовій гравітації ми маємо узагальнене геометричне об'єднання гравітаційних і квантових ефектів матерії; інтерпретація Бома показує, що форма квантового потенціалу та його відношення до конформаційного ступеня вільності просторово-часової метрики можуть бути отримані з рівнянь руху.