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## DYNAMICS OF PARTICLES WITH HIGH INITIAL TRANSVERSE VELOCITIES IN A MAGNETIC FIELD WITH CUSP GEOMETRY

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PACS 28.60.+s; 29.90.+r  
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The motion of charged particles with high initial transverse energies ( $E_{\perp} \sim E_{\parallel}$ ) injected into a region occupied by a magnetic field with cusp geometry has been studied numerically. The dependence of the particle penetration depth on the initial conditions and the criterion for a particle to pass through the system are obtained. It is shown that the particles with the same initial transverse energy can either pass through the system or be reflected, depending on the ratio between their initial radial and azimuthal velocities. Some parameters of the particle flow – in particular, the particle velocity, the radial displacement of particles with respect to the starting radius, and the flow direction – at the system center, where the magnetic field vanishes, and their dependences on the initial radial velocity are analyzed. Possible applications of the results of studies are discussed.

### 1. Introduction

An axially symmetric system with oppositely directed magnetic fields was considered as one of the possible variants of a magnetic trap for the plasma confinement. To elucidate the efficiency of the holding of charged particles in such a system, both theoretical and experimental works were carried out (see, e.g., works [1, 2] and references therein). In work [2], the motion of particles injected at the end face from a position that is shifted with respect to the axis was calculated numerically for some configurations of the magnetic field. The results obtained allowed the particle trajectories to be classified. Some particles moved along the magnetic field lines, not crossing the plane with zero magnetic field. Other particles crossed the system and left the trap through the opposite plug. One more part of particles was captured by the trap and oscillated inside the system, the time of

oscillations being longer than the transit one. In calculations, it was supposed that particles entering into the magnetic field had only a velocity component oriented along the field line, whereas the transverse components were absent. The transmission of particles through the system was demonstrated to be determined by the parameter  $\eta = r_0/r_L$ , where  $r_0$  is the initial position of a particle,  $r_L = v_0/\omega_c$  is the Larmor radius,  $v_0$  is the initial particle velocity,  $\omega_c = eH_0/Mc$  is the cyclic frequency,  $H_0$  the magnetic field at the particle starting position, and  $M$  the particle mass. The particles passed through the system, if the condition  $\eta < \eta^{cr} < 1$  was obeyed, where  $\eta^{cr}$  is the value of parameter  $\eta$ , at which the particle reflection begins.

When this configuration of a magnetic field was applied to analyze the isotope separation [3], it turned out that the initial transverse components of the particle velocity should be taken into account. The purity degree of separated isotopes depended on the initial transverse velocity. In the majority of examined cases, the complete separation was attained, when the maximum initial radial velocity of injected particles did not exceed  $0.05v_0$ . Particles with larger radial velocities inserted a contamination into the isolated isotope. The corresponding calculations [3] showed that, after particles with nonzero initial radial velocities having crossed the plane without magnetic field, the radial dimension of the region, in which the particles move, increases. When passing through the system, the particles with initial radial velocities  $\dot{r}_0 > |0.05|$  become decelerated more strongly, so that such particles—first of all, particles with  $\dot{r}_0 < 0$ —start to be reflected. The results of those calculations allowed a conclusion to be drawn that the selection of

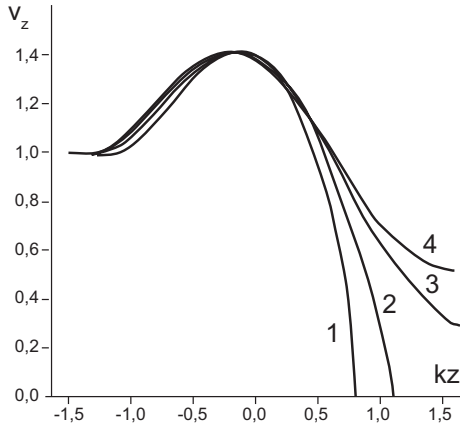


Fig. 1. Distributions of the longitudinal velocity of a particle at its motion through the magnetic system for various initial radial velocities.  $v_{\perp} = v_0$ ;  $\dot{r}_0 = -0.8$  (1), 0.3 (2), 0.6 (3), and 0.7 (4)

separated isotopes has to be made after the particles have crossed the whole system, in the region at the system output, which cannot be reached by particles with higher transverse velocities. The range of initial radial velocities analyzed in the cited work did not exceed 0.1.

However, the beam also includes particles with larger transverse components of the velocity. Therefore, the motion of such particles has to be considered in a wider interval of transverse velocities to estimate their contribution to the total balance of separated and reflected particles. It is necessary to take also into account that particles with the same transverse energy, when entering the magnetic configuration, may have different distributions of their energies between the azimuthal and radial motions.

## 2. Calculation Procedure

In this work, we report the results of our calculations obtained in the case where the initial transverse energy of particles is comparable with the energy of their directed motion. We solved a system of equations, which, in the cylindrical coordinate system  $(r, z, \varphi)$ , looks like [2]

$$\ddot{r} - r\dot{\varphi}^2 = -\omega_c r \dot{\varphi} I_0(kr) \sin kz,$$

$$\ddot{z} = -\omega_c r \dot{\varphi} I_1(kr) \cos kz,$$

$$\dot{\varphi} = \omega_c \left[ \frac{\sin(kr) I_1(kr)}{kr} + \frac{1}{\omega_c} \left( \frac{r_0}{r} \right)^2 \dot{\varphi}_0 + \frac{kr_0}{(kr)^2} I_1(kr_0) \right].$$

The components of the magnetic field  $H_z$  and  $H_r$  were taken in the form of the first harmonics of a solution

presented in work [2]. Before the numerical solution, the equations were transformed into a dimensionless form. The dimensionless longitudinal and radial coordinates are  $kz$  and  $kr$ , where  $k = \pi/2L$ , and  $2L$  is the distance between the plugs, i.e. the longitudinal size of the system. As characteristic values, the initial velocity  $v_0$  and the magnetic field  $H_0$  at the initial position of the incident particle were chosen. The dimensionless longitudinal and radial velocities are  $v'_z = v_z/v_0$  and  $r' = \dot{r}/v_0$ , respectively. Below, the unprimed dimensionless quantities will be used.

The total transverse energy of a particle can be written down as a sum of radial and azimuthal energies,  $E_{\perp} = M(\dot{r}^2 + r^2 \dot{\varphi}^2)/2$ . It was supposed that the particle, when moving in a constant longitudinal magnetic field and reaching the left boundary of a magnetic field configuration, possesses at least one, radial or azimuthal, velocity component different from zero. The initial conditions at this boundary, i.e. at the cross-section  $kz = -1.57$ , were taken as follows:  $r = r_0$ ,  $v_z = v_0$ ,  $\dot{r} = \dot{r}_0$ , and  $\dot{\varphi} = \dot{\varphi}_0$ . At a given total transverse energy, the radial velocity of the particle can range from  $\dot{r}_{0\max}$  to  $-\dot{r}_{0\max}$  (the limiting values  $\pm \dot{r}_{0\max}$  correspond to the cases where the total transverse energy is contained in the radial motion). Depending on the value of radial velocity selected from this interval, the magnitude of azimuthal velocity also changes. The value of parameter  $\eta$  was selected to be less than  $\eta^{cr}$ , so that the transmission conditions were satisfied for particles, whose transverse velocity components were absent or small.

## 3. Results of Numerical Calculations

In Fig. 1, the distributions  $v_z(z)$  of the particle velocity component oriented along the axis of the system are depicted for a particle, half an energy of which is contained in the transverse motion, when the particle enters into the system. The curves are parametrized by the initial radial velocity of the particle,  $\dot{r}_0$ . One can see that the transverse energy is almost completely transformed into that of longitudinal motion, when the particle moving in a decaying magnetic field reaches the center of the system. The divergences in the magnitude of longitudinal velocity at  $kz = 0$  associated with nonzero initial radial velocities are small, amounting to about 5%. The longitudinal velocity is  $v_z = 1.39$  at  $\dot{r}_0 = -0.8$ , and 1.361 at  $\dot{r}_0 = 0.6$ . The maximum velocity calculated from the total energy of the particle at this cross-section is  $v_z = 1.414$ .

At a subsequent motion, the penetration depth of the particle depends on the initial radial velocity. As is seen

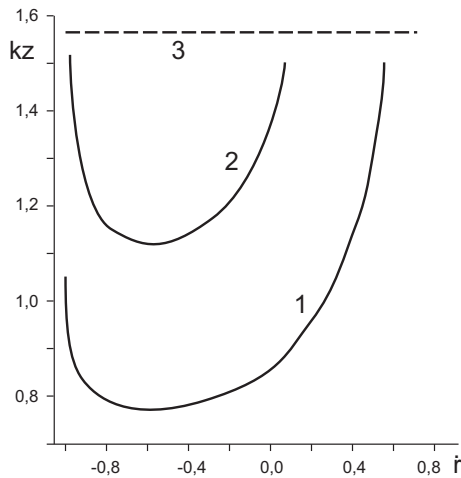


Fig. 2. Depth of penetration of particles into the magnetic system as a function of their initial radial velocities for various starting radii  $kr = 0.14$  (1) and  $0.18$  (2)

from Fig. 1, the particle can be either reflected from (curves 1 and 2) or transmitted through the system (curves 3 and 4). More detailed results are exhibited in Fig. 2. The curves in the figure denote the reflection points for particles, the initial transverse energy of which was equal to the longitudinal energy and which crossed the plane of zero magnetic field, as functions of the initial radial velocity for two starting radii. The most decelerated are those particles, whose initial radial velocities fall within the interval from  $-0.5$  to  $0$ , i.e. which have rather high azimuthal velocity components. Particles with high initial radial velocities directed toward the axis of the system penetrate farther, but they are also reflected. The least decelerated are the particles with high initial transverse velocities  $\dot{r}_0 > 0$  directed along the radius. As is seen from Fig. 2, the particles start to pass through the system, when their initial radial velocities become positive, which depends on the starting radius.

In Fig. 3, the velocities of particles that have been transmitted through the system under the same conditions are depicted for the case  $kz = 1.57$ . If a particle starts from a small radius (curve 1), it is not reflected, but only decelerated depending on the magnitude of its initial radial velocity. An increase of the starting radius (the conditions correspond to plots 2 and 3 in Fig. 2) results in that particles begin to pass the system only if  $\dot{r}_0 > 0$ . At a larger starting radius, the particle can be transmitted, if  $\dot{r}_0 > 0.5$ .

Analogous calculations were carried out for a particle, the initial transverse energy of which is either higher or lower than the energy of directed motion. As an exam-

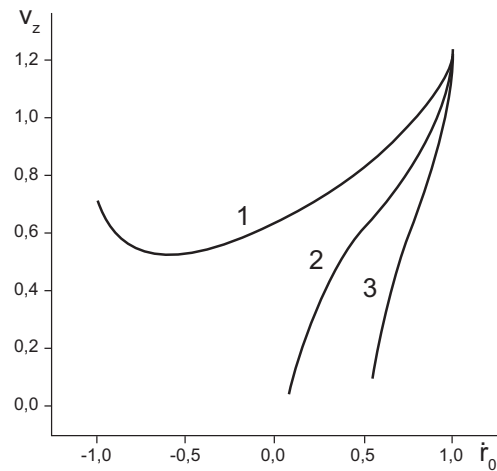


Fig. 3. Longitudinal velocity of a particle at the exit from the system ( $kz = 1.57$ ) as a function of its initial radial velocity for various starting radii  $kr = 0.1$  (1),  $0.14$  (2), and  $0.18$  (3)

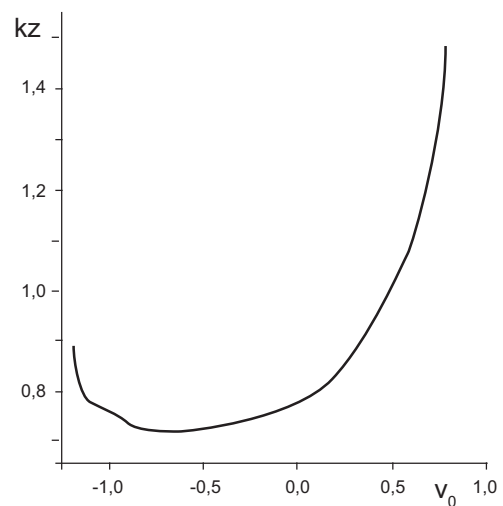


Fig. 4. Depth of penetration of particles into the magnetic configuration as a function of the initial radial velocity for  $v_{\perp} = 1.2v_0$

ple, Fig. 4 exhibits a curve, which denotes the reflection points for a particle, the transverse velocity of which exceeds the longitudinal one by a factor of 1.2. As was in the previous case, particles pass through the system, if  $\dot{r}_0 > 0$ . However, in this case, the transmission begins only if  $\dot{r}_0 > 0.75$ . In general, the dependence of the particle penetration depth on the initial radial velocity behaves similarly to the previous one. The most decelerated are particles with high initial azimuthal velocities. Particles with high positive initial radial velocity penetrate farther and can pass through the system. The magnitude of initial radial velocity, at which the passage begins, depends on the starting radius.

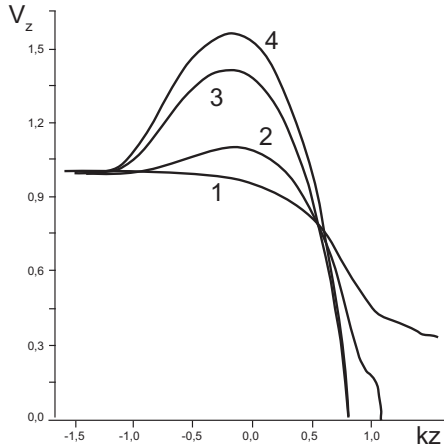


Fig. 5. Distributions of longitudinal velocities along the direction of motion for particles with various initial transverse energies  $E_{\perp} = 0.0025$  (1), 0.25 (2), 1.0 (3), and 1.44 (4)

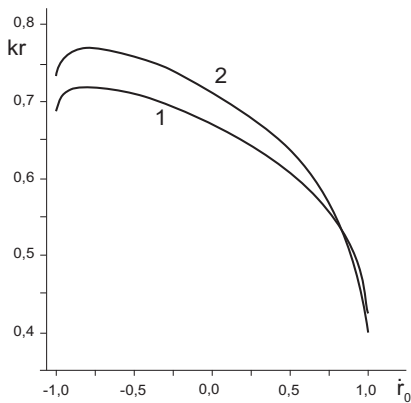


Fig. 6. Radial displacement of particles as a function of their initial radial velocity for various starting ( $kz = 0$ ) radii.  $kr_0 = 0.14$  (1) and 0.18 (2)

In work [4], a possibility of applying a periodic structure with the acute-angled (cusp) geometry of a magnetic field to select a separated isotope in the framework of the ionic cyclotron resonance (ICR) method used for the isotope separation was discussed. The results of numerical calculations carried out in work [5] showed that, if the flow incident onto the system contains particles with  $v_{\perp} \sim v_0$  and  $v_{\perp} \ll v_0$ , then, as the particles move toward the center of the system (the plane of zero magnetic field), they become so shifted along the radius that they turn out isolated near the zero-magnetic-field plane. Particles with low initial transverse velocities move closer to the axis, whereas particles with higher initial transverse velocities are shifted toward the periphery. The treatment of the results of calculation allowed us to supplement the dependences presented in work [5].

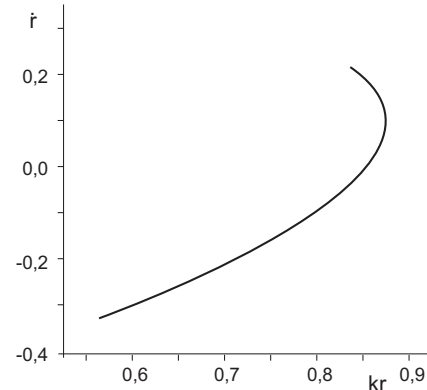


Fig. 7. Radial velocity of a particle as a function of its radial displacement at  $kz = 0$

In Fig. 5, the distributions of the longitudinal velocity component  $v_z$  along the coordinate  $kz$  are shown for various initial transverse energies. If the transverse energy is low, the longitudinal velocity at the cross-section  $kz = 0$  is lower than the initial one. If the initial transverse energy increases, the longitudinal velocity grows in such a way that the practically whole transverse energy transforms into the energy of longitudinal motion. The flow of particles moving at large radii has rather a good orientation. As the calculation shows, the longitudinal velocity reaches its maximum at  $kz = -0.25 \div 0$  and then starts to fall down, whereas the transverse velocity starts to grow up. This fact is responsible for the spread of the transverse velocities over the cross-section at  $kz = 0$ , where the maximum angle of a velocity deflection from the axis of the system is about 10%.

In Fig. 6, the dependences of the radial displacement of a particle at the trap center,  $kz = 0$ , on the initial radial velocity are depicted for various initial radii. The dependences demonstrate that the majority of particles starting from the indicated interval of radii are shifted and fall within the interval of radii 0.55–0.75. The smallest displacement is observed for particles with the highest initial radial velocities,  $\dot{r}_0 > 0$ . The interval of radii  $kr < 0.5$  includes particles with initial radial velocities of about 1. This region can also contain particles, which had a low initial transverse component of the velocity, but started from a large enough radius. Therefore, in this region, the latter particles cannot be separated spatially from particles, which had high initial transverse velocities.

Some capabilities for such a separation follow from Fig. 7, where the radial velocity of a particle at the cross-section  $kz = 0$  is shown as a function of its radial displacement for particles with high initial components

of velocity. One can see that only particles, which are maximally shifted along the radius, possess positive radial components of their velocity. Particles at smaller radii at this cross-section have radial velocities directed toward the axis of the system. The smaller the particle displacement along the radius, the larger is the velocity. The radial velocities of particles with low initial radial velocities are always positive in this region.

The Table exhibits the radial velocities for such particles at various values of parameters. One can see that the region of motion for particles with low initial transverse velocities in a vicinity of  $kz = 0$  is not overlapped with the region, where particles with high initial radial velocities move. If we distinguish the interval of radii, where the radial positions of particles with high and low initial radial velocities overlap, then, bearing in mind the direction of radial velocity, we may expect that such a separation will take place after this part of flow has undergone a certain displacement along the direction of motion.

Hence, if the particles that enter the system possess high transverse components of velocities, the latter insert appreciable variations to the motion of those particles in the considered magnetic configuration. Provided that the condition  $\eta < \eta^{cr}$  is obeyed, particles with an identical initial transverse energy can either pass through the system or be reflected, which depends on the ratio between the initial radial and azimuthal components of their velocities. The fraction of transmitted or reflected particles depends also on the starting radius of a particle. The most decelerated are particles, which have the radial velocity directed toward the axis of the system and a high enough azimuthal velocity component. Such particles do not pass through the system. Particles with higher radial components of their velocities are decelerated to a much less extent, so that they can be transmitted through the system. This result should be taken into account, when selecting the optimum conditions for the particle injection. The optimum radius for the particle

**Radial displacements and the radial velocities of particles in the plane  $kz = 0$**

$\dot{r}_0$	$\dot{\varphi}_0$	$r_0$	$\dot{r}$	$kr$
-0.05	0	0.14	0.1545	0.257
0	0.089	0.14	0.144	0.254
-0.03	0.077	0.14	0.15	0.262
0.03	0.077	0.14	0.126	0.238
0.05	0	0.25	0.283	0.410
0.05	0	0.18	0.198	0.284
0.03	0.078	0.18	0.168	0.315
0	0.062	0.20	0.209	0.357

injection must be so chosen that the fraction of transmitted particles with high velocities would be as small as possible. Our calculations demonstrate that, at the center of the examined magnetic configuration, the larger part of energy is concentrated in the longitudinal motion of particles. At the same time, a well-directed flow of particles moves at larger radii in this region, which is not overlapped with the flow of particles with low transverse velocities. The parameters of the particle flow at the center of the system testify that this configuration of the magnetic field can be applied to separate a required isotope making use of the ICR technique.

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Received 01.08.11.

Translated from Ukrainian by O.I. Voitenko

#### ДИНАМІКА ЧАСТИНОК З ВЕЛИКИМИ ПОЧАТКОВИМИ ПОПЕРЕЧНИМИ ШВИДКОСТЯМИ У МАГНІТНОМУ ПОЛІ ГОСТРОКУТОВОЇ ГЕОМЕТРІЇ

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#### Резюме

Розглянуто рух заряджених частинок, що інжектуються з великими початковими енергіями поперечного руху ( $E_{\perp} \sim E_{\parallel}$ ), у системі з гострокутовою геометрією магнітного поля. Отримано залежності глибини проникнення і проходження через систему таких частинок за різних умов на вході. Показано, що частинки з однаковою початковою енергією поперечного руху можуть пройти через систему або відбитися залежно від співвідношення між початковою радіальною та азимутальною швидкостями. Розглянуто характеристики потоку частинок у центрі системи в перетині, де магнітне поле дорівнює нулю (швидкість частинок, радіальний зсув частинок щодо стартового радіуса, спрямованість потоку), залежно від початкової радіальної швидкості. Запропоновано можливі застосування результатів дослідження.