ON THE INTERPLAY BETWEEN Q^2 AND t DEPENDENCES IN EXCLUSIVE DIFFRACTIVE PRODUCTION OF REAL PHOTONS AND VECTOR MESONS IN ep COLLISIONS

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We show how the familiar phenomenological way of combining the Q^2 (photon virtuality) and t (squared momentum transfer) dependences of the scattering amplitude in Deeply Virtual Compton Scattering (DVCS) [1, 2] and Vector Meson Production (VMP) [2] processes can be understood in an off-mass-shell generalization of dual amplitudes with Mandelstam analyticity [3]. By comparing different approaches, we managed also to constrain the numerical values of the free parameters.

1. Introduction

Measurements of exclusive deep inelastic processes such as the production of a real photon or a vector meson (the processes known as Deeply Virtual Compton Scattering (DVCS) and Vector Meson Production (VMP), respectively) opened a new window in the study of the nucleon structure (SF) in three dimensions (3D), namely in the virtuality Q^2, the energy $W = \sqrt{s}$ in the center of mass of the $\gamma^* p$ system, and the squared momentum transfer t. The construction of scattering amplitudes depending simultaneously on these variables is a challenge for the theory, and its knowledge is necessary for the deconvolution of the relevant Generalized Parton Distributions.

We earlier published an explicit phenomenological mode for the DVCS amplitude [1], which was later generalized for the VMP [2], treating these processes on the same footing. This phenomenological amplitude satisfies the Regge behavior and the scaling behavior. It is compatible with the quark counting rules and fits the experimental data on DVCS and VMP. On the other hand, this amplitude is based on several assumptions, that were not well justified in the original works. We will come back to this point in Section 2.

Quite generally, two possible ways of combining $\tilde{Q}^2$ and t dependences in DVCS and VMP exist: one is "additive", $z = t - \tilde{Q}^2$ as in Refs. [1, 2], and the other one is multiplicative, or scaling, $f(t/\tilde{Q}^2)$ called "Reggeometric" in [4].

An alternative approach is known, however, in the literature. In a number of papers [3, 5–8], the authors made attempts to build a generalized 3D dual amplitude $A(s, t, Q^2)$, by starting from the classical Dual Amplitude with Mandelstam Analyticity (DAMA) [9], and then related it to 3D SF. The main problem on this way is how the photon virtuality Q^2 enters the scattering amplitude. In Ref. [11], the Q^2 dependence is described via a generalization of the vector dominance model. According to Donnachie and Landshoff [12, 13], the Q^2 evolution can be effectively mimicked by a properly chosen factor in front of the Regge-pole terms. Based on this idea, these authors built a Regge-dual model with Q^2-dependent form factors in [7, 8], inspired by the pole series expansion of DAMA [9], which fits the SF data in

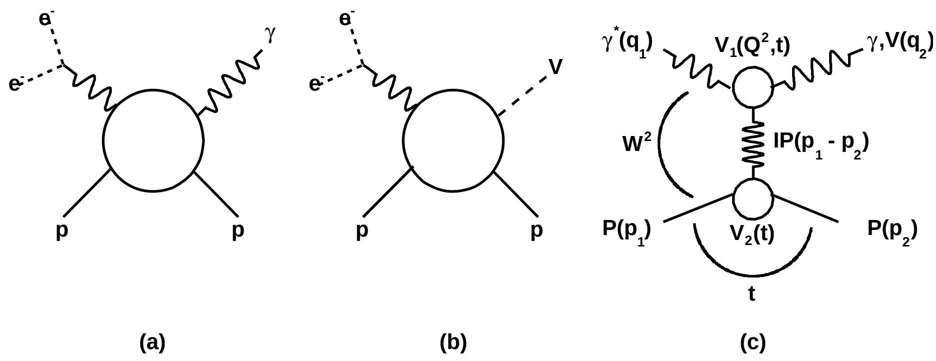

Fig. 1. Diagrams of DVCS (*a*) and VMP (*b*); (*c*) DVCS (VMP) amplitude in a Regge-factorized form

the resonance region. It is important that DAMA not only allows, but rather requires nonlinear complex Regge trajectories [9]. Then the trajectory with restricted real part leads to a limited number of resonances.

A consistent treatment of the problem requires the account for the spin dependence. It was done in [8], and a substantial improvement of the fit, in comparison to the earlier works [7] ignoring the spin dependence, was found. Nevertheless, the applicability range of the above model [8] is limited to the resonance region, as it was actually discussed by the authors. For the sake of simplicity, we ignore the spin dependence in this paper. Our goal is rather to check qualitatively the proposed new ways of constructing 3D dual amplitudes and/or SFs (see Ref. [10]).

Probably, the most successful way of introducing Q^2 in the scattering amplitude was proposed in Ref. [3], where a new, modified DAMA model with Q^2-dependence, thereafter referred to as M-DAMA, was suggested. M-DAMA preserves the attractive features of DAMA, such as its pole structure in s and t and the Regge asymptotic behavior. An additional feature is that its Q^2-dependent form factors have the correct $Q^2 \to \infty$ limit when compared with the structure function (at $t = 0$) at large-x. One of the main virtues of the M-DAMA model is its applicability to physical processes over a wide kinematic region and connections imposed by the duality conditions. Recently, the DAMA and M-DAMA models were successfully applied to the detailed study of the J/Ψ photoproduction and electroproduction processes [14, 15]. The electroproduction process is similar to photoproduction, but with virtual photons, carrying virtuality Q^2. Thus, the J/Ψ photoproduction was studied in the framework of DAMA [14]; while the authors used a generalized Q^2-dependent M-DAMA model to describe J/Ψ electroproduction [15], and only the new parameters, governing the Q^2 dependence in the model, were allowed to vary; the others were kept the same as in Ref. [14].

We will see that, in the M-DAMA framework, we can naturally justify the model ansatz proposed in Refs. [1, 2], and it helps us to substantially reduce the number of free parameters of the model.

The paper is organized as follows. In Section 2, we recall the main features of the model developed in Refs. [1, 2]. In Section 3, we will find the explicit expression for the DVCS amplitude in M-DAMA and determine some of the model parameters. Section 4 contains our conclusions.

2. The Model

2.1. Kinematics

The diagrams of the reactions in question, DVCS and VMP processes, with a single-photon exchange are shown in Fig. 1. Since we are interested in the nucleon structure, the precisely calculable electroweak vertex $e^-\gamma e^-$ of Fig. 1, *a, b* can be factorized out. In the remaining sub-process $\gamma^* p \to \gamma(V)p$, where γ^* is the incoming virtual photon and the outgoing vector particle is a real photon γ (Fig. 1, *a*) or a vector meson V (Fig. 1, *b*), at high energies typical of the HERA experiments, the amplitude is dominated by Regge exchanges, as shown in Fig. 1, *c*. In the center of mass of the $\gamma^* p$ system, the three independent variables of the reactions are, as mentioned above, the virtuality $Q^2 = -q_1^2$, whose physical values are positive, the energy $W = (p_1 + q_1)$, and squared momentum transfer $t = (q_2 - q_1)^2$. In the study of VMP, it is customary to combine the virtuality Q^2 and the squared mass of the produced vector particle M_V^2 as $\tilde{Q}^2 = Q^2 + M_V^2$.

2.2. The Amplitude

In Refs. [1, 2], a simple factorized Regge-pole model for DVCS and VMP was suggested and successfully fitted to the HERA data. Note that, at the HERA energies, the sub-leading (secondary Reggeon) contributions are negligible, so that a Pomeron exchange can account for the whole dynamics of the reaction. The Pomeron pole contribution was defined in Refs. [1, 2] on the following grounds:
– It is a single factorable Regge pole;
– The dependence on the mass and virtuality of the external particles enters via the relevant residue functions, which means that the virtuality Q^2 and the produced vector meson mass enter only via the upper residue in Fig. 1(c), V_1, while the Pomeron trajectory $\alpha(t)$ is universal and Q^2-independent;
– Following the dual models (see, e.g., Ref. [16]), we introduce the t dependence in the residues that enter solely in terms of the trajectory;
– The Pomeron trajectory has logarithmic form

$$\alpha_t(t) = \alpha_0 - \alpha_1 \ln(1 - \alpha_2 t), \qquad (1)$$

and the parameters α_i, $i = 0-2$ have to be determined from the data fitting. At small $|t|$, the Pomeron trajectory has nearly linear behavior with a slope $\alpha'_t = \alpha_1\alpha_2$. At large $|t|$, the amplitude and the cross-section obey the scaling behavior governed by the quark counting rule. In fact, the logarithmic asymptotics of the trajectory is required by the scaling of the fixed angle scattering amplitude (see Refs. [1, 17]). Moreover, it follows from perturbative Quantum Chromodynamics (pQCD) calculations (consider, e.g., the BFKL theory [18]).

Figure 2 shows the comparison of our logarithmic trajectory with a linear one, $\alpha_0 + \alpha' t$, where $\alpha_0 = 1.09$ and $\alpha' = 0.25$ GeV^{-2} for the intercept and the slope, respectively, typical of the soft processes [12], have been used. The logarithmic asymptotics are important for physical reasons: at large $|t|$, the amplitude and the cross-sections obey the scaling behavior governed by the quark counting rules, as seen in hadronic reactions [17], where sufficiently large values of $|t|$ have been reached in pp and $\bar{p}p$ scattering, confirming the quark counting rules. More arguments in favor of the logarithmic behavior in Q^2 can be found in Ref. [22]. This is expected in future measurements [19] and should be implied in DVCS and VMP as well. The high Q^2 region is governed by the QCD evolution, and it is beyond the scope of the Regge-pole models. In any case, according to the DGLAP evolution equation [20], the high Q^2 behavior must be tempered with respect to that given by the linear trajectory and be closer to the logarithmic one or, maybe, even slower (double logarithmic?). This behavior was studied in Ref. [21].

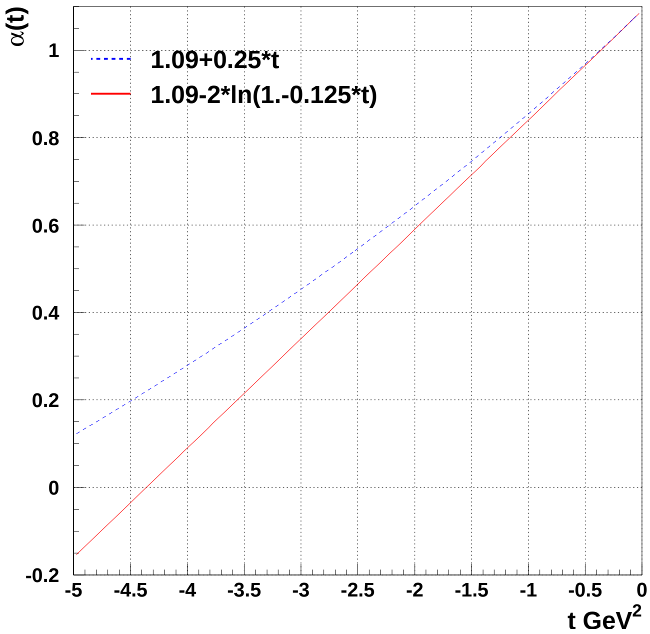

Fig. 2. Logarithmic vs linear trajectory as a function of t

Neglecting spin, the invariant scattering amplitude with a simple Regge pole exchange, as shown in Fig. 1, c, can be written as

$$A(s,t,\tilde{Q}^2)_{\gamma^* p \to \gamma(V)p} = -A_0 V_1(t,\tilde{Q}^2) V_2(t) (-is/s_0)^{\alpha(t)}\,. \qquad (2)$$

Here, A_0 is a normalization factor, $V_1(t,\tilde{Q}^2)$ is the $\gamma^* I\!P \gamma$ vertex, and $V_2(t)$ is the $pI\!Pp$ vertex.

Similarly to Ref. [23], only the helicity conserving amplitude was considered in Ref. [1] for DVCS. For not too large Q^2, the contribution from longitudinal photons is small (it vanishes for $Q^2 = 0$). Moreover, at high energies typical of the HERA collider, the amplitude is dominated by the helicity conserving Pomeron exchange, and, since the final photon is real and transverse, the initial one is also transverse. Electroproduction of vector mesons, discussed in the present paper, requires to take both the longitudinal and transverse cross-sections into account.

Following the arguments based on duality (see Ref. [1] and references therein), the t dependence of the $pI\!Pp$ vertex V_2 is introduced via the trajectory as

$$V_2(t) = e^{b_1 \alpha_t(t)}\,. \qquad (3)$$

In Refs. [1, 2], a generalization of this concept was applied also to the $\gamma^* I\!P \gamma$ vertex V_1, which depends, apart from t, also on $\tilde{Q}^2$: first, a new Fazio–Fiore–Jenkovszky–Lavorini (FFJL) variable $z_{\rm FFJL}$, combining t and Q^2 dependences was introduced as

$$z_{\rm FFJL} = t - \tilde{Q}^2\,. \tag{4}$$

Then the $\gamma^* I\!P\gamma$ was approximated through the trajectory

$$\delta(z) = \delta_0 - \delta_1 \ln(1-\delta_2 z) \tag{5}$$

as follows:

$$V_1(t,\tilde{Q}^2) = V_1(z) = e^{b_2\delta(z)}\,. \tag{6}$$

Furthermore, it was also assumed in Refs. [1, 2] that the parameters of the $\delta(z)$-trajectory are equal to those of the Pomeron trajectory: $\delta_i = \alpha_i\,,\ i = 0-2$.

Hence, the FFJL scattering amplitude in Eq. (2) can be written in the form

$$A(s,t,\tilde{Q}^2)_{\gamma^*p\to\gamma(V)p} = -A_0 e^{b_2\alpha(z)} e^{b_1\alpha(t)} (-is/s_0)^{\alpha(t)}\,. \tag{7}$$

In the next section, we will try to justify the above proposed ansatz for the DVCS and VMP amplitudes in the framework of M-DAMA [3].

3. Explicit DVCS and VMP Amplitude from a Modified DAMA Model

Technically on the model construction level, the only difference in the VMP amplitude with respect to the DVCS one is the use of $\tilde{Q}^2$ instead of just Q^2. So for simplicity, we will consider only DVCS case in this section; a generalization for VMP is rather straightforward.

3.1. M-DAMA amplitude and its Regge assymptotics

The Dual Model with Mandelstam Analyticity appeared as a generalization of narrow-resonance (e.g., Veneziano) dual models, intended to overcome the manifestly nonunitarity of the latter [9]. Contrary to narrow-resonance dual models, DAMA requires non-linear, complex trajectories. The dual properties of DAMA were studied in Ref. [2].

The DAMA amplitude [9] is given by

$$D(s,t) = c\int\limits_0^1 dz \left(\frac{z}{g}\right)^{-\alpha_s(s')-1} \left(\frac{1-z}{g}\right)^{-\alpha_t(t'')}, \tag{8}$$

where $\alpha_s(s)$ and $\alpha_s(t)$ are Regge trajectories in the s and t channels, correspondingly; $x' = x(1-z)$, $x'' = xz$ $(x = s,t,u)$; g and c are parameters, $g>1$, $c>0$.

To extend our model off-mass shell, we need to construct the Q^2-dependent dual amplitude. To this aim, we use the so-called Modified DAMA (M-DAMA) formalism developed in Ref. [3]. The scattering amplitude is given by

$$D(s,t,Q^2) = c\int\limits_0^1 dz \left(\frac{z}{g}\right)^{-\alpha_s(s')-\beta(Q^{2''})-1} \times$$

$$\times \left(\frac{1-z}{g}\right)^{-\alpha_t(t'')-\beta(Q^{2'})}, \tag{9}$$

where $\beta(Q^2)$ is a monotonically decreasing dimensionless function of Q^2.

It has been shown in Ref. [3] that, by choosing the β function in the form

$$\beta(Q^2) = -1 - \frac{\alpha_t(0)}{\ln g} \ln\left(\frac{Q^2+Q_0^2}{Q_0^2}\right), \tag{10}$$

all asymptotics of the amplitude at large s remain valid, and the Q^2 behavior of the amplitude is in qualitative agreement with the experiment.

Regge assymptotics of M-DAMA amplitude reads (see [3] for details)

$$D|_{s\to-\infty} \approx -s^{\alpha_t(t)+\beta(0)} g^{\alpha_t(t)+\alpha_s(a)+\beta(Q^2)+\beta(0)+2} \times$$

$$\times\ a^{-\alpha_t(t)-\beta(0)-1} \sqrt{\frac{2\pi}{-\alpha_s''(0)\ln g - \frac{\alpha_s'(0)\ln g}{a}}}, \tag{11}$$

where

$$a = \frac{\alpha_t(t)+\beta(0)+1}{\alpha_s'(0)\ln g} = \frac{(\alpha_t(t)+\beta(0)+1)s_0}{\alpha_{1,s}\ln g}\,, \tag{12}$$

assuming that the Regge trajectory is approximately linear for small s: $\alpha_s(s) = \alpha_s(0) + \alpha_{1,s}s/s_0$, and s_0 is some characteristic scale. Now putting $\beta(0) = -1$ [3] and ignoring the slow dependences, we have:

$$D(s,t,Q^2) \sim \left(\frac{s}{s_0}\right)^{\alpha_t(t)-1} \left(\frac{\alpha_t(t)}{\alpha_{1,s}\ln g}\right)^{-\alpha_t(t)} g^{\beta(Q^2)+\alpha_t(t)},$$

$$\text{for}\|s|\to\infty\,. \tag{13}$$

The minimal model for the total scattering amplitude is a sum:

$$A(s,t,Q^2) = c(s-u)(D(s,t,Q^2) - D(u,t,Q^2)). \tag{14}$$

In the Regge limit, $t, Q^2 = \text{const}$, $s \to \infty$, $u = -s$, the total amplitude behaves itself as

$$A(s,t,Q^2)|_{s\to\infty} \sim \left(\frac{s}{s_0}\right)^{\alpha_t(t)} \left(\frac{\alpha_t(t)}{\alpha_{1,s}\ln g}\right)^{-\alpha_t(t)} \times$$

$$\times g^{\beta(Q^2)+\alpha_t(t)}\,. \tag{15}$$

3.2. Explicit DVCS amplitude from M-DAMA

Comparing Eqs. (7) and (15), we see that, in order to combine the t and Q^2 dependences in the $g^{\beta(Q^2)+\alpha_t(t)}$ term of Eq. (15), the following connection between the parameters $\beta(Q^2)$, Eq. (10), and $\alpha_t(t)$, Eq. (1), is required:

$$\alpha_1 = \alpha_t(0)/\ln g \quad \Rightarrow \quad \ln g = \alpha_t(0)/\alpha_1\,. \tag{16}$$

This requirement will fix the M-DAMA model parameter g.

Now, based on Eq. (16), we obtain

$$\beta(Q^2)+\alpha_t(t) = \alpha_t(0)-1-\alpha_1 \ln\left((1-\alpha_2 t)(1+\frac{Q^2}{Q_0^2})\right) =$$

$$= \alpha_t(0) - 1 - \alpha_1 \ln\left(1-\alpha_2\left(t - \frac{Q^2}{\alpha_2 Q_0^2} + t\frac{Q^2}{Q_0^2}\right)\right) =$$

$$= \delta(0) - \alpha_1 \ln(1-\alpha_2 z) = \delta(z), \tag{17}$$

where

$$z = t - \frac{Q^2}{\alpha_2 Q_0^2} + t\frac{Q^2}{Q_0^2}\,. \tag{18}$$

Please, note that $\delta(0) = \alpha_t(0)-1$, but this shift is always possible to do without any problem for the model. So, we can assume that parameters of $\delta(z)$ are just the same as parameters of $\alpha_t(t)$ (if Eq.([16]) is satisfied).

Thus, three main assumptions of Refs. [1, 2], namely that

A) Q^2 and t enter into the $\gamma^* I\!P \gamma$ vertex V_1 via one variable z;

B) the contribution of this vertex can be given in the form $e^{b_2\delta(z)}$;

C) the new z-dependent trajectory has exactly the same form and the same parameters as the Pomeron trajectory;

can be justified in M-DAMA, fixing its only free parameter g as it is given in Eq. ([16]).

The resulting M-DAMA scattering amplitude looks like

$$A(s,t,Q^2)|_{s\to\infty} \sim \left(\frac{s}{s_0}\right)^{\alpha_t(t)} \left(\frac{\alpha_t(t)}{\alpha_{1,s}\ln g}\right)^{-\alpha_t(t)} g^{\delta(z)} =$$

$$= \left(\frac{s}{s_0}\right)^{\alpha_t(t)} e^{\ln\left(\frac{\alpha_t(0)}{\alpha_t(t)}\frac{\alpha_{1,s}}{\alpha_1}\right)\alpha_t(t)} e^{\ln g\,\delta(z)}\,. \tag{19}$$

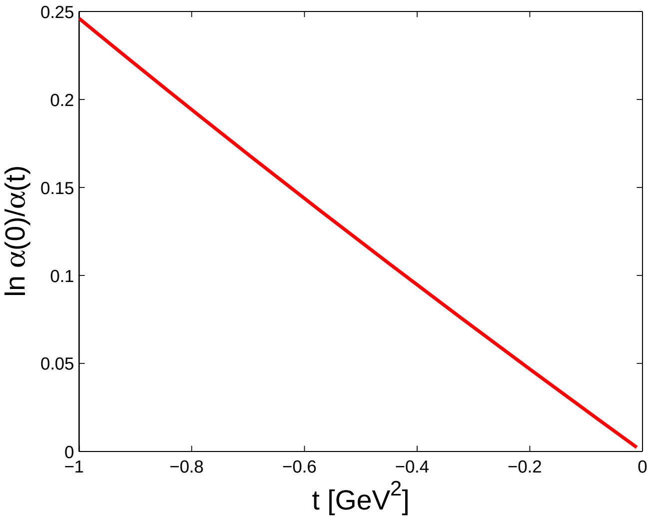

Fig. 3. Correction to the b_1 parameter. We assume that the slopes of the trajectories are approximately the same plotted in $\ln\left(\frac{\alpha_t(0)}{\alpha_t(t)}\right)$. The parameters of the $\alpha_t(t)$ trajectory are taken as quoted in the text. We expect that the parameter b_1 close to 8, and, thus, the correction amounts at most to 3%

3.3. Comparison of the DVCS amplitude from M-DAMA with the FFJL ansatz

Comparing the FFJL amplitude, Eq. (7), with our expression, Eq. (19), we identify:

$$z = t - \frac{Q^2}{\alpha_2 Q_0^2} + t\frac{Q^2}{Q_0^2}. \tag{20}$$

Note that our z coincides with the $z_{\text{FFJL}} = t - Q^2$, introduced in [2], in the region $Q^2/Q_0^2 \ll 1$ and if the characteristic scales are the same, i.e. $Q_0^2 = 1/\alpha_2$.

The further identification gives

$$b_2 = \ln g = \alpha(0)/\alpha_1\,, \tag{21}$$

and, finally,

$$b_1 = \ln\left(\frac{\alpha_t(0)}{\alpha_t(t)}\frac{\alpha_{1,s}}{\alpha_1}\right) = \tilde{b}_1 + \ln\left(\frac{\alpha_t(0)}{\alpha_t(t)}\right) \tag{22}$$

is not a constant, but a slowly varying function of t. Please, note that the scaling parameter s_0 is not really known – for simplicity, we will fix it to be $s_0 = m_p^2$, and the possible corrections to it (of the type $\ln\tilde{s}_0/s_0$) will contribute to b_1 or, more precisely, to the $\tilde{b}_1$ parameter. Therefore, $\tilde{b}_1$ cannot be fixed from the model and has to be fitted to the data. The functional dependence of $b_1(t)$ is indeed very slow in the region of interest, as illustrated in Fig. 3 (the parameters of the trajectory will be discussed in the next section).

Thus, the final expression for the DVCS amplitude in M-DAMA coincides with that of Refs. [1, 2]:

$$A(s,t,Q^2)|_{s\to\infty} = A_0\left(\frac{s}{s_0}\right)^{\alpha(t)} e^{b_2\alpha(z)}e^{b_1\alpha(t)}, \tag{23}$$

where we use a unifying notation $\alpha(x) = \alpha_t(x) = \delta(x)$. The parameters of the Regge trajectory α (α_i, $i = 0-2$) as well as A_0, b_1, and Q_0^2 are fitting parameters.

3.4. Parameters of the model

Now, we can proceed constructing the physical quantities to be fitted to the experimental data.

The differential cross-section $d\sigma(\gamma^*\, p \to \gamma(V)\, p)/dt$ is defined as

$$\frac{d\sigma}{dt}(\tilde{Q}^2, s, t) = \frac{\pi}{s^2}|A(\tilde{Q}^2, s, t)|^2. \tag{24}$$

This integral can be calculated analytically, as it was done in Ref. [2], and the result is

$$\sigma(s,\tilde{Q}^2) = \frac{\pi|A_0|^2(s/s_0)^{2\alpha_0}}{s^2\left(1+\frac{\tilde{Q}^2}{Q_0^2}\right)^{2b_2\alpha_1}}\frac{e^{2\alpha_0(b_1+b_2)}}{2\alpha_1\left[b_1+b_2+\ln(s/s_0)\right]-1}. \tag{25}$$

Although the model has many parameters, most of them are constrained by plausible assumptions. First, we can fix the intercept of $\alpha_t(t)$ (and, correspondingly, $\delta(z)$) to some typical value, for example 1.08 [2]. The hardening of the dynamics with increasing $\tilde{Q}^2$ may be accounted for either by letting the intercept to be $\tilde{Q}^2$-dependent, unacceptable by Regge-factorization, or by introducing one more, hard component in the Pomeron (still unique!) with a $\tilde{Q}^2$-dependent residue (as suggested, e.g., in Refs. [13] and [2]). In any case, the trajectories and their parameters are the same for DVCS and for VMP. The other two parameters of the trajectories, α_1 and α_2 (δ_1 and δ_2) can be fixed in the following way: their product $\alpha' = \alpha_1\alpha_2$, which is the forward slope of the trajectory, can be set equal to some typical value, like $\alpha' = 0.25$ GeV^{-2}. Furthermore, since $\alpha_1 \approx 2$ from the quark counting rules (see Ref. [1]), we get $\alpha_2 = \alpha'/\alpha_1 = 0.125$ GeV^{-2}.

With these parameters we obtain, using eq. (21): $b_2 = \ln g = \alpha(0)/\alpha_1 = 0.545 \Rightarrow g = 1.725$. Please, note that, in the original FFJL papers [1, 2], b_2 was a free fitting parameter of the model. In the fits were obtained the following results for this parameter [2]: $0.65 < b_2 < 0.77$ from the fits to DVCS data; $1.08 < b_2 < 1.15$ from the fits to VMP data. Taking into account that the z variable appearing in the M-DAMA approach is a bit different from $z_{\rm FFJL}$ suggested in the FFJL model, we see the obtained value $b_2 = 0.545$ as a very reasonable number. Note that there are no fitting or extra assumptions – this value of b_2 came directly from the comparison of two expressions for the scattering amplitude: Eq. (7) and Eq. (19).

The parameter s_0 is not fixed by the Regge-pole theory. There is a nice plausible relation $s_0 = 1/\alpha' \approx (1/4)m_p^2$, which follows from the hadronic string model [24]. However, other values, for example $s_0 = m_p^2 \approx 1$ GeV^2 [2], cannot be excluded.

Finally, we can set the parameter b_1 entering the proton vertex (lower vertex of Fig. 1, *c*) as $b_1 = 8.0$. In fact, this (p$I\!\!P$p) vertex is known from the analysis of the pp and $\bar{p}p$ scattering to be of the form $\exp(bt)$, and an estimate of b is $b \approx b_1\alpha_1\alpha_2 \approx 2$ GeV^{-2} (see Ref. [25] and references therein). If the parameter $b_1 \approx 8.0$, the t-dependent correction to it (see Fig. 3) is smaller than 3% in all the region of interest, and, thus, can be safely neglected.

Thus, we remain with the following free parameters: the characteristic virtuality scale Q_0^2 and the squared modulus of the normalization factor $|A_0|^2$.

In Refs. [1, 2], the fitting parameters were the parameter b_2, entering the photon vertex $\gamma^* I\!\!P \gamma(V)$, and $|A_0|^2$. Starting from the M-DAMA, we fix b_2 according to Eq. (21), but introduce a new free parameter Q_0^2. In fact, with an additional requirement that our z should be maximally close to $z_{\rm FFJL}$, we can fix $Q_0^2 = 1/\alpha_2 = 8$ GeV^2. In this case, the two variables will be identical $z \approx z_{\rm FFJL}$ for small Q^2 ($Q^2 \ll 8$ GeV^2), i.e. in the region, where the agreement with experimental data is the best [2].

4. Conclusions

In this paper, we have demonstrated that, interestingly enough, two (DVCS and VMP) models constructed in two completely different manners ultimately give practically the same expression.

The purely phenomenological FFJL model [1, 2] is based on several ansätze, introduced "by hand" to the scattering amplitude, namely that
A) the Q^2 and t enter into the $\gamma^* I\!P \gamma$ vertex V_1 via one variable z;
B) the contribution of this vertex can be given in the form $e^{b_2 \delta(z)}$;
C) the new z-dependent trajectory has exactly the same form and the same parameters as the Pomeron trajectory.
It was shown in Section 3 3.2 that all these assumptions can be naturally understood in the M-DAMA [3] with the correct choice of its only free parameter g (see Eq. (16)).

Furthermore, as it was demonstrated in Sections 3.3 and 3.4, the comparison of the FFJL and M-DAMA scattering amplitudes fixes some free parameters of the model, namely $b1$, $b2$, and Q_0^2, if we require that z and $z_{\rm FFJL}$ be as close as possible.

We would like to stress that this is not just a mathematical coincidence, it has an important consequence. The FFJL model was developed to be used in the Regge limit (see Eq. (2)). On the other hand, the M-DAMA, which shows a similar behavior in the Regge limit, can be used in the whole kinematically allowed region of energies. Technically, this is not always simple, due to the complexity of the M-DAMA integral [3], Eq. (9), but (e.g., in Ref. [15]) the full M-DAMA integral calculation has been performed.

Another interesting result is that the variable z, combining the t and Q^2 dependences in the $\gamma^* IP \gamma$ vertex, is not exactly the same in two approaches. In the low Q^2 limit, $z_{\rm FFJL}$ coincides with z of the M-DAMA under a proper choice of Q_0^2. However, generally, if one uses Q_0^2 as a free parameter, there should be a noticeable difference in the behavior of the scattering amplitudes (7) and (19), especially at large Q^2.

Both the FFJL model and M-DAMA were confronted against the experimental data showing a reasonable agreement; the most recent results can be found in Refs. [2] and [15], correspondingly. The future study will show whether the use of the new variable z, Eq. (20), improves the fits with respect to that of $z_{\rm FFJL}$.

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ПРО ЗАЛЕЖНІСТЬ ВІД Q^2 ТА t ЕКСКЛЮЗИВНОГО ДИФРАКЦІЙНОГО НАРОДЖЕННЯ РЕАЛЬНИХ ФОТОНІВ ТА ВЕКТОРНИХ МЕЗОНІВ В ep ЗІТКНЕННЯХ

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Р е з ю м е

Показано яким чином дуальна амплітуда з мандельстамівською аналітичністю, продовжена за масову поверхню [3], допомогає зрозуміти аддитивну залежність амплітуди розсіяння від віртуальності фотона Q^2 та квадрата переданого імпульсу t [1, 2]. Порівнюючи різні моделі нам вдалося знайти чісельні значення для деяких вільних параметрів цих моделей.