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ON THE LORENTZ-INVARIANCE OF THE DYSON SERIES IN THEORIES WITH DERIVATIVE COUPLINGS

We study the Dyson series for the S-matrix, when the interaction depends on derivatives of the fields. We concentrate on two particular examples: the scalar electrodynamics and the renormalized ϕ^4 theory. By using Wick's theorem, we eventually give evidence that the Lorentz invariance is satisfied, and the usual Feynman rules can be applied to the interaction Lagrangian.

K e y w o r d s: quantum field theory, canonical quantization, Dyson series, Wick's theorem.

1. Introduction

Feynman diagrams and rules are by far the most efficient and convenient way to build theoretical predictions within field theories amenable of a perturbative treatment. They manifestly keep the Lorentz invariance and come naturally if field theories are quantized by means of a functional generator based on the Lagrangian of the theory, which is a Lorentz scalar. The same holds in the canonical approach, which is based on the Dyson series for the S-matrix in the interaction scheme, for theories not featuring derivative couplings. In this case, indeed, the interaction Hamiltonian entering the Dyson series coincides, up to a sign, with the (scalar) interaction Lagrangian, and the Lorentz invariance of the S-matrix and of the ensuing Feynman rules again manifests itself.

The equivalence between the Feynman approach and the Dyson series is not evident if the interaction Lagrangian contains derivatives of the fields. This occurs since the interaction Hamiltonian contains non-invariant terms, which seem to jeopardize the Lorentz invariance of the S-matrix and the derivation of the usual Feynman rules. This problem was known since the late 1940s, when the achievement of a fully covariant formulation of QED free of divergences at any order in perturbation theory stimulated the perturbative investigation and the proof of renormalizability also in other theories. Among them, the *scalar* QED received a special attention, being physically

interesting on its own and posing additional technical problems due to its dependence on derivative couplings. Already in 1950, Rohrlich [1] tackled the problem and showed that, at any perturbative order, non-invariant terms of the interaction Hamiltonian are exactly compensated in the Dyson series by non-covariant terms arising from the time-ordered product of the derivatives of two fields. The argument by Rohrlich is presented at the lowest perturbative order in the textbook [2], whereas, in the textbook [3], a nice general proof is presented for the cancellation of non-invariant terms in the Green functions of the theory.

The problems raised by derivative couplings in other theories were discussed in Refs. [4, 5]. A general solution of these problems in the case of quantum mechanics was then proposed by Nambu [6]. He proved the equivalence of the Dyson series for the Lagrangian, with a modified T-product, in which the time derivatives are performed after the time ordering, with the Dyson series with standard T-products for operators that he eventually proves to be the Hamiltonian. The extension of this proof to QFT was done by Nishijima [7]: in this case, the modified T-product is manifestly covariant, and this property eventually allows one to prove the Lorentz invariance of the Dyson series. We observe that these results, though conclusive, were obtained without using Wick's theorem.

Other mentionable works on the same subject are Refs. [8, 9], where, however, the main focus is on the

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very definition of interaction Hamiltonian in the interaction scheme, the issue of non-invariant terms in the S-matrix being touched laterally. A more direct attack to the problem of derivative couplings by using Wick's theorem can be found in Ref. [10]: in brief, supposing the equivalence of the standard Dyson form for the S-matrix (involving the standard time-ordered product of interaction Hamiltonians) with the 'Wick's form for the S-matrix (involving a modified time-ordered product of interaction Lagrangians) they find a form for the interaction Hamiltonian.

In this scenario, our aim is to give an alternative proof of the equivalence between the Dyson series for the Hamiltonian with standard T-product and the Dyson series for the Lagrangian with a modified covariant T-product. Differently from [7], we will derive, from first principles, the interaction Hamiltonian and start from the usual Dyson series for the Hamiltonian; then, we use Wick's theorem to trade standard T-products with normal ordered products and propagators and to eventually express everything in terms of modified covariant T-products; after discarding the vacuum diagrams, we recognize a Dyson series involving the Lagrangian, with a modified covariant T-product. We believe that the use of Wick's theorem makes plain and pedagogical the proof of the Lorentz invariance of the Dyson series and its coincidence with the perturbative series coming from Feynman diagrams. This is the main advantage of our approach. In addition, with our method, we can emphasize the fact that such coincidence is true, if one discards vacuum diagrams in the original Dyson series for the Hamiltonian. This fact was not explicitly remarked in the previous literature [6, 7].

The plan of the paper is the following. First, we revisit the case of scalar electrodynamics (Section 2) and extend to all orders the analysis given in [2], with the aim of stating the problem in the simplest possible way and to illustrate its solution. Then we move to the case of the renormalized ϕ^4 theory (Section 3), which is taken as a representative of all theories, where the derivative couplings, not originally present in the bare Lagrangian, appear due to the renormalization procedure, as unavoidable counterterms of the kinetic term. In this theory, now at the lowest orders in perturbation theory, we address the problem of cancellation of non-invariant terms and present, as a by-product, a consistent way to define the interaction Hamiltonian in the interaction scheme.

2. Dyson Series of the S-Matrix and Feynman Rules: the Usual Procedure

Let us briefly review how usual perturbative computations based on Feynman diagrams stem from the Dyson series and Wick theorem. Here, the word "usual" refers to the fact that the Lagrangian of the theory $\mathcal{L}$, which depends on the fields ϕ_r and on their derivatives $\partial_\mu \phi_r$, is decomposed as

$$\mathcal{L}(\phi_r, \partial_\mu \phi_r) = \mathcal{L}_0(\phi_r, \partial_\mu \phi_r) + \mathcal{L}'(\phi_r), \qquad (2.1)$$

where $\mathcal{L}_0$, the free Lagrangian, depends on the fields and their derivatives, while $\mathcal{L}'$, the interaction part, depends only on the fields (not on their derivatives). The free Lagrangian is quadratic in the fields, whilst the interaction Lagrangian contains terms at least cubic in the fields and is proportional to a set of real numbers, the coupling constants. All fields ϕ_r in (2.1), and, in general, all the fields throughout the paper without any additional index or subscript, are intended to be in Heisenberg representation.

The definition of the momenta,

$$\pi_r = \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi_r)} = \frac{\partial \mathcal{L}_0}{\partial(\partial_0 \phi_r)}, \qquad (2.2)$$

allows us to introduce the Hamiltonian density

$$\mathcal{H}(\phi_r, \pi_r) = \pi_r \dot{\phi}_r - \mathcal{L} = \pi_r \dot{\phi}_r - \mathcal{L}_0 - \mathcal{L}' = \mathcal{H}_0 + \mathcal{H}', \qquad (2.3)$$

with

$$\mathcal{H}_0(\phi_r, \pi_r) = \pi_r \dot{\phi}_r - \mathcal{L}_0, \quad \mathcal{H}'(\phi_r) = -\mathcal{L}'(\phi_r). \quad (2.4)$$

The next ingredient is the Dyson series for the S-matrix,

$$S = \sum_{n=0}^{+\infty} \frac{(-i)^n}{n!} \int d^4 x_1 \dots d^4 x_n T\left[\mathcal{H}'_I(x_1) \dots \mathcal{H}'_I(x_n)\right], \qquad (2.5)$$

which is written in terms of the interaction Hamiltonian in the so-called interaction representation,

$$\mathcal{H}'_I(t) = U\mathcal{H}'(t)U^{-1}, \quad U = e^{iH_0^{(s)}t}e^{-iH^{(s)}t}, \qquad (2.6)$$

with $H_0^{(s)} = \int d^3x \mathcal{H}_0^{(s)}$ the free Hamiltonian and $H^{(s)} = \int d^3x \mathcal{H}^{(s)}$ the complete Hamiltonian, both in the Schrödinger representation. Since $\mathcal{H}'(\phi_r) = -\mathcal{L}'(\phi_r)$, we get

$$\mathcal{H}'_I = \mathcal{H}'(\phi_{rI}) = -\mathcal{L}'(\phi_{rI}), \qquad (2.7)$$

so that the Dyson series is written as

$$S = \sum_{n=0}^{+\infty} \frac{i^n}{n!} \int d^4x_1 \dots d^4x_n T \times$$
$$\times [\mathcal{L}'(\phi_{rI}(x_1)) \dots \mathcal{L}'(\phi_{rI}(x_n))], \tag{2.8}$$

in terms of the interaction Lagrangian in the Heisenberg scheme, in which all the fields are in the interaction representation. Through the use of the Wick theorem, we eventually find that the perturbative computations can be organized by means of the usual Feynman rules applied to $\mathcal{L}'$.

Clearly, it seems that this picture collapses when $\mathcal{L}'$ contains also derivatives of the fields ϕ_r. What is certainly true in general is that the Dyson series is given by (2.5). What is no more true is the second of (2.4), and (2.7). In addition, the application of the Wick theorem to the S-matrix expansion (2.5), in which the objects inside the T-ordered product depend on derivatives of the fields, is not equivalent to applying Feynman rules, since

$$\langle 0|T[\partial_\mu \phi(x) \partial_\nu \phi(y)]|0\rangle \neq \partial_\mu \partial_\nu \langle 0|T[\phi(x)\phi(y)]|0\rangle. \tag{2.9}$$

The l.h.s. of this expression is what comes from the Wick theorem, the r.h.s. is what comes from the Feynman rules, since in this approach derivatives are attached to vertices, whilst internal lines are associated to propagators $\langle 0|T[\phi(x)\phi(y)|0\rangle$.

However, we will show in two examples that all these problems 'cancel' each other: then, using the Wick theorem in the Dyson series (2.5) is equivalent to applying the Feynman rules to (2.8), which contains $\mathcal{L}'(\phi_{rI})$, *i.e.* the interaction Lagrangian with all the fields in the interaction representation.

The two examples we study are related to the scalar electrodynamics and renormalized ϕ^4 theory in four dimensions[1].

3. Scalar Electrodynamics

As well known, the Lagrangian of the scalar electrodynamics is

$$\mathcal{L} = \left[D_\mu \phi\right]^\dagger D^\mu \phi - m^2 \phi^\dagger \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \tag{3.1}$$

with

$$D_\mu = \partial_\mu - ieA_\mu. \tag{3.2}$$

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[1] The arguments presented below are in fact independent of the space-time dimension.

Then, we have $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$, with

$$\mathcal{L}' = ieA_\mu \phi^\dagger \partial^\mu \phi - ieA^\mu (\partial_\mu \phi^\dagger) \phi + e^2 A_\mu A^\mu \phi^\dagger \phi. \tag{3.3}$$

Defining the conjugate fields

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}^\dagger + ieA_0 \phi^\dagger, \quad \pi^\dagger = \frac{\partial \mathcal{L}}{\partial \dot{\phi}^\dagger} = \dot{\phi} - ieA^0 \phi, \tag{3.4}$$

we introduce the Hamiltonian density

$$\mathcal{H} = \pi \dot{\phi} + \dot{\phi}^\dagger \pi^\dagger - \mathcal{L}, \tag{3.5}$$

which we write $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}'$, where

$$\mathcal{H}_0 = \pi^\dagger \pi + \boldsymbol{\nabla}\phi^\dagger \boldsymbol{\nabla}\phi + m^2 \phi^\dagger \phi + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{3.6}$$

and

$$\mathcal{H}' = -ieA_0 \phi^\dagger (\pi^\dagger + ieA_0 \phi) - ie\mathbf{A}\phi^\dagger \boldsymbol{\nabla}\phi + ieA_0(\pi -$$
$$- ieA_0 \phi^\dagger)\phi + ie\mathbf{A}(\boldsymbol{\nabla}\phi^\dagger)\phi - e^2 A_\mu A^\mu \phi^\dagger \phi - e^2 A_0^2 \phi^\dagger \phi. \tag{3.7}$$

So far, all the expressions above are in the Heisenberg representation. The operators with no subscript are in the Heisenberg representation as well. Moreover, it is understood that all terms in the Lagrangian and Hamiltonian densities are subject to the normal ordering, N. In order to write the Dyson series, we have to pass to the interaction representation. We find the following property to be useful:

$$U\pi^\dagger(x)U^{-1} = \partial^0 \phi_I(x),$$
$$U\pi(x)U^{-1} = \partial^0 \phi_I^\dagger(x), \tag{3.8}$$

where the operator

$$U = e^{iH_0^{(s)}t} e^{-iH^{(s)}t}, \tag{3.9}$$

with the free Hamiltonian $H_0^{(s)} = \int d^3x \mathcal{H}_0^{(s)}$ and the complete Hamiltonian $H^{(s)} = \int d^3x \mathcal{H}^{(s)}$, both in the Schrödinger representation, allows us to pass from the Heisenberg representation to the interaction one. We give a proof of (3.8) in Appendix. Using (3.8), we obtain the following expression for the interaction Hamiltonian in the interaction representation:

$$U\mathcal{H}'U^{-1} = \mathcal{H}_I' = -ieA_{\mu I} \phi_I^\dagger \partial^\mu \phi_I + ieA_I^\mu (\partial_\mu \phi_I^\dagger)\phi_I -$$
$$- e^2 A_{\mu I} A_I^\mu \phi_I^\dagger \phi_I + e^2 A_{0I}^2 \phi_I^\dagger \phi_I. \tag{3.10}$$

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To simplify notations, we remove the subscript $_I$ in the following in all the fields, because, from now on, all the fields are in the interaction representation. However, we keep the subscript $_I$ in the Hamiltonian to stress that it is in the interaction representation. Comparing (3.10) with the interaction Lagrangian (3.3), in which all the fields are promoted to be in the interaction representation [2], we find that

$$\mathcal{H}'_I = -\mathcal{L}' + e^2 A_0^2 \phi^\dagger \phi \equiv -\mathcal{L}' + \mathcal{R}. \qquad (3.11)$$

We did not put the index $_I$ in the Lagrangian in (3.11), because it still has the form of the interaction Lagrangian in the Heisenberg representation: the only caveat, as written before, is that the fields appearing in its expression (3.3) are in the interaction representation.

Now we prove the following equality. The Dyson series

$$S = \sum_{n=0}^{+\infty} \frac{(-i)^n}{n!} \int d^4x_1 \dots d^4x_n T\left[\mathcal{H}'_I(x_1) \dots \mathcal{H}'_I(x_n)\right] \qquad (3.12)$$

can be written as

$$S = \sum_{n=0}^{+\infty} \frac{i^n}{n!} \int d^4x_1 \dots d^4x_n \hat{T}\left[\mathcal{L}'(x_1) \dots \mathcal{L}'(x_n)\right], \quad (3.13)$$

provided that we use a modified definition of the T-product in (3.13). Given

$$\langle 0|T(\phi_1\phi_2^\dagger)|0\rangle \equiv i\Delta_F(x_1 - x_2), \qquad (3.14)$$

the operation $\hat{T}$ satisfies the Wick theorem, but its 'action' on elementary fields is the following [3]:

$$\langle 0|\hat{T}(\phi_1\phi_2^\dagger)|0\rangle = \langle 0|T(\phi_1\phi_2^\dagger)|0\rangle \equiv i\Delta_F(x_1 - x_2), \qquad (3.15)$$

$$\langle 0|\hat{T}((\partial^\mu\phi_1)\phi_2^\dagger)|0\rangle \equiv i\partial_1^\mu\Delta_F(x_1 - x_2), \qquad (3.16)$$

$$\langle 0|\hat{T}((\partial^\mu\phi_1)(\partial^\nu\phi_2^\dagger))|0\rangle \equiv i\partial_1^\mu\partial_2^\nu\Delta_F(x_1 - x_2). \quad (3.17)$$

For the sake of brevity, we have introduced here the notation $\phi_i \equiv \phi(x_i)$ and $\phi_i^\dagger \equiv \phi^\dagger(x_i)$, as well as

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[2] Actually, this procedure is, in our opinion, the correct one to define (interaction) Lagrangians in the interaction representation in a general case.

[3] The $\hat{T}$-product is known in the literature as "Wick T-product", whereas the standard T-product is called also 'Dyson T-product'.

$\partial_i^\mu \equiv \partial/\partial x_{i,\mu}$; below, we will use similarly $A_{\mu,i}$ for $A_\mu(x_i)$ and will extend this notation also to functions of fields, as the Lagrangian density. We remark that the use of the Wick theorem in expansion (3.13) with the operation $\hat{T}$ produces a Dyson series whose terms are all manifestly Lorentz invariant. The Lorentz invariance is not evident, if we use expansion (3.12), which is, however, the *a priori* correct one.

We now give a perturbative proof of this statement. Let us write the first two terms of the Dyson series:

$$S^{(1)} = -i \int d^4x_1 \left[\mathcal{H}'(x_1)\right] = -i \int d^4x_1 (-\mathcal{L}' + \mathcal{R})_1, \qquad (3.18)$$

$$S^{(2)} = -\frac{1}{2} \int d^4x_1 d^4x_2 T\left[\mathcal{H}'(x_1)\mathcal{H}'(x_2)\right] =$$

$$= -\frac{1}{2} \int d^4x_1 d^4x_2 T\left[(-\mathcal{L}' + \mathcal{R})_1(-\mathcal{L}' + \mathcal{R})_2\right] =$$

$$= -\frac{1}{2} \int d^4x_1 d^4x_2 T\left[(-\mathcal{L}')_1(-\mathcal{L}')_2 + \right.$$

$$\left. + \mathcal{R}_1(-\mathcal{L}')_2 + (-\mathcal{L}')_1\mathcal{R}_2 + \mathcal{R}_1\mathcal{R}_2\right]. \qquad (3.19)$$

We remark that, in $S^{(1)}$, there is an extra term with respect to $-\mathcal{L}'$: $e^2(A_{0,1}^2)\phi_1^\dagger\phi_1$, which is not Lorentz invariant. However, this is not the end of the story, since another source of the Lorentz non-invariance comes from the operation of T arising in various terms of $S^{(2)}$ after the application of the Wick theorem. To be precise, we have that, remembering (3.15)–(3.17),

$$\langle 0|T((\partial^\mu\phi_1)\phi_2^\dagger)|0\rangle = i\partial_1^\mu\Delta_F(x_1 - x_2) =$$

$$= \langle 0|\hat{T}((\partial^\mu\phi_1)\phi_2^\dagger)|0\rangle, \qquad (3.20)$$

$$\langle 0|T((\partial^\mu\phi_1)(\partial^\nu\phi_2^\dagger))|0\rangle = i\partial_1^\mu\partial_2^\nu\Delta_F(x_1 - x_2) -$$

$$- i\delta_0^\mu\delta_0^\nu\delta^{(4)}(x_1 - x_2) = \langle 0|\hat{T}((\partial^\mu\phi_1)(\partial^\nu\phi_2^\dagger))|0\rangle -$$

$$- i\delta_0^\mu\delta_0^\nu\delta^{(4)}(x_1 - x_2). \qquad (3.21)$$

We see that a non-covariant term appears in the "contraction" between $\partial^\mu\phi_1$ and $\partial^\nu\phi_2^\dagger$. The use of (3.20) and (3.21) in the term $T\left[(-\mathcal{L}')_1(-\mathcal{L}')_2\right]$ in $S^{(2)}$ eventually gives

$$-\frac{1}{2} \int d^4x_1 d^4x_2 T\left[(-\mathcal{L}')_1(-\mathcal{L}')_2\right] =$$

$$= -\frac{1}{2} \int d^4x_1 d^4x_2 \hat{T}\left[(-\mathcal{L}')_1(-\mathcal{L}')_2\right] + i \int d^4x_1 \mathcal{R}_1. \qquad (3.22)$$

Indeed,

$$T\Big[(-\mathcal{L}')_1(-\mathcal{L}')_2\Big] = T\Big[(ieA_\mu\phi^\dagger\partial^\mu\phi - ieA^\mu(\partial_\mu\phi^\dagger)\phi)_1 \times$$

$$\times (ieA_\nu\phi^\dagger\partial^\nu\phi - ieA^\nu(\partial_\nu\phi^\dagger)\phi)_2\Big] + O(e^3) =$$

$$= T\Big[(ieA_\mu\phi^\dagger\partial^\mu\phi)_1(ieA_\nu\phi^\dagger\partial^\nu\phi)_2\Big] +$$

$$+ T\Big[(ieA_\mu\phi^\dagger\partial^\mu\phi)_1(-ieA^\nu(\partial_\nu\phi^\dagger)\phi)_2\Big] +$$

$$+ T\Big[(-ieA^\mu(\partial_\mu\phi^\dagger)\phi)_1(ieA_\nu\phi^\dagger\partial^\nu\phi)_2\Big] +$$

$$+ T\Big[(-ieA^\mu(\partial_\mu\phi^\dagger)\phi)_1(-ieA^\nu(\partial_\nu\phi^\dagger)\phi)_2\Big] + O(e^3). \tag{3.23}$$

Now, we use the Wick theorem and write the resulting expression in terms of the modified T-product, $\hat{T}$, by means of (3.20), (3.21). We get

$$T\Big[(-\mathcal{L}')_1(-\mathcal{L}')_2\Big] = \hat{T}\Big[(ieA_\mu\phi^\dagger\partial^\mu\phi)_1(ieA_\nu\phi^\dagger\partial^\nu\phi)_2\Big] +$$

$$+ \hat{T}\Big[(ieA_\mu\phi^\dagger\partial^\mu\phi)_1(-ieA^\nu(\partial_\nu\phi^\dagger)\phi)_2\Big] -$$

$$- ie^2N\big[(A_{0,1}^2)\phi_1^\dagger\phi_1\big]\delta^{(4)}(x_1 - x_2) +$$

$$+ \hat{T}\Big[(-ieA^\mu(\partial_\mu\phi^\dagger)\phi)_1(ieA_\nu\phi^\dagger\partial^\nu\phi)_2\Big] -$$

$$- ie^2N\big[(A_{0,1}^2)\phi_1^\dagger\phi_1\big]\delta^{(4)}(x_1 - x_2) +$$

$$+ \hat{T}\Big[(-ieA^\mu(\partial_\mu\phi^\dagger)\phi)_1(-ieA^\nu(\partial_\nu\phi^\dagger)\phi)_2\Big] + O(e^3) =$$

$$= \hat{T}\Big[(-\mathcal{L}')_1(-\mathcal{L}')_2\Big] - 2ie^2N\big[(A_{0,1}^2)\phi_1^\dagger\phi_1\big]\delta^{(4)}(x_1 - x_2) =$$

$$= \hat{T}\Big[(-\mathcal{L}')_1(-\mathcal{L}')_2\Big] - 2i\mathcal{R}_1\delta^{(4)}(x_1 - x_2). \tag{3.24}$$

In view of (3.24), we conclude that

$$S^{(1)} + S^{(2)} = -i\int d^4x_1(-\mathcal{L}')_1 -$$

$$- \frac{1}{2}\int d^4x_1 d^4x_2\hat{T}\Big[(-\mathcal{L}')_1(-\mathcal{L}')_2 +$$

$$+ \mathcal{R}_1(-\mathcal{L}')_2 + (-\mathcal{L}')_1\mathcal{R}_2 + \mathcal{R}_1\mathcal{R}_2\Big], \tag{3.25}$$

where we have used that

$$T(\mathcal{R}_1(-\mathcal{L}')_2) = \hat{T}(\mathcal{R}_1(-\mathcal{L}')_2),$$
$$T((-\mathcal{L}')_1\mathcal{R}_2) = \hat{T}((-\mathcal{L}')_1\mathcal{R}_2),$$
$$T(\mathcal{R}_1\mathcal{R}_2) = \hat{T}(\mathcal{R}_1\mathcal{R}_2),$$

since $\mathcal{R}$ does not contain terms with time derivatives. In other words, as follows from (3.20) and (3.21), the T-product differs from the $\hat{T}$-product, only when it applies to two interaction Lagrangians. This implies that, when applying the Wick theorem to higher order terms of the Dyson S-matrix expansion, the only source of Lorentz non-invariant terms will be the contraction of two interaction Lagrangians.

We observe that, in (3.25), the non-invariant term $\mathcal{R}_1$, originally present in $S^{(1)}$, has been canceled by a non-invariant term generated in $S^{(2)}$ by the contraction of the two interaction Lagrangians – see (3.22).

In the sum $S^{(1)} + S^{(2)}$, there are, however, two terms left which are non-invariant:

$$-\frac{1}{2}\int d^4x_1 d^4x_2\hat{T}\Big[\mathcal{R}_1(-\mathcal{L}')_2 + (-\mathcal{L}')_1\mathcal{R}_2\Big] =$$

$$= -\int d^4x_1 d^4x_2\hat{T}\Big[\mathcal{R}_1(-\mathcal{L}')_2\Big] \tag{3.26}$$

and

$$-\frac{1}{2}\int d^4x_1 d^4x_2\hat{T}\Big[\mathcal{R}_1\mathcal{R}_2\Big]. \tag{3.27}$$

The first of these terms is canceled by the non-invariant contributions which arise in

$$S^{(3)} = \frac{(-i)^3}{3!}\int d^4x_1 d^4x_2 d^4x_3 T\Big[(-\mathcal{L}' + \mathcal{R})_1 \times$$

$$\times (-\mathcal{L}' + \mathcal{R})_2(-\mathcal{L}' + \mathcal{R})_3\Big]$$

from terms with three $\mathcal{L}'$s, two of them being contracted. There are three equivalent such terms, which, recalling (3.24), are easily shown to sum up to

$$\frac{(-i)^3}{3!}\int d^4x_1 d^4x_2 \; 3 \; \hat{T}\Big[-2i\mathcal{R}_1(-\mathcal{L}')_2\Big]$$

and, therefore, exactly cancel (3.26) in $S^{(1)} + S^{(2)} + S^{(3)}$. The other non-invariant term of $S^{(1)} + S^{(2)}$, given in (3.27), is canceled in $S^{(1)} + S^{(2)} + S^{(3)} + S^{(4)}$ by the three equivalent terms in $S^{(3)}$ with one $\mathcal{R}$ and two contracted $\mathcal{L}'$s and by the three equivalent terms in $S^{(4)}$ with four $\mathcal{L}'$s pairwise contracted.

This pattern of cancellations can be generalized. Non-invariant terms containing n factors of the type $\mathcal{R}$ and m factors of the type $\mathcal{L}'$, which can always be put in the form

$$\hat{T}\Big[\mathcal{R}_1 \ldots \mathcal{R}_n(-\mathcal{L}')_{n+1} \ldots (-\mathcal{L}')_{n+m}\Big], \tag{3.28}$$

appear first in $S^{(n+m)}$ and arise also in $S^{(n+m+1)}$ (in terms with $n-1$ factors of the type $\mathcal{R}$ and *one* pair of contracted $\mathcal{L}'$s), then in $S^{(n+m+2)}$ (in terms with $n-2$ factors of the type $\mathcal{R}$ and *two* pairs of contracted $\mathcal{L}'$s), *etc.* The last appearance is in $S^{(2n+m)}$, in terms with no factors of the type $\mathcal{R}$ and n pairs of contracted $\mathcal{L}'$s. To summarize, (3.28) appears in $S^{(n+m+j)}$, $j = 0, 1, ..., n$, in terms with $n-j$ factors $\mathcal{R}$ and j contractions of interaction Lagrangian pairs. Each contraction brings along a Dirac delta which cancels one of the integrations over the space-time, so that all terms end up to be integrated as in $S^{(n+m)}$, *i.e.* over $d^4x_1 ... d^4x_{n+m}$. The combinatorial weight in which term (3.28) appears in $S^{(n+m+j)}$ is given by

$$w_j^{nm} \equiv \frac{(-i)^{n+m+j}}{(n+m+j)!} \binom{n+m+j}{m+2j} \times$$
$$\times \binom{m+2j}{2j} (2j-1)!! \, (-2i)^j,$$

where the first factor comes from the definition of the Dyson series, the second counts the number of (equivalent) terms in $S^{(n+m+j)}$ with $n-j$ factors of type $\mathcal{R}$ and $m+2j$ factors of type $\mathcal{L}'$, the third counts the number of ways to select $2j$ Lagrangians to be contracted out of the $m+2j$ available ones, the fourth is the number of ways $2j$ Lagrangians can be pairwise contracted, the last factor comes from the fact that each of the j contractions of two Lagrangians gives $-2i\mathcal{R}$. The total weight of the non-invariant term (3.28) is, therefore,

$$\sum_{j=0}^{n} w_j^{nm} = \frac{(-i)^{n+m}}{m!} \sum_{j=0}^{n} (-2)^j \frac{(2j-1)!!}{(2j)!(n-j)!} = 0,$$

since, observing that $(2j)! = 2^j j! (2j-1)!!$, we have

$$\sum_{j=0}^{n} (-2)^j \frac{(2j-1)!!}{(2j)!(n-j)!} =$$
$$= \frac{1}{n!} \sum_{j=0}^{n} (-1)^j \binom{n}{j} = \frac{1}{n!}(1-1)^n = 0.$$

We have then proved that the Dyson series (3.12) can be traded for the manifestly Lorentz invariant series (3.13). The perturbative expansion for (3.13) can then be organized according to the usual Feynman rules for the scalar electrodynamics.

4. Renormalized ϕ^4 Theory

We consider here the theory of a massless real scalar field, undergoing a quartic self-interaction, as a simple representative of all field theories which acquire an interaction term depending on derivatives of the fields through the procedure of perturbative renormalization [4].

We will show that a mechanism of cancellation of non-covariant terms takes place on similar grounds as for the scalar electrodynamics, modulo a couple of caveats which make the present case interesting *per sé*.

The Lagrangian of the theory is

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{4!}\phi^4. \tag{4.1}$$

The starting step of the perturbative renormalization is to redefine the field and the coupling as

$$\phi = Z^{1/2}\phi_{\mathrm{R}},$$
$$\lambda = Z_\lambda \lambda_{\mathrm{R}},$$

leading to the following expression for the Lagrangian:

$$\mathcal{L} = \frac{Z}{2}\partial_\mu \phi_{\mathrm{R}} \partial^\mu \phi_{\mathrm{R}} - \frac{Z^2 Z_\lambda \lambda_{\mathrm{R}}}{4!}\phi_{\mathrm{R}}^4, \tag{4.2}$$

which can be recast in the form

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_{\mathrm{R}} \partial^\mu \phi_{\mathrm{R}} - \frac{\lambda_{\mathrm{R}}}{4!}\phi_{\mathrm{R}}^4 +$$
$$+ \frac{Z-1}{2}\partial_\mu \phi_{\mathrm{R}} \partial^\mu \phi_{\mathrm{R}} - \frac{(Z^2 Z_\lambda - 1)\lambda_{\mathrm{R}}}{4!}\phi_{\mathrm{R}}^4. \tag{4.3}$$

The first two terms in $\mathcal{L}$ have the same form as in the original Lagrangian, but they are written through the renormalized field and coupling; the remaining two terms are the so-called "counterterms". For the purposes of perturbative calculations and the related renormalization procedure, all terms but the first one in (4.3) must be considered as interaction terms, so that we can write $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$, with

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi \tag{4.4}$$

and

$$\mathcal{L}' = -\frac{\lambda}{4!}\phi^4 + \frac{Z-1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{(Z^2 Z_\lambda - 1)\lambda}{4!}\phi^4, \tag{4.5}$$

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[4] The actual perturbative renormalizability of the ϕ^4 theory and the triviality issue are inessential in this context.

where we have omitted, for brevity, the subscript R, understanding that, from now on, the field and coupling are always the renormalized ones. We can see that $\mathcal{L}'$ contains an interaction term depending on the field derivatives in spite of the fact that the original 'bare' theory had a derivative-free interaction. Moreover, $\mathcal{L}'$ depends on the renormalized coupling λ both explicitly and through the renormalization constants Z and Z_λ, which in perturbation theory must take the form of a power series in λ, the constant term being equal to one. In the following, it will prove convenient to consider $(Z-1)$ and $(Z^2 Z_\lambda - 1)$ as additional, independent couplings, their relation to λ, *i.e.* the fact that they are both $O(\lambda)$, being used only to justify their smallness and, therefore, their suitability as expansion parameters.

To introduce the Hamiltonian, we have to define the conjugate field:

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = Z\dot{\phi}. \tag{4.6}$$

We stress that ϕ is the renormalized field, therefore π, after the quantization, will implicitly enter the canonical commutation relations together with ϕ. It can be easily shown that the equations of motion for ϕ and π, as derived from their commutators with the Hamiltonian (to be written below), are equivalent to the equation of motion for the bare field, *i.e.* the Euler–Lagrange equation for the bare field ϕ as derived from the original Lagrangian (4.1). This is in marked contrast with Ref. [11], where, instead, the canonical commutation relations were imposed at the level of the bare fields, and an *ad hoc* modification of the Hamiltonian had to be performed to obtain the equation of motion of the bare field from the Hamiltonian dynamics.

The Hamiltonian density is defined in the usual way:

$$\mathcal{H} = \pi\dot{\phi} - \mathcal{L} = \frac{\pi^2}{2} + \frac{1}{2}(\boldsymbol{\nabla}\phi)^2 + \frac{\lambda}{4!}\phi^4 - \frac{\pi^2(Z-1)}{2Z} +$$
$$+ \frac{(Z-1)}{2}(\boldsymbol{\nabla}\phi)^2 + \frac{(Z^2 Z_\lambda - 1)\lambda}{4!}\phi^4, \tag{4.7}$$

which we can split as $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}'$, with

$$\mathcal{H}_0 = \frac{\pi^2}{2} + \frac{1}{2}(\boldsymbol{\nabla}\phi)^2 \tag{4.8}$$

and

$$\mathcal{H}' = \frac{\lambda}{4!}\phi^4 - \frac{\pi^2(Z-1)}{2Z} + \frac{(Z-1)}{2}(\boldsymbol{\nabla}\phi)^2 +$$

$$+ \frac{(Z^2 Z_\lambda - 1)\lambda}{4!}\phi^4. \tag{4.9}$$

So far, all the fields are in the Heisenberg representation and, again, all terms in the Lagrangian and Hamiltonian densities are implicitly assumed to be subjected to the normal ordering, N. In order to write the Dyson series, we pass to the interaction representation and use the property

$$U\pi(x)U^{-1} = \partial^0\phi_I(x), \tag{4.10}$$

which is analogous to (3.8) for a real scalar field. Using (4.10), we obtain the following expression for the interaction Hamiltonian in the interaction representation:

$$U\mathcal{H}'U^{-1} = \mathcal{H}'_I = \frac{\lambda}{4!}\phi^4 - \frac{\dot{\phi}^2(Z-1)}{2Z} +$$
$$+ \frac{(Z-1)}{2}(\boldsymbol{\nabla}\phi)^2 + \frac{(Z^2 Z_\lambda - 1)\lambda}{4!}\phi^4, \tag{4.11}$$

where all the fields are to be intended in the interaction representation. In the following, we remove the subscript $_I$ in all the fields, because, from now on, all the fields are in the interaction representation. However, we keep the subscript $_I$ in the Hamiltonian, to stress that it is in the interaction representation. Comparing (4.11) with the interaction Lagrangian (4.5), in which all the fields are promoted to be in the interaction representation, we find that

$$\mathcal{H}'_I = -\mathcal{L}' + \frac{(Z-1)^2}{2Z}\dot{\phi}^2. \tag{4.12}$$

We did not put the subscript $_I$ in the Lagrangian in (4.12), because it still has the form of the interaction Lagrangian in the Heisenberg representation: the only caveat, as written before, is that the fields appearing in its expression (4.5) are in the interaction representation. We observe that $\mathcal{H}'_I$ is not Lorentz-invariant, due to the presence of the term depending on $\dot{\phi}$. This expression for $\mathcal{H}'_I$ agrees, *mutatis mutandis*, with the one found in Ref. [11].

The stage now is set to prove that, also in the present case, the Dyson series (3.12) can be written as in (3.13), provided that a modified definition of the T-product, $\hat{T}$, is used: as before, $\hat{T}$ satisfies the Wick theorem and

$$\langle 0|\hat{T}(\phi_1\phi_2)|0\rangle = \langle 0|T(\phi_1\phi_2)|0\rangle \equiv$$
$$\equiv i\Delta_F(x_1 - x_2), \tag{4.13}$$

$$\langle 0|\hat{T}\big((\partial^\mu \phi_1)\phi_2\big)|0\rangle \equiv i\partial_1^\mu \Delta_F(x_1 - x_2), \qquad (4.14)$$

$$\langle 0|\hat{T}\big((\partial^\mu \phi_1)(\partial^\nu \phi_2)\big)|0\rangle \equiv i\partial_1^\mu \partial_2^\nu \Delta_F(x_1 - x_2). \quad (4.15)$$

We present a sketch of the perturbative proof of the validity of expansion (3.13). The two main ingredients are, as in the case studied in the previous section, the Wick theorem and the following relations between the standard T-product and the modified one, $\hat{T}$:

$$\langle 0|T\big((\partial^\mu \phi_1)\phi_2\big)|0\rangle = i\partial_1^\mu \Delta_F(x_1 - x_2) =$$
$$= \langle 0|\hat{T}\big((\partial^\mu \phi_1)\phi_2\big)|0\rangle, \qquad (4.16)$$
$$\langle 0|T\big((\partial^\mu \phi_1)(\partial^\nu \phi_2)\big)|0\rangle = i\partial_1^\mu \partial_2^\nu \Delta_F(x_1 - x_2) -$$
$$- i\delta_0^\mu \delta_0^\nu \delta^{(4)}(x_1 - x_2) = \langle 0|\hat{T}\big((\partial^\mu \phi_1)(\partial^\nu \phi_2)\big)|0\rangle -$$
$$- i\delta_0^\mu \delta_0^\nu \delta^{(4)}(x_1 - x_2). \qquad (4.17)$$

We now write the first term of the Dyson series:

$$S^{(1)} = -i \int d^4 x_1 \Big[\mathcal{H}'(x_1) \Big] =$$
$$= -i \int d^4 x_1 \Big(-\mathcal{L}' + \frac{(Z-1)^2}{2Z}\dot{\phi}^2 \Big)_1. \qquad (4.18)$$

Consider that

$$\frac{(Z-1)^2}{2Z} = \frac{(Z-1)^2}{2}\frac{1}{1+(Z-1)} =$$
$$= \frac{(Z-1)^2}{2}\Big[1 - (Z-1) + (Z-1)^2 - (Z-1)^3 + ... \Big],$$

where each term is proportional to an integer power of the 'coupling' $(Z-1)$, starting from $(Z-1)^2$. This means that, to cancel all non-invariant terms in $S^{(1)}$, one needs to consider the non-invariant terms arising from the operation of T through the Wick theorem in *all* other pieces $S^{(n)}$ of the Dyson expansion. Let us work this out explicitly for the lowest-order contribution, proportional to $(Z-1)^2$, which requires considering, in addition to $S^{(1)}$, just $S^{(2)}$:

$$S^{(2)} = -\frac{1}{2}\int d^4 x_1 d^4 x_2 T\Big[\mathcal{H}'(x_1)\mathcal{H}'(x_2) \Big] =$$
$$= -\frac{1}{2}\int d^4 x_1 d^4 x_2 T\Big[\Big(-\mathcal{L}' + \frac{(Z-1)^2}{2Z}\dot{\phi}^2 \Big)_1 \times$$
$$\times \Big(-\mathcal{L}' + \frac{(Z-1)^2}{2Z}\dot{\phi}^2 \Big)_2 \Big]. \qquad (4.19)$$

Restricting to contributions at most of order $(Z-1)^2$, we notice that

$$T\Big[(-\mathcal{L}')_1 (-\mathcal{L}')_2 \Big] =$$

$$= T\Big[\Big(-\frac{\lambda}{4!}\phi^4 + \frac{(Z-1)}{2}\partial_\mu\phi\partial^\mu\phi - \frac{(Z^2 Z_\lambda - 1)\lambda}{4!}\phi^4 \Big)_1 \times$$
$$\times \Big(-\frac{\lambda}{4!}\phi^4 + \frac{(Z-1)}{2}\partial_\nu\phi\partial^\nu\phi - \frac{(Z^2 Z_\lambda - 1)\lambda}{4!}\phi^4 \Big)_2 \Big] =$$
$$= T\Big[\Big(-\frac{\lambda}{4!}\phi^4 - \frac{(Z^2 Z_\lambda - 1)\lambda}{4!}\phi^4 \Big)_1 \times$$
$$\times \Big(-\frac{\lambda}{4!}\phi^4 - \frac{(Z^2 Z_\lambda - 1)\lambda}{4!}\phi^4 \Big)_2 \Big] +$$
$$+ T\Big[\Big(-\frac{\lambda}{4!}\phi^4 - \frac{(Z^2 Z_\lambda - 1)\lambda}{4!}\phi^4 \Big)_1 \Big(\frac{(Z-1)}{2}\partial_\nu\phi\partial^\nu\phi \Big)_2 \Big] +$$
$$+ T\Big[\Big(\frac{(Z-1)}{2}\partial_\mu\phi\partial^\mu\phi \Big)_1 \Big(-\frac{\lambda}{4!}\phi^4 - \frac{(Z^2 Z_\lambda - 1)\lambda}{4!}\phi^4 \Big)_2 \Big] +$$
$$+ T\Big[\Big(\frac{(Z-1)}{2}\partial_\mu\phi\partial^\mu\phi \Big)_1 \Big(\frac{(Z-1)}{2}\partial_\nu\phi\partial^\nu\phi \Big)_2 \Big]. \quad (4.20)$$

We rewrite such expression by applying the Wick theorem and then expressing the resulting terms by means of the modified T-product using (4.16), (4.17). If we neglect terms with two contractions contributing to vacuum diagrams, we get

$$T\Big[(-\mathcal{L}')_1 (-\mathcal{L}')_2 \Big] = \hat{T}\Big[\Big(-\frac{\lambda}{4!}\phi^4 - \frac{(Z^2 Z_\lambda - 1)\lambda}{4!}\phi^4 \Big)_1 \times$$
$$\times \Big(-\frac{\lambda}{4!}\phi^4 - \frac{(Z^2 Z_\lambda - 1)\lambda}{4!}\phi^4 \Big)_2 \Big] +$$
$$+ \hat{T}\Big[\Big(-\frac{\lambda}{4!}\phi^4 - \frac{(Z^2 Z_\lambda - 1)\lambda}{4!}\phi^4 \Big)_1 \Big(\frac{(Z-1)}{2}\partial_\nu\phi\partial^\nu\phi \Big)_2 \Big] +$$
$$+ \hat{T}\Big[\Big(\frac{(Z-1)}{2}\partial_\mu\phi\partial^\mu\phi \Big)_1 \Big(-\frac{\lambda}{4!}\phi^4 - \frac{(Z^2 Z_\lambda - 1)\lambda}{4!}\phi^4 \Big)_2 \Big] +$$
$$+ \hat{T}\Big[\Big(\frac{(Z-1)}{2}\partial_\mu\phi\partial^\mu\phi \Big)_1 \Big(\frac{(Z-1)}{2}\partial_\nu\phi\partial^\nu\phi \Big)_2 \Big] +$$
$$+ N\Big[\frac{(Z-1)^2}{4}(-i)\, 4\, (\dot{\phi}_1)^2 \delta^{(4)}(x_1 - x_2) \Big] =$$
$$= \hat{T}\Big[(-\mathcal{L}')_1 (-\mathcal{L}')_2 \Big] +$$
$$+ N\Big[(Z-1)^2(-i)(\dot{\phi}_1)^2 \delta^{(4)}(x_1 - x_2) \Big], \qquad (4.21)$$

where the last, non-Lorentz-invariant term cancels exactly the non-invariant term in $S^{(1)}$ of order $(Z-1)^2$, so that

$$S^{(1)} + S^{(2)} = -i \int d^4 x_1 (-\mathcal{L}')_1 -$$

$$-\frac{1}{2}\int d^4x_1 d^4x_2 \hat{T}\Big[(-\mathcal{L}')_1(-\mathcal{L}')_2\Big] + O((Z-1)^3).$$
(4.22)

The procedure can be repeated also for terms proportional to $(Z-1)^3 \sim \lambda^3$, which requires considering $S^{(1)}$, $S^{(2)}$ and $S^{(3)}$, the latter being given by

$$S^{(3)} = \frac{i}{6}\int d^4x_1 d^4x_2 d^4x_3 T\Big[\mathcal{H}'(x_1)\mathcal{H}'(x_2)\mathcal{H}'(x_3)\Big] =$$
$$= \frac{i}{6}\int d^4x_1 d^4x_2 d^4x_3 T\Bigg[\left(-\mathcal{L}' + \frac{(Z-1)^2}{2Z}\dot{\phi}^2\right)_1 \times$$
$$\times\left(-\mathcal{L}' + \frac{(Z-1)^2}{2Z}\dot{\phi}^2\right)_2\left(-\mathcal{L}' + \frac{(Z-1)^2}{2Z}\dot{\phi}^2\right)_3\Bigg]. \quad (4.23)$$

A straightforward, but tedious calculation, based on the application of the Wick theorem and then of Eqs. (4.16) and (4.17) leads to

$$S^{(2)} = -\frac{1}{2}\int d^4x_1 d^4x_2 \hat{T}\Bigg[\left(-\mathcal{L}' + \frac{(Z-1)^2}{2Z}\dot{\phi}^2\right)_1 \times$$
$$\times\left(-\mathcal{L}' + \frac{(Z-1)^2}{2Z}\dot{\phi}^2\right)_2\Bigg] + i\frac{(Z-1)^2}{2Z^2}\int d^4x_1 \dot{\phi}_1^2 \quad (4.24)$$

and also to

$$\frac{i}{6}\int d^4x_1 d^4x_2 d^4x_3 T\Big[(-\mathcal{L}')_1(-\mathcal{L}')_2(-\mathcal{L}')_3\Big] =$$
$$= \frac{i}{6}\int d^4x_1 d^4x_2 d^4x_3 \hat{T}\Big[(-\mathcal{L}')_1(-\mathcal{L}')_2(-\mathcal{L}')_3\Big] +$$
$$+ \frac{(Z-1)^2}{2}\int d^4x_1 d^4x_2 \hat{T}\Big[(-\mathcal{L}')_1\dot{\phi}_2^2\Big] +$$
$$+ i\frac{(Z-1)^3}{2}\int d^4x_1 \dot{\phi}_1^2. \quad (4.25)$$

In getting this last expression, we neglected double contractions insisting on the same couple of variables, which contribute to disconnected graphs, containing a vacuum diagram, and a triple contraction, which produces a vacuum diagram.

Then, using the fact that both $Z-1$ and $\mathcal{L}'$ are $O(\lambda)$, we get

$$S^{(1)} + S^{(2)} + S^{(3)} = -i\int d^4x_1(-\mathcal{L}')_1 -$$
$$- \frac{1}{2}\int d^4x_1 d^4x_2 \hat{T}\Big[(-\mathcal{L}')_1(-\mathcal{L}')_2\Big] +$$
$$+ \frac{i}{6}\int d^4x_1 d^4x_2 d^4x_3 \hat{T}\Big[(-\mathcal{L}')_1(-\mathcal{L}')_2(-\mathcal{L}')_3\Big] + O(\lambda^4).$$
(4.26)

We stress that the equivalence (4.26) holds, if one neglects diagrams containing a vacuum to vacuum process. This subtlety, even if not relevant for practical applications, was apparently overlooked by [6, 7].

5. Conclusions

In this short note, we have given the evidence that the perturbative Dyson series for the S-matrix enjoys the relativistic invariance even in the case where the interaction depends on derivatives of the fields. This problem is usually overlooked, since people almost always resort to the Feynman diagrams and rules which manifestly keep the Lorentz invariance and which come naturally, if field theories are quantized by means of a functional generator. However, the equivalence between the Feynman approach and the more traditional Dyson series is not *a priori* evident, if the interaction Lagrangian contains derivatives of the fields. We have tackled this problem in the case of scalar electrodynamics and renormalized ϕ^4 theory, giving simple perturbative arguments based on the Wick theorem that the Dyson series for a Hamiltonian is Lorentz-invariant and – after discarding vacuum diagrams – coincides with the perturbative series coming from applying the Feynman rules to the Lagrangian. More in general, we are confident that, by similar techniques, the same coincidence can be proven for all other renormalized quantum field theories, in particular, for QED.

APPENDIX.
Proof of (3.8) and (4.10)

We give a proof of relations (3.8) and (4.10). We start from the definition of a field in the interaction representation:

$$\phi_I = U\phi U^{-1}, \quad U = e^{iH_0^{(s)}t}e^{-iH^{(s)}t}, \quad (A1)$$

with ϕ in the Heisenberg representation. Then, we have

$$\partial_0 U = -ie^{iH_0^{(s)}t}H'^{(s)}e^{-iH^{(s)}t} = -iUH', \quad (A2)$$

with

$$H' = e^{iH^{(s)}t}H'^{(s)}e^{-iH^{(s)}t} \quad (A3)$$

the interaction Hamiltonian in the Heisenberg representation. As an immediate consequence, we have

$$\partial_0 U^{-1} = iH'U^{-1}, \quad (A4)$$

In addition, one has

$$\partial_0 \phi = -i[\phi, H], \quad (A5)$$

for fields in the Heisenberg representation.

Putting everything together, we have

$$\partial_0 \phi_I = -iU[\phi, H_0]U^{-1}, \quad (A6)$$

with the 'free' Hamiltonian H_0 in the Heisenberg representation. For the scalar electrodynamics, it equals

$$H_0 = \int d^3x \mathcal{H}_0,$$

$$\mathcal{H}_0 = \pi^\dagger \pi + \boldsymbol{\nabla}\phi^\dagger \boldsymbol{\nabla}\phi + m^2 \phi^\dagger \phi + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{A7}$$

For the renormalized ϕ^4 theory, it is

$$\mathcal{H}_0 = \frac{\pi^2}{2} + \frac{1}{2}(\boldsymbol{\nabla}\phi)^2. \tag{A8}$$

Since, in both cases, we have the fundamental equal time commutation relation (in the case of renormalized fields, the canonical commutation relations have to be imposed on renormalized fields and momenta)

$$[\phi(x), \pi(y)] = i\delta^3(\mathbf{x} - \mathbf{y}), \tag{A9}$$

we have

$$\partial_0 \phi_I = -iU[\phi, H_0]U^{-1} = U\pi^\dagger U^{-1}, \tag{A10}$$

which gives immediately relations (3.8) and (4.10), since, for the renormalized ϕ^4 theory, $\pi^\dagger = \pi$.

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ПРО ЛОРЕНЦ-ІНВАРІАНТНІСТЬ РЯДУ
ДАЙСОНА В ТЕОРІЯХ ІЗ ВЗАЄМОДІЄЮ,
ЩО ЗАЛЕЖИТЬ ВІД ПОХІДНИХ

Ми вивчаємо ряд Дайсона для S-матриці, коли взаємодія містить похідні від полів. Ми зосередилися на двох конкретних прикладах: скалярній електродинаміці та перенормованій теорії ϕ^4. Використовуючи теорему Віка, ми доводимо, що лоренц-інваріантність виконується, і що звичайні правила Фейнмана можна застосувати до лагранжіана взаємодії.

К л ю ч о в і с л о в а: квантова теорія поля, канонічне квантування, ряд Дайсона, теорема Віка.