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MIXED SYMMETRY STATES IN ^{92}Zr AND ^{94}Mo NUCLEI

Mixed-symmetry states of ^{92}Zr and ^{94}Mo isotopes are investigated with the use of the collective models, Interacting Boson Model-2 (IBM-2) and Quasiparticle Phonon Model (QPM). The energy spectra and electromagnetic transition rates for these isotopes are calculated. The results of IBM-2 and QPM are compared with available experimental data. We have obtained a satisfactory agreement, and IBM-2 gives a better description. In these isotopes, we observe a few states having a mixed symmetry such as 2_2^+, 2_3^+, 3_1^+, and 1_s^+. It is found that these isotopes have a vibrational shape corresponding to the U(5) symmetry.

Keywords: IBM-2, QPM, mixed symmetry states, B(E2), B(M1), mixing ratios.

1. Introduction

The focus of this work is on an other kind of mixed-symmetry states. In 1984, Hamilton [1] suggested the first example of a $(n, n'\gamma)$ weakly collective 2^+ mixed-symmetry state in vibrational nuclei based on the analysis of $E2/M1$ multipole mixing ratios. Its main experimental signatures is a strong $B(M1)$-strength to the 2_1^+ and a weakly collective $B(E2)$-strength to the ground state [2]. In general, detecting a transition between two excited states exhibits a challenging task and requires the combination of complementary experimental techniques. Hence, the knowledge about this mode stayed sparse in the 1980s and 1990s. There were only a few examples based on the absolute transition strength reported in [3–8]. In the 2000s, the situation changed with the improvement of several experimental techniques likes the successful-reaction which allows determining the decay pattern of states far off the yrast band. The prime example of 2^+ mixed-symmetry states was identified by Pietralla *et al.* in ^{94}Mo [9]. Not only the $B(M1)$-strength in this nucleus is very high indicating a very "clean" mixed-symmetry state (MSS), but also a multiphonon structure was observed [9, 10] which is formed by the symmetric and th mixed-symmetric 2^+ phonons. This observation proves that both phonons can be considered as building blocks of a collective nuclear structure in nearly spherical nuclei. Nowadays, a large amount of data on this excitation mode is available [2]. Unfortunately, no example in radioactive nuclei has been identified so far. This can be traced back to the difficulties in measuring the $B(M1)$ strength.

As stressed by Heyde and Sau [11], the properties of the 2^+ mixed-symmetry states are directly sensitive to the effective proton-neutron interaction. Near the closed shells, one observes highly exciting phenomena: The structure of both 2^+ phonons contains part-large single particle contributions in addition to the collective one [12, 13]. This opens the opportunity to study, in detail, the interplay between the collective and single-particle degrees of freedom, when going from spherical to more collective nuclei driven by the proton-neutron interaction. Since usually only two large single-particle components – one proton and one neutron – are present, one can draw conclusions about the shell structure and the strengths of the proton-neutron interaction, by investigating the configuration mixing between these large components. The $B(M1)$-strength between the MSS 2_1^+ and the g-factors of both states are excellent experimental observation for measuring the degree of mixing. This is highlighted by comparing the 2_1^+ and MSS structures in ^{92}Zr and ^{94}Mo.

Since ^{92}Zr lies at a proton subshell gap, the proton two-quasiparticle state is located at much higher energy causing a drastically larger energy difference between the lowest proton and neutron two- quasiparticle configurations than in 94Mo. As a result, the proton-neutron interaction mediates a smaller configuration mixing in ^{92}Zr than in ^{94}Mo resulting in a smaller $B(M1)$-strength and a negative g-factor of the [9, 14]. Clearly, 2^+ mixed-symmetry states constitute a very fine probe for exploring the nuclear structure sensitive to tiny shell effects and the detailed strength of the proton-neutron interaction.

The typical strategy how to learn more about the nucleus and its constituents is to make use of external fields and to analyze the response. In order to obtain the complementary experimental information about the nature of an excitation mode, it is important to use different probes interacting strongly, electromagnetically or weakly. So far,the gamma spectroscopy has been the main experimental technique for investigating the properties of 2^+ mixed-symmetry states. This method gives access to absolute electromagnetic transition strengths, i.e., only the structure of the proton wavefunction is tested. This work investigates the 2^+ mixed-symmetry states in ^{92}Zr and ^{94}Mo in the electron and proton scatterings. Since protons interact at medium energies isoscalarly [15] with the nucleus, a detailed test of the neutron wavefunction is possible. In addition, the energy and momentum transfers are decoupled in the proton and electron scattering experiments allowing one to explore the nuclear structure at different momenta transferred, which is impossible in the gamma spectroscopy.

The aim of this work is on another kind of mixed-symmetry states like ^{92}Zr and ^{94}Mo nuclei.

2. Theoretical Considerations

2.1. The Interacting Boson Model (IBM)

The IBM-1 [16] is a purely phenomenological model. Its basic assumption is that the collective low-lying states in even-even nuclei can be described by a fixed number of bosons having the angular momentum and parity $J = 0^+$ (s-boson) or $J = 2^+$ (d-boson). The restriction to s-boson and d-boson stems from the observation that the residual interaction between like nucleons is strongest in the $J = 0^+$ and $J = 2^+$ channels. So, the microscopic counterparts of s- and d-boson are correlated fermion pairs in the shell model. It is possible to enlarge the model space and to consider, e.g., g-bosons. Fixing the boson number is the fundamental difference to the geometrical model causing several predictions, where both approaches differ seriously. In the IBM-1 – the simplest version of the interacting boson mode – no distinction is made between protons and neutrons, and the nucleus is considered as an one component system. This restriction is lifted in the IBM-2 which explicitly distinguishes between proton and neutron bosons, as described in the next section. The IBM considers only single boson energies for s-boson, as well as for d-bosons. So clearly, it can not account for any shell effects. Therefore, the model space of the IBM-1 is a six-dimensional one spanned by the single substate of the s-boson and the five magnetic substates of the d-boson. To construct a suitable Hamiltonian, the following points have to be taken into account:

• the Hamiltonian must fulfill the rotational symmetry, hermicity, and Pauli principle,

• the interaction between bosons is assumed to have a two-body character,

• since the Hamiltonian must conserve the boson number, every creation-operator must be combined with a destruction-operator.

The IBM-2 distinguishes between proton and neutron bosons. The Hamiltonian of IBM-2 can be written as [17, 19]

$$H = H_\pi + H_v + H_{\pi v}, \qquad (1)$$

$$H = \epsilon(n_{d\pi} + n_{dv}) + KQ_\pi Q_v + V_{\pi K} + V_{VV} + M_{\pi V}, \qquad (2)$$

where $n_{d\rho}$ (where $\rho = \pi$ or v) are the d-boson number operators for protons and neutrons with the respective d-boson energies ϵ_ρ. The symbol Q_ρ denotes the quadrupole operator for proton-bosons and neutron-bosons. The last term of Eq. (2) denotes the so-called Majorana interaction force, this parameter fixes the state location with the mixed proton bosons – neutron bosons symmetry with respect to totally symmetric

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states and is defined as [18, 19]

$$M_{\pi v} = \frac{1}{2}\xi_2(s_\pi^+ d_v^+ - d_\pi^+ s_v^+)\,(s_\pi d_v^- - d_v^- s_\pi) - - \sum_{K=1,3} \xi_K \left(\left[d_\pi^+ d_v^+\right]^{(K)} \left[d_\pi^- d_v^-\right]^{(K)} \right). \quad (3)$$

The operator of quadrupole moment in the IBM-2 model for proton and neutron boons takes the form [7]:

$$Q_\pi^{\chi_\pi} = (d_\pi^+ d_\pi^\sim)^{(2)} \chi_\pi (s_\pi^+ d_\pi^\sim + d_\pi^+ s_\pi)^{(2)}$$

and

$$Q_\pi^{\chi_v} = \left(d_v^+ d_v^\sim\right)^{(2)} \chi_v (s_v^+ d_v + d_v^+ s_v)^{(2)}. \quad (4)$$

The terms $V_{\pi\pi}$ is the interaction of proton-proton bosons, and V_{vv} is the interaction of neutron-neutron bosons only and is given by [20]:

$$V_{\pi\pi} = \sum_{J=0.2.4} C_{L\rho} \left[\left(d^+ d^+\right)_\pi^{(L)} (d^\sim d^\sim)_\pi^{(L)} \right]^{(0)}$$

and

$$V_{\pi\pi} = \sum_{J=0.2.4} C_{L\rho} \left[\left(d^+ d^+\right)_v^{(L)} (d^\sim d^\sim)_v^{(L)} \right]^{(0)}. \quad (5)$$

On the nucleonic level, the isospin is approximately a good quantum number and presents a useful symmetry to describe nuclear systems and to simplify calculations. Protons and neutrons are treated as different states of one particle: the nucleon. On the bosonic level, the F-spin quantum number was introduced in Ref. [19] as an analog to the isospin concept. The F-spin quantum numbers for proton and neutron bosons are given by:

$$b_\pi^+ \mid 0\rangle = \left[\begin{array}{l} F = 1/2, \\ F_Z = 1/2, \end{array}\right. \quad b_v^+ \mid 0\rangle = \left[\begin{array}{l} F = 1/2, \\ F_Z = 1/2. \end{array}\right. \quad (6)$$

The treatment of proton and neutron bosons as an F-spin doublet imposes the SU(2) group structure; therefore, the isospin and F-spin are mathematically identical. The generator of the SU(2) group can be written as

$$\begin{aligned} F_+ &= d_\pi^+ d_v^\sim + s_\pi^+ s_v^\sim, \\ F_- &= d_v^+ d_\pi^\sim + s_v^+ s_\pi^\sim, \\ F_Z &= \frac{1}{2}[d_\pi^+ d_\pi^\sim + s_\pi^+ s_\pi^\sim + d_v^+ d_v^\sim - s_v^+ s_v^\sim], \\ F_Z &= \frac{1}{2}[N_\pi - N_v], \end{aligned} \quad (7)$$

F_+ and F_- enhance or lower F_Z – being one half of the difference between the numbers of proton and neutron bosons – by 1. Since F_+, F_-, and F_Z form a Lie algebra, they are closed under the commutation. As introduced in Eq. (7), it is possible to define a Casimir operator for this algebra commuting with each generator:

$$F^2 = F_- F_+ + F_Z(F_Z + 1). \quad (8)$$

Conveniently, F_Z is chosen to label the states together with the corresponding eigenvalue of $F^2 = F_Z(F_Z + 1)$. For a given number of proton and neutron bosons, the F-spin can take values between $F_{\min} = [N_\pi - N_v]/2$, $F_{\max} = [N_\pi + N_v]/2$. F-spin is a useful quantum number to classify the boson states with respect to their symmetry under pairwise proton and neutron exchange. The basis states that are characterized by a maximum F-spin quantum number $F = F_{\max}$ can be transformed by the successive action of the F-spin raising operator $\mathbf{F}_+$ into a state that consists of proton bosons only. Obviously, such a state is unchanged under the pairwise exchange of proton and neutron labels, since it does not contain any neutron bosons. Therefore, IBM-2 states with maximum F-spin quantum number are called Fully-Symmetric States (FSSs). All states with $F < F_{\max}$ contain at least one pair of proton and neutron bosons which behave antisymmetrically under the exchange of proton and neutron labels. This class of states is investigated in this work and is referred to as Mixed-Symmetric States (MSSs). The IBM-2 transition operators have a simple form with the use of the multipole operators. The M1 transition operator is given by:

$$T(M1) = g_\pi L_\pi^{(1)} + g_v^{(1)}, \quad (9)$$

where $L_\pi^{(1)}$ and $L_v^{(1)}$ are the proton and neutron bosons angular momentum operators which are given as

$$L_\pi^{(1)} = (10)^{1/2} (d_\pi^+ d_\pi^\sim)^{(1)} \text{ and } L_v^{(1)} = (10)^{1/2} (d_v^+ d_v^\sim)^{(1)}, \quad (10)$$

$$T(M1) = \sqrt{\frac{3}{4\pi}} \left(g_\pi L_\pi^{(1)} + g_v L_\pi^{(1)} \right). \quad (11)$$

The g_π and g_v are the boson g-factors which are measured in nuclear magnetons (μ_n) units. The $T^{(M1)}$ operator can be written as [20]:

$$T(M1) = 0.77 \left[(d^+ d^\sim)_\pi - (d^+ d^\sim)_v \right]^{(1)} (g_\pi - g_v). \quad (12)$$

Typically, $g_\pi = 1$ and $g_v = 0$ are chosen in calculations, $L_\rho^{(1)}$ denotes the total angular momentum which is, by construction, a good quantum number and cannot connect different states. This term can only induce $M1$-transitions between states which differ by one unit of the F-spin [2]. Therefore, $M1$-transitions between two fully symmetric states are exactly forbidden. Since, no other $M1$ transitions are allowed, except for the one between a fully symmetric state and a mixed-symmetry state, it can be used as an unique experimental signature for identifying the mixed-symmetry states. The difference between the boson g-factors amounts to $\sim 1\mu_N$. Consequently, one can expect a $M1$ transition matrix element of the order of $\langle FSS\|T(M1)\|MSS\rangle \approx 1\mu_N$. The $E2$ transition operator is given by

$$T(E2) = e_\pi Q_\pi^{\chi_\pi} + e_v Q_v^{\chi_v}. \qquad (13)$$

The $e_\pi(e_v)$ are the effective charges for proton (neutron) bosons, respectively, in eb units. The effective charges e_π and e_v depend on the numbers of proton bosons and neutron bosons. The quadrupole operators $Q_\pi^{\chi_\pi}$ and $Q_v^{\chi_v}$ are defined in Eq. (13). The reduced electric quadrupole transition rates between two states are given by [20]:

$$B(E2;\, i \to f) = \frac{|\langle I_i\|T(E2)\|I_f\rangle|^2}{2I_i + 1}. \qquad (14)$$

The quadrupole moment for a state characterized by the angular momentum I of a nucleus is defined as [20]:

$$Q_{2_1} = \sqrt{\frac{16\pi}{175}}\,\langle 2_1^+\|T(E2)\|2_1^+\rangle. \qquad (15)$$

2.2. The Quasiparticle Phonon Model

The Quasiparticle Phonon Model (QPM) is a phenomenological, microscopic model. It uses a separable force in the particle-hole channel making it possible to include all relevant single-particle states for describing the collective excitation, i.e., no effective charges are necessary to reproduce electromagnetic transition strengths. The golden horse of the QPM is the coupling of one-phonon states to two- and three-phonon states – a feature which is unique to the QPM. The next section describes technical aspects how the QPM tackles the nuclear many-body problem followed by a section about transition operators with a special focus on $M1$-transitions being important for MSS. The phenomenological Hamiltonian used in QPM calculations contains four parts

$$H_{\rm qpm} = H_{\rm sp} + H_{\rm pair} + H_{\rm m} + H_{\rm sm} \qquad (16)$$

where $H_{\rm sp}$ is the single-particle Hamiltonian usually taken as a Wood–Saxon potential, $H_{\rm pair}$ absorbs the short-range pairing correlations in the particle-particle channel, $H_{\rm m}$ represents a separable multipole interaction in the particle-hole channel, $H_{\rm sm}$ is a separable spin-multipole interaction in the particle-hole channel. The QPM equations are obtained by a step-by-step diagonalization of the Hamiltonian. In what follows, each of these steps is examined in detail, and realistic examples are provided for the case of ^{92}Zr. The discussion is limited to even-even nuclei and natural parity states. Therefore, the last term $H_{\rm sm}$, being only important for unnatural parity states, is not considered here. The additional information can be found in Ref. [21]. First, an appropriate mean-field potential is chosen separately for protons and neutrons- to account for parts of the long-range interaction. The common choice in the case of QPM is a Wood–Saxon potential of the form

$$U^\tau(r) = \frac{V_0^\tau}{1 + e^{(r-R_0^\tau)/a_0^\tau}} - \frac{\hbar^2}{\mu^2 c^2}\frac{1}{r} \times$$
$$\times \frac{d}{dr}\left(\frac{V_{lS}^\tau}{1 + e^{(r-R_0^\tau)/a_{lS}^\tau}}\, l.S\right) + V_{\rm C}(r). \qquad (17)$$

$V_{\rm C}$ represents the coulomb potential and μ the reduced mass. All parameters are fitted to obtain a suitable description of the properties of nuclei in a given mass region with the restrictions $R_{lS}^\tau = R_0^\tau$, $a_{lS}^\tau = a_0^\tau$, and $R_{\rm C} = R_0^p$. Of course, this treatment reflects the phenomenology of the QPM approach. However, in principle, it would be possible to use a mean-field potential obtained in a self-consistent way for the Hatree–Fock and Skyrme forces [21]. This thesis involves the nuclei ^{92}Zr and ^{94}Mo.

The main theoretical tool to account for this effect is the BCS theory developed by Bardeen, Cooper, and Schrieffer in 1957. As an ansatz for the nuclear ground state, a wave function is chosen as such that reflects the superfluid character of nuclei

$$|{\rm BCS}\rangle = \prod_{K>0}(u_k + V_k a_k^+ a_{k'}^+)|0\rangle, \qquad (18)$$

where k runs over the whole single-particle basis, $|0\rangle$ is the vacuum state, and k' represents the time-reversed state of k, *i.e.* in a spherical basis $k = (n, j, l, m)$, and $k' = (n, j, l, -m)$. The square of the coefficients u_k and v_k can be interpreted as the probability that the state k is either empty or occupied by a nucleon pair. In QPM calculations, the pairing correlations are absorbed in the second term $H_{\rm pair}$ of Eq. (18)

$$H_{\rm pair} = -\sum_{\tau}^{n,p} G_0^{(\tau)} \sum_{jj'} \sqrt{(2j+1)(2j'+1)} \times$$
$$\times [a^+_{jm} a^+_{j-m}]_{00} \, [a^+_{j'-m'} a^+_{j'm'}]_{00}, \qquad (19)$$

where

$$[a^+_j a^+_{j'}]_{\lambda\mu} = \sum_{mm'} C^{\lambda\mu}_{jmj'm'} \, a^+_{jm} a^+_{j'm'}. \qquad (20)$$

$C^{\lambda\mu}_{jmj'm'}$ is the common Clebsch–Gordan coefficient. The structure of the pairing Hamiltonian is very simple and assumes that the monopole pairing is of zero-range and state-independent, as indicated by the constant matrix element $G_0^{(\tau)}$. In principle, the last assumption is not fully justified, and more refined treatments are recommended like using a density-dependent pairing force [22]. However, the QPM is able to account for the main properties of spherical nuclei. Therefore, this treatment seems to be acceptable.

As for the particle-hole excitation spectrum in the independent shell model, one can introduce a quasiparticle excitation spectrum relative to the BCS-ground state with the pleasant feature that the pairing correlations are already included. The quasiparticle energies can be calculated from the coefficients u_k and v_k with the equation below:

$$\epsilon_j = \sqrt{\triangle_j^2 + (E_j - \lambda_\tau)^2}, \qquad (21)$$

where E_j is the single-particle energy from the Wood–Saxon potential, and $\triangle_\tau$ is the so-called pairing gap

$$\triangle_\tau = G^{(\tau)} \sum_j u_j v_j. \qquad (22)$$

In the third step of the diagonalization procedure, $H_{\rm m}$ is included, being responsible for the mixing of quasiparticle states. In the quasiparticle representation, Hamiltonian (18) can be written as

$$H_{\rm qpm} = \sum_{\tau}^{n,p} \sum_{\tau\rho}^{\pm 1} \sum_{j,m} \epsilon_j \alpha^+_{jm} \alpha_{jm} +$$
$$+ \left(\sum_{\lambda\mu} \sum_{\tau\rho}^{\pm 1} (\kappa_0^{(\lambda)} + \rho \kappa_1^{(\lambda)}) M^+_{(\lambda\mu)}(\tau) M^+_{(\lambda\mu)}(\rho\tau) \right), \qquad (23)$$

where the multipole operator is given by

$$M^+_{(\lambda\mu)} = \sum_{jj'}^{\tau} \left\{ \frac{u_{jj'}}{2} \left([\alpha^+_j \alpha^+_{j'}]_{\lambda\mu} + \right. \right.$$
$$\left. \left. + (-1)^{\lambda+1} [\alpha^+_j \alpha^+_{j'}]_{\lambda-\mu} \right) - v^{(-)}_{jj'} B_\tau(jj'; \lambda\mu) \right\}, \qquad (24)$$

where

$$B_\tau(jj' : \lambda\mu) = \sum_{mm} (-1)^{j'-m'} c^{\lambda\mu}_{jmj'm'} \, \alpha^+_{jm} \alpha^+_{j'm'}. \qquad (25)$$

Besides the excitation energies, the electromagnetic decay properties are an excellent to test the model predictions and to deepen our understanding of nuclear structures, e.g., they can give important informations about the collective phenomena signaled by large transition strengths. In the following, the expressions for transitions being important for MSS are discussed. In the quasiparticle and phonon representations, the electric transition operator transforms into

$$M(E\lambda\mu) = \sum_{\tau}^{n,p} e_\tau^{(\lambda)} \sum_{jj'} \frac{\langle j \| E\lambda \| j' \rangle}{\sqrt{2\lambda+1}} \left\{ \frac{u_{jj'}}{2} \times \right.$$
$$\times \sum (\Psi^{\lambda_i}_{jj'} - \phi^{\lambda_i}_{jj'}) (Q^+_{\lambda\mu i} + (-)^{\lambda-\mu} Q_{\lambda-\mu i}) +$$
$$\left. + v^{(-)}_{jj'} \sum_{mm'} C^{\lambda\mu}_{jmjm'} (-)^{j'-m'} \alpha^+_{j'm'} \alpha_{j'm'} \right\}, \qquad (26)$$

where the single-particle transition matrix element $\langle j \| E\lambda \| j' \rangle = \langle j \| i^\lambda Y_\lambda r^\lambda \| j' \rangle$. The first term corresponds to a one-phonon exchange term between the initial and final states, while the second one is the so-called boson-forbidden transition, i.e., in a pure boson picture neglecting the inner fermion structure of the Q-operators, this transition would be forbidden. The quantity e_r represents the effective charges to account for states outside the chosen model space. In the shell model, the typical values are $e_n = 0.5$, $e_p = 1.5$. Since the QPM uses a drastically larger model space containing all necessary states contributing to the transition of interest, it is possible to take the bare values $e_n = 0$ and $e_p = 1.0$. The explicit reduced matrix element for a ground state transition of a one-phonon state is:

$$\langle Q_{\lambda j} \| \mu(E\lambda) \| 0^+_{g.s} \rangle = \sum_{\tau}^{n,p} e_\tau^{(\lambda)} \times$$
$$\times \sum_{jj'} \frac{u^+_{jj'}}{2} \langle j \| (E\lambda) \| j' \rangle (\Psi^{\lambda_i}_{jj'} + \phi^{\lambda_i}_{jj'}). \qquad (27)$$

Table 1. **The IBM-2 Hamiltonian for ^{92}Zr and ^{94}Mo isotopes, all Hamiltonian parameters in MeV units and are dimensionless**

Isotopes	N_v	N_π	ϵ	κ	χ_v	χ_π	$\xi_1 = \xi_3$	ξ_2	$C_{L\pi}(L = 0, 2, 4)$
Zr-92	1	5	1.000	–3.366	–0.70	–0.580	–0.70	0.22	0.025
Mo-94	1	5	1.020	–3.364	–0.72	–0.580	–0.72	0.24	0.026

Table 2. **Parameters of the Wood–Saxon potential used to calculate the properties of ^{92}Zr and ^{94}Mo**

Particles	V_0, MeV	R_0, fm	a_0, fm	V_{ls}, fm
Neutron	–44.70	5.802	0.6200	–9.231
Proton	–56.70	5.577	0.6301	–9.306

The expression for the magnetic transitions with multipolarity λ_1 between two RPA-one-phonon states with multipolarities λ_2 and λ_3 is given by

$$\langle Q_{3\lambda i}\|\mu(E\lambda_1)\|Q\lambda_3 i'\rangle =$$
$$= \sum_{\tau}^{n,p} e_\tau^{(\lambda)} \sum_{j_1 j_2 j_3} v_{jj'}^{+} \langle j_1\|M\lambda_1\|j_2'\rangle \times$$
$$\times \begin{bmatrix} \lambda_3 & \lambda_2 & \lambda_1 \\ j_1 & j_2 & j_3 \end{bmatrix} \left(\Psi_{j_2 j_3}^{\lambda_3}\Psi_{j_3 j_1}^{\lambda_3} + \phi_{j_2 j_3}^{\lambda_3}\phi_{j_3 j_1}^{\lambda_3}\right). \quad (28)$$

3. Results and Discussion

3.1. Energy spectra

The parameters of IBM-2 Hamiltonian for ^{92}Zr and ^{94}Mo are extrapolated in Table 1. The ϵ, κ, χ_v and the Majorana parameter $\xi_1 = \xi_3, \xi_2$ are treated as free parameters which are functions of the numbers of neutron and proton bosons or function of the number of neutron bosons such as $\epsilon = \epsilon(N_\pi, N_v)$, $\kappa = \kappa(N_\pi, N_v)$, $\chi_v = \chi_v(N_v)$, $\xi_K = \xi_K(N_\pi, N_v)$. The other parameters depend only on N_v or N_π, the parameter $\chi_\pi = \chi_\pi(N_\pi)$, $C_{L\pi} = C_{L\pi}(N_\pi)$ and $C_{Lv} = C_{Lv}(N_v)$. The number of proton bosons is accounted from the nearest major shell ($Z = 50$). The Zr isotope has 40 protons; therefore„ we have $N_\pi = 5$ proton bosons, while the number of neutron bosons is accounted from the nearest neutron closed shell $N = 50$. Therefore, the number of neutron bosons $N_v = 1$. The ^{94}Mo has 42 protons; therefore, we have $N_\pi = 5$ proton bosons. The number of neutron bosons for this isotope $N_v = 1$. An increase of the interaction strength κ will start to mix the unperturbed proton and neutron boson states and will finally cause the collective FSS and MSS connected by a strong $M1$-transition, as seen in experiment. This parameter is usually fixed empirically to experimental data without considering its microscopic origin which determines its strength.

The parameter χ_π is a constant for ^{92}Zr and ^{94}Mo isotopes. This is due to the numbers of proton bosons that are constant in these isotopes, whereas we include $C_{0\pi}$, $C_{2\pi}$, and $C_{4\pi}$ terms in the proton-proton bosons interaction parameter $V_{\pi\pi}$ and don't include V_{vv}, because $N_\pi > N_v$ for the ^{92}Zr and ^{94}Mo isotopes.

The IBM-2 Hamiltonian parameters for ^{92}Zr and ^{94}Mo isotopes are estimated by fitting the experimental values. We varied one parameter, while keeping the others to be constant to get the best fit with experimental data. We can use these parameters to evaluate the energy levels and electromagnetic transition rates using the computer codes NPBOS and NPBTRAN program [23].

The Wood–Saxon parameters used to calculate the properties of these nuclei are shown in Table 2. To obtain the ground state configuration, we simply fill the available number of nucleons from the bottom to the top accounting for the Pauli exclusion principle. At this stage, the model is called the independent shell model and can describe only some properties of magic nuclei. For giving a realistic description of nuclei, the residual interaction codified in the last three terms of Eq. (18) has to be taken into account.

The energy spectra for ^{92}Zr and ^{94}Mo in IBM-2 and QPM are given in Figs. 1 and 2, respectively, together with the experimental values, the agreement being quite good for low-lying levels having a collective nature. One can observe the discrepancies between our IBM-2 and QPM calculations and experimental data for high-spin states. This is due to that these states are outside of the IBM-2 space. The calculated yrast states 6^+, 8^+, and 4^+ are slightly higher than the

Table 3. **The energy ratios for ^{92}Zr and ^{94}Mo**

Isotopes	$R_1 = E(4_1^+)/E(2_1^+)$			$R_2 = E(6_1^+)/E(2_1^+)$			$R_3 = E(8_1^+)/E(2_1^+)$		
	QPM	IBM-2	Exp.	QPM	IBM-2	Exp.	QPM	IBM-2	Exp.
^{92}Zr	3.228	3.172	3.165	1.585	1.586	1.600			
^{94}Mo	3.693	3.440	3.392	2.761	2.694	2.665	1.675	1.803	1.805
SU(5)		2			3			4	
O(6)		2.5			4.5			7	
SU(3)		3.3			7			12	

experimental values. The calculations within IBM-2 are closer to experimental data more than those for QPM, which is due to the parameters of the IBM-2 Hamiltonian.

The energy ratios for ^{92}Zr and ^{94}Mo isotopes are given in Table 3. The energy ratio $R_1 = 1.600$ for ^{92}Zr isotope and 1.805 for ^{94}Mo isotope. This indicates that these isotopes show an anharmonic vibratory shape character (almost spherical shape) corresponding to the SU(5) symmetry.

One of the important features of IBM-2 is the prediction of mixed-symmetry states 1^+, 2_2^+ and 3_1^+ in these isotopes. These states are obtained by choosing the suitable values of Majorana parameters. In this study, we found that the values of energies of 0_2^+ in two theoretical models are close to experimental values [24, 25].

The QPM calculations were performed following the procedure of Section 2. The parameters of the Wood–Saxon potential used to obtain the single-particle basis were fixed with regard for the properties of neighboring nuclei and are given in Table 2. The strength of the pairing force was fitted to odd-even mass differences, and the strength of the residual interaction was determined to describe the B(E2) value and the excitation energy of 2_1^+. No additional parameters are necessary to include the coupling to multiphonon states. Since the QPM uses, in contrast to the shell model, a single-particle basis being sufficiently large to fulfill the energy-weighed sum rules. The 2_1^+ and 2_2^+ states are nearly pure one-phonon states, while 2_3^+ is dominantly a two-phonon state with noticeable one-phonon contributions.

The wave functions of three lowest 2^+ states are shown in Tables 4 and 5. The 2_1^+ and 2_2^+ states are dominated by the first and second RPA-phonons, respectively. The contributions of other one-, two-, and

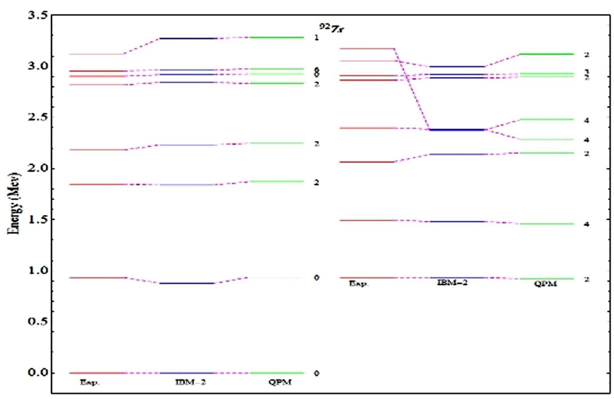

Fig. 1. Comparison between experimental data [24] and IBM-2 and QPM calculated energy levels for ^{92}Zr isotope

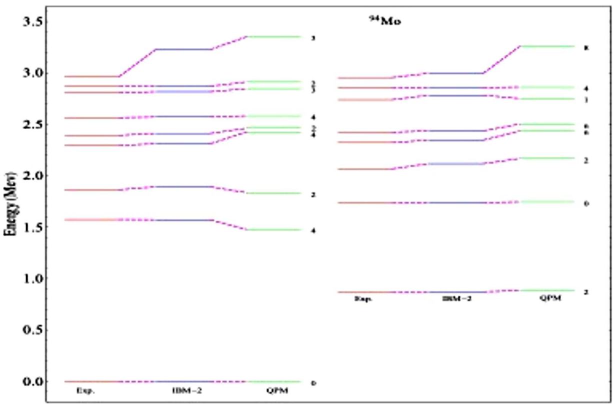

Fig. 2. Comparison between experimental data [26] and IBM-2 and QPM calculated energy levels for ^{94}Mo isotope

three-phonon states are less than 10%. The first and second RPA-phonons are mainly formed by the same two two-quasiparticle components $(2d_{5/2} \otimes 2d_{5/2})_n$ and $(1g_{9/2} \otimes 1g_{9/2})_p$. In the case of $[2_1^+]_{[\mathrm{RPA}]}$, both components are in-phase and, for $[2_2^+]_{[\mathrm{RPA}]}$, out-ofphase forming the microscopic analogs of the symmetric and mixed-symmetry one-quadrupole phonon states defined in the framework of the IBM-2.

The 2_3^+ state has a different structure. Its main amplitude is a two-phonon component formed by the first RPA-phonon. However, the contributions of $[2_4^+]_{[\mathrm{RPA}]}$ and $[2_5^+]_{[\mathrm{RPA}]}$ phonons are appreciable and indicate deviations from a harmonic phonon picture in IBM-2 (Table 5) and QPM (Table 4). Figures 1 and 2, present a comparison between experimental and calculated excitation energies of five lowest states. The IBM-2 and QPM reproduces the experimental energies with a reasonable accuracy of ∼0.1 MeV.

3.2. Electromagnetic transition probabilities

3.2.1. Electric transition probability B(E2)

In IBM-2, to calculate the reduced electric transition probability $B(E2)$ from Eq. (13), we note that $B(E2)$ depends on the effective charges of proton and neutron bosons. The effective charge values for proton and neutron bosons are estimated from the experimental $B(E2; 2_1^+ \to 0_1^+)$ values.

The effective charges of proton bosons for ^{92}Zr and ^{94}Mo isotopes are given as $e_\pi = 1.5$ and $e_v = 0.5$ in QPM. The effective charges are $e_p = 1.0$ and $e_n = 0.0$. The $B(E2)$ values for ^{92}Zr and ^{94}Mo isotopes and experimental values are listed in Table 6, the agreement between them is a quite good.

Table 4. **The structure of three lowest 2^+ states in ^{92}Zr**

States	Exp. (MeV)	QPM (MeV)	Structure
2_1^+	0.934	0.922	$91\%[2_1^+]_{[\mathrm{RPA}]}$
2_2^+	1.847	1.872	$91\%[2_2^+]_{[\mathrm{RPA}]}$
2_3^+	2.066	2.153	$17\%[2_4^+]_{[\mathrm{RPA}]} + 13\%[2_5^+]_{[\mathrm{RPA}]} + 54\%[2_1^+ \otimes 2_1^+]_{[\mathrm{RPA}]}$

Table 5. **The structure of three lowest 2^+ states in ^{94}Mo**

States	Exp. (MeV)	QPM (MeV)	Structure
2_1^+	0.871	0.871	$92\%[2_1^+]_{[\mathrm{RPA}]}$
2_2^+	1.864	1.892	$92\%[2_2^+]_{[\mathrm{RPA}]}$
2_3^+	2.067	2.117	$16\%[2_4^+]_{[\mathrm{RPA}]} + 14\%[2_5^+]_{[\mathrm{RPA}]} + 52\%[2_1^+ \otimes 2_1^+]_{[\mathrm{RPA}]}$

In $E2$ transitions, the probability rates between symmetric and mixed- symmetry states are proportional to the quantity $(e_\pi \chi_p i - e_v \chi_v)^2$ [26] and depend on the nature of the states. The comparison between experimental and calculated $B(E2)$ transition rates is shown in Table 6. Again, the QPM describes well the decay properties being important for the wave functions of the symmetric and mixed-symmetry states. The $E2$ reduced probability between isovector to isoscalar states is low, differing by an even number of phonons.

The electric quadrupole moments for the first excited states for ^{92}Zr and ^{94}Mo isotopes are presented in Table 6, the values of $Q(2_1^+)$ decrease, by indicating a prolate shape in the first excited state. The theoretical values are in agreement with experimental values in magnitude and sign.

3.2.2. Magnetic transition probability B(M1)

In order to calculate the $M1$ transition probabilities in IBM-2, we have to estimate the g-factor of bosons for proton bosons and neutron bosons in Eq. (11). We used the Sambataro relation [27]:

$$g = g_\pi \frac{N_\pi}{N_\pi + N_v} + g_v \frac{N_v}{N_\pi + N_v}. \tag{29}$$

Equation (29) is used to estimate the g-factor for the first excited 2_1^+ state. The magnetic dipole moment value for ^{92}Zr isotope, $\mu = -0.360(20)\mu_N$ [24], and the mixing ratio value for ^{92}Zr isotope to the transition $\delta(E2/M1; 2_2^+ \to 2_1^+) = 0.016(12)\ eb/\mu_N$ [28], are used to evaluate the g-factor of bosons. The value for proton and neutron bosons is $g_\pi = 0.46\mu_N$, $g_v = 0.24\mu_N$. For ^{94}Mo isotope, we follow the same procedure in ^{92}Zr isotope. The values are $g_\pi = 0.47\mu_N$, $g_v = 0.23\mu_N$. In QPM, the estimated g-factor for ^{92}Zr is equal to $(-0.18\mu_N)$ and, for ^{94}Mo, is $0.274\mu_N$.

To evaluate the magnetic transition probability in $B(M1)$ in IBM-2 and QPM, we used Eqs. (11) and (27), respectively. The theoretical results and available experimental values are tabulated in Table 7. Unfortunately, the experimental data on the $M1$ transition probability are very rare, and the approximate theory does not make it possible to settle the question of nuclear nonaxiality. They show a reasonable agreement with the available experimental

Table 6. **Comparison of the calculated and experimental E2 transition strengths in 2^+ states in ^{92}Zr ^{94}Mo isotopes in W.U.**

Transitions	^{92}Zr			^{94}Mo		
	Exp. [24]	IBM-2	QPM	Exp. [25, 29]	IBM-2	QPM
$2_1^+ \to 0_1^+$	0.64(6)	0.66	0.87	0.203 (4)	0.314	0.31
$2_2^+ \to 0_1^+$	3.4(4)	4.22	5.22	0.33 (11)	0.52	0.58
$2_3^+ \to 0_1^+$	<0.005	0.33	0.0031	–	0.45	0.11
$2_2^+ \to 2_1^+$	–	23	11	60^{+20}_{-30}	57	32
$4_1^+ \to 2_1^+$	4.05 (11)	5.2	5.25	26.0 (42)	25.43	17
$4_2^+ \to 2_1^+$	6.1 (8)	7.4	8.23	5.9 (10)	5.1	3.5
$3_1^+ \to 2_1^+$	–	11	8.2	$2.5^{+3.10}_{-2.1}$	3.11	0.876
$3_1^+ \to 2_2^+$	–	0.854	1.2	$1.5^{+1.2}_{-0.6}$	0.99	1.414
$Q(2_1^+)$	–0.23 (5)	–0.26	–0.30	–0.13 (8)	–0.14	–0.21

Table 7. **Comparison of the calculated and experimental M1 transition strengths in ^{92}Zr ^{94}Mo isotopes in μ_N^2 Units**

Transitions	^{92}Zr			^{94}Mo		
	Exp. [24]	IBM-2	QPM	Exp. [25, 29]	IBM-2	QPM
$2_1^+ \to 0_1^+$	0.37 (4)	0.53	0.0011	0.37 (4)	0.007	0.0009
$3_1^+ \to 2_1^+$	–	0.087	0.0076	$0.01^{+0.102}_{-0.06}$	0.006	0.071
$3_1^+ \to 2_2^+$	–	0.004	0.005	$0.24^{+0.14}_{-0.07}$	0.0021	0.0006
$1_1^+ \to 0_1^+$	–	0.96	1.43	0.16 (1)	1.100	0.89
$1_1^+ \to 2_1^+$	–	0.0056	0.0087	0.007^{+6}_{-2}	0.0057	0.95
$2_3^+ \to 2_1^+$	–	0.00034	0.00011	0.003	0.009	0.126
$1_1^+ \to 2_2^+$	–	0.55	0.409	0.43 (12)	0.641	0.853
$2_3^+ \to 2_1^+$	–	0.41	0.431	0.35 (15)	0.329	0.356
$3_1^+ \to 4_1^+$	–	0.12	0.011	$0.074^{+0.044}_{-0.019}$	0.11	0.0071
$\mu(2_1^+)$	–0.360 (20)	–0.35	–0.41	–	–0.371	0.45

Table 8. **Comparison of the calculated and experimental $\delta(E2/M1)$ in ^{92}Zr ^{94}Mo isotopes in eb/μ_N Units**

Transitions	^{92}Zr			^{94}Mo		
	Exp. [29]	IBM-2	QPM	Exp. [29]	IBM-2	QPM
$2_2^+ \to 2_1^+$	0.016 (2)	0.022	1.22	–2.0 (3)	–2.5	6
$3_1^+ \to 2_1^+$	2.13 (7)	2.2	3.78	1.36 (9)	1.56	3.4
$3_1^+ \to 2_2^+$	–	10	16	–	8	0.22
$2_3^+ \to 2_1^+$	–1.04 (11)	–1.55	–2.34	$-1.9^{+0.5}_{-0.4}$	–2.0	4.5
$4_1^+ \to 2_1^+$	–	–1.414	2.45	–	2.98	13
$3_1^+ \to 4_1^+$	–	6.7	9.88	–	4.21	8.9
$4_2^+ \to 4_1^+$	–	0.14	11	–	3.90	11
$1_1^+ \to 2_1^+$	–	23	20	–	17	22

values. In addition, it is questionable, if the $B(M1)$ strength is a suitable yool to identify a MSS in heavier nuclei, since it is decreasing, as the square of the deformation parameter increases. It is not clear whether this decrease can be attributed to a washing out of the MSS, or, may be, a structural change in 2_1^+ is the cause. The $B(M1)$ strength depends on the structure of both states and cannot answer this question.

The $M1$ transition theoretical scheme is in a reasonable agreement with the experimental data. We get strong transitions between members of the isoscalar and isovector groups having an equal number of phonons. We also obtain weak transitions between different isospin states, but with different phonon numbers and between states belonging to the same isospin group.

The magnetic dipole moment for the first excited state $\mu(2_1^+)$ has been calculated in two collective models, the results agree satisfactorily with the experimental values.

3.2.3. Mixing ratios

To estimate the mixing ratios for ^{92}Zr ^{94}Mo isotopes, we apply the following equation [28]:

$$\delta(E2/M1; J_i \to J_f) = 0.835 E_\gamma \text{ (in MeV)} \times \times \frac{\langle J_f \| \mu(E2) \| J_i \rangle}{\langle J_f \| \mu(M1) \| J_i \rangle} \frac{eb}{\mu_N}. \quad (30)$$

The mixing ratio values are listed in Table 8. wWe can see a generally good agreement with experiments values with the results of IBM-2- and QPM-based calculations. The scheme of $\delta(E2/M1$ rates is consistent with the picture provided by IBM-2 more than QPM in magnitude and sign.

4. Conclusions

The systematic investigation of the mixed-symmetry states for ^{92}Zr ^{94}Mo isotopes is carried out by the collective models, namely, Interacting Boson Model-2 (IBM-2) and Quasiparticle Phonon Model (QPM). The energy spectra and electromagnetic transitions are presented in this work, the agreement with the experimental values is quite well. Now, we summarize the conclusions of the main results in this study.

1. From the energy ratios, we show that the ^{92}Zr and ^{94}Mo isotopes possess the U(5) symmetry and have a vibrational shape (close to the spherical one).

2. The calculations of energy levels within IBM-2 and QPM, indicates that 2_2^+, 2_3^+, and 1^+ are mixed-symmetry states. They have the same character as those proposed by Hamilton *et al.*, [1] in the U(5) limit.

3. The rates of electric transitions $B(E2)$ between symmetric and mixed- symmetry states are proportional to the quantity $(e_\pi \chi_\pi - e_v \chi_v)^2$ [26] and depend on the nature of the states.

4. For the magnetic transition $B(M1)$, one can observe the strong transitions between members of the isoscalar and isovector groups having an equal number of phonons.

5. For $M1$ transitions, we obtained a weak transition between different isospin states, but with different phonon umbers and between states belonging to the same isospin group.

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СТАНИ ЗІ ЗМІШАНОЮ
СИМЕТРІЄЮ ЯДЕР ^{92}Zr І ^{94}Mo

Вивчаються стани зі змішаною симетрією ізотопів ^{92}Zr та ^{94}Mo із застосуванням колективних моделей: модель-2 взаємодіючих бозонів (IBM-2) і модель фононів-квазічастинок (QPM). Розраховані спектри енергії та швидкості електромагнітних переходів для цих ізотопів добре узгоджуються з наявними експериментальними даними. Модель IBM-2 надає більш точний опис даних. В цих ізотопах ми виявили кілька станів зі змішаною симетрією, таких як 2_2^+, 2_3^+, 3_1^+ і 1_s^+. Встановлено, що ці ізотопи мають вібраційну форму, що відповідає U(5) симетрії.

Ключові слова: модель-2 взаємодіючих бозонів, модель фононів-квазічастинок, стани зі змішаною симетрією, B(E2), B(M1), коефіцієнти змішування.