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THREE-PARTICLE FIELDS AS A METHOD TO DESCRIBE BARYONS IN SCATTERING PROCESSES

A model of three-particle fields has been proposed to describe baryons in elastic and inelastic scattering processes. The model makes it possible to describe the confinement of quarks in a hadron and, simultaneously, the interaction of quarks in various hadrons, when the latter collide. Such an interaction is provided by the exchange of bound states between two gluons that are also in the confinement state.

Keywords: multi-particle fields, subset of simultaneity, confinement.

1. Introduction

In our opinion, there is a fundamental problem in the quantum field theory. The essence of this problem consists in that operators that can be used to construct the Fock state of a system of interacting particles [1–4] differ from the operators of quantized fields. This happens, because the field function operators are defined in the Minkowski space and, therefore, depend on time in an arbitrary reference frame. As a result, such operators, when acting on the Fock state, change not only the occupation numbers of single-particle states, but also the time dependence of those states. Furthermore, this change occurs independently of the time evolution operator. At the same time, for a system of interacting particles, the time dependence of its state cannot be reduced to the time dependence of single-particle states, being determined exclusively by the time evolution operator. In this case, the determination of the occupation numbers of single-particle states requires the separation of the time dependence of the system state from its dependences from other dynamical variables, with the expansion of the latter in the products of single-particle states [5]. In so doing, we obtain operators whose action changes the state dependence on the occupation numbers, but does not directly change the state dependence on time. That is, they change the state dependence on the occupation numbers, and, owing to the change in this dependence, by acting on such a changed state with the time evolution operator, we obtain a change in its dependence on time. As a consequence, the new operators differ from the generated elements of the algebra of field operators, and the corresponding commutation rules for them are not clear. Therefore, the result of the action of the field operators and the time evolution operator on the elements in the Fock space of a system of interacting particles becomes uncertain. This situation can be well illustrated by the example of the well-known Tamm–Dancoff method [6–9]. In work [6], the problem concerned was ignored, because the time dependences of both the Fock state and the field operators were ignored. In this case, the operators applied to express the state and the operators included in the Hamiltonian turn out to be functions of the momentum in both cases. In work [6], they are permuted according

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to equal-time commutation relationships [4], which formally makes it possible to determine the action of the Hamiltonian on the state function and solve the corresponding eigenvalue problem. However, in our opinion, this approach is erroneous. It is so, because the creation operators, with the help of which the state is constructed, generate a particle with a certain three-dimensional momentum. That is, they increase the corresponding occupation number in the expansion of the coordinate part of the state function over single-particle state momentum eigenfunctions. The field creation operators, which also depend on the three-dimensional momentum in the interaction representation, generate a particle not only with the indicated momentum, but also with a certain energy that is completely determined by this three-dimensional momentum. Therefore, they are different operator-valued functions of the three-dimensional momentum, and there is no reason to permute them according to the commutation relationships for the operators of the same field. In general, by applying the Fourier transformation to any field operator written in the coordinate representation, it is easy to see that this operator creates or destroys a particle not only with a certain momentum but also with a certain energy. This can be clearly seen in work [10] where multi-particle effects were considered using ordinary single-particle field operators. The mentioned problems manifested themselves in works [7, 9] where the Tamm–Dancoff method was tried to be formulated in a Lorentz-covariant way. This immediately leads to the consideration of multi-time probability amplitudes. Our viewpoint that it is impossible to apply the multi-time description in the relativistic quantum theory was explained in detail in work [11].

In the problems, where the initial and final states can be composed from free quanta of interacting fields, as it was done in the problems of lepton scattering, the indicated difficulty does not arise, because, if the particles do not interact, their single-particle states can be shifted in time separately and independently of one another. Then such a state can be obtained by acting on the vacuum with field operators and permutating them with the same field operators entering the time evolution operator. Following this way, we arrive at a dynamic model that leads to the conservation of the sum of the single-particle energy-momenta, which corresponds to the experiment in this case.

However, in hadron scattering problems, quarks in hadrons strongly interact both in the initial and final states. Therefore, the considered problem becomes essential. Nowadays, when describing processes with hadrons, this problem is circumvented by dividing the scattering process into stages [12, 13] and describing each stage separately making use of a specially developed approach. In particular, initial hadrons are usually represented using the parton model [14]. This approach cannot be considered a satisfactory solution to the specified problem. Firstly, the hadron is replaced by a system of non-interacting particles, and, in such a way, single-particle energy-momenta are artificially introduced into the problem. Secondly, the parton model does not operate with amplitudes but with probabilities [15, 16]. Therefore, such a construction as the parton creation operator or the Lagrangian of the parton field cannot exist in principle in the framework of this model. Hence, the parton model is rather a way to phenomenologically bypass the considered problem than to solve it. The same can be said about the hadronization stage [12, 13]. The field interaction operator contains a delta function that ensures the preservation of the sums of single-particle energy-momenta [17, 18]. As a consequence, the time evolution operator ensures the preservation of the sum of single-particle energy-momenta only, whereas there are no single-particle energy-momenta for hadrons, and experimentally only the four-vector of the total energy-momentum of hadrons is preserved. Probably, this conclusion does not depend on the calculation method for the time evolution operator. Indeed, the time dependence of the generating functional of this operator [3] manifests itself only through its one-particle arguments [19]. If we apply the Fourier transform to those arguments and change from the time evolution operator to the scattering one, then the scattering operator kernels describing the mappings between the subspaces of the Fock space corresponding to different particles [3] will contain delta functions of the difference between the sums of single-particle energy-momenta in the initial and final states. Therefore, even if it were possible to accurately calculate the continuous integral in the time evolution operator, it would probably not help us to describe the hadronization of quarks and gluons. From this point of view, it is clear that lattice calculations will not help us in this sense [20–23]. Actually, such calculations comprise a method for the approximate calculation of the continuous integral that enters the expression for the scattering operator associated with the transition amplitude from one state of non-interacting particles into another state of also non-interacting particles.

In work [11], a method of multi-particle fields was proposed to describe two-particle bound states (mesons) and bound states of gauge bosons. In the cited work, as well as in works [24, 25], the motivation of this method was presented, and its differences from other approaches aimed at describing the bound states of quarks and gluons in hadron scattering processes were analyzed. The description of scattering experiments where accelerated protons are used at the initial stage requires the construction of a similar model for three-quark systems. A similar model was already considered in work [24], and it was even applied in work [26] to describe the elastic scattering of protons. However, in the mentioned work [11], a more consistent version of the multi-particle field method was proposed. The aim of this work is to describe a three-quark system in the framework of this approach. In all calculations below, we use a system of units where the action and the limiting rate of interaction transfer c are dimensionless, and all other quantities are multiplied by such combinations of Planck's constant $\hbar$ and the rate c that the corresponding products are some powers of length.

2. The Subset of Simultaneity and the Scalar Product on This Subset

Let us firstly consider a system of three non-interacting fermions, and afterwards consider the interaction between them, which leads to the formation of a bound state. All events that can occur with such a system can be presented as a set of twelve-component columns

$$z^a = \begin{pmatrix} x_1^0 \\ x_1^1 \\ x_1^2 \\ x_1^3 \\ x_2^0 \\ x_2^1 \\ x_2^2 \\ x_2^3 \\ x_3^0 \\ x_3^1 \\ x_3^2 \\ x_3^3 \end{pmatrix}. \tag{1}$$

Here, the lower subscripts mean the numbers of particles, and the upper subscripts mean the corresponding temporal or spatial coordinates in the Minkowski space of every particle. The fact that the particles are identical and actually not enumerated can be taken into account in the standard way by imposing appropriate symmetry conditions on the dependences on those coordinates.

Let us consider the linear space of columns (1) and introduce the scalar product on it,

$$\langle z|\, z\rangle = \frac{1}{3}\left(g^{\mathrm{Minc}}_{a_1a_2} x_1^{a_1} x_1^{a_2} + g^{\mathrm{Minc}}_{a_1a_2} x_2^{a_1} x_2^{a_2} + \right. \\ \left. + g^{\mathrm{Minc}}_{a_1a_2} x_3^{a_1} x_3^{a_2}\right). \tag{2}$$

Here, $g^{\mathrm{Minc}}_{a_1a_2}$ is the Minkowski tensor. In what follows, it is convenient to consider this linear space in the Jacobi coordinates,

$$\begin{aligned} x_1^a &= X^a - \frac{1}{3}y_1^a - \frac{1}{2}y_2^a, \\ x_2^a &= X^a - \frac{1}{3}y_1^a + \frac{1}{2}y_2^a, \\ x_3^a &= X^a + \frac{2}{3}y_1^a. \end{aligned} \tag{3}$$

In these coordinates, the scalar product (2) takes the form

$$\langle z|z\rangle = g^{\mathrm{Minc}}_{a_1a_2}\left(X^{a_1}X^{a_2} + \frac{2}{9}y_1^{a_1}y_1^{a_2} + \frac{1}{6}y_2^{a_1}y_2^{a_2}\right). \tag{4}$$

The quantum state of the system under consideration is described by a column in the Fock space, with only the three-particle component being nonzero. The square of the absolute value of this component has the meaning of the combined probability density for the results of the measurements of the dynamic variables of three particles performed simultaneously in the reference frame. The principal character of the issue concerning the simultaneity of measurements was discussed in detail in works [11, 28]. Thus, the Fock state is not considered on the linear space of columns (1) but on its subset determined by the relationships

$$y_1^0 = 0, \quad y_2^0 = 0, \tag{5}$$

which will be called the subset of simultaneity. We also introduce the notations

$$\mathbf{y}_1 = \left(y_1^1, y_1^2, y_1^3\right), \quad \mathbf{y}_2 = \left(y_2^1, y_2^2, y_2^3\right). \tag{6}$$

The set of those quantities will be called the internal coordinates of the dynamic system under consideration. The three-particle component of the Fock column will be denoted as $\Psi_3(X, \mathbf{y}_1, \mathbf{y}_2)$, where X stands for four numbers X^0, X^1, X^2, and X^3.

A point of the subset of simultaneity can be characterized by the ten-dimensional column

$$q = \begin{pmatrix} X^0 \\ X^1 \\ X^2 \\ X^3 \\ y_1^1 \\ y_1^2 \\ y_1^3 \\ y_2^1 \\ y_2^2 \\ y_2^3 \end{pmatrix}. \tag{7}$$

For such columns, let us introduce the scalar product in such a way that it coincides with formula (4) in the case when the simultaneity condition (5) is fulfilled,

$$\langle q|\, q\rangle = g_{a_1 a_2} q^{a_1}\ q^{a_2}, \tag{8}$$

where

$$g_{a_1 a_2} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{9}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{9}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{9}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6 \end{pmatrix}.$$

Of course, the subset of simultaneity cannot be isolated in the Lorentz-invariant way, and every inertial observer has his own subset of this kind. As was shown in detail in work [11], this circumstance does not contradict the principles of the theory of relativity. It occurs because, on the basis of the principle of relativity, if an inertial observer carries out measurements in a certain quantum system and must make them simultaneously with respect to himself, then every other inertial observer must make his own measurements also simultaneously with respect to himself. Therefore, different inertial observers cannot use the same system of events for their measurements. It is known that the Lorentz transformations relate the coordinates of the same event in different inertial reference frames. Since different observers must realize different events when performing measurements, the coordinates of those events should not be obligatory related via the Lorentz transformations.

It was also shown in works [11, 27] that the dependence of the Fock state on internal variables remains the same in various inertial reference frames, and the dependence on the coordinates X^0, X^1, X^2, X^3 transforms when changing from one inertial reference frame to another one according to the law of the ordinary scalar function

$$\Psi'_3(X', \mathbf{y}_1, \mathbf{y}_2) = \Psi_3\left(X = \hat{\Lambda}^{-1}(X'), \mathbf{y}_1, \mathbf{y}_2\right). \tag{9}$$

Here $\Psi'_3(X', \mathbf{y}_1, \mathbf{y}_2)$ is the three-particle component of the Fock state in the reference frame obtained from the initial reference frame, with respect to which the three-particle component equals $\Psi_3(X, \mathbf{y}_1, \mathbf{y}_2)$, using the Lorentz transformation $\hat{\Lambda}$.

Note that the relationship $X = \hat{\Lambda}^{-1}(X')$ appears in Eq. (9) due not to the Lorentz transformations, but to the form transformation of the dependence of the function $\Psi_3(X, \mathbf{y}_1, \mathbf{y}_2)$ on the arguments X^0, X^1, X^2, X^3 [4,27]. Let us explain this in more detail. Suppose that we have two inertial observers who use an "unprimed" reference system and a "primed" one. Suppose that the "unprimed" observer, using his ensemble of three-particle systems, measured the coordinates of three particles $\mathbf{x}_1 = \left(x_1^1, x_1^2, x_1^3\right)$, $\mathbf{x}_2 = \left(x_2^1, x_2^2, x_2^3\right)$, and $\mathbf{x}_3 = \left(x_3^1, x_3^2, x_3^3\right)$ at various time moments X^0; each time he did the measurements simultaneously for himself. On the basis of the results of those measurements, he calculated the location of the center of mass of the three-particle system using the vector $\mathbf{X} = \left(X^1, X^2, X^3\right)$, as well as the internal coordinates $\mathbf{y}_1$ and $\mathbf{y}_2$ using formulas (3). Suppose now that the "primed" observer did the same but using his ensemble of analogous three-particle systems. As required by the principle of relativity, the "primed" observer made similar measurements simultaneously for himself. The moments of time when he did it are denoted as X'^0 and the measurement results as $\mathbf{x}'_1 = \left(x'^1_1, x'^2_1, x'^3_1\right)$, $\mathbf{x}'_2 = \left(x'^1_2, x'^2_2, x'^3_2\right)$, and $\mathbf{x}'_3 = \left(x'^1_3, x'^2_3, x'^3_3\right)$; accordingly, $\mathbf{X}' = \left(X'^1, X'^2, X'^3\right)$ and $\mathbf{y}'_1 = \left(y'^1_1, y'^2_1, y'^3_1\right)$,

$\mathbf{y}'_2 = \left(y'^1_2, y'^2_2, y'^3_2\right)$. For the "unprimed" observer, the events consist in that he simultaneously detects particles at the time moment X^0 in vicinities of the points $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ in his reference frame. On the other hand, the "primed" observer simultaneously, at the time moment X'^0, registers the particles in vicinities of the points $\mathbf{x}'_1$, $\mathbf{x}'_2$, and $\mathbf{x}'_3$ in a different reference frame. Therefore, those triplets of events are different. Hence, their coordinates cannot be related to each other. However, the arguments presented in works [11, 27] bring us to the conclusion that, for every measurement carried out by the "unprimed" observer at the moment X^0 with a result in a vicinity of $(\mathbf{X}, \mathbf{y}_1, \mathbf{y}_2)$, there is a measurement carried out by the "primed" observer at the moment X'^0 with the result in a vicinity of $(\mathbf{X}', \mathbf{y}_1, \mathbf{y}_2)$, and the quantities $\left(X'^0, X'^1, X'^2, X'^3\right)$ are related to the quantities $\left(X^0, X^1, X^2, X^3\right)$ by the relationships $X'^a = \Lambda^a_b X^b$, where Λ^a_b are the matrix elements of the Lorentz transformation that transforms the "unprimed" reference frame into the "primed" one. Formula (9) has exactly this sense, and this is exactly what we mean, when we say that the relationship between the coordinates X does not arise due to the Lorentz transformations but due to the transformation of the state form [4, 27].

The problem indicated in the previous section arises because the field operators are defined on the Minkowski space, whereas the Fock state, which they must act on, is defined on the subset of simultaneity. In our opinion, the way out of this situation is to construct a field model with field operators that are defined not on the Minkowski space but on the subset of simultaneity. In what follows, such fields are called multi-particle. The field operators of such multi-particle fields do not change the occupation numbers of single-particle states, but they change the occupation numbers of multi-particle (in our case, three-particle) states. Let us consider the construction of this model.

3. Three-Particle Bispinor Field on the Subset of Simultaneity

A subset of simultaneity with a defined scalar product is a domain of definition of the three-particle field. Let us consider the range of values of this field. We are interested in constructing operators that change the occupation numbers of three-quark states. In the case of non-interacting quarks, a required operator can be obtained by multiplying three bispinor field operators and then transiting on the subset of simultaneity,

$$\hat{\Psi}_{s_1 s_2 s_3, f_1 f_2 f_3, c_1 c_2 c_3}\left(x_1, x_2, x_3\right)\Big|_{x_1^0 = x_2^0 = x_3^0} = \\ = \hat{\Psi}_{s_1, f_1, c_1}\left(x_1\right) \hat{\Psi}_{s_2, f_2, c_2}\left(x_2\right) \times \\ \times \hat{\Psi}_{s_3, f_3, c_3}\left(x_3\right)\Big|_{x_1^0 = x_2^0 = x_3^0}. \quad (10)$$

Here, s_1, s_2, s_3 are bispinor indices; f_1, f_2, f_3 are flavor indices, and c_1, c_2, c_3 are color indices. Therefore, the range of values of the three-particle field is a linear tensor space, where the tensor product of three bispinor representations of the Lorentz group and the tensor product of three fundamental representations of the flavor and color $SU(3)$ groups are implemented.

Below, we are going to consider only strong interaction between quarks, which results in their confinement and the existence of the hadron as a bound state of three quarks. Therefore, we will ignore the flavor structure of the three-particle field tensor and will not write out the flavor indices.

We want to construct operators that will change the occupation numbers of the proton three-particle states, i.e., the states corresponding to particles with spin $\frac{1}{2}$. For this purpose, from the linear space of tensors $\hat{\Psi}_{s_1 s_2 s_3}$, we can select an invariant subspace on which the bispinor representation of the Lorentz group is realized. This selection was discussed in detail in works [11, 24, 25]. The tensors from this linear space will be denoted as $\hat{\Psi}_{s_1, c_1, c_2, c_3}$. Since any hadron is colorless, it is necessary to select, from the linear space $\hat{\Psi}_{s_1, c_1, c_2, c_3}$, an invariant subspace of tensors of the form $\hat{\Psi}_{s_1, c_1, c_2, c_3} = \hat{\Psi}_{s_1} \varepsilon_{c_1, c_2, c_3}$ (here $\varepsilon_{c_1, c_2, c_3}$ is the three-dimensional Levi-Civita symbol), which transform according to the trivial representation of the $SU_c(3)$ group. However, it is convenient to make this selection later, when the interaction between quarks will be taken into account.

Hence, let us consider a three-particle operator-valued field function $\hat{\Psi}_{s_1, c_1, c_2, c_3}(q)$, where q is an arbitrary column (7) on the subset of simultaneity. As an action for this field, we can take the quantity

$$S = \int d^{10} q \left(g^{a_1 a_2} \frac{\partial \bar{\Psi}_{s_1, c_1 c_2 c_3}(q)}{\partial x^{a_1}} \frac{\partial \Psi_{s_1, c_1 c_2 c_3}(q)}{\partial x^{a_2}} - \right. \\ \left. - \left(3 m_q\right)^2 \bar{\Psi}_{s_1, c_1 c_2 c_3}(q) \Psi_{s_1, c_1 c_2 c_3}(q) \right). \quad (11)$$

Here $\bar{\Psi}_{s_1,c_1,c_2,c_3}(q)$ denotes the Dirac conjugate field, summation over the repeating indices is assumed, and m_q denotes the mass of the constituent quark (due to the neglect of all interactions, this parameter is adopted to be the same for quarks of all flavors).

The corresponding Lagrange–Euler equations can be written in the form

$$-\left(g^{\mathrm{Minc}}\right)^{ab}\frac{\partial^2\hat{\Psi}_{s_1,c_1c_2c_3}(X,\mathbf{y}_1,\mathbf{y}_2)}{\partial X^a\partial X^b}-\left((3m_q)^2+2(3m_q)\times\left(-\frac{1}{2\left(\frac{2}{3}m_q\right)}\Delta_{\mathbf{y}_1}-\frac{1}{2\left(\frac{1}{2}m_q\right)}\Delta_{\mathbf{y}_2}\right)\right)\times\hat{\Psi}_{s_1,c_1c_2c_3}(X,\mathbf{y}_1,\mathbf{y}_2)=0. \quad (12)$$

Here, the following notation was used:

$$\Delta_{\mathbf{y}_a}=\frac{\partial^2}{\partial(y_a^1)^2}+\frac{\partial^2}{\partial(y_a^2)^2}+\frac{\partial^2}{\partial(y_a^3)^2},\quad a=1,2. \quad (13)$$

The expression $\left(-\frac{1}{2((2/3)m_q)}\Delta_{\mathbf{y}_1}-\frac{1}{2((1/2)m_q)}\Delta_{\mathbf{y}_2}\right)$ on the right-hand side of Eq. (12) coincides with the internal Hamiltonian of a system of three non-interacting non-relativistic particles. Since the system with a fixed number of particles is considered, the expansion of the dependence of the field $\hat{\Psi}_{s_1,c_1c_2c_3}(X,\mathbf{y}_1,\mathbf{y}_2)$ on the internal variables $\mathbf{y}_1$ and $\mathbf{y}_2$ in a series of the eigenfunctions of this internal Hamiltonian must have non-zero terms only with those eigenfunctions for which the eigenvalues are smaller than m_q. Then, with an accuracy up to the squared ratio between the eigenvalues of the internal Hamiltonian and $3m_q$, we can rewrite Eq. (12) in the form

$$-\left(g^{\mathrm{Minc}}\right)^{ab}\frac{\partial^2\hat{\Psi}_{s_1,c_1c_2c_3}(X,\mathbf{y}_1,\mathbf{y}_2)}{\partial X^a\partial X^b}-\left(\hat{H}^{\mathrm{int},0}(\mathbf{y}_1,\mathbf{y}_2)\right)^2\hat{\Psi}_{s_1,c_1c_2c_3}(X,\mathbf{y}_1,\mathbf{y}_2)=0, \quad (14)$$

where

$$\hat{H}^{\mathrm{int},0}(\mathbf{y}_1,\mathbf{y}_2)=3m_q\hat{E}-\frac{1}{2\left(\frac{2}{3}m_q\right)}\Delta_{\mathbf{y}_1}-\frac{1}{2\left(\frac{1}{2}m_q\right)}\Delta_{\mathbf{y}_2}, \quad (15)$$

and $\hat{E}$ is the unary operator. Since the operators $\hat{P}_a = i(\partial/\partial X^a)$ and $\hat{H}^{\mathrm{int}}(\mathbf{y}_1,\mathbf{y}_2)$ commute, then the "factorization" procedure [4] can be applied to Eq. (14), which gives rise equation

$$i\hat{\gamma}^a_{s_1s_2}\frac{\partial\hat{\Psi}_{s_2,c_1c_2c_3}(X,\mathbf{y}_1,\mathbf{y}_2)}{\partial X^a}-\hat{H}^{\mathrm{int},0}(\mathbf{y}_1,\mathbf{y}_2)\hat{\Psi}_{s_1,c_1c_2c_3}(X,\mathbf{y}_1,\mathbf{y}_2)=0, \quad (16)$$

where $\hat{\gamma}^a_{s_1s_2}$ are the elements of the Dirac matrices. It is natural to call this equation the three-particle Dirac equation. Equation (16) is generated by the Lagrangian

$$L^{(0)}=\frac{i}{2}\left(\sum_{b=0}^{3}\left(\hat{\bar{\Psi}}_{s_1,c_1c_2c_3}(q)\,\gamma^b_{s_1s_2}\frac{\partial\hat{\Psi}_{s_2,c_1c_2c_3}(q)}{\partial q^b}-\frac{\partial\hat{\bar{\Psi}}_{s_1,c_1c_2c_3}(q)}{\partial q^b}\gamma^b_{s_1s_2}\hat{\Psi}_{s_2,c_1c_2c_3}(q)\right)\right)-(3m_q)\,\hat{\bar{\Psi}}_{s_1,c_1c_2c_3}(q)\hat{\Psi}_{s_1,c_1c_2c_3}(q)+\frac{1}{2(3m_q)}\sum_{b=4}^{9}\sum_{d=4}^{9}g^{bd}\frac{\partial\hat{\bar{\Psi}}_{s_1,c_1c_2c_3}(q)}{\partial q^b}\times\frac{\partial\hat{\Psi}_{s_1,c_1c_2c_3}(q)}{\partial q^d}. \quad (17)$$

4. Gauge Fields on the Subset of Simultaneity

Now, we can account for the strong interaction between quarks in the usual way, i.e., by passing from the globally $SU_c(3)$-symmetric expression (17) to the corresponding locally symmetric Lagrangian via substituting the derivatives by the covariant differentiation operators

$$\begin{aligned}\hat{D}_b\left(\hat{\Psi}_{s_1,c_1c_2c_3}(q)\right)&=\frac{\partial\hat{\Psi}_{s_1,c_1c_2c_3}(q)}{\partial q^b}-ig\hat{A}^{(1)}_{b,g_1}(q)\,\lambda^{g_1}_{c_1c_4}\hat{\Psi}_{s_1,c_4c_2c_3}(q)-ig\hat{A}^{(2)}_{b,g_2}(q)\,\lambda^{g_2}_{c_2c_4}\hat{\Psi}_{s_1,c_1c_4c_3}(q)-ig\hat{A}^{(3)}_{b,g_3}(q)\,\lambda^{g_3}_{c_3c_4}\hat{\Psi}_{s_1,c_1c_2c_4}(q),\\ \overline{D_b\Psi_{s_1,c_1c_2c_3}(q)}&=\frac{\partial\hat{\bar{\Psi}}_{s_1,c_1c_2c_3}(q)}{\partial q^b}+ig\hat{A}^{(1)}_{b,g_1}(q)\,\hat{\bar{\Psi}}_{s_1,c_4c_2c_3}(q)\,\lambda^{g_1}_{c_4c_1}+ig\hat{A}^{(2)}_{b,g_2}(q)\,\hat{\bar{\Psi}}_{s_1,c_1c_4c_3}(q)\,\lambda^{g_2}_{c_4c_2}+ig\hat{A}^{(3)}_{b,g_3}(q)\,\hat{\bar{\Psi}}_{s_1,c_1c_2c_4}(q)\,\lambda^{g_3}_{c_4c_3}.\end{aligned} \quad (18)$$

Here g is the strong interaction constant; $\lambda^{g_1}_{c_1c_2}$ $(g_1 = 1, 2, ..., 8; c_1,\ c_2 = 1, 2, 3)$ are elements of the Gell-Mann matrices; and $\hat{A}^{(1)}_{b,g_1}(q)$, $\hat{A}^{(2)}_{b,g_1}(q)$, and $\hat{A}^{(3)}_{b,g_1}(q)$ are operators of gauge fields. Since the space of internal indices of gluon fields is Euclidean, there is no difference between the upper and lower indices, so, we write the internal indices of Gell-Mann matrices as upper ones, and the internal indices of gluon fields as lower ones only for the convenience of notation. At the local $SU_c(3)$ transformation, each of those fields is transformed according to the usual law, which allows the compensation of the terms arising owing to the dependence of the transformation parameters on coordinates.

We attract attention to the presence of exactly three gauge fields rather than one, as it takes place in the one-particle quantum field theory. The local invariance condition for the Lagrangian determines only the transformation law for the gauge field at the corresponding local transformation of the fermion fields. As a result, the Lagrangian and the dynamic equations for the gauge field become determined. The three introduced gauge fields must have the same transformation law and the same dynamic equations. However, these identical equations can be associated with different boundary conditions, which leads to their three different solutions. Since we are going to describe the bound state of three quarks, the boundary conditions with respect to the internal variables have to be taken into account substantially.

In addition, we can consider that the baryon, whose creation and annihilation are expected to be described by a three-particle bispinor field, must be colorless. Therefore, from the linear space of three-index tensors $\hat{\Psi}_{s_1,c_1c_2c_3}(q)$, we select an invariant subspace on which the trivial representation of the $SU_c(3)$ group is realized. For this purpose, let us put

$$\hat{\Psi}_{s_1,c_1c_2c_3}(q) = \hat{\Psi}_{s_1}(q)\varepsilon_{c_1c_2c_3}. \tag{19}$$

It means that we zero the projections of the field $\hat{\Psi}_{s_1,c_1c_2c_3}(q)$ onto other invariant subspaces composing the linear space of three-index tensors, thus expressing the absence of the baryon's color.

Substituting the derivatives in Eq. (17) by operators (18) and taking Eq. (19), into account, we obtain

$$\frac{L}{6} = \frac{i}{2}\left(\sum_{b=0}^{3}\left(\hat{\bar{\Psi}}_{s_1}(q)\ \gamma^b_{s_1s_2}\frac{\partial\hat{\Psi}_{s_2}(q)}{\partial q^b} - \frac{\partial\hat{\bar{\Psi}}_{s_1}(q)}{\partial q^b}\gamma^b_{s_1s_2}\hat{\Psi}_{s_2}(q)\right)\right) - (3m_q)\,\hat{\bar{\Psi}}_{s_1}(q)\,\hat{\Psi}_{s_1}(q) + \frac{1}{2\,(3m_q)}\sum_{b=4}^{9}\sum_{d=4}^{9}g^{bd}\frac{\partial\hat{\bar{\Psi}}_{s_1}(q)}{\partial q^b}\frac{\partial\hat{\Psi}_{s_1}(q)}{\partial q^d} +$$
$$+ \frac{g^2}{(9m_q)}\hat{\bar{\Psi}}_{s_1}(q)\,\hat{\Psi}_{s_1}(q)\sum_{b=4}^{9}\sum_{d=4}^{9}g^{bd}\times \Big(\hat{A}^{(1)}_{b,g_1}(q)\,\hat{A}^{(1)}_{d,g_1}(q) + \hat{A}^{(2)}_{b,g_1}(q)\,\hat{A}^{(2)}_{d,g_1}(q) + \hat{A}^{(3)}_{b,g_1}(q)\,\hat{A}^{(3)}_{d,g_1}(q) - \hat{A}^{(1)}_{b,g_1}(q)\,\hat{A}^{(2)}_{d,g_1}(q) - \hat{A}^{(1)}_{b,g_1}(q)\,\hat{A}^{(3)}_{d,g_1}(q) - \hat{A}^{(2)}_{b,g_1}(q)\,\hat{A}^{(3)}_{d,g_1}(q)\Big). \tag{20}$$

Here, for convenience, we divided the Lagrangian by a factor of 6, which arises when summating the components of the Levy-Civita symbol.

Let us introduce new gauge fields $\hat{A}^{(+)}_{b,g_1}(q)$, $\hat{A}^{(-,1)}_{b,g_1}(q)$, and $\hat{A}^{(-,2)}_{b,g_1}(q)$ using the following relationships, which are similar to those that determine the three-particle Jacobian coordinates:

$$\begin{aligned}\hat{A}^{(+)}_{b,g_1}(q) &= \frac{1}{3}\left(\hat{A}^{(1)}_{b,g_1}(q) + \hat{A}^{(2)}_{b,g_1}(q) + \hat{A}^{(3)}_{b,g_1}(q)\right),\\ \hat{A}^{(-,1)}_{b,g_1}(q) &= \hat{A}^{(3)}_{b,g_1}(q) - \frac{1}{2}\left(\hat{A}^{(1)}_{b,g_1}(q) + \hat{A}^{(2)}_{b,g_1}(q)\right),\\ \hat{A}^{(-,2)}_{b,g_1}(q) &= \hat{A}^{(2)}_{b,g_1}(q) - \hat{A}^{(1)}_{b,g_1}(q).\end{aligned} \tag{21}$$

After substitution (21), Lagrangian (20) can be written in the form

$$\frac{L}{6} = \frac{i}{2}\left(\sum_{b=0}^{3}\left(\hat{\bar{\Psi}}_{s_1}(q)\gamma^b_{s_1s_2}\frac{\partial\hat{\Psi}_{s_2}(q)}{\partial q^b} - \frac{\partial\hat{\bar{\Psi}}_{s_1}(q)}{\partial q^b}\gamma^b_{s_1s_2}\hat{\Psi}_{s_2}(q)\right)\right) - (3m_q)\,\hat{\bar{\Psi}}_{s_1}(q)\hat{\Psi}_{s_1}(q) + \frac{1}{2\,(3m_q)}\sum_{b=4}^{9}\sum_{d=4}^{9}g^{bd}\frac{\partial\hat{\bar{\Psi}}_{s_1}(q)}{\partial q^b}\frac{\partial\hat{\Psi}_{s_1}(q)}{\partial q^d} +$$
$$+ \frac{g^2}{(9m_q)}\left(\sum_{b=4}^{9}\sum_{d=4}^{9}g^{bd}\left(\hat{A}^{(-,1)}_{b,g_1}(q)\hat{A}^{(-,1)}_{d,g_1}(q) + \frac{3}{4}\hat{A}^{(-,2)}_{b,g_1}(q)\hat{A}^{(-,2)}_{d,g_1}(q)\right)\right)\hat{\bar{\Psi}}_{s_1}(q)\hat{\Psi}_{s_1}(q). \tag{22}$$

5. Generalization of the Method Used for Achieving Gauge Invariance

Note that the method applied to obtain the Lagrangian that is invariant with respect to the local $SU_c\,(3)$ transformation is not the most general. Really, if we consider the expression obtained from the product $\left(\partial\hat{\bar{\Psi}}_{s_1,c_1c_2c_3}/\partial q^b\right)\left(\partial\hat{\Psi}_{s_1,c_1c_2c_3}/\partial q^d\right)$ by replacing the derivatives with covariant differentiation operators, then we obtain summands of three types: (i) which do not contain the Gell-Mann matrices, (ii) which contain matrix elements $\lambda^{g_1}_{c_1c_2}$ of one of the matrices, and (iii) which contain the products $\lambda^{g_1}_{c_1c_2}\lambda^{g_2}_{c_3c_4}$ of matrix elements of two matrices. The first powers of the matrix elements of the Gell-Mann matrices enter the Lagrangian in the form of their convolutions with the gauge fields, $\hat{A}^{(n)}_{g_1}(q)\lambda^{g_1}_{c_1c_2}$ ($n = 1, 2, 3$); and the second powers, in the form of the convolutions $\hat{A}^{(n_1)}_{g_1,b}(q)\hat{A}^{(n_2)}_{g_2,d}(q)\lambda^{g_1}_{c_1c_2}\lambda^{g_2}_{c_3c_4}$. In this case, to achieve invariance with respect to the local $SU_c\,(3)$ transformation, the transformation law of these coefficients rather than their explicit expression is essential. Therefore, if the product $\hat{A}^{(n_1)}_{g_1,b}(q)\hat{A}^{(n_2)}_{g_2,d}(q)$ is substituted by the tensor $\hat{A}^{(n_1,n_2)}_{bd,g_1g_2}(q)$ with the same transformation law as the product $\hat{A}^{(\,n_1)}_{g_1,b}(q)\hat{A}^{(n_2)}_{g_2,d}(q)$ has, then a more general expression is obtained for the Lagrangian, which satisfies the requirement of local $SU_c\,(3)$ invariance.

Under the local $SU_c\,(3)$ transformation

$$\begin{aligned}&\hat{\Psi}'_{s_1,c_1c_2c_3}(q) = \hat{U}_{c_1c_4}(q)\,\hat{U}_{c_2c_5}(q)\times\\&\times\hat{U}_{c_3c_6}(q)\,\hat{\Psi}_{s_1,c_4c_5c_6}(q),\\&\hat{\bar{\Psi}}'_{s_1,c_1c_2c_3}(q) = \hat{\bar{\Psi}}_{s_1,c_4c_5c_6}(q)\,\hat{U}^{-1}_{c_4c_1}(q)\times\\&\times\hat{U}^{-1}_{c_5c_2}(q)\,\hat{U}^{-1}_{c_6c_3}(q),\end{aligned}\tag{23}$$

where $\hat{U}(q) = \exp\left(i\hat{\lambda}^{g_1}\theta_{g_1}(q)\right)$, and $\theta_{g_1}(q)$ are coordinate-dependent transformation parameters ($g_1 = 1, 2, ..., 8$), the gauge fields transform according to the law

$$\hat{A}'^{(n)}_{b,g_1}(q) = D_{g_1g_2}(q)\,\hat{A}^{(n)}_{b,g_2}(q) + \frac{\partial\theta_{g_1}(q)}{\partial q^b}.\tag{24}$$

Here $D_{g_1g_2}(q)$ are elements of the matrices of the adjoint representation of the $SU_c\,(3)$ group. Accordingly, the product of two components of the gauge field is transformed according to the law

$$\begin{aligned}&\hat{A}'^{(n_1)}_{b,g_1}(q)\,\hat{A}'^{(n_2)}_{d,g_3}(q) =\\&= D_{g_1g_2}(q)\,D_{g_3g_4}(q)\,\hat{A}^{(n_1)}_{b,g_2}(q)\,\hat{A}^{(n_2)}_{d,g_4}(q) +\\&+ D_{g_1g_2}(q)\,\hat{A}^{(n_1)}_{b,g_2}(q)\,\frac{\partial\theta_{g_3}(q)}{\partial q^d} +\\&+ \frac{\partial\theta_{g_1}(q)}{\partial q^b}D_{g_3g_4}(q)\,\hat{A}^{(n_2)}_{d,g_4}(q) + \frac{\partial\theta_{g_1}(q)}{\partial q^b}\frac{\partial\theta_{g_3}(q)}{\partial q^d}.\end{aligned}\tag{25}$$

Therefore, if the transformation law

$$\begin{aligned}&\hat{A}'^{(n_1n_2)}_{bd,g_1g_3}(q) = D_{g_1g_2}(q)\,D_{g_3g_4}(q)\,\hat{A}^{(n_1n_2)}_{bd,g_2g_4}(q) +\\&+D_{g_1g_2}(q)\,\hat{A}^{(n_1)}_{b,g_2}(q)\,\frac{\partial\theta_{g_3}(q)}{\partial q^d} +\\&+ \frac{\partial\theta_{g_1}(q)}{\partial q^b}D_{g_3g_4}(q)\,\hat{A}^{(n_2)}_{d,g_4}(q) + \frac{\partial\theta_{g_1}(q)}{\partial q^b}\frac{\partial\theta_{g_3}(q)}{\partial q^d},\end{aligned}\tag{26}$$

holds for the tensor $\hat{A}^{(n_1,\,n_2)}_{bd,g_1g_3}(q)$, then the Lagrangian where the product of the gauge field components is replaced by a tensor is invariant with respect to the local $SU_c\,(3)$ transformation. The field described by the tensor $\hat{A}^{(n_1,\,n_2)}_{bd,g_1g_3}(q)$ will be called the two-gluon field. Below, on the basis of the results of work [11], we will show that this field describes the creation and annihilation processes of the bound state of two gluons interacting with three quarks. Such bound states of gluons are known as glueballs.

As one can see from transformation law (26), this law for the two-gluon field contains operators of the one-particle gluon field. However, as can be seen from the previous calculations in the case when the color state of a three-quark system is described by the Levi-Civita tensor, there are no terms containing the first powers of the elements of the Gell-Mann matrices in the expression for the Lagrangian because the convolution on the color indices leads to the trace $\mathrm{Sp}(\hat{\lambda}^{g_1})$. Since, in this case, single-particle fields enter the expression for the Lagrangian in the form $\hat{A}^{(n)}_{g_1}(q)\mathrm{Sp}(\hat{\lambda}^{g_1})$, they are also absent from the Lagrangian. That is, in the model considered, in the case of the colorless baryon state, such fields become unphysical because, since they do not enter the Lagrangian, their values are not determined by the system dynamics. Furthermore, since single-particle gluon fields do not enter the Lagrangian, for the invariance of the latter with respect to local $SU_c\,(3)$ transformations, there is no need to transform these fields according to law (24). Therefore, in all possible gauges, these fields can be set equal to zero. Then, instead of Eq. (26), we get

$$\hat{A}'^{(n_1n_2)}_{bd,g_1g_3}(q) = D_{g_1g_2}(q)\, D_{g_3g_4}(q)\, \hat{A}^{(n_1n_2)}_{bd,g_2g_4}(q) + \\ + \frac{\partial\theta_{g_1}(q)}{\partial q^b}\frac{\partial\theta_{g_3}(q)}{\partial q^d}. \quad (27)$$

The corresponding expression for the Lagrangian, instead of formula (20), looks like

$$\frac{L}{6} = \frac{i}{2}\left(\sum_{b=0}^{3}\left(\hat{\bar{\Psi}}_{s_1}(q)\,\gamma^b_{s_1s_2}\frac{\partial\hat{\Psi}_{s_2}(q)}{\partial q^b} - \right.\right. \\ \left.\left. - \frac{\partial\hat{\bar{\Psi}}_{s_1}(q)}{\partial q^b}\gamma^b_{s_1s_2}\hat{\Psi}_{s_2}(q)\right)\right) - (3m_q)\,\hat{\bar{\Psi}}_{s_1}(q)\,\hat{\Psi}_{s_1}(q) + \\ + \frac{1}{2\,(3m_q)}\sum_{b=4}^{9}\sum_{d=4}^{9} g^{bd}\frac{\partial\hat{\bar{\Psi}}_{s_1}(q)}{\partial q^b}\frac{\partial\hat{\Psi}_{s_1}(q)}{\partial q^d} - \\ - \frac{g^2}{(9m_q)}\hat{\bar{\Psi}}_{s_1}(q)\,\hat{\Psi}_{s_1}(q)\,\times \\ \times\sum_{b=4}^{9}\sum_{d=4}^{9} g^{bd}\delta^{g_1g_2}\left(\hat{A}^{(1,1)}_{bd,g_1g_2}(q) + \hat{A}^{(2,2)}_{bd,g_1g_2}(q) + \right. \\ + \hat{A}^{(3,3)}_{bd,g_1g_2}(q) - \hat{A}^{(1,2)}_{bd,g_1g_2}(q) - \hat{A}^{(1,3)}_{bd,g_1g_2}(q) - \\ \left. - \hat{A}^{(2,3)}_{bd,g_1g_2}(q)\right). \quad (28)$$

The Kronecker delta symbol $\delta^{g_1g_2}$ originates from the trace of the product of two Gell-Mann matrices.

Let us introduce the following notations:

$$\hat{A}^{(+)}_{bd,g_1g_2}(q) = \hat{A}^{(1,1)}_{bd,g_1g_2}(q) + \hat{A}^{(2,2)}_{bd,g_1g_2}(q) + \\ + \hat{A}^{(3,3)}_{bd,g_1g_2}(q), \\ \hat{A}^{(-)}_{bd,g_1g_2}(q) = \hat{A}^{(1,2)}_{bd,g_1g_2}(q) + \hat{A}^{(1,3)}_{bd,g_1g_2}(q) + \\ + \hat{A}^{(2,3)}_{bd,g_1g_2}(q), \\ \hat{V}_{bd,g_1g_2}(q) = \hat{A}^{(+)}_{bd,g_1g_2}(q) - \hat{A}^{(-)}_{bd,g_1g_2}(q), \\ \hat{V}(q) = \sum_{b=4}^{10}\sum_{d=4}^{10} g^{bd}\delta^{g_1g_2}\hat{V}_{bd,g_1g_2}(q). \quad (29)$$

As one can see from Eq. (27), the inhomogeneous terms in the transformation law for the tensor fields are identical for all types of those fields. Therefore, the field $\hat{V}_{bd,g_1g_2}(q)$ is transformed as the tensor product of two adjoint representations of the $SU_c(3)$ group, in contrast to the one-gluon field, which, owing to the presence of an inhomogeneous term, is not transformed according to any specific transformation of this group. The range of values of the field $\hat{V}(q)$ is a projection of the linear tensor space $\hat{V}_{bd,g_1g_2}$ onto an invariant subspace, on which the scalar representation of the group of transformations at changing from one inertial reference frame to another is realized, and the scalar representation of the group of local $SU_c(3)$ transformations. As was discussed in work [11], the group of transformations at changing the reference frame differs from the Lorentz group in that the boosts are replaced by identical transformations on the subspace of internal variables (the field $\hat{V}_{bd,g_1g_2}(q)$ and the fields from which it was constructed have nonzero components only on this subspace). Now formula (28) for the Lagrangian takes the form

$$\frac{L}{6} = \frac{i}{2}\left(\sum_{b=0}^{3}\left(\hat{\bar{\Psi}}_{s_1}(q)\,\gamma^b_{s_1s_2}\frac{\partial\hat{\Psi}_{s_2}(q)}{\partial q^b} - \right.\right. \\ \left.\left. - \frac{\partial\hat{\bar{\Psi}}_{s_1}(q)}{\partial q^b}\gamma^b_{s_1s_2}\hat{\Psi}_{s_2}(q)\right)\right) - (3m_q)\,\hat{\bar{\Psi}}_{s_1}(q)\,\hat{\Psi}_{s_1}(q) + \\ + \frac{1}{2\,(3m_q)}\sum_{b=4}^{9}\sum_{d=4}^{9} g^{bd}\frac{\partial\hat{\bar{\Psi}}_{s_1}(q)}{\partial q^b}\frac{\partial\hat{\Psi}_{s_1}(q)}{\partial q^d} + \\ + \frac{g^2}{(9m_q)}\hat{\bar{\Psi}}_{s_1}(q)\,\hat{\Psi}_{s_1}(q)\,\hat{V}(q). \quad (30)$$

6. Dynamic Model of Interaction between the Three-Particle Bispinor Field and the Two-Gluon Field

It is obvious that Lagrangian (30) has to be supplemented with the field Lagrangian $\hat{V}(q)$. For this purpose, let us first consider the tensor field $\hat{V}_{bd,g_1g_2}(q)$. Since this field transforms according to a certain representation of the local $SU_c(3)$ group, it is analogous to "matter fields", and the standard method of Lagrangian consrtuction can be applied. The Lagrangian of the free field $\hat{V}_{bd,g_1g_2}(q)$ can be chosen in the form

$$L_V = \frac{1}{2}g^{bb_1}g^{dd_1}g^{ll_1}\frac{\partial\hat{V}_{bd,g_1g_2}(q)}{\partial q^l}\frac{\partial\hat{V}_{b_1d_1,g_1g_2}(q)}{\partial q^{l_1}} - \\ - \frac{1}{2}M_G^2 g^{bb_1}g^{dd_1}\hat{V}_{bd,g_1g_2}(q)\,\hat{V}_{b_1d_1,g_1g_2}(q). \quad (31)$$

Here M_G denotes the mass of each of the particles (glueballs), whose creation and annihilation are described by the field operators $\hat{V}_{bd,g_1\ g_2}(q)$. This Lagrangian is not invariant with respect to the local field transformation as the tensor product of two adjoint representations of the $SU_c(3)$ group. Such invariance can be achieved using considerations similar to those that led to Lagrangian (28).

First, let us substitute the derivatives in the Lagrangian by covariant differentiation operators. The latter, for the field transforming as the tensor product of two adjoint representations of the $SU_c(3)$ group, have the form

$$\hat{D}_l\left(\hat{V}_{bd,g_1g_2}(q)\right)=\frac{\partial\hat{V}_{bd,g_1g_2}(q)}{\partial q^l}-$$
$$-ig\hat{A}^{(\mathrm{I})}_{l,g_3}(q)\,\hat{I}^{g_3}_{g_1g_4}\hat{V}_{bd,g_4g_2}(q)-$$
$$-ig\hat{A}^{(\mathrm{II})}_{l,g_5}(q)\,\hat{I}^{g_5}_{g_2g_6}\hat{V}_{bd,g_1g_6}(q). \qquad (32)$$

Here $\hat{I}^{g_1}_{g_2g_3}$ are matrix elements of the generators of the adjoint representation of the $SU_c(3)$ group, and $\hat{A}^{(\mathrm{I})}_{l,g_1}(q)$ and $\hat{A}^{(\mathrm{II})}_{l,g_1}(q)$ are gauge field operators. For $\hat{A}^{(\mathrm{I})}_{l,g_1}(q)$ and $\hat{A}^{(\mathrm{II})}_{l,g_1}(q)$, any fields with the transformation law (24) can be used. In particular, it can be the single-particle gauge fields $\hat{A}^{(n)}_{l,g_1}(q)$, $n=1,2,3$, which were considered earlier. Now, however, they are "spanned" by the generators of the adjoined representation rather than the generators of the fundamental representation of the $SU_c(3)$ group. The Lagrangian summand containing the convolution of covariant derivatives reads

$$g^{bb_1}g^{dd_1}g^{ll_1}\hat{D}_l\left(\hat{V}_{bd,g_1g_2}(q)\right)\hat{D}_{l_1}\left(\hat{V}_{b_1d_1,g_1g_2}(q)\right)=$$
$$=g^{bb_1}g^{dd_1}g^{ll_1}\left(\frac{\partial\hat{V}_{bd,g_1g_2}(q)}{\partial q^l}\,\frac{\partial\hat{V}_{b_1d_1,g_1g_2}(q)}{\partial q^{l_1}}-\right.$$
$$-i\hat{A}^{(\mathrm{I})}_{l_1,g_3}(q)\,\frac{\partial\hat{V}_{bd,g_1g_2}(q)}{\partial q^l}\hat{I}^{g_3}_{g_1g_4}\hat{V}_{b_1d_1,g_4g_2}(q)-$$
$$-i\hat{A}^{(\mathrm{II})}_{l_1,g_3}(q)\,\frac{\partial\hat{V}_{bd,g_1g_2}(q)}{\partial q^l}\hat{I}^{g_3}_{g_2g_4}\hat{V}_{b_1d_1,g_1g_4}(q)-$$
$$-i\hat{A}^{(\mathrm{I})}_{l,g_3}(q)\,\hat{I}^{g_3}_{g_1g_4}\hat{V}_{bd,g_4g_2}(q)\,\frac{\partial\hat{V}_{b_1d_1,g_1g_2}(q)}{\partial q^{l_1}}-$$
$$-i\hat{A}^{(\mathrm{II})}_{l,g_3}(q)\,\hat{I}^{g_3}_{g_2g_4}\hat{V}_{bd,g_1g_4}(q)\,\frac{\partial\hat{V}_{b_1d_1,g_1g_2}(q)}{\partial q^{l_1}}+$$
$$+g^2\hat{A}^{(\mathrm{I})}_{l,g_3}(q)\,\hat{A}^{(\mathrm{I})}_{l_1,g_5}(q)\,\hat{I}^{g_3}_{g_4g_1}\hat{I}^{g_5}_{g_1g_6}\times$$
$$\times\hat{V}_{bd,g_4g_2}(q)\,\hat{V}_{b_1d_1,g_6g_2}(q)+$$
$$+g^2\hat{A}^{(\mathrm{II})}_{l,g_5}(q)\,\hat{A}^{(\mathrm{II})}_{l,g_3}(q)\,\hat{I}^{g_5}_{g_6g_2}\hat{I}^{g_3}_{g_2g_4}\times$$
$$\times\hat{V}_{bd,g_1g_6}(q)\,\hat{V}_{b_1d_1,g_1g_4}(q)-$$
$$-2g^2\hat{A}^{(\mathrm{I})}_{l,g_3}(q)\,\hat{A}^{(\mathrm{II})}_{l_1,g_5}(q)\,\hat{I}^{g_3}_{g_1g_4}\hat{I}^{g_5}_{g_2g_6}\times$$
$$\left.\times\hat{V}_{bd,g_4g_2}(q)\,\hat{V}_{b_1d_1,g_1g_6}(q)\right). \qquad (33)$$

Here we took advantage of the fact that the matrix elements of the generators of the adjoint representation coincide with the structural constants and, therefore, are antisymmetric with respect to permutations of an arbitrary pair of indices. Now, without violating the Lagrangian invariance with respect to the local $SU_c(3)$ transformation, we can replace the products of single-particle fields $\hat{A}^{(\mathrm{I})}_{l,g_1}(q)\hat{A}^{(\mathrm{I})}_{l_1,g_2}(q)$, $\hat{A}^{(\mathrm{II})}_{l,g_1}(q)\hat{A}^{(\mathrm{II})}_{l_1,g_2}(q)$, and $\hat{A}^{(\mathrm{I})}_{l,g_1}(q)\hat{A}^{(\mathrm{II})}_{l_1,g_2}(q)$ by the tensors $\hat{A}^{(\mathrm{I,I})}_{ll_1,g_1g_2}(q)$, $\hat{A}^{(\mathrm{II,II})}_{ll_1,g_1g_2}(q)$, and $\hat{A}^{(\mathrm{I,II})}_{l\ l_1,g_1g_2}(q)$, which describe two-particle fields with the same transformation law as the indicated products have. Since the transformation law of a one-particle field coincides with Eq. (24), the transformation law of those tensors also coincides with Eq. (26).

Since the field $\hat{V}(q)$ in Lagrangian (30) is a projection of the linear tensor space $\hat{V}_{bd,g_1g_2}$ onto the invariant subspace where the scalar representations of the transformation group for changing from one reference frame to another are implemented, and the $SU_c(3)$ group, then it is convenient to distinguish the same projection in the Lagrangian as well for the field $\hat{V}_{bd,g_1g_2}(q)$. Expanding the linear tensor space $\hat{V}_{bd,g_1g_2}$ into a direct sum of invariant subspaces, we obtain

$$\hat{V}_{bd,g_1g_2}(q)=kg_{bd}\,\delta_{g_1g_2}\hat{V}(q)+..., \qquad (34)$$

where k is a normalizing coefficient, and "..." denotes the projections on the remaining invariant subspaces. Since these projections do not enetr the interaction Lagrangian (30), then, in order to obtain the simplest model at the initial stage, let us put them equal to zero. The coefficient k can be found by substituting expansion (34) into Eq. (29):

$$\hat{V}(q)=k\left(\sum_{b=4}^{9}\sum_{d=4}^{9}g^{bd}g_{bd}\delta^{g_1g_2}\delta_{g_1g_2}\right)\hat{V}(q). \qquad (35)$$

Making allowance for the form of metric tensor (8) and the fact that the indices g_1 and g_1 take values from 1 to 8, we get

$$k=\frac{1}{1350}. \qquad (36)$$

Taking the discussed transformations into account, the Lagrangian of the field $\hat{V}(q)$ looks like

$$L_V = \frac{1}{2}g^{ll_1}k\frac{\partial\hat{V}(q)}{\partial q^l}\frac{\partial\hat{V}(q)}{\partial q^{l_1}} - \frac{k}{2}M_G^2\hat{V}^2(q) +$$
$$+\frac{1}{2}k^2 g^{bb_1}g_{b_1d_1}g_{bd}g^{dd_1}g^{ll_1}g^2\hat{A}^{(\mathrm{I,I})}_{ll_1,g_3g_5}(q)\times$$
$$\times\hat{I}^{g_3}_{g_2g_1}\hat{I}^{g_5}_{g_1g_2}\hat{V}^2(q)+$$
$$+\frac{1}{2}k^2 g^{bb_1}g_{bd}g_{b_1d_1}g^{dd_1}g^{ll_1}g^2\hat{A}^{(\mathrm{II,II})}_{ll_1,g_5g_3}(q)\times$$
$$\times\hat{I}^{g_5}_{g_1g_2}\hat{I}^{g_3}_{g_2g_1}\hat{V}^2(q)-$$
$$-k^2 g^{bb_1}g_{bd}g_{b_1d_1}g^{dd_1}g^{ll_1}g^2\hat{A}^{(\mathrm{I,II})}_{ll_1,g_3g_5}(q)\times$$
$$\times\hat{I}^{g_3}_{g_1g_2}\hat{I}^{g_5}_{g_2g_1}\hat{V}^2(q). \tag{37}$$

As was shown above, single-particle gauge fields do not enter the Lagrangian and can be put equal to zero. In this case, the transformation law for two-particle fields coincides with Eq. (26). By direct calculations, it is possible to verify that the following identity holds for the generators of the adjoint representation:

$$\hat{I}^{g_3}_{g_1g_2}\hat{I}^{g_4}_{g_2g_1} = 2\delta^{g_3g_4}. \tag{38}$$

Taking into account this identity, the Lagrangian of the field $\hat{V}(q)$ takes the form

$$L_V = \frac{1}{2}g^{ll_1}k\frac{\partial\hat{V}(q)}{\partial q^l}\frac{\partial\hat{V}(q)}{\partial q^{l_1}} - \frac{k}{2}M_G^2\hat{V}^2(q) +$$
$$+g^2kg^{ll_1}\delta^{g_3g_5}\left(\hat{A}^{(\mathrm{I,I})}_{ll_1,g_3g_5}(q)+\hat{A}^{(\mathrm{II,II})}_{ll_1,g_5g_3}(q)-\right.$$
$$\left.-2\hat{A}^{(\mathrm{I,II})}_{ll_1,g_3g_5}(q)\right)\hat{V}^2(q). \tag{39}$$

Due to the mutual compensation of inhomogeneous terms in law (26), the field $\hat{A}^{(\mathrm{I,I})}_{ll_1,g_3g_5}(q)+\hat{A}^{(\mathrm{II,II})}_{ll_1,g_5g_3}(q)-2\hat{A}^{(\mathrm{I,II})}_{ll_1,g_3g_5}(q)$ transforms as the tensor product of two adjoint representations of the $SU_c(3)$ group. But the field $\hat{V}_{bd,g_1g_2}(q)$ also transforms according to the same law. All previous considerations did not determine the gauge fields themselves but only the law of their transformation, in order to achieve local $SU_c(3)$ invariance of the Lagrangian. Since the field $\hat{V}_{bd,g_1g_2}(q)$ has the required transformation law, then by using it as the field $\hat{A}^{(\mathrm{I,I})}_{ll_1,g_3g_5}(q)+\hat{A}^{(\mathrm{II,II})}_{ll_1,g_5g_3}(q)-2\hat{A}^{(\mathrm{I,II})}_{ll_1,g_3g_5}(q)$, we obtain the following $SU_c(3)$ locally invariant Lagrangian for the field $\hat{V}(q)$:

$$L_V = \frac{1}{2}g^{ll_1}k\frac{\partial\hat{V}(q)}{\partial q^l}\frac{\partial\hat{V}(q)}{\partial q^{l_1}} -$$
$$-\frac{k}{2}M_G^2\hat{V}^2(q)+g^2k\hat{V}^3(q). \tag{40}$$

Summing up this Lagrangian with Eq. (30), we arrive at the Lagrangian for the dynamic model of interacting multiparticle fields,

$$\frac{L}{6}=\frac{i}{2}\left(\sum_{b=0}^{3}\left(\hat{\bar{\Psi}}_{s_1}(q)\gamma^b_{s_1s_2}\frac{\partial\hat{\Psi}_{s_2}(q)}{\partial q^b}-\right.\right.$$
$$\left.\left.-\frac{\partial\hat{\bar{\Psi}}_{s_1}(q)}{\partial q^b}\gamma^b_{s_1s_2}\hat{\Psi}_{s_2}(q)\right)\right)-(3m_q)\hat{\bar{\Psi}}_{s_1}(q)\hat{\Psi}_{s_1}(q)+$$
$$+\frac{1}{2(3m_q)}\sum_{b=4}^{9}\sum_{d=4}^{9}g^{bd}\frac{\partial\hat{\bar{\Psi}}_{s_1}(q)}{\partial q^b}\frac{\partial\hat{\Psi}_{s_1}(q)}{\partial q^d}+$$
$$+\frac{g^2}{(9m_q)}\hat{\bar{\Psi}}_{s_1}(q)\hat{\Psi}_{s_1}(q)\hat{V}(q)+$$
$$+\frac{1}{2}g^{ll_1}\frac{k}{6}\frac{\partial\hat{V}(q)}{\partial q^l}\frac{\partial\hat{V}(q)}{\partial q^{l_1}}-\frac{1}{2}\frac{k}{6}M_G^2\hat{V}^2(q)+$$
$$+g^2\frac{k}{6}\hat{V}^3(q). \tag{41}$$

Instead of the field $\hat{V}(q)$, let us introduce a new field $\hat{u}(q)$ according to the relationship

$$\hat{V}(q)=-\sqrt{\frac{6}{k}}\hat{u}(q). \tag{42}$$

The choice of the sign in this expression will be explained below. After this substitution, we get

$$\frac{L}{6}=\frac{i}{2}\left(\sum_{b=0}^{3}\left(\hat{\bar{\Psi}}_{s_1}(q)\gamma^b_{s_1s_2}\frac{\partial\hat{\Psi}_{s_2}(q)}{\partial q^b}-\right.\right.$$
$$\left.\left.-\frac{\partial\hat{\bar{\Psi}}_{s_1}(q)}{\partial q^b}\gamma^b_{s_1s_2}\hat{\Psi}_{s_2}(q)\right)\right)-(3m_q)\hat{\bar{\Psi}}_{s_1}(q)\hat{\Psi}_{s_1}(q)+$$
$$+\frac{1}{2(3m_q)}\sum_{b=4}^{9}\sum_{d=4}^{9}g^{bd}\frac{\partial\hat{\bar{\Psi}}_{s_1}(q)}{\partial q^b}\frac{\partial\hat{\Psi}_{s_1}(q)}{\partial q^d}-$$
$$-\frac{g^2}{(9m_q)}\sqrt{\frac{6}{k}}\hat{\bar{\Psi}}_{s_1}(q)\hat{\Psi}_{s_1}(q)\hat{u}(q)+$$
$$+\frac{1}{2}g^{ll_1}\frac{\partial\hat{u}(q)}{\partial q^l}\frac{\partial\hat{u}(q)}{\partial q^{l_1}}-\frac{1}{2}M_G^2\hat{u}^2(q)-g^2\sqrt{\frac{6}{k}}\hat{u}^3(q). \tag{43}$$

Next, it is convenient to change from the variables q to new variables z according to the relationship

$$q = \begin{pmatrix} z^0 \\ z^1 \\ z^2 \\ z^3 \\ \sqrt{\frac{2}{9}}z^4 \\ \sqrt{\frac{2}{9}}z^5 \\ \sqrt{\frac{2}{9}}z^6 \\ \sqrt{\frac{1}{6}}z^7 \\ \sqrt{\frac{1}{6}}z^8 \\ \sqrt{\frac{1}{6}}z^9 \end{pmatrix}. \quad (44)$$

For further analysis, it is convenient to change to dimensionless quantities. As the characteristic mass, it is natural to choose the proton mass, which will be denotes as M_P. Then, the characteristic length equals M_P^{-1}. Since the action is a dimensionless quantity, the Lagrangian has a dimensionality of M_P^{10}. Next, we introduce the dimensionless parameters μ_q and m_G,

$$\mu_q = \frac{m_q}{M_P}, \quad m_G = \frac{M_G}{M_P}. \quad (45)$$

We also introduce the dimensionless coordinates ρ^a and the dimensionless fields $\hat{\bar{\psi}}_{s_1}(\rho)$, $\hat{\psi}_{s_1}(\rho)$, $\hat{v}(\rho)$:

$$\begin{aligned} &\rho^a = M_p z^a, \quad \hat{\bar{\Psi}}_{s_1}(\rho) = M_p^{2,5}\hat{\bar{\psi}}_{s_1}(\rho), \\ &\hat{\Psi}_{s_1}(\rho) = M_p^{2,5}\hat{\psi}_{s_1}(\rho), \quad \hat{u}(\rho) = M_p^2\hat{v}(\rho). \end{aligned} \quad (46)$$

Then, the Lagrangian dimensionality equals M_P^6. To achieve the required dimensionality, we have to multiply the whole Lagrangian by M_P^4. Since below we are interested in dynamic equations only, it is convenient to consider the dimensionless Lagrangian

$$\begin{aligned} l = \frac{i}{2}\Bigg(\sum_{b=0}^{3}\Bigg(\hat{\bar{\psi}}_{s_1}(\rho)\,\gamma^b_{s_1s_2}\frac{\partial\hat{\psi}_{s_2}(\rho)}{\partial\rho^b} - \\ -\frac{\partial\hat{\bar{\psi}}_{s_1}(\rho)}{\partial\rho^b}\gamma^b_{s_1s_2}\hat{\psi}_{s_2}(\rho)\Bigg)\Bigg) - \\ -\Bigg(\frac{1}{2\,(3\mu_q)}\sum_{b=4}^{10}\frac{\partial\hat{\bar{\psi}}_{s_1}(\rho)}{\partial\rho^b}\frac{\partial\hat{\psi}_{s_1}(\rho)}{\partial\rho^b} + \\ +\frac{g^2}{(9\mu_q)}\sqrt{\frac{6}{k}}\hat{\bar{\psi}}_{s_1}(\rho)\,\hat{\psi}_{s_1}(\rho)\,\hat{v}(\rho) + \\ +(3\mu_q)\,\hat{\bar{\psi}}_{s_1}(\rho)\,\hat{\psi}_{s_1}(\rho)\Bigg) + \\ +\frac{1}{2}\sum_{l_1=0}^{3}\sum_{l_2=0}^{3}g^{l_1l_2}\frac{\partial\hat{v}(\rho)}{\partial\rho^{l_1}}\frac{\partial\hat{v}(\rho)}{\partial\rho^{l_2}} - \\ -\frac{1}{2}\sum_{l=4}^{9}\left(\frac{\partial\hat{v}(\rho)}{\partial\rho^l}\right)^2 - \frac{1}{2}m_G^2\hat{v}^2(\rho) - g^2\sqrt{\frac{6}{k}}\,\hat{v}^3(\rho). \end{aligned} \quad (47)$$

For further consideration, it is convenient to distinguish the dimensionless coordinates of the center of mass and the dimensionless internal coordinates. Therefore, let us introduce the notations

$$\begin{aligned} &\rho_X^b = \rho^b, \quad b = 0,1,2,3, \\ &\rho_y^b = \rho^b, \quad b = 4,5,...,9. \end{aligned} \quad (48)$$

For the indicated variables, we will also use the notations ρ_X and ρ_y when talking about the whole set of certain dimensionless coordinates.

Let $v_0(\rho_y)$ be some function whose numerical value depends on the internal coordinates. Let us represent the field $\hat{v}(\rho)$ in the form

$$\hat{v}(\rho) = v_0(\rho_y)\,\hat{E} + \hat{v}_1(\rho). \quad (49)$$

where $\hat{v}_1(\rho)$ is a new dynamic variable (the operator-valued field function), and $\hat{E}$ is the unit operator. We also present the three-particle bispinor field in the form

$$\begin{aligned} &\hat{\psi}_{s_1}(\rho) = \hat{\Psi}_{s_1}(\rho_X)\,\phi(\rho_y), \\ &\hat{\bar{\psi}}_{s_1}(\rho) = \hat{\bar{\Psi}}_{s_1}(\rho_X)\,\phi^*(\rho_y). \end{aligned} \quad (50)$$

where $\hat{\bar{\Psi}}_{s_1}(\rho_X)$ and $\hat{\Psi}_{s_1}(\rho_X)$ are also new operator-valued field functions, and $\phi^*(\rho_y)$ and $\phi(\rho_y)$ are numerical, mutually complex conjugate functions. Using Eqs. (49) and (50), and performing integration by parts over the internal coordinates, Lagrangian (47) can be represented as the sum of three terms:

(i) the Lagrangian of the three-particle bispinor field,

$$\begin{aligned} l_\Psi = \frac{i}{2}\Bigg(\sum_{b=0}^{3}\Bigg(\hat{\bar{\Psi}}_{s_1}(\rho_X)\,\gamma^b_{s_1s_2}\frac{\partial\hat{\Psi}_{s_2}(\rho_X)}{\partial\rho_X^b} - \\ -\frac{\partial\hat{\bar{\Psi}}_{s_1}(\rho_X)}{\partial\rho_X^b}\gamma^b_{s_1s_2}\hat{\Psi}_{s_2}(\rho_X)\Bigg)\Bigg)\,\phi^*(\rho_y)\,\phi(\rho_y) - \\ -\phi^*(\rho_y)\Bigg(-\frac{1}{2\,(3\mu_q)}\sum_{b=4}^{10}\frac{\partial^2\phi(\rho_y)}{\left(\partial\rho_y^b\right)^2} + \end{aligned}$$

$$+\frac{g^2}{(9\mu_q)}\sqrt{\frac{6}{k}}v_0(\rho_y)\phi(\rho_y)+(3\mu_q)\phi(\rho_y)\Bigg)\times$$
$$\times\hat{\bar{\Psi}}_{s_1}(\rho_X)\hat{\Psi}_{s_1}(\rho_X),\qquad(51)$$

(ii) the Lagrangian of the two-particle gauge field,

$$l_v=\frac{1}{2}\sum_{l_1=0}^{3}\sum_{l_2=0}^{3}g^{l_1l_2}\frac{\partial\hat{v}_1(\rho)}{\partial\rho^{l_1}}\frac{\partial\hat{v}_1(\rho)}{\partial\rho^{l_2}}-$$
$$-\frac{1}{2}\sum_{l=4}^{9}\left(\frac{\partial v_0(\rho_y)}{\partial\rho^l}\hat{E}+\frac{\partial\hat{v}_1(\rho)}{\partial\rho^l}\right)^2-$$
$$-\frac{1}{2}m_G^2\Big(v_0(\rho_y)\hat{E}+\hat{v}_1(\rho)\Big)^2-$$
$$-g^2\sqrt{\frac{6}{k}}\Big(v_0(\rho_y)\hat{E}+\hat{v}_1(\rho)\Big)^3,\qquad(52)$$

(iii) and the Lagrangian of the interaction of those two fields,

$$l_{\text{int}}=-\frac{g^2}{(9\mu_q)}\sqrt{\frac{6}{k}}\hat{\bar{\Psi}}_{s_1}(\rho_X)\hat{\Psi}_{s_1}(\rho_X)\times$$
$$\times\phi^*(\rho_y)\phi(\rho_y)\hat{v}_1(\rho).\qquad(53)$$

Now we can consider the dynamics of the field system with this Lagrangian in the interaction representation with respect to the Lagrangian l_{int}. In other words, the field operators can be considered as solutions of the dynamical equations for the Lagrangian that is the sum $l_\Psi+l_v$, and the dynamics of the system state can be considered as if it is determined by the Hamiltonian generated by the interaction Lagrangian l_{int}. This way allows us to consider the dynamics of operators generated by the Lagrangian l_Ψ separately from the dynamics of operators generated by the Lagrangian l_v.

Let us first consider the Lagrangian l_Ψ. It can be rewritten in the form

$$l_\Psi=\frac{i}{2}\left(\sum_{b=0}^{3}\left(\hat{\bar{\Psi}}_{s_1}(\rho_X)\gamma^b_{s_1s_2}\frac{\partial\hat{\Psi}_{s_2}(\rho_X)}{\partial\rho^b_X}-\right.\right.$$
$$\left.\left.-\frac{\partial\hat{\bar{\Psi}}_{s_1}(\rho_X)}{\partial\rho^b_X}\gamma^b_{s_1s_2}\hat{\Psi}_{s_2}(\rho_X)\right)\right)\phi^*(\rho_y)\phi(\rho_y)-$$
$$-\phi^*(\rho_y)\hat{H}^{\text{int}}(\phi(\rho_y))\hat{\bar{\Psi}}_{s_1}(\rho_X)\hat{\Psi}_{s_1}(\rho_X),\qquad(54)$$

where the internal Hamiltonian $\hat{H}^{\text{int}}$ of the three-particle system is introduced,

$$\hat{H}^{\text{int}}(\phi(\rho_y))\equiv-\frac{1}{2(3\mu_q)}\sum_{b=4}^{10}\frac{\partial^2\phi(\rho_y)}{\left(\partial\rho^b_y\right)^2}+$$
$$+\frac{g^2}{(9\mu_q)}\sqrt{\frac{6}{k}}v_0(\rho_y)\phi(\rho_y)+(3\mu_q)\phi(\rho_y).\qquad(55)$$

As one can see from this expression, the product $\left(g^2/(9\mu_q)\sqrt{6/k}\right)v_0(\rho_y)$ before the function $\phi(\rho_y)$ in the second term plays the role of the potential energy of interaction between the quarks. Accordingly, the function $\phi(\rho_y)$ itself can be considered as the coordinate part of the internal state of the baryon. Since the dependence of the field functions on their arguments in the chosen interaction representation has to express the dynamics of a free baryon, the function $\phi(\rho_y)$ has to be an eigenfunction of the internal Hamiltonian (55) that corresponds to the smallest eigenvalue, i.e., the baryon mass. Aiming at describing the proton, let us put it equal to the proton mass M_P, i.e.,

$$M_p\phi(\rho_y)=-\frac{1}{2(3\mu_q)}\sum_{b=4}^{10}\frac{\partial^2\phi(\rho_y)}{\left(\partial\rho^b_y\right)^2}+$$
$$+\frac{g^2}{(9\mu_q)}\sqrt{\frac{6}{k}}v_0(\rho_y)\phi(\rho_y)+(3\mu_q)\phi(\rho_y).\qquad(56)$$

If the eigenfunction $\phi(\rho_y)$ is chosen to be normalized to unity, then the action for the Lagrangian l_Ψ is reduced to the ordinary expression for the bispinor fields, and the dynamic equations to a system of Dirac equations. Accordingly, the general solution of this system is a linear combination of negative- and positive-frequency solutions. In the framework of the standard quantization procedure [4], the negative- and positive-frequency coefficients describe the creation and annihilation, respectively, of particles with the proton mass, spin $\frac{1}{2}$, and the internal state that transforms according to the trivial representation of the $SU_c(3)$ group. In addition, since we are only interested in strong interaction, we did not consider explicitly the flavor state of the three-quark system, but it can be chosen to correspond to the proton state. That is, the creation and annihilation operators corresponding to the field $\hat{\Psi}_s(\rho_X)$ $(s=1,2,3,4)$ describe the creation and annihilation of protons.

Let us now consider the Lagrangian l_v (Eq. (52)). From its expression, one can see that the dynamic Lagrange–Euler equations can be obtained for the field $\hat{v}(\rho)=v_0(\rho_y)\hat{E}+\hat{v}_1(\rho)$ (see Eq. (49)). This means that the function $v_0(\rho_y)$ is a partial solution of those equations if $\hat{v}_1(\rho)=0$. Then the representation $\hat{v}(\rho)=v_0(\rho_y)\hat{E}+\hat{v}_1(\rho)$ can be considered in such a way that we have a “grand solution”

$v_0(\rho_y)$ that describes the interaction of quarks inside the baryon, and we quantize small fluctuations $\hat{v}_1(\rho)$ around it. On the basis of this circumstance and using Lagrangian (52), we obtain the following dynamic equation for the function $v_0(\rho_y)$:

$$\sum_{l=4}^{9} \frac{\partial^2 v_0(\rho_y)}{\left(\partial \rho_y^l\right)^2} - m_G^2 v_0(\rho_y) - 3g^2 \sqrt{\frac{6}{k}} (v_0(\rho_y))^2 = 0. \tag{57}$$

The dynamics of fluctuations $\hat{v}_1(\rho)$, with regard for Eq. (57), is described by the Lagrangian

$$l_{v_1} = \frac{1}{2} \sum_{l_1=0}^{3} \sum_{l_2=0}^{3} g^{l_1 l_2} \frac{\partial \hat{v}_1(\rho)}{\partial \rho^{l_1}} \frac{\partial \hat{v}_1(\rho)}{\partial \rho^{l_2}} - \frac{1}{2} \sum_{l=4}^{9} \left(\frac{\partial \hat{v}_1(\rho)}{\partial \rho^l}\right)^2 - \frac{1}{2} m_G^2 (\hat{v}_1(\rho))^2 - 3g^2 \sqrt{\frac{6}{k}} (v_0(\rho_y)) (\hat{v}_1(\rho))^2 - g^2 \sqrt{\frac{6}{k}} (\hat{v}_1(\rho))^3. \tag{58}$$

The last, cubic term $-g^2\sqrt{6/k}\,(\hat{v}_1(\rho))^3$ can be added to the interaction Lagrangian (53). Then, taking into account that we use the interaction representation with respect to the interaction Lagrangian

$$l_{\rm int}^1 = -\frac{g^2}{(9\mu_q)} \sqrt{\frac{6}{k}} \hat{\bar{\Psi}}_{s_1}(\rho_X) \hat{\Psi}_{s_1}(\rho_X) \times \phi^*(\rho_y) \phi(\rho_y) \hat{v}_1(\rho) - g^2 \sqrt{\frac{6}{k}} (\hat{v}_1(\rho))^3 \tag{59}$$

the dependence of the field $\hat{v}_1(\rho)$ on its arguments is described by a Lagrangian that, after integration by parts, can be written in the form

$$l_{v_1}^{(0)} = \frac{1}{2} \sum_{l_1=0}^{3} \sum_{l_2=0}^{3} g^{l_1 l_2} \frac{\partial \hat{v}_1(\rho)}{\partial \rho^{l_1}} \frac{\partial \hat{v}_1(\rho)}{\partial \rho^{l_2}} - \hat{v}_1(\rho) \left(-\frac{1}{2} \sum_{l=4}^{9} \left(\frac{\partial^2 \hat{v}_1(\rho)}{\partial (\rho^l)^2}\right) + \frac{1}{2} m_G^2 \hat{v}_1(\rho) + 3g^2 \sqrt{\frac{6}{k}} (v_0(\rho_y)) \hat{v}_1(\rho)\right). \tag{60}$$

The expression

$$\hat{H}_v^{\rm int}(\hat{v}_1(\rho)) = -\frac{1}{2} \sum_{l=4}^{9} \left(\frac{\partial^2 \hat{v}_1(\rho)}{\partial (\rho^l)^2}\right) + \frac{1}{2} m_G^2 \hat{v}_1(\rho) + 3g^2 \sqrt{\frac{6}{k}} (v_0(\rho_y)) \hat{v}_1(\rho) \tag{61}$$

formally looks like a result of the action of the internal Hamiltonian of a three-particle system with the potential energy $3g^2(6/k)\,v_0(\rho_y)$. Therefore, by representing the field $\hat{v}_1(\rho)$ in the form

$$\hat{v}_1(\rho) = \hat{V}_1(\rho_X)\, \phi_v(\rho_y), \tag{62}$$

where $\hat{V}_1(\rho_X)$ is a new operator-valued field function, and $\phi_v(\rho_y)$ is the eigenfunction of operator (61) that corresponds to the eigenvalue that we denote as $\mu_G^2/2$, i.e.,

$$\frac{\mu_G^2}{2} \phi_v(\rho_y) = -\frac{1}{2} \sum_{l=4}^{9} \left(\frac{\partial^2 \phi_v(\rho_y)}{\partial \left(\rho_y^l\right)^2}\right) + \frac{1}{2} m_G^2 \phi_v(\rho_y) + 3g^2 \sqrt{\frac{6}{k}} (v_0(\rho_y)) \phi_v(\rho_y), \tag{63}$$

we obtain

$$l_{v_1}^{(0)} = \left(\frac{1}{2} \sum_{l_1=0}^{3} \sum_{l_2=0}^{3} g^{l_1 l_2} \frac{\partial \hat{V}_1(\rho_X)}{\partial \rho_X^{l_1}} \frac{\partial \hat{V}_1(\rho_X)}{\partial \rho_X^{l_2}} - \frac{\mu_G^2}{2} \left(\hat{V}_1(\rho_X)\right)^2\right) (\phi_v(\rho_y))^2. \tag{64}$$

If the eigenfunction $\phi_v(\rho_y)$ of the operator $\hat{H}_v^{\rm internal}$ [Eq. (61)] is normalized to unity, then substituting Lagrangian (64) into the expression for the action and integrating over the variables ρ_y, which are internal for the field $\hat{V}_1(\rho_X)$, we get the usual Lagrangian of a real scalar field, for which the quantization procedure leads to the appearance of the creation and annihilation operators for particles with the mass μ_G.

As one can see from Eq. (34), the field $\hat{V}_1(\rho_X)$ is associated with the expansion of the linear space of two-index tensors in a direct sum of its invariants subspaces. Therefore, the creation and annihilation operators corresponding to this field must change the occupation numbers of two-gluon states. At the same time, operator (61) and its eigenfunction $\phi_v(\rho_y)$ are three-particle. Therefore, we cannot interpret operator (61) as an internal Hamiltonian of the two-gluon state, and its eigenfunction as the function describing this state.

This situation can be explained by the fact that we consider both single-particle gluon fields, which were introduced when extending derivatives (18), and two-particle fields, which were introduced in Eq. (28) as the fields interacting with the internal state of quarks in the baryon. Such an interaction can provide information about the measurement result concerning the locations of three quarks in a baryon, but it cannot serve as an information source about the location of two gluons inside a two-particle state. This happens because at least one gluon must interact simultaneously with at least two quarks, which can be detected at two different points when performing the measurement. Therefore, under such conditions, interaction with quarks cannot transform a pair of gluons into a state that is an eigenstate for their relative coordinates. As a result, the eigenfunction $\phi_v(\rho_y)$ of operator (61) does not describe the state of a gluon pair. However, since the two-gluon fields interact with the baryon through their interactions with the quarks inside it, the interaction vertex (it becomes nonlocal) must take into account the internal structure of the baryon.

Taking into account Eq. (62), the interaction Lagrangian (59) can be rewritten in the form

$$l^1_{\text{int}} = -\frac{g^2}{(9\mu_q)}\sqrt{\frac{6}{k}}\hat{\bar{\Psi}}_{s_1}(\rho_X)\hat{\Psi}_{s_1}(\rho_X)\times \\ \times \hat{V}_1(\rho_X)\phi^*(\rho_y)\phi(\rho_y)\phi_v(\rho_y) - \\ - g^2\sqrt{\frac{6}{k}}\left(\hat{V}_1(\rho_X)\right)^3(\phi_v(\rho_y))^3. \quad (65)$$

Whence one can see that the function $\phi_v(\rho_y)$ describes both the vertex of the proton interaction with the two-gluon field and the self-action vertex for the two-gluon field interacting with the three-particle proton field.

Note that the square of the absolute value of the function $\phi(\rho_y)$, which is an eigenfunction of the internal Hamiltonian (55) and is considered by us as the coordinate part of the eigenfunction for the energy of the internal state of the system of quarks in the proton, has a probabilistic meaning. At the same time, the interaction vertex in the chosen interaction representation affects the time dependence of the state of the relativistic quantum system and has no direct probabilistic meaning. Therefore, the non-local vertex is not described by the function $\phi(\rho_y)$, and this fact explains the appearance of a new function $\phi_v(\rho_y)$ in the Lagrangian vertices.

At the same time, as was shown in papers [11, 24], on the basis of speculations analogous to those used in the presented paper for the three-quark field, the two-gluon field can be considered independently of the three-quark one. In so doing, similarly to the appearance of the internal Hamiltonian of the three-quark system in the previous considerations, there arises the internal Hamiltonian of the two-gluon system, and its eigenfunctions can already be interpreted as characterizing the eigenstates of the two-gluon system. By separating the dependence of the field functions of the three-quark field on the Jacobian coordinates (they correspond to the center of mass) and the dependence on the internal variables, one can reach a coincidence between the Lagrangian of the operator-valued field function of the coordinates of the center of mass, after its integration over the internal variables, and Lagrangian (64), also after its integration over the internal variables. This fact makes it possible to consider only one field $\hat{V}_1(\rho_X)$ rather than two different two-gluon fields. The field $\hat{V}_1(\rho_X)$ interacts with protons and mesons [11, 24], and acts on itself according to the law $\sim\left[\hat{V}_1(\rho_X)\right]^3$. This circumstance can be used to describe the processes of elastic and inelastic proton scattering.

The previous relationships include the function $v_0(\rho_y)$ in the form of potential energy (with an accuracy to the coefficient). This function is defined by Eq. (57). Let us consider a spherically symmetric solution of this equation, i.e., such a solution where the independent variables are included in the combination

$$r = \sqrt{\sum_{a=4}^{9}(\rho^a)^2}. \quad (66)$$

In this case, Eq. (57) takes the form

$$\frac{d^2v_0(r)}{dr^2} + \frac{5}{r}\frac{dv_0(r)}{dr} - m_G^2 v_0(r) - \\ - 3g^2\sqrt{\frac{6}{k}}(v_0(r))^2 = 0. \quad (67)$$

The analysis of the properties of the solutions of this equation was made in detail in work [28]. The conclusion was made that under certain boundary conditions, the solution of this equation tends to infinity as the argument $r \to \infty$. This result can be physically interpreted as the confinement of quarks in the three-quark system under consideration. Again, under certain boundary conditions, Eq. (67) gives a potential energy with negative eigenvalues in the discrete spectrum, which correspond to bound states with no confinement. A possible physical interpretation of this case is discussed in the next section. A similar case of the bound state of two gauge bosons (it was considered in works [11, 29]) describes the mechanism of spontaneous symmetry breaking.

7. Discussion of the Results and Conclusions

The model of multi-particle fields proposed in this and previous works can be used when attempting to describe experiments on elastic and inelastic hadron scattering. In the case of proton-proton collisions, we have a three-particle bispinor field corresponding to protons and antiprotons, and this field interacts with a global two-particle field. This global field is self-acting owing to the self-action of the non-Abelian gauge field and can interact with a two-particle meson field or other three-particle fields. Those components can be used to construct diagrams corresponding to elastic and inelastic scattering processes. In so doing, non-perturbative effects will be described by internal Hamiltonians of multi-particle fields. Such a description turned out non-relativistic, and this fact can be explained on the basis of the arguments presented in work [11].

The essence of those arguments consists in that quantum mechanics is an inherently non-local theory. If we consider a measurement in the coordinate representation, then the process of interaction with a measurement device must be implemented in such a way that the particles in the system could interact with that device at any point of some region. That is, the device is not localized at some point, but it is distributed over the region. As a result, when the device interacts with the particles of a multi-particle system in some region, the state of such a system changes instantaneously because every particle can interact with the device at any point, and, therefore, the change of the state does not need the propagation of interaction from one point in the space-time to another. Accordingly, the finiteness of the propagation velocity ceases to be significant in this case, and, therefore, the description becomes similar to the non-relativistic one.

At the same time, in this paper, we considered the simplest version of the multi-particle model in order to obtain a description that would be the closest to the standard one-particle theory. Therefore, when discussing the meaning of the quantized multi-particle field, we assumed that its dependence on the coordinates of the center of mass can be separated from the dependence on the internal variables. This assumption is not critically important for the many-particle field quantization. Indeed, to give the field operators the meaning of the creation and annihilation ones, only the law of their transformation at a space-time shift is essential [4]. But such a transformation does not affect the internal variables and changes only the space-time coordinates of the center of mass. Therefore, how the internal variables enter the dependence of the field operator on its arguments does not play a substantial role in the quantization procedure.

One can try to obtain other solutions to the dynamic equations considered in this paper and interpret them physically. In particular, the most obvious generalization of the model considered in this work is to expand the multi-particle operators in the eigenstates of the internal Hamiltonian rather than confine the consideration to the ground state contribution only, as we did in this work. It is also of interest to consider models, where the number of particles in the internal states of interacting hadrons change and where multi- and single-particle fields interact. However, for now, it is still unclear how the relevant terms could be introduced into the internal Hamiltonians, proceeding from the gauge-based principle of introducing interactions.

In effect, it can be said that the problem discussed in this paper is a consequence of the contradiction between the non-local character of quantum mechanics and the local character of the quantum field theory. Perhaps this contradiction manifests itself in the well-known fact that the integrals corresponding to some Feynman diagrams diverge. As is known [4], those divergences arise because of the uncertainty in the chronological pairing of the field operators when their time arguments coincide, i.e., just on the subset of simultaneity, which was considered above. Those divergences arise on the light cone, i.e., on the subset of points of the tensor product of two Minkowski spaces that separates the region of this product containing points in common with the subset of simultaneity from the region with no such points. On the other hand, those divergences arise for some diagrams with loops, i.e., multi-particle intermediate states. However, for such states, another dynamics, different from that described by single-particle Green's functions, is possible on the subset of simultaneity. Perhaps, the solutions of multi-particle equations, which were considered in paper [28], mentioned in the previous section, and do not lead to confinement, could be useful in this case. Then, for the multi-particle problem, we will obtain a continuous spectrum, and the states of this spectrum should be taken into account when integrating over intermediate multi-particle states.

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ТРИЧАСТИНКОВІ ПОЛЯ ЯК МЕТОД
ОПИСУ БАРІОНІВ У ПРОЦЕСАХ РОЗСІЯННЯ

Запропоновано модель тричастинкових полів для опису баріонів у процесах пружного і непружного розсіяння. Модель дозволяє описати утримання кварків усередині адрону і їх конфайнмент, і одночасно вона описує взаємодію кварків різних адронів у процесі їх зіткнення. Така взаємодія забезпечується обміном зв'язаними станами двох глюонів, які теж знаходяться в стані конфайнменту.

Ключові слова: багаточастинкові поля, підмножина одночасності, конфайнмент.