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DAMPING OF MAGNETOELASTIC WAVES

A general method for constructing a model of the dissipative function describing the relaxation processes induced by the damping of coupled magnetoacoustic waves in magnetically ordered materials has been developed. The obtained model is based on the symmetry of the magnet and describes both exchange and relativistic interactions in the crystal. The model accounts for the contributions of both the magnetic and elastic subsystems to the dissipation, as well as the relaxation associated with the magnetoelastic interaction. The dispersion law for coupled magnetoelastic waves is calculated in the case of a uniaxial ferromagnet of the "easy axis" type. It is shown that the contribution of the magnetoelastic interaction to dissipative processes can play a significant role in the case of magnetoacoustic resonance.

Keywords: magnetoelastic interaction, dissipative function, dispersion law, uniaxial ferromagnet, relaxation.

1. Introduction

Coupled magnetoelastic oscillations comprise a complicated natural phenomenon, which is a result of the interaction between the magnetic subsystem of a crystal and the crystal lattice. Vibrations of this type have been a subject of extensive researches for many years [1, 2], and a number of important fundamental results have been obtained for them [3–5].

The influence of the magnetoelastic interaction on the spectra of spin and elastic oscillations is rather small, as a rule. However, this interaction reveals itself substantially in the case of magnetoacoustic resonance, when the spin-wave frequency approaches the sound frequency, and the quasispin and quasisound branches in the wave spectrum begin to repulse each other [3, 4]. The results of relevant studies also demonstrate that the magnetoelastic interaction increases, if magnetically ordered systems approach the point of spin-reorientation phase transition [4] or structural phase transition in the lattice [6, 7].

The study of the phenomena arising owing to the interaction between the magnetic and elastic subsystems has gained a new impetus recently. This is a consequence of numerous experimental studies [8–11] carried out with magnetically ordered systems, where the magnetoelastic interaction can be rather strong [10, 11]. The results of modern researches, both experimental [11] and theoretical [12], testify to the creation of special conditions for the propagation of oscillations under the influence of the magnetoelastic interaction. The latter can lead to the non-consistent propagation of both the spin and elastic waves in multilayer structures of various types [11, 12]. The corresponding results may testify that the influence of the magnetoelastic interaction can be used in modern functional elements applied for the quantum information processing and based on the magnonics and magnetic spintronics principles [13, 14].

A complete description of collective magnetoelastic oscillations is evidently impossible, if their damping is not taken into account. Unfortunately, the scope of few studies dealing with the dissipation of magnetoelastic oscillations is confined to the consideration of the damping of only spin waves on the basis of the relaxation term in the Gilbert form [15]. It should be noted that the consideration of this issue in the framework of the Landau–Lifshitz [16] or Gilbert [15] model using the corresponding relaxation term is not correct [17, 18]. Those models do not take the symmetry of a crystal into account. This factor directly affects the propagation of elastic waves. Furthermore, in many cases, the presence of spatial inhomogeneities testifies that the relaxation processes of the exchange origin must be taken into consideration. Of course, an approach in which the damping of only spin waves is considered cannot be correct as well, because it absolutely ignores the relaxation processes that may be associated with the magnetoelastic interaction. A simultaneous account for the purely elastic relaxation is also important in this situation.

Earlier, we have developed a phenomenological theory to describe relaxation phenomena in magnets. This theory makes it possible to consider the exchange relaxation [19, 20]. In this work, proceeding from the principles expounded in the cited works, we propose a model that describes the dissipation of coupled magnetoelastic waves, as well as a mechanism for constructing the dissipative function for such oscillations in the general form.

2. Dissipative Function for Magnetoelastic Waves

While constructing the dissipative function that would describe the damping of collective magnetoacoustic waves, we proceed from the expression for the total energy (the quasiequilibrium thermodynamic potential) of a ferromagnet,

$$F = \int f\left(\mathbf{M},\ \frac{\partial \mathbf{M}}{\partial x_i}\right) dV, \tag{1}$$

where $\mathbf{M} = \mathbf{M}(\mathbf{r}, t)$ is the magnetization vector, and $f(\mathbf{M},\ \partial\mathbf{M}/\partial x_i)$ is the total energy density. In our case, the latter parameter consists of the magnetic, f_m, elastic, f_e, and magnetoelastic, f_{me}, components:

$$f(\mathbf{M},\ \partial\mathbf{M}/\partial x_i) = f_m + f_e + f_{me}. \tag{2}$$

The construction of the required quasiequilibrium thermodynamic potential for a ferromagnet is a fundamental result of work [16]. This construction is based on the crystal-symmetry considerations and the grouping of interactions in a ferromagnet into two classes: weak relativistic interactions and strong exchange interactions. A not less fundamental result is the derivation of the equation for the dynamics of the magnetic moment, which was called the Landau–Lifshitz equation,

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}^{\text{eff}} + \mathbf{R}, \tag{3}$$

and the introduction of the concept of effective magnetic field as a variational derivative of the thermodynamic potential of the ferromagnet with respect to the magnetization,

$$\mathbf{H}^{\text{eff}} = -\frac{\delta F}{\delta \mathbf{M}} = -\frac{\partial F}{\partial \mathbf{M}} + \frac{\partial}{\partial x_i}\frac{\partial F}{\partial \frac{\partial \mathbf{M}}{\partial x_i}} - \\ - \frac{\partial^2}{\partial x_i^2}\frac{\partial F}{\partial \frac{\partial^2 \mathbf{M}}{\partial x_i^2}} + \ldots + (-1)^{n+1}\frac{\partial^n}{\partial x_i^n}\frac{\partial F}{\partial \frac{\partial^n \mathbf{M}}{\partial x_i^n}}. \tag{4}$$

The term $\mathbf{R}$ in Eq. (3) is responsible for the magnetization relaxation. It was proposed by Landau on the basis of general physical ideas concerning dissipative processes [16]. Later, Gilbert constructed the dissipative function of a ferromagnet that corresponds to the Landau–Lifshitz relaxation and proposed to express the relaxation term in terms of the time derivative of the magnetization [15]. Despite the vector character of the equation of motion, the Landau–Lifshitz–Gilbert relaxation term is characterized by a single relaxation constant, which corresponds to an isotropic medium. A consideration of the expression for the relaxation term in the framework of the models in [15, 16] shows that it does not make allowance for the symmetry of a magnetic material. As a result, there arise a lot of physical contradictions [17, 18]. It is also important to note that the relaxation term in the Landau–Lifshitz or Gilbert form corresponds to the spin-spin and spin-orbit interactions, so that there is no opportunity to consider the dissipative processes associated with the exchange interaction in a crystal, which are important in many cases [19, 20].

In their classical works [21, 22], L.D. Landau and E.M. Lifshitz proposed to describe the relaxation processes by introducing the corresponding dissipative function into the equations of motion. This function must be a positive quadratic form. According to the basic phenomenological principles, the dissipative function $Q = \int q dV$ can be constructed following the same rules as for the quasiequilibrium thermodynamic potential, and it must include the terms of the same origin, as the total energy of the crystal does [21, 22]. Therefore, it is quite reasonable to represent the dissipative function density q similarly to expression (2), i.e. as the sum of three terms, each describing the relaxation processes of the magnetic, elastic, or magnetoelastic origin,

$$q = q_m + q_e + q_{me}. \tag{5}$$

The procedure of constructing the dissipative function for magnetic oscillations was described in our works [19, 20]. In particular, the term q_m can be expressed as a quadratic form of the effective magnetic field and its spatial derivatives,

$$q_m = \frac{1}{2}\lambda^r_{ik}\, H^{\rm eff}_i H^{\rm eff}_k + \frac{1}{2}\,\lambda^{\rm ex}_{ik}\frac{\partial \mathbf{H}^{\rm eff}}{\partial x_i}\frac{\partial \mathbf{H}^{\rm eff}}{\partial x_k} + ..., \tag{6}$$

where the tensors λ^r_{ik} and $\lambda^{\rm ex}_{ik}$ characterize relativistic and exchange, respectively, dissipative processes.

The elastic component q_e of the dissipative function density (5) must depend on the time derivatives of the strain tensor and also must be quadratic [21]. So, the most general form for this component is

$$q_e = \frac{1}{2}\eta_{ij,sp}\,\frac{\partial E_{ij}}{\partial t}\frac{\partial E_{sp}}{\partial t}. \tag{7}$$

The fourth-rank tensor $\eta_{ij,sp}$ is called the viscosity tensor, and its components are determined by the crystal symmetry, like the components of the elasticity tensor that enters the expression for the elastic energy [21].

The magnetoelastic component q_{me} of the dissipative function is constructed on the basis of similar principles and considerations. From expressions (6) and (7), it follows that q_{me} must consist of the time derivatives of the strain tensor and the components of the effective magnetic field. It is known that the dissipative function must be invariant with respect to the transformations of the crystal symmetry group. Therefore, the magnetoelastic component of the dissipative function has to be constructed from the invariant products of the time derivatives of the strain tensor and the gradients of the effective magnetic field in the form of the quadratic form

$$q_{me} = \frac{1}{2}\beta_{ij,sp}\,\frac{\partial E_{ij}}{\partial t}\left(\frac{\partial H^{\rm eff}_s}{\partial x_p} + \frac{\partial H^{\rm eff}_p}{\partial x_s}\right). \tag{8}$$

Here, the tensor $\beta_{ij,sp}$ characterizes the contribution of the magnetoelastic interaction to the energy dissipation of coupled oscillations and, by analogy with the viscosity tensor, has to be of the fourth rank. Hence, the effective magnetic field cannot enter the component q_{me} linearly. The dissipative function must also be invariant with respect to the time reflection operation. Therefore, q_{me} cannot also include even power exponents of the effective magnetic field (or its gradients).

Let us demonstrate the procedure of constructing the dissipative function for a ferromagnet with uniaxial symmetry following the proposed method. For this purpose, expressions for the components of the total energy density (2) have to be written down at first. The magnetic part of the energy of the examined uniaxial ferromagnet in an external magnetic field $\mathbf{H}$ looks like

$$f_m = \frac{(\boldsymbol{\mu}^2-1)^2}{8\chi} + \frac{1}{2}\alpha_{ik}\frac{\partial \boldsymbol{\mu}}{\partial x_i}\frac{\partial \boldsymbol{\mu}}{\partial x_k} - \\ - \frac{1}{2}K_1\mu_z^2 - \frac{1}{2}K_2\mu_z^4 - \mathbf{MH}, \tag{9}$$

where χ is the longitudinal magnetic susceptibility, α_{ik} is the tensor that characterizes the inhomogeneous exchange interaction (for the simplicity, the case $\alpha_{ik} = \mathrm{diag}(\alpha,\alpha,\alpha)$ will be considered), K_1 and K_2 are the uniaxial anisotropy constants (all the constants have the energy dimensionality), $\boldsymbol{\mu} = \mathbf{M}/M_0$ is the normalized magnetization vector, and M_0 is the saturation magnetization. The first term in expression (9) makes allowance for the homogeneous exchange interaction, which can make an essential contribution to the exchange dissipative processes. This contribution is especially important when describing the dissipation of magnetic solitons [23–26], including domain walls [25] and Bloch points [26].

The elastic energy of the uniaxial crystal can be written in the form [21]

$$f_e = \frac{1}{2}C_{11}\left(E_{xx}+E_{yy}\right)^2 + \frac{1}{2}C_{33}E_{zz}^2 + \\ + C_{13}\left(E_{xx}+E_{yy}\right)E_{zz} + 2C_{44}\left(E_{xz}+E_{yz}\right)^2 + \\ + \frac{1}{2}C_{66}(E_{xx}^2 + E_{yy}^2 + 2E_{xy}^2), \tag{10}$$

where E_{ik} are the components of the strain tensor, and C_{ik} are the elastic moduli of the second order (the components of the elasticity tensor) for the crystal concerned.

Finally, the last term in expression (2) describes the interaction between the magnetic and elastic subsystems. In the case of uniaxial crystal symmetry, it looks like [22, 27]

$$f_{me} = \frac{1}{2}B_{11}\left(\mu_x^2+\mu_y^2\right)(E_{xx}+E_{yy}) + \frac{1}{2}B_{33}\mu_z^2 E_{zz} + \\ + \frac{1}{2}B_{13}\mu_z^2(E_{xx}+E_{yy}) + \frac{1}{2}B_{31}\left(\mu_x^2+\mu_y^2\right)E_{zz} + \\ + \frac{1}{2}B_{44}(\mu_x\mu_z E_{xz} + \mu_y\mu_z E_{yz}) + \\ + \frac{1}{2}B_{66}(\mu_x^2 E_{xx} + \mu_y^2 E_{yy} + 2\mu_x\mu_y E_{xy}), \quad (11)$$

where B_{ik} are the constants of magnetoelastic interaction corresponding to this case.

Following the results of works [19, 20], let us write down the tensors that characterize the relaxation of spin waves in a ferromagnet with uniaxial symmetry in the form $\lambda_{ik}^r = \mathrm{diag}(\lambda^r, \lambda^r, 0)$ and $\lambda_{ik}^{\mathrm{ex}} = \mathrm{diag}(\lambda^{\mathrm{ex}}, \lambda^{\mathrm{ex}}, \lambda^{\mathrm{ex}})$. Then, for the magnetic component (6) of the dissipative function, we obtain

$$q_m = \frac{1}{2}\lambda^r\left(\left(H_x^{\mathrm{eff}}\right)^2 + \left(H_y^{\mathrm{eff}}\right)^2\right) + \frac{1}{2}\lambda^{\mathrm{ex}}\left(\frac{\partial \mathbf{H}^{\mathrm{eff}}}{\partial x_i}\right)^2. \quad (12)$$

The elastic component (7) of the dissipative function should be constructed analogously to the corresponding component (10) of the total energy. Hence, for a uniaxial crystal, it looks like

$$q_e = \frac{1}{2}\eta_{11}\left(\frac{\partial E_{xx}}{\partial t} + \frac{\partial E_{yy}}{\partial t}\right)^2 + \frac{1}{2}\eta_{33}\left(\frac{\partial E_{zz}}{\partial t}\right)^2 + \\ + \eta_{13}\left(\frac{\partial E_{xx}}{\partial t} + \frac{\partial E_{yy}}{\partial t}\right)\frac{\partial E_{zz}}{\partial t} + 2\eta_{44}\left(\frac{\partial E_{xz}}{\partial t} + \frac{\partial E_{yz}}{\partial t}\right)^2 + \\ + \frac{1}{2}\eta_{66}\left(\left(\frac{\partial E_{xx}}{\partial t}\right)^2 + \left(\frac{\partial E_{yy}}{\partial t}\right)^2 + 2\left(\frac{\partial E_{xy}}{\partial t}\right)^2\right). \quad (13)$$

The magnetoelastic component q_{me} of the dissipative function is constructed proceeding from the general form (8) and similarly to the corresponding componemt of the total ferromagnet energy,

$$q_{me} = \beta_{11}\left(\frac{\partial E_{xx}}{\partial t} + \frac{\partial E_{yy}}{\partial t}\right)\left(\frac{\partial H_x^{\mathrm{eff}}}{\partial x} + \frac{\partial H_y^{\mathrm{eff}}}{\partial y}\right) + \\ + \beta_{33}\frac{\partial E_{zz}}{\partial t}\frac{\partial H_z^{\mathrm{eff}}}{\partial z} + \beta_{13}\left(\frac{\partial E_{xx}}{\partial t} + \frac{\partial E_{yy}}{\partial t}\right)\frac{\partial H_z^{\mathrm{eff}}}{\partial z} + \\ + \beta_{31}\frac{\partial E_{zz}}{\partial t}\left(\frac{\partial H_x^{\mathrm{eff}}}{\partial x} + \frac{\partial H_y^{\mathrm{eff}}}{\partial y}\right) + \\ + 2\beta_{44}\frac{\partial E_{xz}}{\partial t}\left(\frac{\partial H_x^{\mathrm{eff}}}{\partial z} + \frac{\partial H_z^{\mathrm{eff}}}{\partial x}\right) + \\ + 2\beta_{44}\frac{\partial E_{yz}}{\partial t}\left(\frac{\partial H_y^{\mathrm{eff}}}{\partial z} + \frac{\partial H_z^{\mathrm{eff}}}{\partial y}\right) + \\ + \beta_{66}\left(\frac{\partial E_{xx}}{\partial t}\frac{\partial H_x^{\mathrm{eff}}}{\partial x} + \frac{\partial E_{yy}}{\partial t}\frac{\partial H_y^{\mathrm{eff}}}{\partial y}\right) + \\ + 2\,\beta_{66}\frac{\partial E_{xy}}{\partial t}\left(\frac{\partial H_x^{\mathrm{eff}}}{\partial y} + \frac{\partial H_y^{\mathrm{eff}}}{\partial x}\right). \quad (14)$$

Thus, formulas (12), (13), and (14) allow us to obtain a complete expression for the dissipative function. On its basis, the frequencies of the coupled magnetoelastic oscillations making allowance for the oscillation damping can be calculated.

3. Dispersion Law for Magnetoelastic Waves in a Uniaxial "Easy-Axis" Ferromagnet

On the basis of the results obtained above, let us calculate the dispersion law for coupled magnetoelastic waves in a uniaxial "easy-axis" ferromagnet in the ground state. In this case, the magnetic moment of the ferromagnet, μ, is directed along the easy-magnetization axis Oz, and the condition for this state to exist is $K_1 + K_2 > 0$ [28]. In accordance with the standard method used to phenomenologically describe the dynamics of the magnetic moment [3, 20], let us consider small adiabatic fluctuations of the magnetic moment density in the ferromagnet. Then we can write

$$\boldsymbol{\mu}(\mathbf{r}, t) = \boldsymbol{\mu}_0 + \mathbf{m}(\mathbf{r}, t), \quad (15)$$

where $\boldsymbol{\mu}_0 = (0, 0, 1)$ is the vector of the "easy-axis" phase magnetization in the equilibrium state, and $\mathbf{m}(\mathbf{r}, t)$ are small deviations of the fluctuation origin from $\boldsymbol{\mu}_0$.

Analogously, the components of the strain tensor E_{ik} can be expressed as the sums of the equilibrium values E_{ik0} and small deviations ε_{ik} from them,

$$E_{ik}(\mathbf{r}, t) = E_{ik}^0 + \varepsilon_{ik}(\mathbf{r}, t). \quad (16)$$

The equilibrium values of the strain tensor components in the ground state of a uniaxial ferromagnet can be easily found from the condition $\partial f/\partial E_{ik} =$

0. In this case, only the following equilibrium components differ from zero [5]:

$$E^0_{xx} = E^0_{yy} = \frac{B_{13}C_{33} - B_{33}C_{13}}{2(2C^2_{13} - C_{33}(2C_{11} + C_{66}))},$$
$$E^0_{zz} = \frac{-B_{13}C_{13} - B_{33}(2C_{11} + C_{66})}{2(2C^2_{13} - C_{33}(2C_{11} + C_{66}))}. \tag{17}$$

The inhomogeneous term of the elastic strain tensor, $\varepsilon_{ik}(\mathbf{r}, t)$, can be expressed in terms of the vector of particle displacements $\mathbf{U}$ using the formula [4]

$$\varepsilon_{ik}(\mathbf{r}, t) = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right). \tag{18}$$

The dispersion laws for coupled magnetoelastic waves can be calculated using the dynamic equation for the magnetization vector (3), which, making allowance for definition (15), takes the form

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma\mu \times \mathbf{H}^{\text{eff}} + \mathbf{R}_m, \tag{19}$$

as well as the dynamic equation for the particle displacement vector $\mathbf{U}$ [4, 21],

$$\rho\frac{\partial^2 \mathbf{U}}{\partial t^2} = -\frac{\delta F}{\delta \mathbf{U}} + \mathbf{R}_e. \tag{20}$$

Here, $\gamma \approx \frac{2|\mu_B|}{\hbar}$ is the gyromagnetic ratio, and ρ is the magnet material density.

The relaxation terms in the dynamic equations (19) and (20) can be obtained as the variations of the magnet dissipative function [20, 21],

$$\mathbf{R}_m = \frac{\delta Q}{\delta \mathbf{H}^{\text{eff}}}, \quad \mathbf{R}_e = \frac{\delta Q}{\delta\left(\frac{\partial \mathbf{U}}{\partial t}\right)}. \tag{21}$$

Let us consider the oscillations, whose wave vector $\mathbf{k}$ is directed along the magnetic moment of the crystal, $\mathbf{k} \parallel \mu_0 \parallel Oz$. In this case, only the following components of the elastic-strain tensor and the gradients of the effective magnetic field remain different from zero:

$$\varepsilon_{zz} = \frac{\partial U_z}{\partial z}, \quad \varepsilon_{xz} = \varepsilon_{zx} = \frac{1}{2}\frac{\partial U_x}{\partial z},$$
$$\varepsilon_{yz} = \varepsilon_{zy} = \frac{1}{2}\frac{\partial U_y}{\partial z}, \tag{22}$$

and

$$\frac{\partial H^{\text{eff}}_x}{\partial z}, \quad \frac{\partial H^{\text{eff}}_y}{\partial z}, \quad \frac{\partial H^{\text{eff}}_z}{\partial z}.$$

By applying relations (22) to components (12), (13), and (14) of the dissipative function and taking advantage of definitions (21), the following components are obtained:

for the magnetic relaxation term,

$$R_{mx} = \lambda^r H^{\text{eff}}_x - \lambda^{\text{ex}}\frac{\partial^2 H^{\text{eff}}_x}{\partial z^2} - \beta_{44}\frac{\partial}{\partial t}\frac{\partial^2 U_x}{\partial z^2},$$
$$R_{my} = \lambda^r H^{\text{eff}}_y - \lambda^{\text{ex}}\frac{\partial^2 H^{\text{eff}}_y}{\partial z^2} - \beta_{44}\frac{\partial}{\partial t}\frac{\partial^2 U_y}{\partial z^2}, \tag{23}$$
$$R_{mz} = -\lambda^{\text{ex}}\frac{\partial^2 H^{\text{eff}}_z}{\partial z^2} - \beta_{33}\frac{\partial}{\partial t}\frac{\partial^2 U_z}{\partial z^2},$$

and for the elastic relaxation term,

$$R_{\text{ex}} = -\eta_{44}\frac{\partial}{\partial t}\left(\frac{\partial^2 U_x}{\partial z^2} + \frac{\partial^2 U_y}{\partial z^2}\right) - \beta_{44}\frac{\partial^2 H^{\text{eff}}_x}{\partial z^2},$$
$$R_{ey} = -\eta_{44}\frac{\partial}{\partial t}\left(\frac{\partial^2 U_x}{\partial z^2} + \frac{\partial^2 U_y}{\partial z^2}\right) - \beta_{44}\frac{\partial^2 H^{\text{eff}}_y}{\partial z^2}, \tag{24}$$
$$R_{ez} = -\eta_{33}\frac{\partial}{\partial t}\frac{\partial^2 U_z}{\partial z^2} - \beta_{33}\frac{\partial^2 H^{\text{eff}}_z}{\partial z^2}.$$

For further calculations, let us expand the total energy density (2) in a power series in small deviations $\mathbf{m}(\mathbf{r}, t)$ and ε_{ik}. Then we should substitute the result and the relaxation terms (23) and (24) into the dynamic equations (19) and (20) and linearize them. Afterward, we should pass in the resulting equations to the Fourier components of the small deviations $\mathbf{m}(\mathbf{r}, t) \sim \exp(-i(\omega t - \mathbf{kr}))$ and $\mathbf{U}(\mathbf{r}, t) \sim \exp(-i(\omega t - \mathbf{kr}))$ with respect to the time t and the coordinates r, where ω is the frequency and $\mathbf{k}$ the wave vector of collective waves. As a result, Eqs. (8) and (9) bring about a system of six equations for the components of the vectors $\mathbf{m}$ and $\mathbf{U}$. From the condition that this system of equations has a solution (the determinant of the system has to be equal to zero), it is possible to obtain the dispersion law for coupled magnetoacoustic oscillations taking their damping into account. In the linear approximation in the dissipative constants, the dispersion law looks like

$$\omega^2\left(\omega^2 - \frac{C_{33}}{\rho}k^2\right)\left(\omega^2 - \frac{2C_{44}}{\rho}k^2\right)\left(\omega^2 - \gamma^2 M_0^2 \omega^2_{m\parallel}\right) -$$
$$-\left(\omega^2 - \frac{C_{33}}{\rho}k^2\right)\left(\omega^2 - \frac{C_{44}}{\rho}k^2\right)\frac{\gamma^2 B^2_{44} k^2 \omega_{m\parallel}}{8\rho} +$$
$$+ i\omega\left(\omega^2 - \frac{C_{33}}{\rho}k^2\right)\left(\omega^2 - \frac{2C_{44}}{\rho}k^2\right) \times$$

$$\times\,\omega^2\omega_{m1}\left(\lambda^r+\lambda^{\rm ex}k^2\right)+$$
$$+\,i\omega\left(\omega^2-\frac{C_{33}}{\rho}k^2\right)\left(\omega^2-\frac{2C_{44}}{\rho}k^2\right)\times$$
$$\times\left(\omega^2-\gamma^2M_0^2\omega_{m\|}^2\right)\left(\omega_{m2}\lambda^{\rm ex}k^2\right)-$$
$$-\,i\omega\left(\omega^2-\frac{2C_{44}}{\rho}k^2\right)\left(\omega^2-\gamma^2M_0^2\omega_{m\|}^2\right)\frac{\eta_{33}\omega^2k^2}{\rho}-$$
$$-\,i\omega\left(\omega^2-\frac{C_{33}}{\rho}k^2\right)\left(\omega^2-\gamma^2M_0^2\omega_{m\|}^2\right)\frac{\eta_{44}\omega^2k^2}{\rho}+$$
$$+\,i\left(\omega^2-\frac{C_{33}}{\rho}k^2\right)\left(\omega^2-\frac{C_{44}}{\rho}k^2\right)\times$$
$$\times\,\frac{\gamma^2B_{44}\beta_{44}\omega_{m\|}\omega_{m3}k^3}{2\rho}=0. \tag{25}$$

Here, the following notations were introduced:

$$\omega_{m1}=\frac{\alpha k^2}{M_0^2}+\frac{(B_{11}+B_{66})E_{xx}^0}{M_0^2}+\frac{B_{11}E_{yy}^0}{M_0^2}+$$
$$+\frac{B_{31}E_{zz}^0}{M_0^2}+\frac{(M_z^2-M_0^2)}{2M_0^2\chi},$$
$$\omega_{m2}=\frac{\alpha k^2}{M_0^2}+\frac{B_{13}\left(E_{xx}^0+E_{yy}^0\right)}{M_0^2}+\frac{B_{33}E_{zz}^0}{M_0^2}-\frac{K_1}{M_0^2}-$$
$$-\frac{3K_2}{M_0^2}+\frac{(3M_z^2-M_0^2)}{2M_0^2\chi},$$
$$\omega_{m3}=\frac{H}{M_0}-\frac{B_{13}\left(E_{xx}^0+E_{yy}^0\right)}{M_0^2}-\frac{B_{33}E_{zz}^0}{M_0^2}+$$
$$+\frac{K_1}{M_0^2}+\frac{K_2}{M_0^2}+\frac{(M_0^2-M_z^2)}{2M_0^2\chi},$$
$$\omega_{m\|}=\omega_{m1}+\omega_{m3}.$$

Thus, the general method presented in this work to describe the dissipative processes makes it possible to calculate the spectrum of coupled magnetoacoustic waves taking their damping into account. Let us analyze the obtained result.

4. Discussion and Conclusions

The calculated dispersion law (25) consists of seven terms. Each of them characterizes the corresponding dynamic process in the ferromagnet. If the relaxation processes and the magnetoelastic interaction in the magnet are neglected, only the first term remains in expression (25). Then the latter decays into three independent spectral equations; one for spin waves [28],

$$\omega_{\rm sw}^2=\gamma^2M_0^2\omega_{m\|}^2$$

and two for acoustic waves [21],

$$\omega_{\rm ph2}^2=\frac{2C_{44}}{\rho}k^2,\quad \omega_{\rm ph5}^2=\frac{C_{33}}{\rho}k^2.$$

The combination of first and second terms gives the spectrum of coupled magnetoelastic waves without taking their damping into account [5]. The third and fourth terms characterize the damping of spin waves and, together with the first term, give the corresponding dispersion law [18–20]. The fifth and sixth terms describe the energy dissipation in the $\omega_{\rm ph2}$ and $\omega_{\rm ph5}$, respectively, sound modes. The seventh term is responsible for the influence of the magnetoelastic interaction on the energy dissipation of coupled oscillations.

It is well known that the magnetoelastic interaction makes a substantial contribution to the dispersion law of coupled oscillations, when their frequency approaches the magnetoacoustic resonance. In work [5], it was shown that, for the "easy-axis" ground state and the wave vector direction $\mathbf{k}\parallel\mu$, this occurs, when $\omega^2\to\frac{2C_{44}}{\rho}k^2$. In this case, only the first, second, fifth, and seventh terms in expression (25) survive. The terms responsible for the spin contribution to the dissipation become of the second order of smallness and can be omitted. So, the relaxation of magnetoelastic waves makes the magnetoelastic contribution (seventh term) that is comparable with the purely elastic contribution (fifth term). This means that the contribution of the magnetoelastic interaction to dissipative processes can play an essential role in the case of magnetoacoustic resonance.

The result obtained for a uniaxial ferromagnet has a general character and, no doubt, can also be valid for magnetically ordered materials of other types.

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ЗАТУХАННЯ МАГНІТОПРУЖНИХ ХВИЛЬ

Р е з ю м е

Представлено загальний метод побудови моделі дисипативної функції, що описує релаксаційні процеси, зумовлені затуханням зв'язаних магнітоакустичних хвиль у магнітовпорядкованих матеріалах. Отримана модель дисипативної функції базується на врахуванні симетрії магнетика та описує як обмінну, так і релятивістичну взаємодію в кристалі. При цьому враховано внески в дисипацію як магнітної і пружної підсистеми, так і релаксацію, пов'язану з магнітопружною взаємодією. Розраховано закон дисперсії зв'язаних магнітопружних хвиль для одноосного феромагнетика типу "легка вісь". Показано, що внесок магнітопружної взаємодії в дисипативні процеси може відігравати суттєву роль у випадку магнітоакустичного резонансу.