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CHIRAL ASYMMETRY IN RELATIVISTIC MATTER IN A MAGNETIC FIELD

In this mini review, we consider chiral asymmetry in the normal ground state of magnetized relativistic matter in the NJL model with local four-fermion interaction and QED. It is shown that the chiral shift parameter associated with the relative shift of the longitudinal momenta (along the direction of the magnetic field) in the dispersion relations for opposite chirality fermions is dynamically generated in the normal ground state. This contribution affects fermions in all Fermi levels, including those around the Fermi surface, and contributes to the non-dissipative axial current taking place in relativistic matter in a magnetic field. The chiral asymmetry of the normal ground state in QED matter in a magnetic field is characterized by an additional chiral structure. It formally looks like that of the chiral chemical potential, but is an odd function of the longitudinal component of momentum along the magnetic field. The origin of this parity-even chiral structure is directly connected with the long-range character of the QED interaction. The leading radiative corrections to the chiral separation effect in QED are calculated, and the form of the Fermi surface in the weak magnetic field is determined.

Keywords: dense relativistic matter, magnetic field, axial current.

CONTENTS

1. Introduction

Dense relativistic matter in strong magnetic fields naturally exists in compact stars. The central regions of neutron stars are characterized by the highest matter densities that occur in nature and may exceed up to ten times the density of the nuclear matter. This may lead to the appearance of quark matter in the cores of neutron stars. If such a quark matter exists, then it will be a strongly coupled relativistic matter. It is also very important that neutron stars are usually characterized by very strong magnetic fields that could reach up to 10^{15} G in magnetars [1, 2] (for a recent review of theoretical developments in studies of dense matter in compact stars, see Ref. [3]). Many physical properties of the stellar matter under extreme conditions realized inside compact stars are understood theoretically and could be tested to some extent through observational data.

Relativistic matter in a strong magnetic field is created too in heavy ion collisions [4]. Such a matter is actively studied both experimentally and theoretically. The dynamics of fermions and the chiral asymmetry of magnetized relativistic matter has attracted much attention during the last years. In relativistic matter, chemical potential is much larger than the mass of particles. In such a case, helicity, which is the projection of the spin of particles on their momenta, is related to chirality. Consequently, unlike non-relativistic matter, where spin polarization in a magnetic field is studied usually, it is more appropriate to investigate the chiral asymmetry in magnetized relativistic matter. At present, this is an active research area in particle physics with important developments also in condensed matter physics.

It was suggested in Refs. [5, 6] that the topological charge changing transitions in QCD during heavy ion collisions may result in the appearance of metastable domains with $\mathcal{P}$ and $\mathcal{CP}$ breaking with chirality induced in quark-gluon plasma by the axial anomaly [7]. Phenomenologically, to mimic the effect of topological charge changing transitions, it was proposed in [8] to introduce a chiral chemical potential μ_5. This chemical potential couples to the difference between the number of left- and right-handed fermions and enters the Lagrangian density through the term $\mu_5\bar{\psi}\gamma^0\gamma^5\psi$. This produces a chiral asymmetry in magnetized relativistic matter and leads to a non-dissipative electric current $\mathbf{j} = e^2\mathbf{B}\mu_5/(2\pi^2)$ in the presence of an external magnetic field $\mathbf{B}$ [6, 8, 9]. This phenomenon is known in the literature as the chiral magnetic effect (CME) (for a recent review, see Ref. [10]). Moreover, the charge-dependent correlations and flow, observed in heavy-ions collisions at RHIC [11–14] and LHC [15], appear to be in a qualitative agreement with the predictions of the CME [16, 17] (see, however, the recent discussion in Ref. [18]).

Unlike the chiral chemical potential, which is a rather exotic quantity and not so well defined theoretically, the chemical potential μ (associated, for example, with conserved electric or baryon charge) is common in many physical systems. Therefore, in this mini review, we shall consider only systems with ordinary chemical potential. (This paper is based on a series of recent papers with V.A. Miransky, I.A. Shovkovy, and Xinyang Wang.) It was shown in Refs. [19–21] that a non-dissipative axial current $\mathbf{j}_5 = e\mathbf{B}\mu/(2\pi^2)$ exists in the equilibrium state of noninteracting massless fermion matter in a magnetic field. This effect is known as the chiral separation effect (CSE) in the literature (for a brief review, see Sec. 2 in Ref. [10]). In fact, as suggested in Refs. [22, 23], the CSE may lead to a chiral charge separation (i.e., effectively inducing a nonzero chiral chemical potential μ_5) and, thus, trigger the CME even in the absence of topological fluctuations in the initial state.

The physical and mathematical reasons for the chiral asymmetry in relativistic matter in a magnetic field are quite transparent (for a recent elegant exposition, see Ref. [24]). In a free theory, the magnetic field $\mathbf{B}$ projects the spins of fermions on the lowest Landau level (LLL) along the direction of the magnetic field. Since fermions can freely propagate in a magnetic field only along or opposite to the direction of $\mathbf{B}$, magnetized relativistic matter responds chirally asymmetrically to the magnetic field. This leads to the appearance of a non-dissipative axial current $\mathbf{j}_5 = e\mathbf{B}\mu/(2\pi^2)$ [19, 20].

It was argued in Refs.[20, 21] that non-dissipative currents in magnetized relativistic matter are completely determined by the topological currents induced only in the LLL and intimately connected with the chiral anomaly. This fact is directly connected with the well known result that the chiral anomaly is also generated in a magnetic field only in the LLL [25].

However, the dense relativistic matter in a strong magnetic field may hold some new theoretical surprises. In particular, it was shown in Ref. [26] that the normal ground state of such matter is characterized by a dynamically generated chiral shift parameter Δ. It enters the effective Lagrangian density through the following quadratic term: $\Delta\bar{\psi}\gamma^3\gamma^5\psi$. The meaning of this parameter is most transparent in the chiral limit: it determines a relative shift of the longitudinal momenta in the dispersion relations of opposite chirality fermions, $k^3 \to k^3 \pm \Delta$, where the momentum k^3 is directed along the magnetic field. This suggests a possible connection between the parameter Δ and the axial current along the direction of the magnetic field. Taking into account that fermions in *all* Landau levels, including those around the Fermi surface, are affected by Δ, the corresponding matter may have unusual transport and/or emission properties.

It is instructive to discuss the symmetry properties of the chiral shift parameter Δ. Just like the external magnetic field, the Δ term, being symmetric with respect to parity transformations $\mathcal{P}$, breaks time reversal $\mathcal{T}$ and the rotational symmetry $SO(3)$ down to $SO(2)$ (i.e., the rotations about the axis set by the magnetic field). Moreover, since the Δ term is even under charge conjugation $\mathcal{C}$, it breaks $\mathcal{CPT}$ symmetry, which is also broken by the fermion density. Therefore, the absence of the chiral shift parameter is not protected by any symmetry, which, in turn, suggests that such a term should be dynamically generated even by perturbative dynamics.

The dynamics responsible for the generation of the chiral shift parameter in the Nambu–Jona-Lasinio model was studied at finite temperature beyond the chiral limit in Ref. [27]. It was shown that Δ is rather insensitive to temperature when $T \ll \mu$, where μ is the chemical potential, and *increases* with T when $T > \mu$. The first regime is appropriate for stellar matter, and the second one is realized in heavy ion collisions.

The important question is whether the induced axial current coincides with that in the theory of noninteracting fermions in a magnetic field [20] or whether it is affected by interactions (for related discussions, see Refs. [22, 26–31]) through, for example, the dynamically generated chiral shift parameter. The chiral anomaly is exact as an operator relation, but it contains the divergence of the axial current rather than the current itself. Consequently, to get the axial current from the chiral anomaly, one should "integrate" the anomaly and calculate the ground state expectation value of the corresponding operator. Then, the question concerning an "integration constant" in the induced axial current and its dependence on interactions naturally arises. Till now, no conclusive answer to this question was given (e.g., see the discussion in Ref. [10]). The first studies of interaction effects on the chiral asymmetry of relativistic matter in a magnetic field were done in Refs. [26, 27, 31] by using Nambu–Jona-Lasinio (NJL) like models with local interaction. In particular, by using the Schwinger–Dyson (gap) equation, it was found that the interaction unavoidably generates a chiral shift parameter Δ [26, 27], when the fermion density is nonzero. Furthermore, as shown in Refs. [22, 26, 27], the chiral shift Δ is responsible for an additional contribution to the axial current.

It was recently shown in Ref. [22] that while the dynamics responsible for the generation of the chiral shift Δ essentially modifies the form of this current, it does *not* affect the form of the chiral anomaly. Moreover, while the topological contribution to the axial current is generated in the infrared kinematic region (at the LLL), the contribution of Δ to this current is mostly generated in the ultraviolet region, which implies that higher Landau levels are important in this case. The chiral asymmetry for noninteracting fermions exists only in the LLL. On the other hand, the chiral shift parameter found in Refs. [26, 27] in the NJL model is the same for all Landau levels. This means that the whole Fermi surfaces of the left- and right-handed fermions are shifted relative to each other by 2Δ in the longitudinal direction in the momentum space. Such an unusual ground state closely resembles that in Weyl semimetals in condensed matter physics [32, 33], in which quasiparticles are described by the Weyl equation.

The study of Dirac and Weyl semimetals is at present the very active area of research in condensed matter physics. A relativistic-like electron spectrum in these materials results in their unique electronic and transport properties [34–38] intimately related to the chiral anomaly. A different property of Dirac and Weyl semimetals was considered in Ref. [39]: a dynamical rearrangement of their Fermi surfaces in a magnetic field. It was shown that the manifestation of this effect in Dirac semimetals is quite spectacular: a Dirac semimetal is transformed into a Weyl one. The resulting Weyl semimetal has a pair of Weyl nodes for each of the original Dirac points. Each pair of the nodes is separated by a dynamically induced chiral shift, whose direction coincides with the direction of the magnetic field. The magnitude of the chiral shift is determined by the strengths of the magnetic field, the quasiparticle charge density, and the strength of the interaction.

Since the NJL model is nonrenormalizable and the chiral anomaly is intimately connected with ultraviolet divergencies, in order to reach a solid conclusion about the presence or absence of higher-order radiative corrections to the axial current, one should consider them in a renormalizable model. In the recent paper [40], assuming that the magnetic field $\mathbf{B}$ is weak and using the expansion in powers of $\mathbf{B}$ up to linear order, the leading radiative corrections to the axial current were calculated in dense QED in a magnetic field. It was found that, like in the NJL model, the axial current is not fixed by the chiral anomaly relation and does not coincide with the expression in the free theory, solely provided by the LLL. Instead, it receives nontrivial radiative corrections produced at all Landau levels. Because of that, it is natural to expect that, like in the NJL model, the chiral asymmetry in QED will be induced by interactions in higher Landau levels too. These expectations were confirmed in Ref. [41].

The paper is organized as follows. The chiral asymmetry of the normal ground state of relativistic matter in a magnetic field in the Nambu–Jona-Lasinio model is studied in Sec. 2. The general properties of the model and the gap equation are considered in Subsec. 2.1. The dynamical generation of the chiral shift parameter and induced axial current are studied in Subsec. 2.2. In Subsec. 2.3, it is shown that although the chiral shift parameter contributes to the axial current, it does not modify the chiral anomaly. The leading radiative corrections to the chiral separation effect in QED are considered in Sec. 3. The chiral asymmetry of the normal state of magnetized relativistic matter in QED is studied in Sec. 4. The results are summarized and conclusions are given in Sec. 5.

2. Chiral Asymmetry in Nambu–Jona-Lasinio Model

In this section, we study chiral asymmetry of the normal ground state in magnetized relativistic matter in the Nambu–Jona-Lasinio model.

2.1. Model and gap equation

The Lagrangian of the NJL model with one fermion flavor reads

$$\mathcal{L} = \bar{\psi}\left(iD_\nu + \mu_0\delta^0_\nu\right)\gamma^\nu\psi - m_0\bar{\psi}\psi + \frac{G_{\rm int}}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\psi\right)^2\right], \quad (1)$$

where m_0 is the bare fermion mass and μ_0 is the chemical potential. By definition, $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$. The covariant derivative $D_\nu = \partial_\nu + ieA_\nu$ includes the vector potential A_ν, which describes the external magnetic field B pointing in the z-direction in the Landau gauge, $A^\nu = xB\delta^\nu_2$.

In the chiral limit, $m_0 = 0$, this model possesses the chiral $U(1)_L \times U(1)_R$ symmetry, which is known to be spontaneously broken in the vacuum state ($\mu_0 = 0$) because of the magnetic catalysis phenomenon [42,43] due to the enhanced pairing dynamics of fermions and antifermions in the infrared region. The enhancement results from the non-vanishing density of states in the LLL that is subjected to an effective dimensional reduction $D \to D-2$. At a sufficiently large value of the chemical potential, the chiral symmetry is expected to be restored. As we shall see below, the corresponding normal ground state is characterized by a nonzero chiral shift parameter Δ.

We shall analyze model (1) in the mean field approximation, which is reliable in the weakly coupled regime when the dimensionless coupling constant $g \equiv G_{\rm int}\Lambda^2/(4\pi^2) \ll 1$, where Λ is an ultraviolet cutoff. We consider the following ansatz for the inverse full propagator:

$$iG^{-1}(u,u') = \Big[(i\partial_t + \mu)\gamma^0 - (\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) + i\tilde{\mu}\gamma^1\gamma^2 + \Delta\gamma^3\gamma^5 - m\Big]\delta^4(u-u'), \quad (2)$$

where $u = (t, \boldsymbol{r})$, $\pi^k = i(\partial^k + ieA^k)$ is the canonical momentum, m is the dynamical fermion mass, μ is an effective chemical potential in the fermion dispersion relation, $\tilde{\mu}$ is an anomalous magnetic moment, and Δ is the chiral shift parameter.

In the mean field approximation, the gap equation reduces to the following set of equations (one may check that $\tilde{\mu} = 0$ is a self-consistent solution of the gap equation in the mean field approximation):

$$\mu - \mu_0 = -\frac{1}{2}G_{\rm int}\langle j^0\rangle, \quad (3)$$

$$\Delta = -\frac{1}{2}G_{\rm int}\langle j^3_5\rangle, \quad (4)$$

$$m - m_0 = -G_{\rm int}\langle\bar{\psi}\psi\rangle, \quad (5)$$

where the chiral condensate $\langle\bar{\psi}\psi\rangle$, vacuum expectation values of the fermion density j^0, and axial current density j^3_5 are

$$\langle j^0\rangle = -{\rm tr}\left[\gamma^0 G(u,u)\right], \quad (6)$$

$$\langle j^3_5\rangle = -{\rm tr}\left[\gamma^3\gamma^5 G(u,u)\right], \quad (7)$$

$$\langle\bar{\psi}\psi\rangle = -{\rm tr}\left[G(u,u)\right]. \quad (8)$$

We shall consider only the normal phase with $m = m_0 = 0$ and $\langle\bar{\psi}\psi\rangle = 0$. This phase is realized when the chemical potential $\mu_0 > m_{\rm dyn}/\sqrt{2}$ [26], where $m_{\rm dyn}$ is a dynamical fermion mass in a magnetic field at zero chemical potential and zero temperature.

It is very instructive to analyze Eqs. (3) and (4) in perturbation theory in the coupling constant g. In the zeroth approximation, we have a theory of free fermions in a magnetic field with $\mu = \mu_0$ and $\Delta = 0$. However, even in this case, the fermion density $\langle j^0\rangle$ and the axial current density $\langle j^3_5\rangle$ are nonzero. The former can be presented as a sum over the Landau levels:

$$\langle j^0\rangle_0 = \frac{\mu_0|eB|}{\pi^2}\left[\frac{1}{2} + \sum_{n=1}^{\infty}\sqrt{\mu_0^2 - 2n|eB|}\,\theta\left(|\mu_0| - \sqrt{2n|eB|}\right)\right], \quad (9)$$

and the latter is [20]

$$\langle j^3_5\rangle_0 = \frac{eB}{2\pi^2}\mu_0. \quad (10)$$

In the next order in the coupling constant, one finds from Eq. (4) that $\Delta \propto G_{\rm int}\langle j_5^3\rangle_0 \neq 0$. Thus, in the normal phase of this theory, there *necessarily* exists a nonzero chiral shift parameter Δ. In fact, this is one of the main results of Ref. [26]. Let us also emphasize that the result that Δ is generated in perturbation theory is directly connected with the fact that the vanishing Δ is not protected as we discussed in the Introduction by any symmetry. Thus, a nonzero Δ is an unavoidable consequence in interacting systems.

2.2. Induced axial current

In this subsection, using the gauge invariant point-splitting regularization (for a review of this regularization, see Refs. [44, 45]), we study the influence of the chiral shift parameter Δ on the form of the axial current in the NJL model, where as Eq.(4) implies $\Delta = \text{const}$ in all Landau levels. Our main conclusion is that Δ essentially changes the form of the axial current. Moreover, while the contribution of the chemical potential in the axial current is generated in the infrared kinematic region (at the LLL) [20], the contribution of Δ in the current is mostly generated in the ultraviolet region (at all Landau levels).

As it is well known, the general form of the fermion propagator in a constant magnetic field is [46]

$$G(u,u') = e^{i\Phi(u,u')}\bar{G}(u-u') \tag{11}$$

with the Schwinger phase

$$\Phi(u,u') = e\int\limits_{u'}^{u} dx^\nu A_\nu, \tag{12}$$

where the integration is performed along the straight line. The translation invariant part $\bar{G}(u-u')$ depends only on the field strength $F_{\mu\nu}$.

In the normal phase with $m = \tilde{\mu} = 0$, the inverse propagator (2) can be rewritten as

$$iG^{-1} = iD_\nu\gamma^\nu + \mu\gamma^0 + \Delta\gamma^3\gamma^5 = \left(iD_\nu\gamma^\nu + \mu\gamma^0 - - \Delta s_\perp\gamma^3\right)\mathcal{P}_5^- + \left(iD_\nu\gamma^\nu + \mu\gamma^0 + \Delta s_\perp\gamma^3\right)\mathcal{P}_5^+, \tag{13}$$

where $s_\perp \equiv \text{sign}(eB)$, $D_\nu = \partial_\nu + ieA_\nu$, and $\mathcal{P}_5^\mp = (1 \mp s_\perp\gamma^5)/2$. This equation implies that the effective electromagnetic vector potential equals $A_\nu^- = A_\nu + (s_\perp\Delta/e)\delta_\nu^3$ and $A_\nu^+ = A_\nu - (s_\perp\Delta/e)\delta_\nu^3$ for the $-$ and $+$ chiral fermions, respectively. Since the field strength $F_{\mu\nu}$ for $A_\nu^\mp$ is the same as for A_ν, Δ affects only the Schwinger phase (12):

$$\Phi_\Delta^-(u,u') = \Phi(u,u') + s_\perp\Delta(u^3 - u'^3), \tag{14}$$

$$\Phi_\Delta^+(u,u') = \Phi(u,u') - s_\perp\Delta(u^3 - u'^3). \tag{15}$$

Thus, we find

$$G(u,u') = \exp[is_\perp\Delta(u^3-u'^3)]\,\mathcal{P}_5^-\,G_0(u,u') + + \exp[-is_\perp\Delta(u^3-u'^3)]\,\mathcal{P}_5^+\,G_0(u,u'), \tag{16}$$

where G_0 is the propagator with $\Delta = 0$. Note that Δ appears now only in the phase factors.

By making use of Eq. (7), the axial current density in the point-splitting regularization equals

$$\langle j_5^\mu\rangle = -\text{tr}\left[\gamma^\mu\gamma^5\,G(u,u+\epsilon)\right]_{\epsilon\to 0}. \tag{17}$$

The fermion propagator in an electromagnetic field has the following singular behavior for $u'-u = \epsilon \to 0$ [44, 45]:

$$G_0(u,u+\epsilon) \simeq \frac{i}{2\pi^2}\left[\frac{\hat{\epsilon}}{\epsilon^4} - \frac{1}{16\epsilon^2}eF_{\mu\nu}\left(\hat{\epsilon}\sigma^{\mu\nu} + \sigma^{\mu\nu}\hat{\epsilon}\right)\right], \tag{18}$$

where $\hat{\epsilon} = \gamma_\mu\epsilon^\mu$. Then, using Eqs. (16)–(18), we find

$$\langle j_5^\mu\rangle_{\rm singular} = \frac{i\epsilon^\mu s_\perp}{\pi^2\epsilon^4}\left(e^{-is_\perp\Delta\epsilon^3} - e^{is_\perp\Delta\epsilon^3}\right) + + \frac{ieF_{\lambda\sigma}\epsilon_\beta\epsilon^{\beta\mu\lambda\sigma}}{8\pi^2\epsilon^2}\left(e^{-is_\perp\Delta\epsilon^3} + e^{is_\perp\Delta\epsilon^3}\right)\Big|_{\epsilon\to 0}. \tag{19}$$

Taking into account that the limit $\epsilon \to 0$ should be taken in this equation symmetrically [44, 45], i.e., $\epsilon^\mu\epsilon^\nu/\epsilon^2 \to \frac{1}{4}g^{\mu\nu}$, and the fact that its second term contains only odd powers of ϵ, we arrive at

$$\langle j_5^\mu\rangle_{\rm singular} = -\frac{\Delta}{2\pi^2\epsilon^2}\delta_3^\mu = \frac{\Lambda^2\Delta}{2\pi^2}\delta_3^\mu, \tag{20}$$

where we used in the last equality that $-1/\epsilon^2$ plays the role of an Euclidean ultraviolet cutoff Λ^2. Eq. (20) agrees with the results obtained in Ref. [26], where it was shown that while the contribution of each Landau level to the axial current density $\langle j_5^\mu\rangle$ is finite at a fixed Δ, their total contribution is quadratically divergent. However, since $G_{int} = 4\pi^2 g/\Lambda^2$, the gap equation (4) implies that the dynamical shift Δ yields

$\Delta \sim g\mu\, eB/\Lambda^2$ and the axial current density is actually finite. The explicit expressions for Δ and $\langle j_5^3 \rangle$ determined by Eqs. (4) and (7) are [26]:

$$\Delta \simeq -g\mu \frac{eB}{\Lambda^2 (1+2ag)}, \tag{21}$$

$$\langle j_5^3 \rangle \simeq \frac{eB}{2\pi^2}\mu + a\frac{\Lambda^2}{\pi^2}\Delta \simeq \frac{eB}{2\pi^2}\frac{\mu}{(1+2ag)}, \tag{22}$$

where a is a dimensionless constant of order one, which is determined by the regularization scheme used. Note that both the topological and dynamical contributions are included in $\langle j_5^3 \rangle$. (Terms of higher order in powers of $|eB|/\Lambda^2$ are neglected in both expressions.)

We conclude that interactions leading to the chiral shift parameter Δ essentially change the form of the induced axial current in a magnetic field. It is important to mention that unlike the topological contribution in $\langle j_5^\mu \rangle$ [20], the dynamical one is generated by all Landau levels.

2.3. Chiral shift parameter and chiral anomaly

In the previous subsection, we showed that the dynamically generated chiral shift contributes to the axial current. Since the chiral anomaly is an exact operator relation, Δ cannot affect the chiral anomaly. In this section, we shall check this explicitly.

In the gauge invariant point-splitting regularization, the divergence of the axial current in the massless theory equals [44, 45]

$$\partial_\mu j_5^\mu = ie\epsilon^\alpha \bar\psi(u+\epsilon)\gamma^\mu\gamma^5\psi(u)\; F_{\alpha\mu}|_{\epsilon\to 0}. \tag{23}$$

By calculating the vacuum expectation value of the divergence of the axial current, we find

$$\langle \partial_\mu j_5^\mu \rangle = -ie\epsilon^\alpha F_{\alpha\mu} \mathrm{tr}\left[\gamma^\mu\gamma^5 G(u,u+\epsilon)\right]_{\epsilon\to 0} =$$

$$= ie\epsilon^\alpha F_{\alpha\mu}\langle j_5^\mu \rangle, \tag{24}$$

where $G(u,u')$ is the fermion propagator (16).

We start our analysis by considering the first term in the axial current density in Eq. (19):

$$\frac{i\epsilon^\mu s_\perp}{\pi^2\epsilon^4}\left(e^{-is_\perp\Delta\epsilon^3} - \text{h.c.}\right) \simeq \frac{2\Delta\epsilon^\mu\epsilon^3}{\pi^2\epsilon^4}\left(1 - \frac{\Delta^2\epsilon_3^2}{6} + ...\right). \tag{25}$$

Its contribution to the right-hand side of Eq. (24) is

$$\frac{2i\Delta\,\epsilon^\alpha\epsilon^\mu\epsilon^3}{\pi^2\epsilon^4}\left(1 - \frac{\Delta^2\epsilon_3^2}{6} + ...\right) eF_{\alpha\mu}. \tag{26}$$

Since this expression contains only odd powers of ϵ, it gives zero contribution after symmetric averaging over space-time directions of ϵ.

Thus, only the second term in Eq. (19) is relevant for the divergence of axial current in Eq. (24), and we obtain

$$\langle \partial_\mu j_5^\mu \rangle = -\frac{e^2\epsilon^{\beta\mu\lambda\sigma}F_{\alpha\mu}F_{\lambda\sigma}\epsilon^\alpha\epsilon_\beta}{8\pi^2\epsilon^2}\left(e^{-is_\perp\Delta\epsilon^3} + \text{h.c.}\right) \to$$

$$\to -\frac{e^2}{16\pi^2}\epsilon^{\beta\mu\lambda\sigma}F_{\beta\mu}F_{\lambda\sigma} \tag{27}$$

for $\epsilon \to 0$ and symmetric averaging over space-time directions of ϵ. Therefore, we conclude that the chiral shift parameter Δ does not affect the chiral anomaly.

3. Radiative Corrections to the Chiral Separation Effect in QED

Since the NJL model is nonrenormalizable and the chiral anomaly is intimately connected with ultraviolet divergencies, in order to reach a solid conclusion about the presence or absence of higher-order radiative corrections to the axial current, one should consider them in a renormalizable model. In this section, assuming that the magnetic field $\mathbf{B}$ is weak and using the expansion in powers of $\mathbf{B}$ up to linear order, the leading radiative corrections to the chiral separation effect in QED are calculated. We find that they do not vanish and attribute this result to the singularities in the fermion propagator at the Fermi surface. On the technical side, the $i\epsilon\,\mathrm{sign}(k_0)$ prescription in the fermion propagator, which distinguishes a chemical potential from the time component A_0 of the photon field, plays a crucial role in deriving this result.

The Lagrangian density of QED in a magnetic field reads

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar\psi\left(i\gamma^\nu\mathcal{D}_\nu + \mu\gamma^0 - m\right)\psi +$$

$$+ \delta_2\bar\psi(i\gamma^\nu\partial_\nu + \mu\gamma^0 + eA_\nu^{\text{ext}}\gamma^\nu)\psi - \delta_m\bar\psi\psi, \tag{28}$$

where μ is the fermion chemical potential, the last two terms are counterterms (we use the notation of Ref. [44], but with the opposite sign of the electric charge, $e \to -e$), and the covariant derivative is

$\mathcal{D}_\mu = \partial_\mu - ieA_\mu - ieA_\mu^{\text{ext}}$. Without loss of generality, we assume like in the previous section that the external magnetic field $\mathbf{B}$ points in the z-direction and is described by the vector potential in the Landau gauge. Note that the counterterms given by the second line in Eq.(28) include the chemical potential μ and the external field A_μ^{ext}.

The renormalization group invariant axial current density is given by

$$\langle j_5^3 \rangle = -Z_2 \text{tr}\left[\gamma^3\gamma^5 G(x,x)\right], \tag{29}$$

where $G(x,y)$ is the full fermion propagator and $Z_2 = 1+\delta_2$ is the wave function renormalization constant of the fermion propagator.

To the first order in the coupling constant $\alpha = e^2/(4\pi)$, the full propagator is expressed through the free propagator $S(x,y)$ in the magnetic field and the one-loop fermion self-energy $\Sigma(u,v)$

$$G(x,y) = S(x,y) + i\int d^4u d^4v S(x,u)\Sigma(u,v)S(v,y) + + i\int d^4u d^4v S(x,u)\Sigma_{\text{ct}}(u,v)S(v,y), \tag{30}$$

where $\Sigma_{\text{ct}}(u,v)$ is the counterterms contribution due to the last two terms in the Lagrangian density (28). The one-loop fermion self-energy is given by

$$\Sigma(x,y) = -4i\pi\alpha\gamma^\mu S(x,y)\gamma^\nu D_{\mu\nu}(x-y), \tag{31}$$

where $D_{\mu\nu}(x-y)$ is the free photon propagator

$$D_{\mu\nu}(q) = -i\frac{g_{\mu\nu}}{q_\Lambda^2} \equiv \equiv -i\left(\frac{g_{\mu\nu}}{q_0^2 - \mathbf{q}^2 - m_\gamma^2 + i\epsilon} - \frac{g_{\mu\nu}}{q_0^2 - \mathbf{q}^2 - \Lambda^2 + i\epsilon}\right). \tag{32}$$

Here, we introduced a nonzero photon mass m_γ which serves as an infrared regulator at the intermediate stages of calculations. Of course, none of the physical observables should depend on this parameter (see discussion below). (Note that since the classical paper of Stueckelberg [47], it is well known that, unlike non-Abelian theories, introducing a photon mass causes no problems in an Abelian gauge theory, such as QED.) As in Ref. [48], we find that the Feynman regularization of the photon propagator (32) with ultraviolet regularization parameter Λ presents the most convenient way of regularizing the theory in the ultraviolet region.

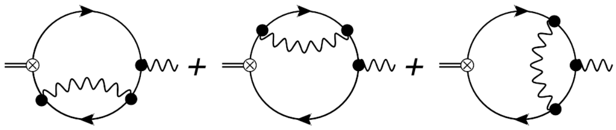

Fig. 1. The leading radiative corrections to the axial current in the approximation linear in magnetic field. Solid and wavy lines correspond to the fermion and photon propagators, respectively. Double solid lines describe the axial current insertions, and the external wavy lines attached to the fermion loops indicate the insertions of the external gauge field

We make use of the weak magnetic field expansion in the calculation of the axial current density. The corresponding diagrams are shown in Fig. 1. For the fermion propagator to linear in B order, we have

$$S(x,y) = \bar{S}^{(0)}(x-y) + i\Phi(x,y)\bar{S}^{(0)}(x-y) + \bar{S}^{(1)}(x-y), \tag{33}$$

where $\bar{S}^{(0)}(x-y)$ and $\bar{S}^{(1)}(x-y)$ are the zeroth and first order terms in powers of B in the translation invariant part of the propagator. Their Fourier transforms equal [40]

$$\bar{S}^{(0)}(k) = i\frac{(k_0+\mu)\gamma^0 - \mathbf{k}\cdot\boldsymbol{\gamma} + m}{(k_0 + \mu + i\epsilon\,\text{sign}(k_0))^2 - \mathbf{k}^2 - m^2} \tag{34}$$

and

$$\bar{S}^{(1)}(k) = -\gamma^1\gamma^2 eB \times \times \frac{(k_0+\mu)\gamma^0 - k_3\gamma^3 + m}{\left[(k_0 + \mu + i\epsilon\,\text{sign}(k_0))^2 - \mathbf{k}^2 - m^2\right]^2}. \tag{35}$$

Using Eqs.(31) and (33), we obtain the following expansion for the fermion self-energy in the approximation linear in magnetic field:

$$\Sigma(u,v) = \bar{\Sigma}^{(0)}(u-v) + i\Phi(u,v)\bar{\Sigma}^{(0)}(u-v) + \bar{\Sigma}^{(1)}(u-v), \tag{36}$$

where the Fourier transforms of $\bar{\Sigma}^{(0)}(x-y)$ and $\bar{\Sigma}^{(1)}(x-y)$ are given by

$$\bar{\Sigma}^{(0)}(p) = -4i\pi\alpha\int\frac{d^4k}{(2\pi)^4}\gamma^\mu\bar{S}^{(0)}(k)\gamma^\nu D_{\mu\nu}(p-k), \tag{37}$$

and

$$\bar{\Sigma}^{(1)}(p) = -4i\pi\alpha\int\frac{d^4k}{(2\pi)^4}\gamma^\mu\bar{S}^{(1)}(k)\gamma^\nu D_{\mu\nu}(p-k). \tag{38}$$

The self-energy $\bar{\Sigma}^{(0)}(p)$ determines the counterterms δ_2 and δ_m in Eq. (28) that are given by the following standard expressions [44]:

$$\delta_2 = -\frac{\alpha}{2\pi}\left(\frac{1}{2}\ln\frac{\Lambda^2}{m^2} + \ln\frac{m_\gamma^2}{m^2} + \frac{9}{4}\right), \quad (39)$$

$$\delta m = m - m_0 = \frac{3\alpha}{4\pi} m\left(\ln\frac{\Lambda^2}{m^2} + \frac{1}{2}\right). \quad (40)$$

Omitting the noninteresting zeroth order in B contribution in Eq. (30), we find the following linear in B contribution to the propagator:

$$G^{(1)}(x,x) = \bar{S}^{(1)}(x,x) + i\int d^4u d^4v \times$$
$$\times\left[\bar{S}^{(1)}(x-u)\bar{\Sigma}^{(0)}(u-v)\bar{S}^{(0)}(v-x) + \right.$$
$$\left. + \bar{S}^{(0)}(x-u)\bar{\Sigma}^{(0)}(u-v)\bar{S}^{(1)}(v-x)\right] +$$
$$+ i\int d^4u d^4v\; \bar{S}^{(0)}(x-u)\bar{\Sigma}^{(1)}(u-v)\bar{S}^{(0)}(v-x) -$$
$$- \int d^4u d^4v\,[\Phi(x,u) + \Phi(u,v) + \Phi(v,x)] \times$$
$$\times \bar{S}^{(0)}(x-u)\bar{\Sigma}^{(0)}(u-v)\bar{S}^{(0)}(v-x). \quad (41)$$

Since

$$\Phi(x,u) + \Phi(u,v) + \Phi(v,x) =$$
$$= -\frac{eB}{2}\,[(x_1-u_1)(v_2-x_2) - (v_1-x_1)(x_2-u_2)]$$

is a translation invariant function, it is convenient to switch to the momentum space on the right-hand side of Eq. (41). We have

$$G^{(1)}(x,x) = \int \frac{d^4p}{(2\pi)^4}\bar{S}^{(1)}(p) +$$
$$+ i\int \frac{d^4p}{(2\pi)^4}\left[\bar{S}^{(1)}(p)\bar{\Sigma}^{(0)}(p)\bar{S}^{(0)}(p) + \right.$$
$$\left. + \bar{S}^{(0)}(p)\bar{\Sigma}^{(0)}(p)\bar{S}^{(1)}(p) + \bar{S}^{(0)}(p)\bar{\Sigma}^{(1)}(p)\bar{S}^{(0)}(p)\right] -$$
$$- \frac{eB}{2}\int \frac{d^4p}{(2\pi)^4}\left[\frac{\partial\bar{S}^{(0)}(p)}{\partial p_1}\bar{\Sigma}^{(0)}(p)\frac{\partial\bar{S}^{(0)}(p)}{\partial p_2} - \right.$$
$$\left. - \frac{\partial\bar{S}^{(0)}(p)}{\partial p_2}\bar{\Sigma}^{(0)}(p)\frac{\partial\bar{S}^{(0)}(p)}{\partial p_1}\right]. \quad (42)$$

By substituting this formula in Eq. (29), we obtain the following expression for the axial current density:

$$\langle j_5^3\rangle = \langle j_5^3\rangle_0 + \langle j_5^3\rangle_\alpha, \quad (43)$$

where

$$\langle j_5^3\rangle_0 = -\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}\left[\gamma^3\gamma^5\bar{S}^{(1)}(p)\right] =$$
$$= -\frac{eB\,\mathrm{sign}(\mu)}{2\pi^2}\sqrt{\mu^2 - m^2} \quad (44)$$

is the contribution to the axial current in the free theory, which coincides, of course, with the very well known topological contribution [20].

The second term on the right-hand side of Eq. (43) defines the leading radiative corrections to the axial current and equals

$$\langle j_5^3\rangle_\alpha = \frac{eB}{2}\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}\left[\gamma^3\gamma^5\frac{\partial\bar{S}^{(0)}(p)}{\partial p_1}\bar{\Sigma}^{(0)}(p)\frac{\partial\bar{S}^{(0)}(p)}{\partial p_2} - \right.$$
$$\left. - \gamma^3\gamma^5\frac{\partial\bar{S}^{(0)}(p)}{\partial p_2}\bar{\Sigma}^{(0)}(p)\frac{\partial\bar{S}^{(0)}(p)}{\partial p_1}\right] - i\int\frac{d^4p}{(2\pi)^4}\times$$
$$\times\,\mathrm{tr}\left[\gamma^3\gamma^5\bar{S}^{(1)}(p)\bar{\Sigma}^{(0)}(p)\bar{S}^{(0)}(p) + \gamma^3\gamma^5\bar{S}^{(0)}(p)\bar{\Sigma}^{(0)}(p)\times\right.$$
$$\left.\times\,\bar{S}^{(1)}(p) + \gamma^3\gamma^5\bar{S}^{(0)}(p)\bar{\Sigma}^{(1)}(p)\bar{S}^{(0)}(p)\right] + \langle j_5^3\rangle_{\mathrm{ct}}, \quad (45)$$

where the counterterms contribution $\langle j_5^3\rangle_{\mathrm{ct}}$ in Eq. (45) contains all the contributions with δ_2 and δ_m. Its explicit form will be given below.

By substituting propagators (34) and (35) into Eq. (45), we find the following leading radiative corrections to the axial current:

$$\langle j_5^3\rangle_\alpha = \langle j_5^3\rangle_{\mathrm{ct}} + 32\pi\alpha eB\int\frac{d^4p\,d^4k}{(2\pi)^8}\frac{1}{(P-K)_\Lambda^2}\times$$
$$\times\left[\frac{(k_0+\mu)[4(p_0+\mu)^2-P^2]-4(p_0+\mu)(\mathbf{p}\cdot\mathbf{k}+2m^2)}{(P^2-m^2)^3(K^2-m^2)} - \right.$$
$$\left. - \frac{(k_0+\mu)[3(p_0+\mu)^2-\mathbf{p}^2+3m^2]-2(p_0+\mu)(\mathbf{p}\cdot\mathbf{k})}{3(P^2-m^2)^2(K^2-m^2)^2}\right]. \quad (46)$$

Here, we use the shorthand notation $K^2 = [k_0 + \mu + i\epsilon\,\mathrm{sign}(k_0)]^2 - \mathbf{k}^2$ and $P^2 = [p_0 + \mu + i\epsilon\,\mathrm{sign}(p_0)]^2 - \mathbf{p}^2$. As for the definition of $(P-K)_\Lambda^2$, it follows from Eq. (32). In the derivation of Eq. (46), the following replacements have been made in the integrand: $p_\perp^2 \to \frac{2}{3}\mathbf{p}^2$, $p_3^2 \to \frac{1}{3}\mathbf{p}^2$, and $p_3k_3 \to \frac{1}{3}(\mathbf{p}\cdot\mathbf{k})$. These replacements are allowed by the rotational symmetry of the other parts of the integrand.

In order to calculate the leading radiative corrections, it is very convenient to integrate by parts in

Eq. (46) using the following identity valid for all integer $n \geq 1$ (for a more detailed consideration, see Ref.[40]):

$$\frac{1}{[[k_0+\mu+i\epsilon\,\mathrm{sign}(k_0)]^2-\mathbf{k}^2-m^2]^n} = \\ = \frac{1}{[(k_0+\mu)^2-\mathbf{k}^2-m^2+i\epsilon]^n} + \frac{2\pi i(-1)^{n-1}}{(n-1)!} \times \\ \times \theta(|\mu|-|k_0|)\theta(-k_0\mu)\delta^{(n-1)}\left[(k_0+\mu)^2-\mathbf{k}^2-m^2\right]. \tag{47}$$

Then we obtain the following leading radiative corrections to the axial current:

$$\langle j_5^3\rangle_\alpha = 64i\pi^2\alpha eB\int\frac{d^4pd^4k}{(2\pi)^8}\times \\ \times\Bigg[\frac{(k_0+\mu)(p_0+\mu)-\mathbf{p}\cdot\mathbf{k}-2m^2}{(P-K)^2_\Lambda(K^2-m^2)}\delta'[\mu^2-m^2-\mathbf{p}^2]\delta(p_0)+ \\ +\frac{3(p_0+\mu)^2-3(k_0+\mu)(p_0+\mu)+\mathbf{p}^2-\mathbf{p}\cdot\mathbf{k}+3m^2}{3(P-K)^2_\Lambda(P^2-m^2)^2}\times \\ \times\delta\left(\mu^2-m^2-\mathbf{k}^2\right)\delta(k_0)\Bigg]+\langle j_5^3\rangle_{\rm ct}. \tag{48}$$

The result in Eq. (48) is quite remarkable for several reasons. Technically, the integration by parts allowed us to reduce the original two-loop expression in Eq. (46) down to a much simpler one-loop form. Indeed, after the integration over one of the momenta in Eq. (48) is performed using the δ-functions in the integrand, the expression will have an explicit one-loop form that makes possible to obtain an analytic result for the leading radiative corrections to the axial current.

In addition, Eq. (48) shows that all nonzero corrections come from the regions of the phase space, where either p or k momentum is restricted to the Fermi surface. The presence of the singular "matter" term on the right-hand side of identity (47) was crucial for obtaining a nonzero result because, in the derivation of Eq. (48), all nonsingular terms are gone after the integration by parts. Therefore, we conclude that the nonzero radiative corrections to the axial current are intimately connected with the precise form of the singularities in the fermion propagator at the Fermi surface.

The calculation of the axial current in Eq. (48) is still technically quite involved. However, it is relatively straightforward to show that the right-hand side in Eq. (48) without the counterterms contribution has a logarithmically divergent contribution when $\Lambda\to\infty$ given by

$$\frac{\alpha eB(2\mu^2+m^2)}{4\pi^3\sqrt{\mu^2-m^2}}\ln\frac{\Lambda}{m}. \tag{49}$$

To cancel this divergence, we should add the contribution due the counterterms in Lagrangian (28). The Fourier transform of the translational invariant part of the counterterm contribution to the self-energy reads

$$\bar{\Sigma}^{(0)}_{\rm ct}(p) = \delta_2[(p_0+\mu)\gamma^0-\mathbf{p}\cdot\boldsymbol{\gamma}]-\delta_m, \tag{50}$$

where $\delta_m = Z_2m_0-m\simeq m\delta_2-\delta m$. Using Eq. (50) and the last term in Eq. (30), we find the following leading order contribution to the axial current density due to counterterms:

$$\langle j_5^3\rangle_{\rm ct} = -\delta_2\langle j_5^3\rangle_0 - 4ieB\int\frac{d^4p}{(2\pi)^4}\frac{\delta_2(p_0+\mu)}{(P^2-m^2)^2}- \\ -8ieB\int\frac{d^4p}{(2\pi)^4}\frac{(p_0+\mu)\left[\delta_2(P^2+2m^2)-2m\delta_m\right]}{(P^2-m^2)^3} = \\ = -\frac{eB}{\pi^2}\sqrt{\mu^2-m^2}\delta_2+\frac{eBm\,(m\delta_2-\delta_m)}{2\pi^2\sqrt{\mu^2-m^2}}. \tag{51}$$

By making use of counterterms (39) and (40), we obtain

$$\langle j_5^3\rangle_{\rm ct} = -\frac{\alpha eB}{2\pi^3}\sqrt{\mu^2-m^2}\left(\frac{1}{2}\ln\frac{\Lambda^2}{m^2}+\ln\frac{m_\gamma^2}{m^2}+\frac{9}{4}\right)- \\ -\frac{3\alpha eBm^2}{4\pi^3\sqrt{\mu^2-m^2}}\left(\frac{1}{2}\ln\frac{\Lambda^2}{m^2}+\frac{1}{4}\right). \tag{52}$$

Calculating the integrals in Eq. (48) and using Eq. (52), we finally find the following leading radiative corrections to the axial current in the case $m \ll |\mu|$:

$$\langle j_5^3\rangle_\alpha = -\frac{\alpha eB\mu}{2\pi^3}\left(\ln\frac{2\mu}{m}+\ln\frac{m_\gamma^2}{m^2}+\frac{4}{3}\right)- \\ -\frac{\alpha eBm^2}{2\pi^3\mu}\left(\ln\frac{2^{3/2}\mu}{m_\gamma}-\frac{11}{12}\right). \tag{53}$$

As expected, this result is independent of the ultraviolet regulator Λ. It does, however, depends on the fictitious photon mass m_γ. The origin of this dependence can be easily traced back to the infrared singularity of the wave function renormalization Z_2 in the Feynman gauge used. Such a singularity is typical of a class of QED observables obtained by perturbative methods. As we discuss in the Conclusion, in the complete physical expression for the axial current, obtained by going beyond the simplest double expansion in the coupling constant and magnetic field, the regulator m_γ^2 will likely be replaced by a physical scale, e.g., such as $|eB|$ or $\alpha\mu^2$.

4. Chiral Asymmetry in Magnetized Relativistic Matter in QED

We showed in Sec. 2 that the chiral shift parameter is generated in magnetized relativistic matter in the NJL model and contributes to the axial current. Since, according to the study in the previous section, the leading radiative corrections to the chiral separation effect do not vanish in QED, this suggests that, like in the NJL model, a nonzero chiral shift parameter should be generated in QED too. We investigate this problem in this section and study the fermion self-energy in QED.

We begin with the dispersion relations for fermion in a weak magnetic field which can be obtained by considering the location of the poles of the fermion propagator. It is convenient to determine the dispersion relations in the limit of large pseudo-momentum or weak magnetic field (i.e., $\mathbf{k}_\perp^2 \gg |eB|$), where the effects of the Schwinger phase can be neglected and the pseudo-momentum can be interpreted as an approximate (or "quasiclassical") fermion's momentum. In this case, the poles of the fermion propagator are defined by the following equation:

$$\det\left[i\bar{S}^{-1}(p) - \bar{\Sigma}^{(1)}(p)\right] = 0, \tag{54}$$

where $\bar{\Sigma}^{(1)}(p)$ is defined in Eq. (38).

To determine the dispersion relations from Eq. (54), we should find the inverse free propagator in the pseudo-momentum representation. The inverse free propagator in the coordinate space equals

$$iS^{-1}(x,y) = \left(i\gamma^\nu \mathcal{D}_\nu + \mu\gamma^0 - m\right)\delta^4(x-y). \tag{55}$$

Representing the Dirac δ-function on the right-hand side of the above equation as the sum over the complete set of Landau levels eigenfunctions ψ_{Np}, we rewrite the inverse free propagator (55) as follows:

$$iS^{-1}(x,y) = \sum_{N=0}^{\infty}\int\frac{dp_0 dp^3 dp\, e^{-ip_0(x_0-y_0)+ip^3(x^3-y^3)}}{(2\pi)^2}\times$$
$$\times\left[(p_0+\mu)\gamma^0 - p^3\gamma^3 - (\boldsymbol{\pi}_\perp\cdot\boldsymbol{\gamma}_\perp) - m\right]\times$$
$$\times\,\psi_{Np}(\mathbf{r}_\perp)\psi^*_{Np}(\mathbf{r}'_\perp). \tag{56}$$

Integrating over p, we find that the result takes the form of a product of the standard Schwinger phase and a translationally invariant function, i.e.,

$$iS^{-1}(x,y) = e^{i\Phi(x,y)} i\bar{S}^{-1}(x-y). \tag{57}$$

The translationally invariant function is given by

$$i\bar{S}^{-1}(x) = \frac{e^{-\xi/2}}{2\pi\ell^2}\sum_{n=0}^{\infty}\int\frac{dp_0 dp^3}{(2\pi)^2}e^{-ip_0x_0+ip^3x^3}\times$$
$$\times\Big\{\left[(p_0+\mu)\gamma^0 - p^3\gamma^3 - m\right]\left[L_n(\xi)\mathcal{P}_- + L_{n-1}(\xi)\mathcal{P}_+\right]+$$
$$+\frac{i}{\ell^2}(\mathbf{r}_\perp\cdot\boldsymbol{\gamma}_\perp)L^1_{n-1}(\xi)\Big\}, \tag{58}$$

where $\xi = \mathbf{r}_\perp^2/(2\ell^2)$. By performing the Fourier transform, we arrive at the following expansion of the translation invariant part of the inverse free propagator over Landau levels:

$$i\bar{S}^{-1}(p) = 2e^{-p_\perp^2\ell^2}\sum_{n=0}^{\infty}(-1)^n\Big\{\left[(p_0+\mu)\gamma^0 - p^3\gamma^3 - m\right]\times$$
$$\times\left[\mathcal{P}_- L_n(2p_\perp^2\ell^2) - \mathcal{P}_+ L_{n-1}(2p_\perp^2\ell^2)\right]+$$
$$+2(\boldsymbol{\gamma}_\perp\cdot\boldsymbol{p}_\perp)L^1_{n-1}(2p_\perp^2\ell^2)\Big\}. \tag{59}$$

Interestingly, by performing the summation over Landau levels and using the well-known formula [49]

$$\sum_{n=0}^{\infty} z^n L_n^\alpha(x) = \frac{1}{(1-z)^{1+\alpha}}\exp\left(\frac{xz}{z-1}\right), \tag{60}$$

we obtain

$$i\bar{S}^{-1}(p) = (p_0+\mu)\gamma^0 - (\boldsymbol{\gamma}_\perp\cdot\mathbf{p}_\perp) - p^3\gamma^3 - m. \tag{61}$$

This is a remarkable result, because it means that the translation invariant part of the inverse free propagator in a magnetic field is identical to the inverse free propagator in the absence of a magnetic field. Consequently, for the inverse free propagator, only the Schwinger phase contains information about the presence of a magnetic field.

For the free propagator in the weak field limit, the dependence on the Landau level index [which is the eigenvalue of the operator $-\frac{1}{2}(\boldsymbol{\pi}_\perp\cdot\boldsymbol{\gamma}_\perp)^2\ell^2$] can be unambiguously replaced by the square of the transverse momentum, i.e., $2n|eB| \to \mathbf{p}_\perp^2$. Therefore, when using the pseudo-momentum representation in Eq. (54), we can interpret $\mathbf{p}_\perp^2$ as a convenient shorthand substitution for $2n|eB|$. This is natural in the weak field limit, when the quantization of Landau levels is largely irrelevant. Indeed, the standard dispersion relation $p_0 = -\mu \pm \sqrt{\mathbf{p}_\perp^2 + p_3^2 + m^2}$ by substituting $\mathbf{p}_\perp^2 \to 2n|eB|$ leads to the fermion energy $p_0 = -\mu \pm \sqrt{2n|eB| + p_3^2 + m^2}$ in the Landau levels.

Calculating $\bar{\Sigma}^{(1)}(p)$ given by Eq. (38), it was found in Ref. [41] that $\bar{\Sigma}^{(1)}(p)$ contains two Dirac structures. One of them is the chiral shift parameter Δ and another one looks like that of the chiral chemical potential but is an odd function of the longitudinal momentum directed along the magnetic field. Note that this dependence on momentum is dictated by the parity symmetry. Since QED in a magnetic field is invariant under parity and the self-energy is obtained in a perturbation theory, parity cannot be broken. The term $\mu_5\bar{\psi}\gamma^0\gamma^5\psi$ is not parity invariant unless μ_5 is an odd function of the momentum along the direction of the magnetic field. Note that same argument also ensures that there is no electric current along the direction of the magnetic field, which would be present due to the chiral magnetic effect if one had $\mu_5 = \text{const}$.

By making use of the chiral representation of the Dirac γ-matrices, the inverse free propagator (61), the general structure of the fermion self-energy found in the weak magnetic field limit in Ref. [41], and calculating the determinant in Eq. (54), we obtain

$$\left[(p_0+\mu-\mu_5)^2 - \mathbf{p}_\perp^2 - (p^3+\Delta)^2\right] \times$$
$$\times \left[(p_0+\mu+\mu_5)^2 - \mathbf{p}_\perp^2 - (p^3-\Delta)^2\right] -$$
$$- 2m^2\left[(p_0+\mu)^2 + \Delta^2 - \mathbf{p}_\perp^2 - p_3^2 - \mu_5^2\right] + m^4 = 0. \quad (62)$$

This expression can be factorized to produce two equations for predominantly left-handed and predominantly right-handed particles:

$$(p_0+\mu)^2 - \mathbf{p}_\perp^2 - p_3^2 - m^2 - \Delta^2 + \mu_5^2 -$$
$$- 2\sqrt{(p^3\Delta + \mu_5(p_0+\mu))^2 + m^2(\Delta^2-\mu_5^2)} = 0, \quad (63)$$

$$(p_0+\mu)^2 - \mathbf{p}_\perp^2 - p_3^2 - m^2 - \Delta^2 + \mu_5^2 +$$
$$+ 2\sqrt{(p^3\Delta + \mu_5(p_0+\mu))^2 + m^2(\Delta^2-\mu_5^2)} = 0. \quad (64)$$

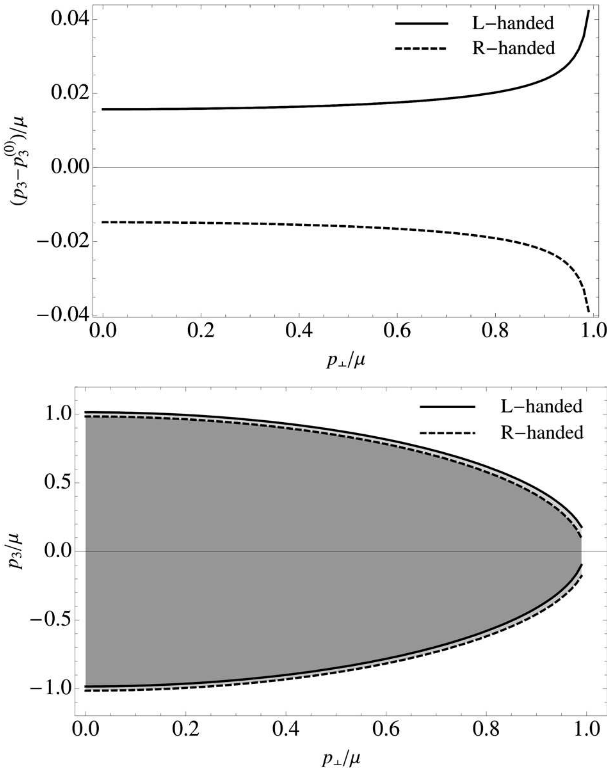

Fig. 2. Asymmetry of the Fermi surface for predominantly left-handed and right-handed particles for $|eB| = 0.1\mu^2$ and $\alpha = 1/137$

Using $\Delta(p)$ and $\mu_5(p)$ obtained in QED in the weak magnetic field limit in Ref. [41] and the dispersion relations that follow from Eqs. (63) and (64), we can easily found the equations for the Fermi surfaces of both types of particles. Namely, we take $p^0 = 0$ and solve for p^3 as a function of $p_\perp$. The results are shown in the lower panel of Fig. 2 in the case of the physical value of the fine structure constant ($\alpha = 1/137$) and the magnetic field $|eB| = 0.1\mu^2$. In order to clearly demonstrate the magnitude of the effect, in the upper panel of Fig. 2 we plot also the difference between the longitudinal momenta with and without the inclusion of the interaction-induced chiral asymmetry.

Fig. 2 implies that the Fermi surface of the predominantly left-handed particles is slightly shifted in the direction of the magnetic field, while the Fermi surface of the predominantly right-handed particles is slightly shifted in the direction opposite of the magnetic field. This is in qualitative agreement with the finding in the NJL model [27]. However, in the case of

QED with its long-range interaction, the chiral asymmetry of the Fermi surfaces comes not only from the chiral shift parameter, but also from the new function $\mu_5(p) \equiv p_3 f(p)$. Also, unlike in the NJL model, both Δ and μ_5 have a non-trivial dependence on the particles' momenta. In particular, they reveal a logarithmic enhancement of the asymmetry near the Fermi surface [41].

5. Conclusion

In this mini review, we studied the chiral asymmetry and induced axial current in the relativistic matter in a magnetic field. Our conclusion is that there are two components in this current, the topological component, induced only in the LLL and intimately connected with the chiral anomaly, and the dynamical one provided by the chiral shift parameter Δ generated in all Landau levels. We showed that the chiral shift parameter contributes to the non-dissipative axial current taking place in relativistic matter in a magnetic field and checked also that the presence of the chiral shift parameter does not affect the standard expression for the chiral anomaly. The expression for the chiral shift parameter, $\Delta \sim g\mu\, eB/\Lambda^2$, obtained in the NJL model implies that both fermion density and magnetic field are necessary for the generation of Δ. Therefore, the chiral shift parameter should also be generated in renormalizable theories that we confirmed in the QED studies.

It has been recently suggested in Refs. [50, 51], that a chiral magnetic spiral solution is realized in the chirally broken phase in the presence of a strong magnetic field. Like the solution with the chiral shift parameter Δ, the chiral magnetic spiral one is anisotropic, but beside that it is also inhomogeneous. It is essential, however, that the solution with the chiral shift is realized in the *normal* phase of matter, in which the fermion density and the axial current density are non-vanishing. It would be interesting to clarify whether there is a connection between these two solutions describing the dynamics in the two different phases of magnetized relativistic matter.

We calculated, too perturbatively in the coupling constant and in linear order in the external magnetic field, the leading radiative corrections to the chiral separation effect in QED and found that they do not vanish. To leading order, these corrections are shown to be directly connected with the Fermi surface singularities in the fermion propagator at nonzero density. This interpretation is strongly supported by another observation: had we ignored the corresponding singular terms in the fermion propagator, the calculation of the two-loop radiative corrections would give a vanishing result.

It is straightforward to trace the origin of the m_γ dependence in Eq. (53) to the calculation of the well known result for the wave function renormalization constant δ_2 given by Eq. (39). In fact, this infrared problem is common for dynamics in external fields in QED (for a thorough discussion, see Sec. 14 in book [52]). The most famous example is provided by the calculation of the Lamb shift, when the electron is in the Coulomb field. The point is that even for a light nucleus with $Z\alpha \ll 1$, one cannot consider the Coulomb field as a weak perturbation in the deep infrared region. The reason is that this field essentially changes the dispersion relation for the electron at low energy and momenta. As a result, its four-momenta are not on the electron mass shell, where the infrared divergence is generated in the renormalization constant Z_2. Because of that, this infrared divergence is fictitious. The correct approach is to consider the Coulomb interaction perturbatively only at high energies, while to treat it nonperturbatively at low energies. The crucial point is matching those two regions that leads to replacing the fictituous parameter m_γ by a physical infrared scale. This is the main subtlety that makes the calculation of the Lamb shift quite involved [52].

In the case of the Lamb shift, the infrared scale is related to the atomic binding energy or equivalently the inverse Bohr radius, where, obviously, the electron wave functions cannot be approximated with plane waves, which is the tacit assumption of the weak field approximation. Almost exactly the same line of arguments applies in the present problem of QED in an external magnetic field. In particular, the fermion momenta perpendicular to the magnetic field cannot be defined with a precision better than $\sqrt{|eB|}$, or equivalently the inverse magnetic length. This implies that the contribution to the axial current, which comes from the low-energy photon exchange between the fermion states near the Fermi surface, should be treated nonperturbatively. Further, while doing the expansion in α and keeping only the leading order corrections, we ignored all screening effects, which formally appear to be of higher order. Since these effects are very important at non-zero density, they could replace the unphysical infrared regulator m_γ^2 with a physical screening mass, i.e., the Debye mass $\sqrt{\alpha}\mu$. This provides another possibility to cure the singularity in Eq. (53).

Studying the fermion self-energy in dense QED in a magnetic field, we confirmed that nonzero radiative corrections to the chiral separation effect in QED are connected with the presence of chiral asymmetry in higher Landau levels induced by interaction. Our result for the fermion self-energy, obtained perturbatively in the coupling constant and in linear order in the external magnetic field, reveals the presence of two chirally asymmetric structures. One of them is the chiral shift parameter, analogous to the one previously obtained in the NJL model. The other one is a new structure that resembles the chiral chemical potential. However, unlike the chiral chemical potential, it preserves parity because it is an odd function of the momentum directed along the magnetic field. It is found that, due to chiral asymmetry of the normal ground state in QED, the Fermi surfaces of the left- and right-handed fermions are shifted relative to each other in momentum space in the direction of the magnetic field.

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КІРАЛЬНА АСИМЕТРІЯ В РЕЛЯТИВІСТСЬКІЙ МАТЕРІЇ У ЗОВНІШНЬОМУ МАГНІТНОМУ ПОЛІ

Р е з ю м е

Розглянуто кіральну асиметрію нормального основного стану релятивістської матерії у зовнішньому магнітному полі в Намбу–Йона-Лазініо моделі з локальною чотирьох ферміонною взаємодією і квантовій електродинаміці. Показано, що параметр кірального зсуву, який пов'язаний з відносним зсувом поздовжних імпульсів (направлених вздовж магнітного поля) в законах дисперсії ферміонів протилежних кіральностей динамічно генерується в нормальному основному стані системи. Цей внесок має місце для ферміонів на всіх рівнях Ландау, включаючи ті, що знаходяться поблизу поверхні Фермі, і дає внесок у бездисипативний аксіальний струм, що має місце в релятивістській матерії у зовнішньому магнітному полі. Кіральна асиметрія нормального основного стану в квантовоелектродинамічній матерії у зовнішньому магнітному полі характеризується додатковою кіральною структурою. Вона формально виглядає як кіральний хімічний потенціал, однак є непарною функцією поздовжної компоненти імпульсу, яка направлена вздовж магнітного поля. Причина появи цієї кіральної структури, яка зберігає парність, прямо пов'язана з далекодіючим характером квантовоелектродинамічної взаємодії. Обчислено лідируючі радіаційні поправки для кірального ефекту розділення в квантовій електродинаміці і визначено форму поверхні Фермі в слабкому магнітному полі.