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ELECTRON-LATTICE ENERGY EXCHANGE AND HOT ELECTRONS IN METAL ISLAND FILMS

Some aspects of the theory of hot electrons in metal island films have been considered. In particular, the influence of the electron-lattice energy exchange, which depends on the system size and on the electronic temperatures of hot electrons is analyzed in detail in the case where the system size approaches its critical value. A high sensitivity of the electronic temperature to the metal-nanoparticle size in a vicinity of this critical value is revealed. The results of computational experiments are presented which confirm the basic concepts of the theory.

K e y w o r d s: thin films, metal nanoparticles, hot electrons, electron-lattice energy exchange, computational experiment.

1. Introduction

In 1965, paper [1] was published in which the cited authors reported about the observation of the electroluminescence and electron emission phenomena in gold and silver island films under the action of an electric field applied to them. Those authors obtained the Diploma for Discovery No. 31 with the following formulation: "A previously unknown phenomenon was found consisting in that an electric current, when passing through thin metal films with an island structure several tens of angströms in thickness (from 40 to 60 nm for gold), excites an emission current, because some of the electrons transferring the charge between the metal islands in the film have a velocity component normal to the film surface". (Priority from June 26, 1963.)

Sometime around 1965, P.G. Borzyak, the supervisor of the authors of work [1], addressed one of the authors of this work (P.M. Tomchuk) and asked him to develop a theory of experimentally observed phenomena in metal island films. The results obtained by P.M. Tomchuk differed from the interpretation proposed by the authors of the discovery. Namely, it was found that the tunnel current heats up electrons and they become "hot". (It should be noted that, besides the heating with the help of a tunnel current, hot electrons in island films can be obtained by means of the laser irradiation.) Hot electrons, due to their inelastic tunneling and inelastic reflection from potential barriers, are responsible for the film glowing. Furthermore, hot electrons stimulate the electron emission (Richardson emission of hot electrons). The concept of hot electrons, in addition to the already mentioned phenomena, also predicts nonlinear current-voltage characteristics. This concept and the nonlinearity of current-voltage characteristics in metal island films were substantiated in works [2,3] published in 1966.

Another consequence of the hot-electron idea consists in that such phenomena as the electron and photon emission can be observed irrespective of the method used to produce hot electrons (electron heating with the help of a current or laser irradiation). In particular, the hot-electron emission from metal island films under the action of a laser radiation was observed in works [4, 5]. Thus, the theory of hot electrons in metal island films turned out to explain the electron and photon emissions (irrespective of the method of electron heating), as well as the emergence of nonlinear current-voltage characteristics. The only question that remained unanswered was: Why can hot electrons be obtained under stationary (quasistationary) conditions and at relatively low electric fields (or low irradiation intensities) only in metal island films, but not in continuous films and massive metals? In continuous metal films and massive metals, hot electrons can be obtained only with the

help of powerful ultrashort laser pulses, i.e., in the pulsed mode.

To understand a relation between the island structure of metal films and the appearance of hot electrons in them, let us consider the following equation that determines the temperature of hot electrons:

$$\frac{\partial}{\partial t}\left(C_e T_e\right) + \alpha(T_e - T) = W. \tag{1}$$

Here, C_e is the heat capacity of the electron gas, T_e is its temperature, T is the lattice temperature, α the constant of electron-lattice energy exchange, and W the energy introduced into a unit volume of the metal island per unit time. Note that if the island size is much larger than the electron free path length, Eq. (1) must also include a gradient term. Furthermore, if the thermal contact of the metal island with the substrate is not sufficiently good, the island lattice is also heated up, and we need to describe this process in another equation. Below, we will show that the peculiarities of the electron heating in small islands are mainly governed by the electron-lattice energy exchange. Therefore, the aforementioned corrections turn out insignificant.

In the stationary case, Eq. (1) brings about the formula

$$T_e - T = \frac{W}{\alpha}. \tag{2}$$

Hence, one can see that if the power W is given, the electronic temperature T_e will be higher for a less intensive electron-lattice energy exchange (the constant α is lower).

The main mechanism of hot-electron relaxation in a massive metal is known: it is the generation of acoustic modes by electrons according to the Cherenkov mechanism [6]. In small metal islands (due to a discrete character of acoustic modes), the Cherenkov mechanism of electron-lattice energy exchange becomes significantly modified, and it can disappear completely for certain island sizes. As a result, according to Eq. (1), the heating of electrons in an island film can occur at much lower input powers than in massive metals and without a thermally induced film destruction.

2. Generation of Acoustic Modes by Hot Electrons

The classical description of the electron-lattice energy exchange can be executed proceeding from the equation of motion for the vector $\mathbf{u}(\mathbf{R}, t)$ of lattice displacements that arise owing to longitudinal acoustic lattice vibrations generated by an electron moving along the trajectory $\mathbf{r}_0(t)$:

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} - s^2 \Delta \mathbf{u} = -\frac{\Lambda}{\rho} \boldsymbol{\nabla} \delta(\mathbf{r}_0(t) - \mathbf{R}). \tag{3}$$

Here, s is the longitudinal sound velocity, ρ the density, and Λ the energy constant of deformation potential. The derivation procedure of Eq. (3) from the first principles can be found, e.g., in work [7]. The energy spent by the electron to generate sound can be calculated as the work fulfilled by the force with which the electron acts on the environment,

$$\frac{\partial \xi}{\partial t} = \frac{\Lambda}{\rho} \int \frac{\partial \mathbf{u}(\mathbf{R}, t)}{\partial t} \boldsymbol{\nabla} \delta(\mathbf{r}_0(t) - \mathbf{R})\, d\mathbf{R}. \tag{4}$$

The difference between the cases of a metal nanoparticle and a massive metal consists in that the electron trajectory in the former case, instead of the rectilinear uniform motion $\mathbf{r}_0(t) = \mathbf{v}t$, where $\mathbf{v}$ is the electron velocity, acquires the oscillatory character, i.e., the electron moves from one potential wall to the other one and backward. If the direction of the electron motion is taken as the axis OZ, the electron trajectory can be described as follows:

$$\mathbf{r}_0(t) = \{0, 0, z_0(t)\},$$
$$z_0(t) = \begin{cases} v\,t & \text{at } t < \tau/2, \\ L - v\,t & \text{at } t \geqslant \tau/2, \end{cases} \tag{5}$$

where $\tau = 2L/v$ is the period of electron oscillations, and L the distance between the potential walls. In the case of infinite metal, we obtain the following expression for the Fourier component of the displacement vector from Eq. (3):

$$\mathbf{u}_{\mathbf{q}} = \frac{i\,\Lambda}{\rho}\,\mathbf{q}\,\frac{e^{-i(\omega\, t - \mathbf{qR})}}{\omega^2 - q^2 s^2}, \tag{6}$$

where $\omega = \mathbf{qv}$. The pole at $\omega = qs$ in Eq. (6) gives the main contribution to the displacement vector magnitude (the Cherenkov mechanism) and, thus, to the electron-lattice energy exchange. From the resonance condition $\omega = qs$ for the wave vector component $q_{||}$ directed along the electron velocity, we obtain

$$q_{||} = \frac{s}{v}\, q. \tag{7}$$

Since $s \ll v$ (for typical metals, $s \sim 10^5$ cm/s and $v \sim 10^8$ cm/s), one can see from Eq. (7) that sound

is mainly generated by the electron in the direction perpendicular to its motion, $q_{||} \ll q_{\perp}$. In addition, Eq. (7) gives us an opportunity to understand how the situation changes in the case of spatially confined systems. For such systems, the minimum magnitude of $q_{||}$ cannot be less than $q_{||}^{\min} = 2\pi/L$. On the other hand, the right-hand side of Eq. (7) cannot exceed $sq_{\rm D}/v \equiv \omega_{\rm D}/v$, where $q_{\rm D}$ is the Debye wave vector, and $\omega_{\rm D}$ the Debye frequency. Therefore, provided that

$$\frac{2\pi}{L} \geqslant \omega_{\rm D}/v, \tag{8}$$

no resonance (at not allowed q-values) becomes possible, and the Cherenkov mechanism is no more active at such dimensions.

Let us dwell on confined systems. In order to deal with a scalar rather than a vector quantity, let us introduce the function χ, so that

$$\mathbf{u} = \boldsymbol{\nabla}\chi. \tag{9}$$

Taking the oscillatory character of the electron motion along the axis OZ [see Eq. (5)] into account, let us expand $\chi(\mathbf{R}, t)$ in the Fourier series in $R_{||}$ and the Fourier integral over $\mathbf{R}_{\perp}$ (the details can be found in works [8, 9]):

$$\chi(\mathbf{R}_{\perp}, R_{||}; t) = \\ = \sum_{l=-\infty}^{\infty} \int d\mathbf{q}_{\perp} \tilde{\chi}(\mathbf{q}_{\perp}, q_{||}^{l}; t)\, e^{i(\mathbf{q}_{\perp}\mathbf{R}_{\perp} + q_{||}^{l} R_{||})}, \tag{10}$$

where

$$q_{||}^{l} = \frac{2\pi l}{L}, \quad l = 1, 2, 3, \ldots . \tag{11}$$

Here, we used the cyclic condition

$$\chi(\mathbf{R}_{\perp}, R_{||}; t) = \chi(\mathbf{R}_{\perp}, R_{||} + L; t). \tag{12}$$

Substituting expansion (10) into Eq. (3), we obtain the following equation to determine $\tilde{\chi}(\mathbf{q}_{\perp}, q_{||}^{l}; t)$:

$$\frac{\partial^2}{\partial t^2}\tilde{\chi}(\mathbf{q}_{\perp}, q_{||}^{l}; t) + q^2 s^2 \tilde{\chi}(\mathbf{q}_{\perp}, q_{||}^{l}; t) = \frac{\Lambda}{\rho} \frac{e^{i\, q_{||}^{l} z_0(t)}}{(2\,\pi)^2}, \tag{13}$$

where $q^2 = q_{\perp}^2 + (q_{||}^{l})^2$.

Therefore, while finding the function $\tilde{\chi}(\mathbf{q}_{\perp}, q_{||}^{l}; t)$, we obtain an equation for an oscillator with the frequency $\omega = qs$ that is subjected to the action of a periodic force from the electron side. Substituting the solution of Eq. (13) into formula (10), we determine the function $\chi(\mathbf{R}_{\perp}, R_{||}; t)$. Then, according to Eq. (9), we can also determine the displacement vector $\mathbf{u}(\mathbf{R}, t)$. In accordance with Eq. (4), the energy spent by the electron on the sound generation (the details of calculation can be found in works [8,9]). For the real part of $\partial\xi/\partial t$, we obtain [8, 9]

$$\mathrm{Re}\left(\frac{\partial \xi}{\partial t}\right) = \frac{\Lambda^2 q_{\max}^2}{16\,\pi\rho\, v} G(L, v). \tag{14}$$

Here, $q_{\max}$ is the magnitude maximum of the acoustic-mode wave vector,

$$q_{\max} = \begin{cases} q_{\rm D} & \text{at } q_{\rm D} < 2\,k_{\rm F}, \\ 2\,k_{\rm F} & \text{at } q_{\rm D} > 2\,k_{\rm F}, \end{cases} \tag{15}$$

where $k_{\rm F}$ is the Fermi wave vector (below, we will assume that $q_{\max} = q_{\rm D}$). In addition, the notation

$$G(L, v) = \begin{cases} (l_{\max}/\eta)^4 (1 + l_{\max}^{-1})^2 & \text{at } \eta \geqslant 1, \\ 0 & \text{at } \eta < 1 \end{cases} \tag{16}$$

was introduced, where

$$l_{\max} = \mathrm{floor}(\eta), \tag{17}$$

the function $\mathrm{floor}(\eta)$ is the greatest integer less than or equal to the number η, and

$$\eta \equiv L/L_{\rm c} = \frac{L}{2\,\pi} \frac{q_{\rm D}}{[(v/s)^2 - 1]^{1/2}} \approx L \frac{q_{\rm D}}{2\,\pi} \frac{s}{v}. \tag{18}$$

Hence,

$$l_{\max} = \mathrm{floor}\,(L/L_{\rm c}); \quad L_{\rm c} = \frac{2\,\pi\, v}{q_{\rm D}\, s}. \tag{19}$$

The expression for $G(L, v)$ depends quasiperiodically on $\eta = L/L_{\rm c}$ (see Fig. 1). The appearance of every new peak in the curve $G(L, v)$ is associated with the inclusion of a new acoustic mode into the electron-lattice energy exchange. [Concerning the resonance of an external force with its own acoustic mode, see Eq. (13).] As one can see from Eq. (16), the electron-lattice energy exchange in the bulk disappears at $L < L_{\rm c}$, i.e., $G(L < L_{\rm c}, v) = 0$, and only the weaker surface energy exchange remains [10].

Expression (14) determines the energy transferred by an electron moving with the velocity v to the lattice per unit time. To find the contribution of all electrons in the volume V (within the energy interval allowed by the Pauli principle) to the electron-lattice

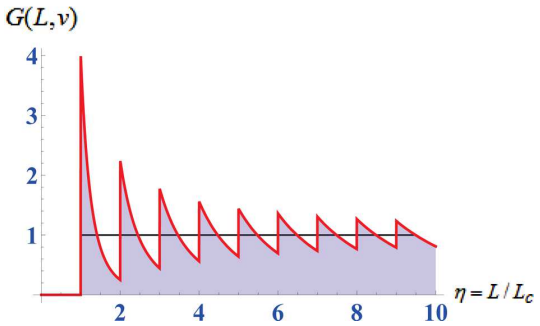

Fig. 1. Quasiperiodic dependence of the function $G(L,v)$ on $\eta = L/L_c$ [see Eq. (14)]

energy exchange, we have to calculate the quantity

$$Q(L,\theta_e) = V\int\limits_{\mu-\theta_e}^{\mu+\theta_e} d\,\varepsilon\left\{2g(\varepsilon)\,f_0(\varepsilon)\mathrm{Re}\left(\frac{\partial\xi}{\partial t}\right)\right\}, \tag{20}$$

where

$$g(\varepsilon) = \frac{\sqrt{2}m^{3/2}}{\pi^2\hbar^3}\sqrt{\varepsilon} \tag{21}$$

is the density of states, ε the electron energy, μ the chemical potential, θ_e the electronic temperature (in energy units), and $f_0(\varepsilon)$ the Fermi function

$$f_0(\varepsilon) = \left[\exp\left(\frac{\varepsilon-\mu}{\theta_e}\right)+1\right]^{-1}. \tag{22}$$

Expression (20) determines the energy spent by hot electrons in the volume V per unit time to excite sound vibrations in the lattice. But, along with the generation of sound vibrations, electrons also absorb sound. At the thermodynamic equilibrium, the energy spent by the electrons on the sound generation is equal to the energy obtained by them at the sound absorption. Therefore, taking the both effects (sound generation and absorption) into account, we see that the energy balance equation from which the electronic temperature is determined – an analog of Eq. (2) – should include the difference $Q(L,\theta_e)-Q(L,\theta)$, where θ is the lattice temperature, rather than $Q(L,\theta_e)$ alone.

Let us change the variable: $u = (\varepsilon-\mu)/\theta_e$. Then, from Eqs. (20) and (14), we obtain

$$Q(L,\theta_e) = \theta_e\frac{m^2\Lambda^2}{2(2\pi\hbar)^3}\frac{V}{\rho}\,q_{\mathrm{D}}^4\int\limits_{-1}^{1}du\,f_0(u)G(L,\theta_e;u). \tag{23}$$

The function $G(L,\theta_e;u)$ in this equation coincides with the function $G(L,v)$, if we put $v = v_{\mathrm{F}}(1+\theta_e u/\mu)^{1/2}$ in the latter (v_{F} is the Fermi velocity). Then, the quantity η in $G(L,\theta_e;u)$ is now equal to

$$\eta = (1+\theta_e u/\mu)^{-1/2}L/L_{\mathrm{F}}, \tag{24}$$

where

$$L_{\mathrm{F}} \equiv \frac{2\pi v_{\mathrm{F}}}{q_{\mathrm{D}}s}. \tag{25}$$

Since $\theta_e \ll 1$, the quantity η, as well as $G(L,u)$, seems to depend weakly on u, so that this dependence can be neglected. This is true as long as the value of L/L_{F} is not an integer number (or a number that is rather close to an integer). The integer values of L/L_{F} correspond to the resonance condition for the force exerted on the "acoustic vibrator" from the electron that oscillates between the potential walls [see Eq. (13)]. Under the resonance conditions (or close to them), both the entrance into the resonance and the exit from it owing to the action of θ_e are possible. Therefore, the dependence on θ_e under those conditions becomes essential.

Now, instead of the heat balance equation (2), which determines the electronic temperature under stationary conditions, we have the equation

$$Q(L,\theta_e) - Q(L,\theta) = VW, \tag{26}$$

where

$$Q(L,\theta_e) = V\alpha\theta_e\int\limits_{-1}^{1}du\,f_0(u)G(L,\theta_e;u) \tag{27}$$

and

$$\alpha \equiv \frac{m^2\Lambda^2}{2(2\pi\hbar)^3}\frac{q_{\mathrm{D}}^4}{\rho}. \tag{28}$$

One can easily see from Eq. (16) that, at $L\to\infty$, the function $G(L\to\infty,\theta_e;u)\to 1$, and, instead of Eq. (27), we obtain the well-known expression [6]

$$\frac{m^2\Lambda^2}{2(2\pi\hbar)^3}\frac{q_{\mathrm{D}}^4}{\rho}(\theta_e-\theta) = W. \tag{29}$$

3. Electron-Lattice Energy Exchange

According to Eq. (16), $G(L,v)=0$ at $L/L_{\mathrm{c}}<1$. This means that the channel of electron-lattice energy exchange that is main in massive metal disappears in

small metal clusters, if the size L of the latter does not exceed $L_{\rm c}$:

$$L < L_{\rm c} \approx L_{\rm F} = \frac{2\pi v_{\rm F}}{q_{\rm D} s}. \tag{30}$$

This condition can be rewritten in the form

$$\omega_{\rm D} < \frac{2\pi v_{\rm F}}{L}. \tag{31}$$

The obtained inequality can be interpreted in the following way. If the electron oscillates between the potential walls with a frequency higher than the maximum of the lattice vibration frequency $\omega_{\rm D}$, it ceases to generate sound vibrations of the lattice, and this channel of the hot electron relaxation becomes closed. Less intensive relaxation mechanisms associated with the electron scattering at the cluster surface come into play [11].

The sharp weakening of the electron-lattice energy exchange under condition (30) [or, equivalently, (31)] can explain the appearance of hot electrons in the stationary state at relatively low energy powers introduced into the nanoparticle. This weakening can also explain why there are no similar effects in continuous films and massive metals. Namely, owing to the intensive electron-lattice energy exchange in those objects, the thermal destruction of the lattice occurs before the electronic temperature substantially exceeds the lattice one. Hot electrons can be obtained in those materials without thermally destructing their lattices only with the help of ultrashort powerful laser pulses.

Let us return to the balance equation that determines the electronic temperature. From Eqs. (26) and (27), we have

$$\int\limits_{-1}^{1} du\, f_0(u)[\theta_e G(L,\theta_e;u) - \theta G(L,\theta;u)] = \frac{W}{\alpha}. \tag{32}$$

Our task in this work consists in a detailed research of the specific features in the size dependence of the electron-lattice energy exchange near the critical size value and in elucidating how those features manifest themselves in the electronic temperature θ_e.

It is worth making the following remark. According to Eqs. (28) and (29), the energy transferred from electrons to the lattice depends linearly on θ_e. If the power W introduced into the nanoparticle increases, this dependence can become nonlinear. (In particular, such a nonlinearity can appear owing to the heating of the lattice as well.) This nonlinearity will automatically reveal itself as a nonlinear dependence of θ_e on W. We will confine the consideration to the nonlinearity associated with the size-related features in the electron-lattice energy exchange, when the size of nanoparticles L is close to the critical value $L_{\rm c}$. It should also be emphasized that, under certain conditions – e.g., if the electrons and the lattice are heated up simultaneously – it is the power introduced into the nanoparticle that may explicitly depend on the electronic temperature.

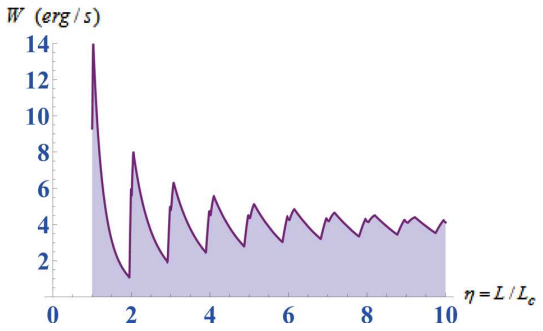

Fig. 2. Dependence of the power W required to obtain the electronic temperature $\theta_e/\theta = 5$ on the nanoparticle size of the nanoparticle size $\eta = L/L_{\rm c}$

After having made these remarks, let us proceed to the discussion of the results obtained. Figure 2 illustrates the relation between the power W introduced into the nanoparticle and the nanoparticle size $\eta = L/L_{\rm c}$ for the fixed value of the electronic temperature $\theta_e/\theta = 5$. This figure clearly demonstrates that the power W required for a given electronic temperature θ_e to be reached strongly depends on the nanoparticle size.

As was noted above, the Cherenkov mechanism of electron-lattice energy exchange is absolutely inactive, if $L < L_{\rm c}$. As the size L grows above $L_{\rm c}$, this mechanism becomes switched-on in a quasiperiodic manner. Every time when a new group of acoustic waves with a longitudinal (relatively to the electron velocity) component $q_{||}$ of the wave vector become engaged in the energy exchange process, there appears a peak in the intensity of the electron-lattice energy exchange. Even this cause alone can provide a wide range of electronic temperatures in the island films, which are characterized by the island size and shape dispersion. An additional contribution to the electronic temperature distribution over nano-

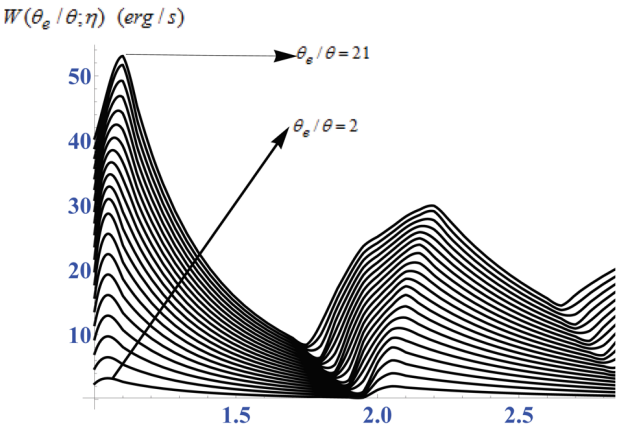

Fig. 3. Dependences of the power $W(\theta_e/\theta;\,\eta)$ required to obtain the identical (fixed) temperature of hot electrons θ_e/θ in nanoparticles with different sizes

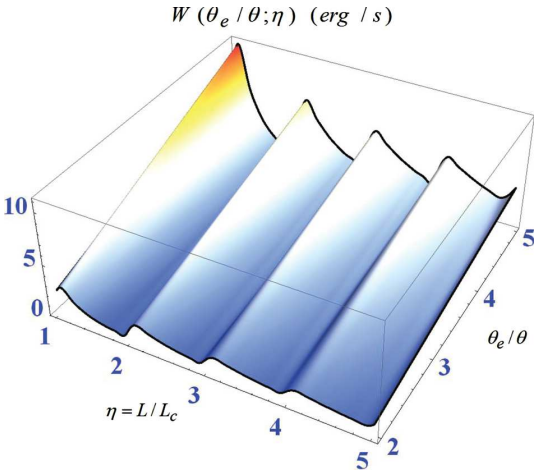

Fig. 4. Dependence between the power introduced into the nanoparticle and the obtained electronic temperature at various nanoparticle sizes

particles can be made by the explicit dependence of W on θ_e.

In Fig. 3, the dependences of the electronic temperature on the power introduced into nanoparticles with various sizes close to the critical value L_c are shown ($2 \leqslant \theta_e/\theta \leqslant 21$ and $\Delta\theta_e/\theta = 1$). The results testify to a high sensitivity of the electronic temperature to the nanoparticle size near the critical value. The same power W introduced into metal nanoparticles with slightly different sizes (near the critical one) may result in substantially different electronic temperatures. An enhanced sensitivity to the nanoparticle size is also observed at nanoparticle sizes that are multiples of the critical value, but this parameter becomes much weaker.

Figure 3 also shows the power W that is required to obtain the same (fixed) temperature of hot electrons θ_e in nanoparticles with different sizes. The curves in Fig. 3 are characterized by different values of the ratioθ/θ_0. One can see that if the sizes of nanoparticles are close to the critical one, substantially different powers have to be introduced into them in order to obtain the same electronic temperature.

Figure 4 shows a three-dimensional graphical representation of the relation between the power introduced into the nanoparticle and the electronic temperature at various nanoparticle sizes.

A possibility of obtaining hot electrons in nanoparticle with definite sizes at relatively low introduced powers explains why hot electrons can be obtained under stationary conditions only in island films, but not in continuous films and massive metals. Owing to the intensive electron-lattice energy exchange in continuous films and massive metals, the thermal destruction of the material occurs earlier than the electronic temperature becomes different from the lattice temperature. Therefore, hot electrons can be produced in those materials without the thermal destruction of the latter only with the help of short and powerful laser pulses. However, in the metal island films with a wide size dispersion, which were used in experiments [1], there always existed metal islands whose sizes were equal (or close) to the critical one. In such islands, the intensity of electron-lattice energy exchange was minimum, so that the electronic temperature was reached at a minimum power. At such relatively low powers, the temperature of the lattice in the island could be easily stabilized at values far from the film destruction temperature.

In works [8, 9], it was shown that the Cherenkov mechanism of sound generation by electrons, which is the main channel for the hot-electron relaxation in massive metals, is no more active in metal nanoparticles whose sizes are smaller than a certain critical value. Furthermore, the decrease in the intensity of electron-lattice energy exchange with the reduction of the particle size and its approach to the critical value was found to be not smooth, but quasiperiodic.

In this work, we analyzed how the size dependence of the electron-lattice energy exchange affects the temperatures of hot electrons when approaching the critical sizes. A high sensitivity of the electronic tem-

perature to the nanoparticle size in vicinities of critical values was demonstrated. The size dependence of the electronic temperature was also observed at nanoparticle sizes that are multiples of the critical size, but it was much weaker.

A wide dispersion of metal islands in the dispersed films [1] is responsible for a wide dispersion of electronic temperatures in different islands and, in such a way, provides a multicolored glowing of the film when a current passes through it.

We considered in detail the size dependence of the electron-lattice energy exchange near the critical size of metal nanoparticle. If the nanoparticle size does not coincide with the critical value $L_{\rm c}$ and the electron velocity changes within the limits

$$(1-\theta_e/\mu)^{1/2} \leqslant v/v_{\rm F} \leqslant (1+\theta_e/\mu)^{1/2},$$

the multiplier $G(L,\theta_e;u)$ in formula (27) can be put out of the integral (at $u=0$), which means, in effect, that we put $L_{\rm c} \approx L_{\rm F}$. Then, from Eq. (26), we obtain the following analytic size dependence:

$$\theta_e \approx \theta + \frac{W}{\alpha}\left(\frac{L}{L_{\rm F}}\right)^4 \Bigg/ \left(\text{floor}\left(\frac{L}{L_{\rm F}}\right)\right)^2 \left\{1+\text{floor}\left(\frac{L}{L_{\rm F}}\right)\right\}^2. \tag{33}$$

From this formula at $L \gg L_{\rm F}$, we obtain the well-known result

$$\theta_e \approx \theta + \frac{W}{\alpha}. \tag{34}$$

4. Conclusions

The results of computational experiments confirmed the fundamental concepts of the theory of hot electrons in metal island films developed by P.M. Tomchuk with co-authors [2, 3, 7–16].

The appearance of hot electrons in metal island films at relatively low input powers is possible owing to the peculiarities of the energy exchange in those objects. In a massive metal, the main channel of the relaxation for nonequilibrium electrons is the Cherenkov mechanism of sound generation. In metal island films, this mechanism is modified owing to the change of the electron trajectory shape (from rectilinear in massive metals to oscillatory in nanoparticles). In this case, the intensity of electron-lattice energy exchange drastically decreases at certain sizes of metal nanoparticles. This circumstance favors the appearance of hot electrons. The size dependences of the energy exchange manifest themselves in the electronic temperature, and the latter manifests itself in various physical phenomena (e.g., electron and photon emissions).

In this work, we considered the sound generation by electrons and the related phenomena that occur in the bulk of metal nanoparticles. Hot electrons in those particles can generate sound in the dielectric matrix, where they are located. The corresponding phenomena were considered, e.g., in works [15–17].

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ЕЛЕКТРОН-ҐРАТКОВИЙ
ЕНЕРГООБМІН І ГАРЯЧІ ЕЛЕКТРОНИ
В ОСТРІВКОВИХ МЕТАЛЕВИХ ПЛІВКАХ

Р е з ю м е

В роботі розглядаються завершальні тези теорії автора, яка описує гарячі електрони в острівкових металевих плівках. Детально досліджено, як розмірні залежності електрон-ґраткового енергообміну впливають на електронні температури гарячих електронів при наближенні системи до критичних розмірів. Виявлено високу чутливість електронної температури до розмірів металевої наночастинки в околі критичних значень. Отримано результати обчислювальних експериментів, які підтверджують основні положення теорії.