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GALAXY ROTATION CURVES IN THE μ -DEFORMATION-BASED APPROACH TO DARK MATTER¹

We elaborate further the μ -deformation-based approach to the modeling of dark matter, in addition to the earlier proposed use of μ -deformed thermodynamics. Herein, we construct μ -deformed analogs of the Lane–Emden equation (for density profiles) and find their solutions. Using these, we plot the rotation curves for a number of galaxies. Different curves describing the chosen galaxies are labeled by respective (different) values of the deformation parameter μ . As a result, the use of μ -deformation leads to the improved agreement with observational data. For all the considered galaxies, the obtained rotation curves (labeled by μ) agree better with data, as compared to the well-known Bose–Einstein condensate model results of T. Harko. Besides, for five of the eight cases of galaxies, we find a better picture for rotation curves, than the corresponding Navarro–Frenk–White (NFW) curves. The possible physical meaning of the parameter μ basic for this version of μ -deformation is briefly discussed.

Keywords: dark matter, μ -deformation, deformed Lane–Emden equation, galaxy rotation curves.

1. Introduction

The model of dark matter as a Bose–Einstein condensate (BEC) of scalar particles arose as an alternative to the cold dark matter (CDM) paradigm. It provides a possibility to resolve several tensions, which CDM faces on the small scales, such as the core-cusp problem and the overabundance of the small-scale structures [1]. We should mention, however, that the BEC model is not unique in this aspect, and models like warm dark matter (warm DM), self-interacting DM are also able to solve CDM problems on the small scales.

The BEC model considers the ultralight DM galaxy halo as a stable “core” solution of the nonlinear Schrödinger (or Gross–Pitaevsky) equation, with the classical Poisson equation for the gravitational potential of the DM halo surrounded by a DM envelope that mimics the CDM halo on the larger distances from the center of a galaxy [2]. The analysis of the luminous matter kinematics in galaxies, like, say, the kinematics of dwarf spheroidal galaxies, indicates

that the coherent state core can represent all required DM in dwarf galaxies, but only some smaller fraction of DM in bigger galaxies [3].

The DM particles within the considered model are ultralight scalar ones with mass $m \sim (1-10) \times 10^{-22}$ eV that is in a good agreement with most observations (except for only the Lyman- α forest). Scalar particles with such extremely small mass² can be considered as axion-like particles, so that their mass is protected against a radiative correction by the nonexact shift symmetry $\phi \rightarrow \phi + C$. It is usually based on a free scalar field with the ϕ^4 self-interaction potential [2].

There exist, however, different extensions of this model, some of which introduce a more complex (than ϕ^4) self-interacting potential. Another interesting way is to consider a non-minimal coupling of the condensate to the gravity, e.g., through potentials

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² Note that in some works, see, e.g., [4], the role of Bose-condensed particles of dark matter is played by gravitons of tiny mass, bound from the above by $m_g \sim 10^{-26}$ eV.

$G_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi$ and $\nabla_\mu\phi\nabla^\mu\phi R$ (here, $G_{\mu\nu}$ – Einstein tensor, R – scalar curvature) [5, 6].

A completely different direction is constituted by the DM models based on a non-standard statistics, like a condensate of particles obeying the infinite statistics [7] and also our preceding work, in which we have proposed the model of dark matter viewed as the condensate of a gas obeying the μ -deformed thermostatics [8].

2. Bose Condensate Dark Matter Model: Gross–Pitaevsky and Lane–Emden Equations

The BEC DM model suggests that DM consists of ultralight bosons of the mass 10^{-22} eV, so their de Broglie wavelength is of astronomical scale (kpc). Within this model, the galaxy DM halo is represented by a halo of such particles, most of which are in the ground state, thus forming a non-relativistic self-gravitating Bose–Einstein condensate. If only particles in the ground state are taken into account, such condensate halo can be described by the Gross–Pitaevsky equation. Here, we give a brief overview of the BEC halo description through the Gross–Pitaevsky equation (see [9] for a more detailed discussion)

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r})+V(\mathbf{r})\Psi(\mathbf{r})+\frac{4\pi\hbar^2a}{m}|\Psi(\mathbf{r})|^2\Psi(\mathbf{r})=\tilde{\mu}\Psi(\mathbf{r}). \quad (1)$$

Herein, $\Psi(\mathbf{r})$ is the wave function of the ground state, $\tilde{\mu}$ denotes the chemical potential, the term $\propto |\Psi(\mathbf{r})|^2\Psi(\mathbf{r})$ is responsible for the self-scattering of condensate particles, and $V(\mathbf{r})$ represents any external potential that will be, in the considered case, the Newtonian gravitational potential of a DM halo that obeys the Poisson equation:

$$\nabla^2V(\mathbf{r})=4\pi G\rho(\mathbf{r}). \quad (2)$$

Particles in the condensate state are suggested to be non-relativistic with almost zero temperature. So, the Thomas–Fermi approximation, where the kinetic term of the equation is neglected, is applicable here. The corresponding equation can be rewritten by introducing the density of particles

$$\rho(\mathbf{r})=m|\Psi(\mathbf{r})|^2$$

instead of the wave-function, and takes a simpler form:

$$\rho(\mathbf{r})=\frac{m^2}{4\pi\hbar^2a}(\tilde{\mu}-mV(\mathbf{r})).$$

Then, similarly to [9], we apply the Laplace operator to the latter equation and use (2), which leads to

$$\Delta_r\rho(r)+k^2\rho(r)=0 \quad \text{with} \quad k^2\equiv\frac{Gm^3}{\hbar^2a}, \quad (3)$$

where a is the scattering length. Thus, we get the Lane–Emden equation with polytropic index $n=1$. This equation admits a simple analytical solution for the DM halo density within the BEC DM model:

$$\rho(r)=\rho_c\frac{\sin kr}{kr}. \quad (4)$$

The solution contains two free parameters: $\rho_c=\rho(0)$, which is the density at the DM halo center, and the parameter k related to the total halo radius R as $k=\pi/R$.

It should be mentioned that this solution involves only particles in the ground state. However, in a more realistic description, other states should also be considered. It is known that the ultralight DM halo consists of the static core (which allows one to solve core/cusp problem) surrounded by an envelope, which mimics the CDM behavior on the larger scales [2]. Solution (4) is responsible only for the core part of a galaxy DM halo. So we expect that it will provide a good explanation of observations on the scales smaller than the core size and, at the same time, will probably meet some tensions with observations of more distant regions of galaxies.

It is also worth to mention that the same approximate equation can be obtained from the Klein–Gordon equation for a scalar field, what makes these models closely connected.

3. Deformation of the Lane–Emden Equation

Since a μ -deformed analog of the Gross–Pitaevsky equation is not available at present (and constitutes a non-trivial problem), we concentrate on performing a μ -deformation of the Lane–Emden equation.

3.1. Elements of μ -calculus

Since our approach exploits the so-called μ -calculus, let us first sketch it briefly (more detailed introduction to the μ -calculus and its application to deformed models is given in [8, 10]). The basic notion of this approach is the μ -bracket (with X being a number or an operator):

$$[X]_\mu=\frac{X}{1+\mu X}, \quad \mu\geq 0.$$

Obviously, $[X]_\mu \rightarrow X$ if $\mu \rightarrow 0$. Using the μ -bracket, we define a μ -deformed (or μ -) derivative such that

$$\mathcal{D}_x^{(\mu)} x^n = [n]_\mu x^{n-1}. \tag{5}$$

The μ -derivative does not satisfy³ the Leibnitz rule, i.e.

$$\mathcal{D}_\mu(f \cdot g) \neq f \cdot (\mathcal{D}_\mu g) + (\mathcal{D}_\mu f) \cdot g.$$

Note that the above action (5) implies the following presentation of the μ -derivative in terms of usual derivative in the form of a formal power series

$$\begin{aligned} \mathcal{D}_x^{(\mu)} &\equiv \left[\frac{d}{dx} \right]_\mu = \frac{\frac{d}{dx}}{1 + \mu \frac{d}{dx}} = \\ &= \frac{d}{dx} \left(1 - \mu \frac{d}{dx} + \mu^2 \frac{d}{dx} \frac{d}{dx} - \dots \right) \end{aligned} \tag{6}$$

that incorporates all higher orders of the derivative $\frac{d}{dx}$. This fact is of basic importance.

Now, one can introduce a deformation in the theory of interest, merely by replacing each derivative in equations by its deformed analog.

The μ -bracket is used to gain deformed versions of known functions. Say, the μ -deformed exponent is

$$\exp_\mu x = \sum_{n=0}^{\infty} \frac{x^n}{[n]_\mu!}.$$

Then we define the deformed sine and cosine functions:

$$\sin_\mu x = \frac{1}{2i} (\exp_\mu(ix) - \exp_\mu(-ix)) =$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{[2n+1]_\mu!},$$

$$\cos_\mu(x) = \frac{1}{2} (\exp_\mu(ix) + \exp_\mu(-ix)) =$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{[2n]_\mu!}.$$

Here the μ -factorial means the product

$$[m]_\mu! = [m]_\mu [m-1]_\mu \dots [2]_\mu [1]_\mu.$$

³ It is nevertheless possible to introduce a μ -analog of the Leibnitz rule, though its definition is rather non-trivial, see [8].

The deformed harmonic functions will arise in the next sections of the present paper. We have to stress the fact that the familiar differential relations between harmonic functions (involving usual derivative) are not valid in the μ -deformed case: $\frac{d}{dx} \sin_\mu(x) \neq \cos_\mu(x)$. However, the deformed counterpart, which uses the μ -derivative, does hold. Namely,

$$\mathcal{D}_x^\mu \sin_\mu(x) = \cos_\mu(x).$$

It is clear that the deformed analog of derivative and deformed functions should reduce to their non-deformed versions in the limiting case of $\mu \rightarrow 0$. It is a simple matter to restore the non-deformed versions of equations of underlying theory at any step of analysis.

3.2. Deforming Laplacian in the LE equation

As already mentioned, the DM halo density in the BEC DM model could be approximately described by the Lane–Emden (LE) equation with polytropic index $n = 1$:

$$\Delta_r \rho(r) + k^2 \rho(r) = 0, \tag{7}$$

where Δ_r is the radial (thus, 1-dimensional) part of the spherical Laplace operator, namely

$$\Delta_r f(r) = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr} f(r) \right).$$

The latter is also equal to

$$\Delta_r f(r) = f''(r) + \frac{2}{r} f'(r),$$

and the LE equation can be written in its more familiar form:

$$\left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} + k^2 \right) \rho(r) = 0. \tag{8}$$

In order to deform, we firstly take the LE equation in the initial form (7) as the starting point. Let us introduce the deformation in the equation by replacing the derivative with respect to r by its μ -deformed analog. Thus, we get a μ -deformed analog of the LE equation:

$$\frac{1}{r^2} \mathcal{D}_r^\mu \left(r^2 \mathcal{D}_r^\mu \rho(r) \right) + k^2 \rho(r) = 0.$$

The equation could be easily rewritten in terms of the dimensionless variable $x = kr$. As a result, we obtain

$$\frac{1}{x^2} \mathcal{D}_x^\mu \left(x^2 \mathcal{D}_x^\mu \rho(x) \right) + \rho(x) = 0. \tag{9}$$

In addition, we adopt the same initial condition, which was valid for the solution of the original equation:

$$\rho(0) = \rho_c, \quad \rho'(0) = 0.$$

In the last relation, the usual or μ -deformed differentiation could be applied. Luckily, as it will become clear later, this does not affect the result.

We are looking for a solution of Eq. (8) in the form

$$\rho(x) = \rho_c \sum_{n=0}^{\infty} a_n x^n.$$

The operators in the equation act on the series as

$$\mathcal{D}_x^\mu \cdot \rho(x) = \sum_{n=1}^{\infty} a_n [n]_\mu x^{n-1},$$

$$\mathcal{D}_x^\mu \cdot (x^2 \rho(x)) = \sum_{n=1}^{\infty} a_n [n]_\mu [n+1]_\mu x^n.$$

Then, from the μ -LE equation we infer

$$\sum_{n=1}^{\infty} a_n [n]_\mu [n+1]_\mu x^{n-2} + \sum_{n=0}^{\infty} a_n x^n = 0. \quad (10)$$

The initial conditions imply that

$$\begin{aligned} \rho(0) = \rho_c = \rho_c a_0 &\rightarrow a_0 = 1, \\ \rho'(0) = 0 = \rho_c a_1 &\rightarrow a_1 = 0. \end{aligned}$$

By changing the summation limits, we have

$$\sum_{n=0}^{\infty} a_{n+2} [n+2]_\mu [n+3]_\mu x^n + \sum_{n=0}^{\infty} a_n x^n = 0.$$

Due to this, we relate the coefficients as

$$a_{n+2} = -a_n \frac{1}{[n+2]_\mu [n+3]_\mu}, \quad n = 2m.$$

Then,

$$\begin{aligned} a_{2n} &= (-1)^n \frac{1}{[2n]_\mu [2n+1]_\mu} \cdots \frac{1}{[3]_\mu [2]_\mu} = \\ &= (-1)^n \frac{1}{[2n+1]_\mu!} [1]_\mu. \end{aligned}$$

The solution then takes the form

$$\rho(kr) = \rho_c [1]_\mu \sum_{n=0}^{\infty} (-1)^n \frac{(kr)^{2n}}{[2n+1]_\mu!} = \rho_c [1]_\mu \frac{\text{sin}_\mu(kr)}{kr}.$$

Thus, denoting $\rho_0 = \rho_c [1]_\mu$, we obtain

$$\rho(kr) = \rho_0 \frac{\text{sin}_\mu(kr)}{kr} \quad (11)$$

as our main result for the DM density distribution.

3.3. Deforming derivatives in the LE equation

As is known for deformed models, various types of the equation of deformation can be proposed. Here, we will present a different version of the μ -deformed LE equation. We start with the LE equation of the polytropic index $n = 1$ in its most common form (8), and introduce a deformation in the equation, by replacing the spatial derivatives d/dr by its deformed analog \mathcal{D}_r^μ :

$$\left(\mathcal{D}_r^\mu \mathcal{D}_r^\mu + \frac{2}{r} \mathcal{D}_r^\mu + k^2 \right) \rho(r) = 0. \quad (12)$$

Again, we are looking for the solution being a power series:

$$\rho(x) = \rho_c \sum_{n=0}^{\infty} a_n x^n.$$

Substituting this in the equation, we have

$$\begin{aligned} &\sum_{n=0}^{\infty} a_{n+2} [n+2]_\mu [n+1]_\mu x^n + 2 \sum_{n=0}^{\infty} a_{n+1} [n+1]_\mu x^{n-1} + \\ &+ \sum_{n=0}^{\infty} a_n x^n = 0. \end{aligned}$$

Let us take the same initial conditions $\rho(0) = \rho_c$, $\rho'(0) = 0$. After similar steps as above, we obtain the result⁴:

$$\rho(r) = \rho_c \sum_{n=0}^{\infty} (-1)^n \frac{(kr)^{2n}}{\prod_{l=1}^n [2l]_\mu ([2l-1]_\mu + 2)}. \quad (13)$$

4. Galaxy Rotation Curves

Now, let us confront the predictions of our model with available observational data. We analyze the rotation curves of low surface brightness (LSB) galaxies, as the kinematics of luminous matter in the galaxy depends on the density distribution within the galaxy. We have chosen those eight LSB galaxies which were analyzed by T. Harko, in order to compare the models. Since these are DM-dominated, we neglect the gravitational contribution of baryonic matter.

⁴ There exists another form of the deformed LE equation also possessing solution (11): its first term is the same as in Eq. (12), but the second and third terms are multiplied with respective functions (of k , r , and μ). This will be explored elsewhere.

Values of the parameters for the rotation curves of galaxies, in the three models

Galaxy	μ -BEC				BEC			NFW	N_{points}
	A, km/s	k, kpc ⁻¹	μ	χ^2	A, km/s	k, kpc ⁻¹	χ^2	χ^2	
DDO 53	32.27	0.97	0.180	1.9	33.16	1.09	5.4	7.8	18
HO I	35.51	1.27	0.151	40.2	34.78	1.27	160.1	109.8	22
HO II	40.11	0.40	0.179	31.8	37.93	0.45	66.8	6.4	35
IC 2574	81.37	0.17	0.179	13.7	73.9	0.24	14.2	117.2	50
NGC 2366	64.68	0.37	0.178	41.2	72.72	0.37	126.1	71.9	41
M81dwB	38.35	2.64	0.180	2.5	38.07	3.15	7.9	4.1	13
DDO 154	51.30	0.38	0.156	199.3	53.81	0.38	549.7	76.8	61
DDO 39	87.52	0.27	0.174	168.2	87.96	0.30	343.0	14.3	21

In the case of disk galaxies, where the trajectories of stars and gas clouds could be assumed circular with a good accuracy, we can apply the simple relation

$$mv(r)^2 = \frac{GM(r)m}{r}.$$

It arises from virial theorem's relation between the kinetic and potential energies $2T = V$ for a stable system in the gravitational potential $V \propto r^{-1}$. Then the velocity $v(r)$ on a circular orbit of radius r could be expressed as

$$v(r) = \sqrt{\frac{GM(r)}{r}}.$$

This equation provides a tool for studying the density distribution of matter in the galaxy through observed rotation curves. We will neglect the gravitational effect of luminous matter. Thus, only the dark matter component $M(r)$ is considered in the previous equation.

Therefore, to define the velocity $v(r)$ on any orbit r , we should calculate the total mass within this orbit

$$M(r) = 4\pi \int_0^r \rho(r')r'^2 dr'.$$

In accordance with the deformed differential relations with radial coordinate r , we have to use the deformed (or μ -) integration⁵. As a final result, we obtain the following expression for the velocity on a circular

⁵ That means applying the operator $(D_r^{(\mu)})^{-1}$ inverse to the one defined in Eqs. (4), (5).

orbit, by using the ‘‘Laplace-deformed’’ density solution (11):

$$v(r) = \sqrt{\frac{4\pi G\rho}{k^2} \sum_{n=0}^{\infty} \frac{(-1)^n (kr)^{2n+2}}{[2n+1]_{\mu}! [2n+3]_{\mu}}}. \quad (14)$$

With the notation

$$A = \sqrt{\frac{4\pi G\rho_0}{k^2}},$$

we also present the expression for the rotation velocity based on the ‘‘derivative-deformation’’ solution (13):

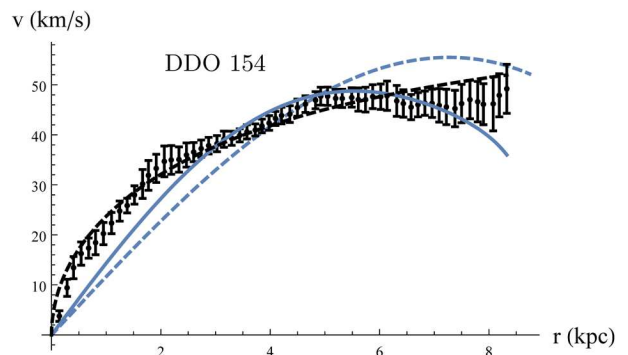
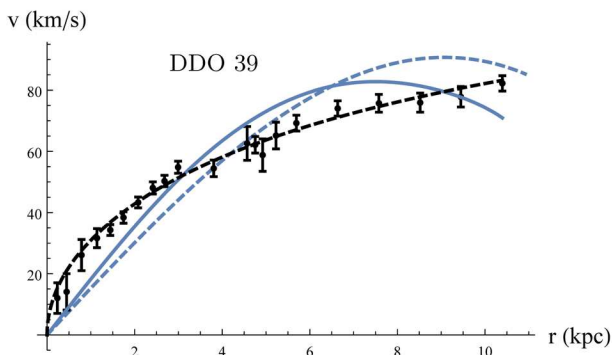
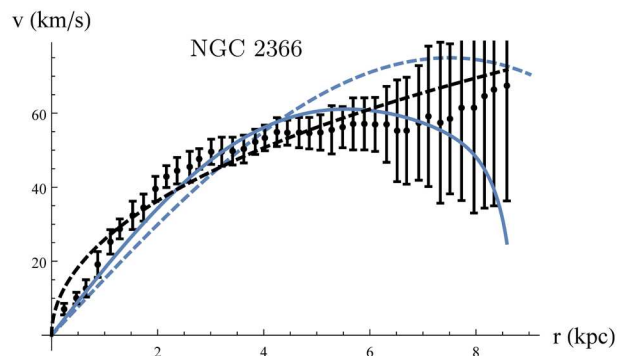
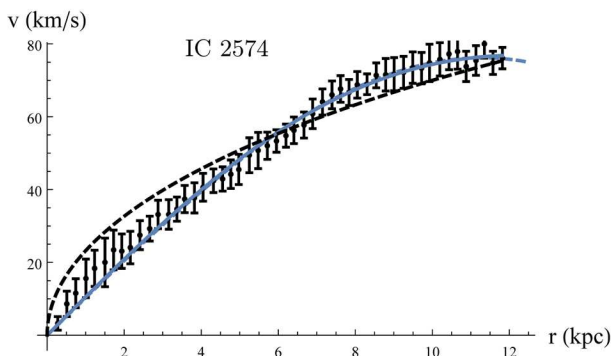
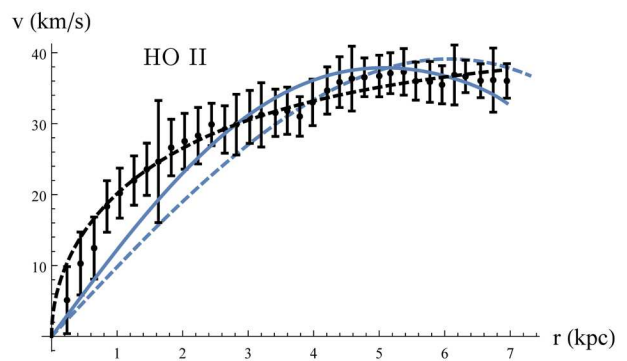
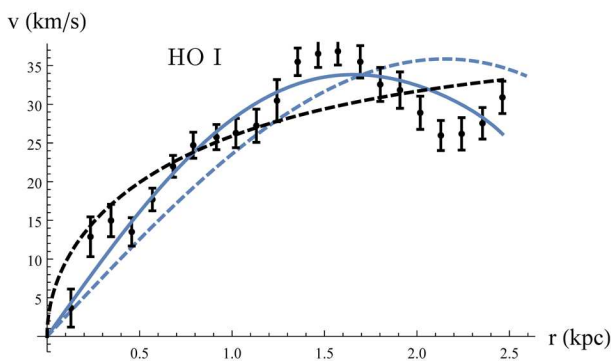
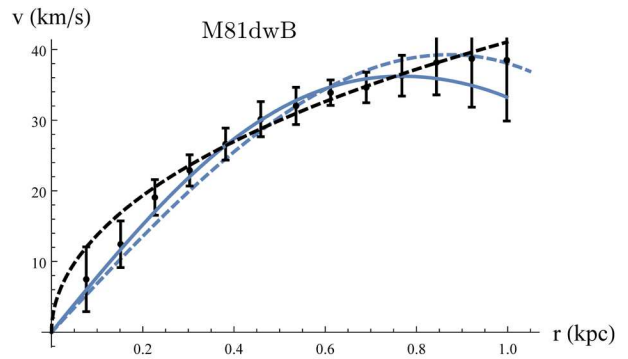
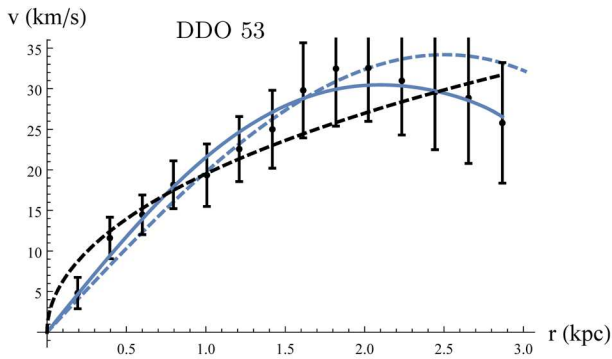
$$v(r) = A \sqrt{\sum_{n=0}^{\infty} \frac{(-1)^n (kr)^{2n+2}}{[2n+3]_{\mu} \prod_{l=1}^n [2l]_{\mu} ([2l-1]_{\mu} + 2)}}. \quad (15)$$

We perform the least-square analysis for the same eight LSB galaxies, which were studied in [9] regarding the classical BEC DM model. The observational rotation curves of these galaxies were taken from [11–13].

In the Table and Figure (solid lines present our curves), we give the results of fitting the rotation curves of eight LSB galaxies by theoretical curves within the μ -deformed ‘‘Laplace deformation’’ rotation curve (14), within the BEC DM stemming from its DM density solution (4), and the Navarro–Frenk–White profile [12] for CDM:

$$\rho_{\text{NFW}}(r) = \frac{\rho_0}{\frac{r}{R_s} \left(1 + \frac{r}{R_s}\right)^2}.$$

The least χ^2 value among three studied models for each of galaxies is denoted by the bold font.



5. Discussion and Concluding Remarks

We explored, in addition to μ -thermodynamics used [8] for the modeling of dark matter, the related approach based on μ -deformed spatial derivative. Two differing μ -deformed analogs of the Lane–Emden equation are studied, and their solutions describing density profiles of DM halos are found. This allowed us to obtain the plots for the rotation curves of a number of galaxies. The corresponding curves for the chosen galaxies involve differing values of the deformation parameter μ . As seen, nice agreement due to the use of μ -deformation is achieved: for all considered galaxies, our results show noticeable improvement as compared to the BEC model results of [9].

Moreover, the used approach provides somewhat better picture (agreement) even with respect to the famous NFW [12] rotation curves, say, in five (of eight) cases, i.e. the curves for the galaxies DDO 53, HO I, IC 2574, NGC 2366, and M81dwB.

The importance and strength of the μ -deformation stems from certain non-locality due to the usage of the deformed spatial μ -derivative (6), that is an extended operator built with the usual derivative in its denominator (that is, all orders of the derivative $\frac{d}{dr}$ are present). This feature resembles such well-known approach as nonlocal modifications of gravity (see, e.g., [14–18] and references therein). There is a rather popular viewpoint that the nonlocal gravity theories are of importance for solving the basic problems of cosmology – that of dark energy and dark matter.

In view of the success of the μ -deformation-based description, let us briefly discuss possible physical sense of the μ -deformation, modifying the (radial) spatial derivative, and the very parameter μ . Being very massive but relatively compact (from the viewpoint of cosmological scales) objects, the DM halos can modify (geometry of) the ambient space, and the employed μ -derivative takes effectively such modification into account, with μ measuring the extent of modification. This agrees with the noticed important feature: if we calculate the total mass of the galaxy DM halo (with fixed proper radius), we find that the bigger the halo mass, the greater the respective value of μ to be taken. At last, let us note that similar conclusions can be drawn basing on the formula (15).

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РОТАЦІЙНІ КРИВІ ГАЛАКТИК
У ПІДХОДІ ДО ТЕМНОЇ МАТЕРІЇ
НА ОСНОВІ μ -ДЕФОРМАЦІЇ

Резюме

В рамках μ -деформації розвинуто модель темної матерії, раніше побудовану у підході, що використовував μ -деформовану термодинаміку. Введено μ -аналоги рівняння Лейна–Емдена (для профілів густини) і знайдено його розв’язки. На їх основі побудовано графіки ротацій-

них кривих для низки галактик. Кожній кривій, які описують вибрані галактики, відповідає своє значення параметра деформації μ . Як наслідок, μ -деформація забезпечує покращене узгодження із спостережуваними даними. Для всіх розглянутих галактик отримані ротаційні криві (марковані значеннями μ) краще узгоджуються з даними порівняно із результатами відомої БЕК-моделі Т. Харко. Для п’яти з восьми галактик картина для ротаційних кривих є кращою навіть у порівнянні з відповідними кривими Наварро–Френка–Вайта (НФВ). Розглянуто можливий фізичний сенс параметра μ .