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# SPINOR MOVING FRAME, POLARIZED SCATTERING EQUATION FOR 11D SUPERGRAVITY, AND AMBITWISTOR SUPERSTRING[1]

*We reveal and discuss the spinor moving frame origin of the formalism of the 11D polarized scattering equation by Geyer and Mason [21]. In particular, we use the spinor moving frame formulation of the 11D ambitwistor superstring [35] considered as a dynamical system in the 11D superspace enlarged by tensorial central charge coordinates to rigorously obtain the expression for the spinor function on a Riemann sphere and the polarized scattering equation which that obeys.*

*Keywords*: supersymmetry, amplitudes, supergravity, higher dimensions, spinor moving frame, ambitwistor string.

## 1. Introduction

An impressive progress in the calculation of the scattering amplitudes of D = 4 maximally supersymmetric theories [1–4] reached with the use of on-shell methods, in particular, of the spinor helicity formalism and its superfield generalization [5–11], naturally led to the search for higher dimensional and curved space generalizations of the latter [12–25]. In particular, in [18, 20], the spinor helicity formalism for 11D supergravity was developed, a new constrained superfield formalism for 10D SYM and 11D SUGRA amplitudes was constructed, and the candidate for a generalization of the BCFW recurrent relations [5] for these constrained superamplitudes was discussed. In [19], an alternative analytic superfield formalism for superamplitudes was proposed. It was also oriented to the use of BCFW-type recurrent relations which are still to be found in this case.

More recently, an apparently different approach to 11D supergravity and 10D SYM amplitudes was proposed in [21]. It is based on the so-called polarized scattering equation, which can be considered as a kind of the square root of the CHY scattering equations [26, 27] (actually present already in [28–30]; see [31] for the recent development and more references). The polarized scattering equation for 6D amplitudes was proposed in [32], while the 11D and 10D polarized scattering equations are among the beautiful findings of [21]. Its relation with ambitwistor string models [33–41], the 11D version of which was considered for the first time in [35], was discussed and especially stressed in [21].

In this contribution based on [42], we revisit the 11D polarized scattering equation formalism of [21] and its ambitwistor superstring origin by using the spinor frame approach. We show that the correct basis to derive the polarized scattering equation is provided by the 11D ambitwistor superstring of [35],

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rather than its modification suggested in [21]. We present a rigorous derivation of the basic equations for the meromorphic spinor function, which is necessary to formulate the polarized scattering equation in its most suggestive form, and derive the polarized scattering equation. To this end, we have used essentially the possibility to formulate the 11D ambitwistor superstring as a system in an enlarged superspace with 528 bosonic coordinates, as well as the $SO(16)$ gauge symmetry of the 11D ambitwistor superstring [35].

Our notations are those of [18–20] and mainly coincide with [42].

## 2. Spinor Frame Approach to the 11D Spinor Helicity Formalism

### 2.1. Scattering data in $D = 11$

The light–like momentum $k_{\mu i}$, $k_{\mu i}k^{\mu}_i = 0$ of a massless particle (consider it to be the $i$-th particle of a scattering process) is expressed in terms of helicity spinors by

$$\begin{aligned} &k_{\mu i}\delta_{qp} = \lambda_{\alpha qi}\tilde{\Gamma}^{\alpha\beta}_{\mu}\lambda_{\beta pi} = \rho^{\#}_i v^{-}_{\alpha qi}\tilde{\Gamma}^{\alpha\beta}_{\mu}v^{-}_{\beta pi}, \\ &2\lambda_{\alpha qi}\lambda_{\beta pi} = 2\rho^{\#}_i v_{\alpha qi}{}^{-}v_{\beta pi}{}^{-} = \Gamma^{\mu}_{\alpha\beta}k_{\mu i}. \end{aligned} \tag{1}$$

Here, $\mu,\nu = 0,1,...,10$ are 11-vector indices, $\alpha,\beta = 1,...,32$ are SO(1,10) spinor indices, $q,p = 1,...,16$ denote the SO(9) spinor indices (below, we will also use these symbols to denote the SO(16) vector indices). In (1), we have used the contractions of the 11D Dirac matrices with the charge conjugation matrices $\Gamma_{\mu\alpha\beta} := \gamma_{\mu\alpha}{}^{\gamma}C_{\gamma\beta}$ and $\tilde{\Gamma}_{\mu}{}^{\alpha\beta} := C^{\alpha\gamma}\gamma_{\mu\gamma}{}^{\beta}$, which are real symmetric and obey

$$\Gamma_{\mu}\tilde{\Gamma}_{\nu} + \Gamma_{\nu}\tilde{\Gamma}_{\mu} = \eta_{\mu\nu}I_{32\times 32}.$$

Equations (1) also describe the essential constraints obeyed by the helicity spinors $\lambda_{\alpha q}$ (denoted by $\kappa_{\mathfrak{a}\alpha}$ in [21]) which can be expressed in terms of spinor harmonics (or spinor frame variables) [2]

$$V^{(\beta)}_{\alpha} = \left(v^{+}_{\alpha q}, v^{-}_{\alpha q}\right) \in \mathrm{Spin}(1,10) \tag{2}$$

by [18]

$$\lambda_{\alpha qi} = \sqrt{\rho^{\#}_i}\, v^{-}_{\alpha qi}. \tag{3}$$

[2] See [20, 43, 44] and refs. therein for details on spinor frame variables which are also called Lorentz harmonics (thus giving credit to the harmonic superspace approach of [45]).

Of course, the above equation describes the real Majorana helicity spinors for the case of momentum with positive energy, $k_{0i} > 0$, in which case also $\rho^{\#}_i > 0$, and $\sqrt{\rho^{\#}_i}$ is well defined. When describing the scattering processes, one usually arrange to consider all the particles as, say, outgoing and assign a momentum with negative energy to incoming particles, say, $k_{0j} < 0$. Although in this case with $\rho^{\#}_j < 0$, one can maintain Eqs. (3) just by setting $\sqrt{\rho^{\#}_j} = i\sqrt{|\rho^{\#}_j|}$ and considering pure imaginary $\lambda_{\alpha qj}$.

Equation (1) implies that the light-like momentum can be associated to the light-like vector from the corresponding moving frame (see [20, 43] and [46, 47])

$$k_{\mu i} = \rho^{\#}_i u^{=}_{\mu i}, \tag{4}$$

$$u^{(a)}_{\mu} = \left(\frac{u^{=}_{\mu} + u^{\#}_{\mu}}{2}, u^{I}_{\mu}, \frac{u^{\#}_{\mu} - u^{=}_{\mu}}{2}\right) \in \mathrm{SO}^{\uparrow}(1,10). \tag{5}$$

The spinor frame variables $v^{-}_{\alpha qi}$ can be considered as a kind of the square root of the light-like frame vector $u^{=}_{\mu}$ in the sense that the following constraints hold:

$$u^{=}_{\mu}\Gamma^{\mu}_{\alpha\beta} = 2v_{\alpha q}{}^{-}v_{\beta q}{}^{-}, \quad v^{-}_{\alpha q}\tilde{\Gamma}_{\mu}{}^{\alpha\beta}v^{-}_{\beta p} = u^{=}_{\mu}\delta_{qp} \tag{6}$$

(see [20, 43] and refs. therein for more details).

The helicity spinors (3) also carry the information about polarizations of the particles. But to make it transparent, we need to endow their space by an additional complex structure (see [19] for the discussion). This can be encoded in the complex polarization vector $U_{\mu i}$ (denoted by $e_{\mu}$ in [21]). It obeys

$$k_{\mu i}U^{\mu}_i = 0, \quad U_{\mu i}U^{\mu}_i = 0 \tag{7}$$

and can be decomposed onto the spacelike vectors of the frame (5) associated to the momentum by (4):

$$U_{\mu i} = u^{I}_{\mu i}U^{I}_i, \quad U^{I}_iU^{I}_i = 0, \quad I = 1,...,9. \tag{8}$$

Using the constraint obeyed by the vector and spinor frame variables (see [19, 20] and refs therein), we find

$$\not{U}_{\alpha\beta} := U_{\mu}\Gamma^{\mu}_{\alpha\beta} = 2v^{-}_{(\alpha|q}\gamma^{I}_{qp}v^{+}_{|\beta)p}U^{I}_i. \tag{9}$$

As was discussed in [19], the complex null polarization nine-vector $U^{I}$ in (8) can be related by

$$\not{U}_{qpi} := U^{I}_i\gamma^{I}_{qp} = 2\bar{w}_{qAi}\bar{w}_{pAi} \tag{10}$$

to the complex $16 \times 8$ matrices obeying "purity conditions" (in terminology of [21])

$$\bar{w}_{qAi}\bar{w}_{qBi} = 0, \quad A, B = 1, ..., 8. \tag{11}$$

Actually, $\bar{w}_{qAi}$ are internal frame variables [19] or $SO(9)/SO(7)\times SO(2)$ harmonics (in the sense of [45]; see [19] and refs therein). This is to say they are 8 complex linear combinations of columns of an SO(9)-valued matrix, schematically

$$(\bar{w}_{qAi}, w_{qi}{}^A) \in SO(9) \tag{12}$$

with $w_q{}^A = (\bar{w}_{qA})^*$. Equation (12) implies that $\bar{w}_{qAi}$ and $w_{qi}{}^A$ obey

$$w_{qi}{}^A\bar{w}_{pAi} + \bar{w}_{qAi}w_{pi}{}^A = \delta_{qp}, \tag{13}$$

$$\bar{w}_{qBi}w_{qi}{}^A = \delta_B{}^A, \quad w_{qi}{}^A w_{qi}{}^B = 0 = \bar{w}_{qAi}\bar{w}_{qBi}, \tag{14}$$

as well as (10) and a few similar relations with other vectors of the SO(9) vector frame (see [19]).

It is convenient to introduce the set of *complex spinor harmonics* (complex spinor frame variables) composed of the real spinor frame variables (2) and the internal harmonics (12), according to [19]:

$$v^{\mp}_{\alpha A} := v^{\mp}_{\alpha q}\bar{w}_{qA}, \quad \bar{v}^{\mp A}_{\alpha} := v^{\mp}_{\alpha q}w_q{}^A. \tag{15}$$

By construction,

$$\begin{aligned} &v^-_{\alpha A}v^{-\alpha}_B = 0, \quad v^-_{\alpha A}v^{-\alpha B} = 0, \\ &v^+_{\alpha A}v^{-\alpha}_B = 0, \quad v^+_{\alpha A}v^{-\alpha B} = \delta_A{}^B, \quad \ldots . \end{aligned} \tag{16}$$

With this notation, Eqs. (9), (10) yield

$$\not{U}_{\alpha\beta} := U_\mu\Gamma^\mu_{\alpha\beta} = 4v^-_{(\alpha|A}v^+_{|\beta)A}. \tag{17}$$

Below, we find convenient to use the SO(1,1) invariant complex helicity spinors

$$\lambda_{\alpha Ai} = \sqrt{\rho^\#_i}v^-_{\alpha Ai}, \quad \lambda^A_{\alpha i} = \sqrt{\rho^\#}\bar{v}^{-A}_{\alpha i} \tag{18}$$

instead of $v^-_{\alpha A}$ and $\bar{v}^{-A}_\alpha$ so that the second equation in (1) can be written in an equivalent form [3]

$$\tilde{\not{k}}^{\alpha\beta} = 4\rho^\# v_A^{-(\alpha}v^{-\beta)A} = 4\lambda_A^{(\alpha}\lambda^{\beta)A} \Leftrightarrow$$

$$\Leftrightarrow \not{k}_{\alpha\beta} = 4\rho^\# v^{-A}_{(\alpha}v^-_{\beta)A} = 4\lambda^A_{(\alpha}\lambda_{\beta)A}. \tag{19}$$

[3] $\lambda_{\alpha Ai}$ were denoted by $\epsilon_{\mathfrak{a}a} = \kappa_{\mathfrak{a}\alpha}\epsilon_{\alpha a}$ in [21], where $\epsilon_{\alpha a}$ is the notation for $\bar{w}_{qA}$.

Equations (17) and (16) imply that the helicity spinors obey

$$\tilde{\not{U}}^{\alpha\beta}_i\lambda_{\beta Ai} = 0, \quad \tilde{\not{U}}^{\alpha\beta}_i\lambda_\beta{}^{Ai} = -2\lambda_{Ai}{}^\alpha. \tag{20}$$

Using (16), it is not difficult to check that

$$\tilde{\not{k}}^{\alpha\beta}_i\lambda_{\beta Ai} = 0, \quad \tilde{\not{k}}^{\alpha\beta}_i\lambda^A_{\beta i} = 0. \tag{21}$$

With Eqs. (16), we also find

$$\lambda_{\alpha A}\lambda_B{}^\alpha = 0, \tag{22}$$

$$\lambda_{\alpha A}\lambda_A{}^\beta := \rho^\# v^-_{\alpha A}v^{-\beta}_A = -\frac{1}{4}k_\mu U_\nu \Gamma^{\mu\nu}{}_\alpha{}^\beta \tag{23}$$

and

$$\lambda_A\Gamma_{\mu\nu}\lambda_B = \rho^\# v^-_A\Gamma_{\mu\nu}v^-_B = k_{[\mu}U_{\nu]}\delta_{AB}. \tag{24}$$

One can recognize the relations from (2.5) of [21] in (23) and (24). Our spinor frame approach is very efficient in the derivation of relations of such type.

## 3. Polarized Scattering Equation of 11D Supergravity and Ambitwistor Superstring

### *3.1. Scattering equations*

The scattering equations [26–29] establishing the relation between scattered particles and points $\sigma_i$ on the Riemann sphere read

$$\sum_i \frac{k^\mu_i k_{j\mu}}{\sigma_i - \sigma_j} = 0. \tag{25}$$

As in [21] (see also refs. therein), we can introduce the meromorphic 11-vector function

$$P_\mu(\sigma) = \sum_i \frac{k_{i\mu}}{\sigma - \sigma_i} \tag{26}$$

and to write the scattering equation (25) in the form

$$k^\mu_i P_\mu(\sigma_i) = 0. \tag{27}$$

Note that, while $P_\mu(\sigma_i)$ diverges, its contraction with $k^\mu_i$ is well defined (if none of $\sigma_{j\neq i}$ coincides with $\sigma_i$, as usually assumed).

One can also write the scattering equation (25)–(27) as the equation for the meromorphic vector function (26) only:

$$Res_{\sigma=\sigma_i}\frac{1}{2}P^2(\sigma) = 0. \tag{28}$$

This equation actually yields [21] the light-likeness of the meromorphic D-vector function (26),

$$P^{\mu}(\sigma)P_{\mu}(\sigma)=0 \tag{29}$$

for any $\sigma$. Thus, we consider (29) with (26) as the third equivalent form of the scattering equation.

Constraint (29) can be generated from the so-called ambitwistor string action [33], and Eq. (26) can be obtained from a deformation of this action by the incorporation of the contribution to the path integral measure from suitable vertex operators. Below, we will describe an 11D supersymmetric generalization of the ambitwistor superstring action proposed in [35] (see [48, 49] for the earlier discussion in the context of a twistor string) and its vertex operator deformation which leads to the polarized scattering equation of [21].

### *3.2. Constrained spinor function on the Riemann sphere*

Equation (29) suggests the existence of a meromorphic function carrying the 11D spinor index which plays the role of square root of the above meromorphic vector function in the same sense, as the helicity spinors can be called the square roots of a light-like momentum, (1),

$$\begin{aligned} &P_{\mu}(\sigma)\delta_{qp}=\lambda_q(\sigma)\tilde{\Gamma}_{\mu}\lambda_p(\sigma),\\ &2\lambda_{\alpha q}(\sigma)\lambda_{\beta q}(\sigma)=\Gamma^{\mu}_{\alpha\beta}P_{\mu}(\sigma). \end{aligned} \tag{30}$$

Furthermore, it is convenient to assume the existence of a spinor frame field $v^{-}_{\alpha q}(\sigma)$ and a (purely gauge or Stückelberg) density $\rho^{\#}(\sigma)$ and to use them to write the general solution of constraints (30) in the form

$$\lambda_{\alpha q}(\sigma)=\sqrt{\rho^{\#}(\sigma)}v^{-}_{\alpha p}(\sigma)\mathcal{S}_{pq}(\sigma), \tag{31}$$

$$\mathcal{S}_{pr}\mathcal{S}_{qr}=\delta_{pq}. \tag{32}$$

Indeed, substituting (31) into (30), we find

$$\begin{aligned} &P_{\mu}(\sigma)\delta_{qp}=\rho^{\#}(\sigma)v^{-}_{q}(\sigma)\tilde{\Gamma}_{\mu}v^{-}_{p}(\sigma),\\ &2\rho^{\#}(\sigma)v^{-}_{\alpha q}(\sigma)v^{-}_{\beta q}(\sigma)=\Gamma^{\mu}_{\alpha\beta}P_{\mu}(\sigma) \end{aligned} \tag{33}$$

which describes the essential constraints on the spinor moving frame functions and their relation to the meromorphic vector function $P_{\mu}(\sigma)=\rho^{\#}(\sigma)u^{=}_{\mu}(\sigma)$.

The presence of the SO(16)-valued matrix field $\mathcal{S}(\sigma)\in\in \mathrm{SO}(16)$ ($\mathcal{S}\mathcal{S}^T=I$) in (31) reflects the invariance of (30) under the SO(16) gauge transformations.

The 11D *polarized scattering equation* is the counterpart of (27) written for the constrained spinor function $\lambda_{\alpha q}(\sigma)$ and the scattering data. It was proposed in [21], where its relation to a modified version of the 11D ambitwistor superstring action of [35] was claimed. Below, we show, following [42], that the appropriate model is actually described by the original action of [35] and obtain the form of a meromorphic spinor function $\lambda_{\alpha q}(\sigma)$ and the polirized counterpart of the scattering equation in the form of Eq. (25) on this basis.

### *3.3. Spinor moving frame formulation of ambitwistor superstring in D=11*

Constraints (30) appear naturally in the spinor moving frame formulation of the 11D ambitwistor superstring, which is based on the action [35]

$$S=\int\limits_{\mathcal{W}^2} d^2\sigma\lambda_{\alpha q}\lambda_{\beta q}\left(\bar{\partial}X^{\alpha\beta}-i\bar{\partial}\theta^{(\alpha}\,\theta^{\beta)}\right), \tag{34}$$

where $\lambda_{\alpha q}=\lambda_{\alpha q}(\sigma)$ is expressed in terms of spinor moving frame functions by (31), $\theta^{\alpha}(\sigma)$ are fermionic 32-component Majorana spinor coordinate functions, $\bar{\partial}\theta\Gamma^{\mu}\theta=\bar{\partial}\theta^{\alpha}\Gamma^{\mu}_{\alpha\beta}\theta^{\beta}$,

$$X^{\alpha\beta}(\sigma)=\frac{1}{32}\tilde{\Gamma}^{\alpha\beta}_{\mu}X^{\mu}(\sigma), \tag{35}$$

and $X^{\mu}(\sigma)$ is an 11-vector bosonic coordinate function.

Actually, it is much more convenient for our purposes to consider action (34) with arbitrary symmetric spin tensor bosonic coordinate functions

$$\begin{aligned} &X^{\alpha\beta}(\sigma)=X^{\beta\alpha}(\sigma)\equiv\frac{1}{32}\tilde{\Gamma}_{\mu}{}^{\alpha\beta}X^{\mu}(\sigma)-\\ &-\frac{1}{64}iZ^{\mu\nu}(\sigma)\tilde{\Gamma}^{\alpha\beta}_{\mu\nu}+\frac{1}{32\times 5!}Z^{\mu_1\cdots\mu_5}(\sigma)\tilde{\Gamma}^{\alpha\beta}_{\mu_1\ldots\mu_5}. \end{aligned} \tag{36}$$

Such a modified action is gauge-equivalent to the original one. Indeed, one can check that the properties of the spinor frame variables guarantee that an arbitrary variation of the $Z^{\mu\nu}(\sigma)$ and $Z^{\mu_1\cdots\mu_5}(\sigma)$ does not change the action. This is the statement of gauge symmetry which can be fixed just by setting $Z^{\mu\nu}(\sigma)=0=Z^{\mu_1\cdots\mu_5}(\sigma)$, thus reducing (36) to (35).

Action (34) can be written as

$$S = \int_{\mathcal{W}^2} d^2\sigma \left(\lambda_{\alpha q}\, \bar{\partial}\mu_q^\alpha - \bar{\partial}\lambda_{\alpha q}\, \mu_q^\alpha - i\bar{\partial}\eta_q\, \eta_q\right), \qquad (37)$$

where $\lambda_{\alpha q}(\sigma) = \sqrt{\rho^{\#}(\sigma)} v_{\alpha p}^{-}(\sigma)\mathcal{S}_{pq}(\sigma)$ (31) and

$$\mu_q^\alpha(\sigma) := X^{\alpha\beta}(\sigma)\lambda_{\beta q}(\sigma) - \frac{i}{2}\theta^\alpha(\sigma)\,\theta^\beta(\sigma)\lambda_{\beta q}(\sigma), \quad (38)$$

$$\eta_q(\sigma) := \theta^\beta(\sigma)\lambda_{\beta q}(\sigma). \qquad (39)$$

These are the 11D generalizations of the four-dimensional Penrose incidence relations. They are imposed on the set of 16 constrained 11D supertwistors

$$\mathcal{Z}_{\Lambda q} = \left(\lambda_{\alpha q},\ \mu_q^\alpha,\ \eta_q\right) \qquad (40)$$

(see [50] and refs. therein for more discussion).

Equations (38) with (36) and (39) describe the general solution of 120 constraints

$$\mathbb{J}_{pq} := 2\lambda_{\alpha[p}\mu_{q]}{}^\alpha + i\eta_p\eta_q = 0 \qquad (41)$$

which can be identified with a generator of the SO(16) gauge symmetry in the Hamiltonian formalism.

The rigid supersymmetry living invariant (34)

$$\delta_\epsilon X^{\alpha\beta} = i\theta^{(\alpha}\epsilon^{\beta)}, \quad \delta_\epsilon\theta^\alpha = \epsilon^\alpha, \qquad (42)$$

is realized on our constrained supertwistor by

$$\delta_\epsilon\lambda_{\alpha q} = 0, \quad \delta_\epsilon\mu_q{}^\alpha = -i\epsilon^\alpha\eta_q, \quad \delta_\epsilon\eta_q = \epsilon^\alpha\lambda_{\alpha q}. \qquad (43)$$

Action (37) is also invariant under the gauge symmetry transformations

$$\delta\mu_q^\alpha = -\frac{1}{64} i\delta Z^{\nu_1\nu_2}(\sigma)\tilde{\Gamma}^{\alpha\beta}_{\nu_1\nu_2}\lambda_{\beta q} + $$
$$+ \frac{1}{32\times 5!}\delta Z^{\nu_1\dots\nu_5}(\sigma)\tilde{\Gamma}_{\nu_1\dots\nu_5}{}^{\alpha\beta}\lambda_{\beta q} \qquad (44)$$

with arbitrary $\delta Z^{\mu\nu}(\sigma) = -\delta Z^{\nu\mu}(\sigma) = \delta Z^{[\mu\nu]}(\sigma)$ and $\delta Z^{\nu_1\dots\nu_5}(\sigma) = \delta Z^{[\nu_1\dots\nu_5]}(\sigma)$. This symmetry allows for the gauge fixing conditions reducing the general solution (38) of the constraints to

$$\mu_q^\alpha := \frac{1}{32} X^\nu \tilde{\Gamma}^{\alpha\beta}_\nu \lambda_{\beta q} - \frac{i}{2}\theta^\alpha\,\theta^\beta\lambda_{\beta q} \qquad (45)$$

which is the incidence relation for the case of ambitwistor superstring considered as a dynamical system in the standard 11D superspace.

The advantage of considering the ambitwistor superstring as a dynamical system in the enlarged superspace $\Sigma^{(528|32)}$, which corresponds to the incidence relations (38) with (36) and (39), is that, in this case, it can be described by action (37) with $\mu_q^\alpha$ variable restricted by (first class) constraints (41) only. Furthermore, we can introduce constraint (41) with Lagrange multiplier into the action,

$$S = \int_{\mathcal{W}^2} d^2\sigma \left(\lambda_{\alpha q}\, \bar{\partial}\mu_q^\alpha - \bar{\partial}\lambda_{\alpha q}\, \mu_q^\alpha - i\bar{\partial}\eta_q\, \eta_q\right) +$$
$$+ \int_{\mathcal{W}^2} d^2\sigma \bar{\mathcal{A}}^{pq}(2\lambda_{\alpha[p}\mu_{q]}^\alpha + i\eta_{[p}\eta_{q]}), \qquad (46)$$

and can consider the variables $\mu_q^\alpha$ as unconstrained.

It is important that action (46) is invariant under the SO(16) gauge symmetry ($\mathcal{O}\mathcal{O}^T = I$)

$$\begin{aligned} &\lambda_{\alpha q}(\sigma) \mapsto \lambda_{\alpha p}(\sigma)\mathcal{O}_{pq}(\sigma), \\ &\mu_q^\alpha(\sigma) \mapsto \mu_p^\alpha(\sigma)\mathcal{O}_{pq}(\sigma) \end{aligned} \qquad (47)$$

provided the Lagrange multiplier $\bar{\mathcal{A}}^{pq} = \bar{\mathcal{A}}^{[pq]}$ is transformed as a gauge field,

$$\bar{\mathcal{A}}^{pq} \mapsto \left(\mathcal{O}^{-1}\bar{\partial}\mathcal{O} + \mathcal{O}^{-1}\bar{\mathcal{A}}\mathcal{O}\right)^{pq}, \quad \mathcal{O}\mathcal{O}^T = I. \qquad (48)$$

Of course, the fields $\lambda_{\alpha q}(\sigma)$ are constrained by algebraic relations which follow from their expression in terms of the spinor moving frame variables (31) (these are actually collected in (30)). However, the fact that $\mu_q^\alpha(\sigma)$ in action (46) can be treated as unconstrained is very useful. In particular, the equations of motion for the constrained spinor functions $\lambda_{\alpha q}(\sigma)$ and for the 16 fermionic functions $\eta_q(\sigma)$, which follow from variations of the unconstrained $\mu_q^\alpha(\sigma)$ and $\eta_q(\sigma)$, can be obtained immediately and have the form of

$$\bar{D}\lambda_{\alpha q} = 0, \quad \bar{D}\eta_q = 0, \qquad (49)$$

where

$$\bar{D}\lambda_{\alpha q} = \bar{\partial}\lambda_{\alpha q} - \lambda_{\alpha p}\bar{\mathcal{A}}^{pq}, \quad \bar{D}\eta_q = \bar{\partial}\eta_q - \eta_p\bar{\mathcal{A}}^{pq} \qquad (50)$$

are SO(16) covariant derivatives constructed with the use of the Lagrange multiplier $\bar{\mathcal{A}}^{pq}$ as an SO(16) gauge field.

To arrive at the equation, whose solution can be related to the meromorphic vector function (26) by (30), we need to include the contribution of a suitable vertex operator into the action, what we are going to describe now.

### 3.4. Vertex operator, equations of motion for effective action, and the form of meromorphic spinor functions

In the spinor frame formalism, the SO(16) gauge invariant generalization of the vertex operator proposed in [21] reads

$$V = \int d^2\sigma_i \delta(k_i\, P(\sigma_i))\mathfrak{W} \exp\Big(2i\mu_q^\alpha(\sigma_i)\lambda_{\alpha Ai} W_{qi}^A(\sigma_i) + + 2\eta_q(\sigma_i)\eta_{Ai} W_{qi}^A(\sigma_i)\Big), \quad (51)$$

where $\mathfrak{W}$ denotes a possible additional worldsheet operator depending on polarization data, the explicit form of which will not be essential for our discussion (see [21] for the references describing its explicit form). In addition, the vertex operator (51) is expressed in terms of fermionic and spinorial bosonic functions describing the ambitwistor string, $\eta_q(\sigma)$ and $\mu_q^\alpha(\sigma)$, $\lambda_{\alpha q}(\sigma)$ (the latter enters $\delta(k_i \cdot P(\sigma_i))$, where $P_\mu(\sigma)$ is assumed to be taken from (30)), and the scattering data of the $i$-th particle. The latter are described by $\lambda_{\alpha Ai}$, which also defines $k_i$ through (1), fermionic matrix function $\eta_{Ai}$ $(= \theta_i^\alpha \lambda_{\alpha Ai})$ and bosonic one $W_{qi}^A(\sigma)$ which obeys the purity conditions

$$W_{qi}^A(\sigma) W_{qi}^B(\sigma) = 0 \quad (52)$$

(this requirement will be motivated below).

Let us stress that, despite the entrance of $W_{qi}^A(\sigma)$ into the set of scattering data, we should consider it as a function of $\sigma_i$, as, otherwise, we break explicitly the local SO(16) symmetry characteristic of the ambitwistor superstring action (37). On the other hand, the entrance of $W_{qi}^A(\sigma_i)$ into the set of scattering data suggests its identification with constant matrices $W_{qi}^A$ up to the universal ($i$-independent) local SO(16) transformations,

$$W_{qi}{}^A(\sigma) = W_{pi}{}^A\, \tilde{\mathcal{O}}_{pq}(\sigma), \quad \tilde{\mathcal{O}}^T \tilde{\mathcal{O}} = I_{16\times 16}. \quad (53)$$

Moreover, this also suggests the identification of the constant matrices $W_{qi}^A$ in (53) with the internal frame matrix variable $w_{pi}^A$ (12) describing the polarization of the scattering particle. Such an identification can be actually made up to a rigid SO(16) rotation only,

$$W_{qi}^A = w_{pi}^A \hat{\mathcal{O}}_{pqi}, \quad \hat{\mathcal{O}}_i^T \hat{\mathcal{O}}_i = I_{16\times 16}, \quad (54)$$

so that (53) becomes

$$W_{qi}^A(\sigma) = w_{pi}^A \tilde{\mathcal{O}}_{pqi}(\sigma), \quad \tilde{\mathcal{O}}_{pqi}(\sigma) = \hat{\mathcal{O}}_{pri}\tilde{\mathcal{O}}_{pq}(\sigma). \quad (55)$$

As $\tilde{\mathcal{O}}_{pqi}(\sigma) = \tilde{\mathcal{O}}_{qpi}^{-1}(\sigma)$ is SO(16)-valued, (55) would imply that $W_{qi}^A(\sigma)$ obeys

$$\begin{aligned} &W_{qi}{}^A \bar{W}_{pAi} + \bar{W}_{qAi} W_{pi}{}^A = \delta_{qp}, \\ &\bar{W}_{qBi} W_{qi}{}^A = \delta_B{}^A, \\ &W_{qi}{}^A W_{qi}{}^B = 0 = \bar{W}_{qAi}\bar{W}_{qBi} \end{aligned} \quad (56)$$

and, hence, describes an SO(16)-valued matrix field.

Equation (53) indicates that $W_{qi}^A(\sigma)$ is essentially a Stückelberg field for the SO(16) gauge symmetry, and its presence implies that this gauge symmetry is actually broken at the points of insertions of the vertex operators, i.e., at $\sigma = \sigma_i$. Clearly, no independent equation can be obtained by varying the Stückelberg field.

The simplest calculations of the path integral with a vertex operator insertions can be done by searching for a saddle point of the exponent of the action multiplied by the exponential factors from the vertex operators. This is to say, the main contribution to the path integral will come from the extrema of the action with the source terms coming from the vertex operator. The essential part of such an action reads

$$\begin{aligned} S + S_V = &\int\limits_{\mathcal{W}^2} d^2\sigma \left(\lambda_{\alpha q}\, \bar{\partial}\mu_q^\alpha - \bar{\partial}\lambda_{\alpha q}\, \mu_q^\alpha - i\bar{\partial}\eta_q\, \eta_q\right) + \\ &+ \int_{\mathcal{W}^2} d^2\sigma \bar{\mathcal{A}}^{[pq]} \left(2\lambda_{\alpha[p}\mu_{q]}^\alpha + i\eta_{[p}\eta_{q]}\right) + \\ &+ \sum_i \int\limits_{\mathcal{W}^2} d^2\sigma \delta(\sigma - \sigma_i) \left(2\mu_q^\alpha(\sigma)\lambda_{\alpha Ai} W_{qi}^A(\sigma) - \right. \\ &\left. - 2i\eta_q(\sigma)\eta_{Ai} W_{qi}^A(\sigma)\right). \end{aligned} \quad (57)$$

This effective action is invariant under the SO(16) gauge symmetry and contains $W_{qi}^A(\sigma)$ which obeys (52) and is assumed to be of the form (53). We would like, however, to discuss action (57) with an arbitrary analytic matrix function $W_{qi}^A(\sigma)$ and to establish the necessary restrictions on it from the consistency of the equations of motion and our assumptions including the scattering equations for the meromorphic vector function (26) related to the constrained spinor functions $\lambda_{\alpha q}(\sigma)$ by (30).

The equations of motion which follow from the variation of action (57) with respect to the unconstrained bosonic and fermionic fields, $\mu_q^\alpha(\sigma)$ and $\eta_q(\sigma)$, have the form

$$\bar{D}\lambda_{\alpha q}(\sigma) = \sum_i \delta(\sigma - \sigma_i)\lambda_{\alpha Ai} W_{qi}^A(\sigma_i), \quad (58)$$

$$\bar{D}\eta_q(\sigma) = \sum_i \delta(\sigma - \sigma_i)\eta_{Ai} W_{qi}^A(\sigma_i), \tag{59}$$

where $\bar{D}$ is defined in (50). Furthermore, in this covariant derivative, $\bar{\mathcal{A}}^{pq}$ is a one-component gauge field associated to the derivative in one (antiholomorphic) complex direction. As such, it can always be gauged away. In other words, this gauge field can be always trivialized, i.e., written in the form

$$\bar{\mathcal{A}}^{pq} = \left(\tilde{\mathcal{O}}^{-1}\bar{\partial}\tilde{\mathcal{O}}\right)^{pq} \tag{60}$$

with SO(16)-valued matrix field $\tilde{\mathcal{O}}_{pq}(\sigma)$ determined up to a constant SO(16) matrix. Then Eqs. (58) and (59) can be written in the equivalent form:

$$\bar{\partial}(\tilde{\mathcal{O}}_{pq}(\sigma)\lambda_{\alpha q}(\sigma)) = \sum_i \delta(\sigma-\sigma_i)\lambda_{\alpha Ai}\tilde{\mathcal{O}}_{pq}(\sigma_i)W_{qi}^A(\sigma_i), \tag{61}$$

$$\bar{\partial}(\tilde{\mathcal{O}}_{pq}(\sigma)\eta_q(\sigma)) = \sum_i \delta(\sigma - \sigma_i)\eta_{Ai}\tilde{\mathcal{O}}_{pq}(\sigma_i)W_{qi}^A(\sigma_i). \tag{62}$$

The solutions of these equations are given by

$$\lambda_{\alpha q}(\sigma) = \sum_{i=1}^n \frac{\lambda_{\alpha Ai}W_{qi}{}^A(\sigma)}{\sigma - \sigma_i}, \tag{63}$$

$$\eta_q(\sigma) = \sum_{i=1}^n \frac{\eta_{Ai}W_{qi}{}^A(\sigma)}{\sigma - \sigma_i}, \tag{64}$$

where

$$W_{qi}{}^A(\sigma) = W_{pi}{}^A\ \tilde{\mathcal{O}}_{pq}(\sigma). \tag{65}$$

This last equation deserves a bit more discussion. Literally, the general solutions of (61) and (62) have the form of (63) and (64) iff

$$W_{qi}^A(\sigma) = W_{pi}{}^A(\sigma_i)\tilde{\mathcal{O}}_{pr}^{-1}(\sigma_i)\tilde{\mathcal{O}}_{rq}(\sigma). \tag{66}$$

This is an algebraic equation for the matrix function $W_{qi}^A(\sigma)$ which is solved by (53).

Of course, Eq. (66) is imposed with the aim to have the solution formulated in terms of the same function $W_{qi}{}^A(\sigma)$ which enters the vertex operator (51). In principle, we could consider the general solution given by (63) and (64) with $W_{qi}{}^A(\sigma)$ substituted by $\tilde{W}_{qi}{}^A(\sigma)$ defined by the r.h.s. of (66) and, thus, leave $W_{qi}{}^A(\sigma)$ in the vertex operator indefinite (up to the purity conditions (52), see below). However, as was already discussed, the special form (65) of $W_{qi}{}^A(\sigma)$ (and even the more specific relation (55)) is actually suggested by the fact that this should describe the scattering data in the vertex operator (51).

### *3.5. Polarized scattering equation*

Now, we are ready to obtain the polarized scattering equation. It appears as a consistency condition for constraint (30) with the meromorphic 11-vector (26) and the meromorphic spinor function (63).

First of all, let us show that the "purity" condition (52) is necessary for such a consistency. Indeed, taking (63), (26), and (19) into account, we can write Eq. (30) as

$$\sum_i \frac{\lambda_{\alpha Ai}}{\sigma - \sigma_i}\sum_j \frac{\lambda_{\beta Bj}}{\sigma - \sigma_j}W_{qj}^B(\sigma)W_{qi}^A(\sigma) = = \sum_i \frac{2\lambda_{(\alpha|Ai}\lambda_{|\beta)i}^A}{\sigma - \sigma_i}. \tag{67}$$

When all $\sigma_i$ are different, the r.h.s. of this equation has first-order poles at $\sigma = \sigma_i$, while the l.h.s generically has second-order poles. These vanish, if we require $W_{qi}^A(\sigma)$ to obey the "purity" conditions (52).

Now, let us observe that the residues of the poles of l.h.s. and r.h.s. of Eq. (67) coincide, if the helicity spinors associated to the scattered particles are related by the condition

$$\sum_j \frac{\lambda_{\alpha Bj}W_{qj}^B W_{qi}^A}{\sigma_i - \sigma_j} = \lambda_{\alpha i}^A. \tag{68}$$

Using (63), we can write this equation in a bit more compact equivalent form

$$\lambda_{\alpha q}(\sigma_i)W_{qi}^A(\sigma_i) = \sqrt{\rho_i^{\#}}v_{\alpha i}^{-A} = \lambda_{\alpha i}^A. \tag{69}$$

This relation basically coincides with the one first introduced in [21] and called there the *11D polarized scattering equation*. Our study has revealed the moving frame nature of both the constrained spinors and constrained spinor functions involved in it and provides a rigorous derivation of the polarized scattering equation from the ambitwistor superstring action. In particular, in such a derivation, we have found that $W_{qi}^A(\sigma_i)$ should also be a value of a(n analytic) matrix function (53) at $\sigma = \sigma_i$, $W_{qi}^A(\sigma_i) = W_{pi}{}^A\ \tilde{\mathcal{O}}_{pq}(\sigma_i)$, rather just a constant matrix $W_{qi}{}^A$.

Equation (68) also can be called a polarized scattering equation. It is imposed on scattering data and provides a polarized counterpart of the scattering equation in the form (25), while (69) is a polarized counterpart of the scattering equation in the form (27).

Furthermore, using (54), we can write the polarized scattering equation (68) in the form

$$\sum_j \frac{\lambda_{\alpha Bj} w_{qj}^B w_{qi}^A}{\sigma_i - \sigma_j} = \lambda_{\alpha i}^A \tag{70}$$

which includes the scattering data described by the helicity spinors and internal harmonics (internal frame variables) (12) only.

## 4. Conclusions

In this contribution, following [42], we have revisited the formalism of the 11D polarized scattering equations in [21] with the use of the spinor frame approach (or the Lorentz harmonic approach), some different applications of which to the description of 11D and 10D amplitudes were searched for in [18–20]. In particular, we have addressed the problem of rigorous derivation of the equations for the spinorial meromorphic function $\lambda_{\alpha q}(\sigma)$ and its fermionic superpartner $\eta_q(\sigma)$ from the spinor moving frame formulation of an 11D ambitwistor superstring [35]. We have shown that, to this end, the (gauge equivalent) formulation of the ambitwistor superstring as a dynamical system in an enlarged 11D superspace $\Sigma^{(528|32)}$ with additional tensor central charge coordinates is very useful. The polarized scattering equation has been then obtained from the consistency of the expression for a meromorphic spinor function obeying the constraints and the scattering equation. Actually, the 11D polarized scattering equation is obtained on this way from the 11D ambitwistor superstring action supplemented by the suitable vertex operator.

An interesting direction for future studies is to apply the spinor frame approach to the construction of an 11D generalization of the 6D rational map and symplectic Grassmannian approach [51–54]. Its relation to the 6D polarized scattering equation approach of [32] was discussed in the very recent work [54].

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СПІНОРНА РУХОМА СИСТЕМА ВІДЛІКУ, РІВНЯННЯ ПОЛЯРИЗОВАНОГО РОЗСІЯННЯ ДЛЯ 11D СУПЕРГРАВІТАЦІЇ ТА АМБІТВІСТОРНА СУПЕРСТРУНА

Р е з ю м е

Ми виявляємо та обговорюємо спінорну рухому систему відліку як основу формалізму 11-вимірного (11D) рівняння поляризованого розсіяння Гайєра та Мейсона [21]. Зокрема, використовується формулювання спінорної рухомої системи відліку для 11D амбітвісторної суперструни [35], яка розглядається як динамічна система в 11D суперпросторі, розширеному координатами, пов'язаними з тензорними центральними зарядами, з метою послідовного отримання виразу для спінорної функції на сфері Рімана, а також рівняння поляризованого розсіяння, якому та задовольняє.