MICROWAVE RESPONSE OF NANOSTRUCTURED HIGH-\(T_c\) SUPERCONDUCTOR THIN FILMS

A model for the microwave response of a nanostructured high-\(T_c\) superconductor (HTS) film, with implanted nanoparticles and nanorods of a dielectric material or point-like and columnar irradiation defects with a nano-sized cross-section is developed. In this case, the microwave surface resistance \(R_s(T, H, \omega)\) is calculated both for the Meissner and mixed states of a superconductor film in an applied dc magnetic field. The obtained results indicate that the implantation of dielectric nanoparticles or point-like radiation defects can significantly improve superconductor characteristics at microwave frequencies. Namely, these nano-sized structural defects can decrease the surface resistance in the Meissner state and eliminate the oscillations of Abrikosov vortices and the related microwave energy losses, thus decreasing the contribution of Abrikosov vortices to the \(R_s\) value in the mixed state of a HTS film.

**Keywords:** high-\(T_c\) superconductor, nanoparticles, nanorods, radiation defects, microwave frequency, surface resistance, Abrikosov vortices.

1. Introduction

The implantation of dielectric phase nanoparticles in the interior of high-temperature superconductors (HTSs) is a modern trend in the fabrication of superconducting materials with high current carrying capability [1–6]. The positive role of implanted dielectric nanoparticles, as well as artificially induced point or extended linear defects produced by the heavy-ion irradiation of superconducting samples [7–9], consists mainly in an enhancement of the pinning of Abrikosov vortices and preventing their dissipative motion under the Lorentz force action. The pinning effect provided by implanted nanoparticles (or point-like radiation defects) under definite technological conditions can be enhanced due to the self-organization of such a zero-dimensional objects in the so-called “nanorods”. Those are extended linear one-dimensional defects inside a superconducting matrix, providing the strong pinning of Abrikosov vortices, when the magnetic field is oriented along their axis [1–4]. Another effective method of production of extended structural linear defects, which can provide the strong pinning of Abrikosov vortices over their whole length in a superconducting thin plate or film is related to the irradiation of superconducting samples by heavy ions with high energies (\(\sim 1\) GeV). Such high energy ions produce long linear radiation tracks with a cross-section about few nanometers (which is comparable with the coherence length value in HTS materials). Those radiation tracks serve as excellent strong pinning sites for Abrikosov vortices, which significantly increase the critical current and irreversibility field values of a superconductor [7–11].

In addition, the dielectric nanoparticles (as well as point-like radiation defects) implanted in the matrix of a HTS film can significantly improve its characteristics at microwave frequencies. Namely, these implanted zero-dimensional nano-sized objects can decrease the surface resistance in the Meissner state [12, 13] and eliminate the oscillations of Abrikosov vortices and related microwave energy losses, thus decreasing the contribution of Abrikosov vortices to the \(R_s\) value in the mixed state of a type-II superconductor [14, 15].
In the present work, we consider the effect of the addition of point-like defects on the microwave surface resistance value $R_s(T)$ in the Meissner state of a superconductor and demonstrate how this treatment can significantly decrease $R_s(T)$ value at least at low temperatures. We also develop a theoretical model for the high-frequency response of Abrikosov vortices in the mixed state of a 3D anisotropic superconductor, which contains: a) separate nanoparticles or 0-dimensional radiation defects acting like strong point-like pinning sites and b) nanorods, which are linear defects for the vortex pinning. We consider vortices as elastic strings and calculate the surface resistance $R_s(n_l, B, H_{rf}, \omega, T)$ dependences for both mentioned cases. Here and in what follows, $n_l$ is the concentration of added nanoparticles (or radiation defects), $B$ is the magnetic induction inside the film due to trapped vortices, $H_{rf}$ is the amplitude of a rf field. We suppose that $B$ is oriented perpendicularly, while $H_{rf}$ is parallel to the film surface. The results indicate that the implantation of nano-sized inclusions such as dielectric nanoparticles or radiation structural defects can decrease the microwave surface resistance in the Meissner state and can significantly improve the superconductor characteristics in the microwave frequency range. This effect was observed in some recent experimental works [9–15].

2. Effect of Superconducting Film Nanostructure on Its Microwave Surface Resistance

The microwave surface resistance of a rather thick superconducting film with $d > 2\lambda$, being in the Meissner state ($d$ is the film thickness, $\lambda$ is the London penetration depth of a weak magnetic field), can be described by the well-known relation [16–18]

$$R_s(T, \omega) = \frac{n_l^2m^*}{\mu}\frac{\lambda^3(T)\sigma_1(T, \omega)}{2};$$

$$\sigma_1(T, \omega) = \frac{m^*n_n(T)}{1 + (\omega\tau(T))^2};$$

$$\sigma_2(T, \omega) = \frac{1}{\mu_0}\frac{\lambda^2(T)}{\omega} = \frac{e^2}{m^*\omega}n_s(T);$$

Here, $\sigma_1(T, \omega)$ is the real part of the complex microwave conductivity $\sigma(T, \omega) = \sigma_1(T, \omega) + i\sigma_2(T, \omega)$. In the framework of the two-fluid model of a superconductor, the expressions for $\sigma_1(T, \omega)$ and $\sigma_2(T, \omega)$ are given by Eqs. (2) and (3); $n_n(T)$ and $n_s(T)$ are the concentrations of normal electrons and those in the superfluid condensate, respectively; $\tau(T)$ is the relaxation time for normal electrons. Within the phenomenological model, $\tau^{-1}(T) = \tau_0^{-1} + \tau_e^{-1} + \tau_{ph}^{-1}(T)$. Here, $\tau_0, \tau_e, \tau_{ph}$ are relaxation times for the electron scattering by natural defects in a pristine film, implanted nanoparticles, and phonons, respectively. From (1), one can see that a decrease in the total relaxation time $\tau(T)$ due to the implantation of dielectric nanoparticles (or radiation defects) in the matrix of a superconductor leads firstly to a decrease in $\sigma_1$ and then, in accordance to (1), to a decrease in the surface resistance $R_s$. This is illustrated by Fig. 1, where results of calculation of $R_s(T)$ with the use of Eq. (1) are demonstrated for different concentrations of nanoparticles at some suitable values of other parameters taken from the corresponding literature (see, e.g., [12, 17]).

This decrease in $R_s$, caused by the implantation of dielectric nanoparticles or nano-sized structural radiation defects in the interior of a superconductor film, should be mostly pronounced at low temperatures and at not too high concentrations $n_l$ of these additional nano-sized inclusions, because an increase in the electron relaxation rate due to the $n_l$ growth also leads to a significant increase in the penetration depth $\lambda$. The latter, according to Eq. (1), leads to the competition of opposite dependences $\lambda$ and $\sigma_1$ on the concentration of nanoparticles. So, there should be an optimal concentration of nano-sized inclusions $n_l$ (correspondingly, $\tau_l$) to get the minimal value of $R_s$ for other fixed parameters (e.g., $T, \omega$).

3. Vortex Oscillations and rf-Losses in Superconductors with Strong Pinning

In the present work, we also develop a theoretical model for the high-frequency response of Abrikosov

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vortices in the mixed state of a 3D anisotropic superconductor with point-like nano-sized inclusions, which act like strong point-like pinning sites. We consider vortices as elastic strings and calculate the surface resistance $R_s(n_i, B, \omega, T)$ in the linear regime (at small amplitudes of rf field $H_{rf}$). At not too high dc magnetic fields $B$, basing on the phenomenological model for rf vortex dynamics [19], one has for the total surface resistance $R_s$: $R_s(n_i, B, \omega, T) = R_{s,e}(n_i, \omega, T) + R_{s,v}(n_i, B, \omega, T)$, where $R_{s,e}$ is given by Eq. (1), while the $R_{s,v}$ term corresponds to the contribution of oscillating vortices.

Using the phenomenological model for rf vortex dynamics [19] and disregarding the collective effects in the vortex ensemble, we have obtained an expression for $R_{s,v}$, which resembles the contribution of vortex oscillations in the cases schematically shown in Figs. 2 and 3. Figure 2 illustrates the case where elastic vortex strings are strongly pinned by point-like pinning sites (dielectric nanoparticles or point-like radiation defects), while the case of extended linear (columnar) pinning sites is illustrated in Fig. 3. In the latter case, one can expect that the main contribution to $R_{s,v}$, at least at high enough temperatures $T \lesssim T_c$, arises due to thermally excited vortex kinks, which connect two parts of the whole vortex line situated on two adjacent neighboring columnar pinning sites and can viscously move along the axis of columns ($z$-axis in Fig. 3) under the Lorentz force action. Below, both these cases are considered.

3.1. Point-like defects

In this case, we suppose that elastic vortex strings are rigidly pinned at discrete points $r_i$ inside a rather thick superconducting plate (thickness $d > 2\lambda$), where nanoparticles or point-like radiation defects are randomly distributed. We suppose the average distance between neighboring nanoparticles $L$ being much less than the London penetration depth $\lambda$ and the intervortex distance $a_\phi \approx \left(\frac{\phi_0}{\pi r_i}\right)^{1/2}$. $L \approx \frac{n_i^{-1/3}}{\approx a_\phi, \lambda}$.

Under these conditions, the vortex segments between adjacent pinning sites along the vortex line in a microwave field behave themselves like nearly independent overdamped oscillators. The role of potential energy for these oscillators is played by the elastic energy of deformed vortex segments. This elastic energy can be easily calculated from the equation for a vortex segment displacement $s(z)$ from its equilibrium position under the Lorentz force action [20]:

$$P \frac{d^2 s}{dz^2} + \phi_0 j(z) = 0; \quad (4)$$

with boundary conditions corresponding to the rigid pinning of a vortex line by point-like pinning sites situated at the distance $L$ from each other: $s(0) =
= s(L) = 0. Here, P is the vortex line tension; \(\phi_0 \equiv \frac{h}{2e}\) is the flux quanta \((2.07 \times 10^{-7} \text{ G-cm}^2)\); \(f(z)\) is the local current density value. The bending of the vortex string by a microwave current, inducing the rf-Lorentz force on vortex line segments between point-like pinning sites, as it is depicted in Fig. 2, causes an increment of the vortex elastic energy. This determines, in turn, the pinning potential well \(U_p\) for oscillations of vortex segments between neighboring pinning sites. This potential well caused by the elastic deformation of vortex segments has a parabolic form (as a function of the average rf vortex displacement \(\langle s \rangle\)) characterized by the specific Labusch parameter \(\alpha_L\) near its bottom:

\[
U_p = \frac{1}{2} \alpha_L \langle s \rangle^2; \langle s \rangle = \frac{1}{L} \int_0^L s(z)dz; \quad (5)
\]

\[
\alpha_L = \frac{12P}{L} = 12P\eta_1^{1/3}. \quad (6)
\]

It should be noted that, in the well-known and usually used theories of rf vortex response in the mixed state of type-II superconductors [21–23], the Labusch parameter \(\alpha_L\) is considered as that phenomenological constant. It is not related neither to the type of pinning sites, nor to their dimensionality and concentration. The model, which we develop here, allows one to specify the Labusch parameter as that depending on the dimensionality of nano-sized inclusions and their concentration. It appears essential (both for point-like and columnar pinning sites) to obtain the value of the contribution of vortices to the surface resistance \(R_{sv}(n_1, B, \omega, T)\) coming from oscillations, as well as its dependence on the suitable parameters: \(n_1, B, \omega, T\). From Eq. (6), it follows that an increase in the concentration of point-like pinning sites \(n_1\) leads to an enhancement of the Labush parameter and to the strengthening of the pinning of vortex segments. In the framework of the well-known Gittleman–Rosenblum model for rf-losses of oscillating vortices and the related surface resistance \(R_{sv}\) [24], the contribution to the microwave surface resistance due to oscillating vortex segments is as follows:

\[
R_{sv} \propto \frac{B}{B_{c2}} \rho_n \frac{\omega^2}{\omega^2 + \omega_p^2}; \quad \omega_p = \frac{12P}{\eta L}. \quad (7)
\]

In Eq. (7), \(\rho_n\) is the the normal state resistivity at \(T > T_c\); \(\omega_p\) is the so-called “pinning frequency” [19]; \(\eta\) is the viscosity coefficient for the vortex motion. The increase in the concentration of nanoparticles leads to a decrease in \(L\) and a growth of \(\omega_p\), thus leading to a decrease in the surface resistance in accordance with Eq. (7).

### 3.2. Columnar defects

In this case, we believe that the main contribution to microwave losses due to vortex oscillations is related to viscous oscillations of vortex kinks, connecting parts of a vortex line pinned on adjacent columnar defects, as it is shown schematically in Fig. 3.

We suppose a rigid pinning exerted on vortices by columnar defects and the free viscous motion of vortex kinks. For the normal orientation of an applied dc magnetic field creating vortices, the vortex kinks arise due to thermal activation processes, being excited from their pinned states on columnar defects. Thus, the concentration of vortex kinks inside a superconductor \(n_k(z, T)\) is determined by the Boltzmann statistics and obeys the barometric formula, by varying near the specimen surface:

\[
n_k(z, T) = \frac{B}{\phi_0} w(z, T) = \frac{B}{\phi_0} w_0 \exp \left(-\frac{E_k(z)}{kT}\right). \quad (8)
\]

Here, \(E_k(z)\) is the kinks energy:

\[
E_k(z) = (\epsilon_\phi - U_p(z)) L;
\]

\[
U_p(z) \approx \frac{\phi_0^2}{8\pi^2\lambda^2} K_0 \left(\frac{2z}{\lambda}\right) \quad (9)
\]

where \(\epsilon_\phi\) is the vortex self-energy: \(\epsilon_\phi = \frac{e^2}{8\pi^2\mu_0\lambda^2} \times [\ln \kappa + 0.5] \) (\(\kappa\) is the Ginzburg–Landau parameter), and \(U_p(z)\) is the potential energy of vortex interaction with the specimen surface evaluated per unit length of a vortex line. We consider this interaction as that of the vortex kink, depicted in Fig. 3, with its image of opposite sign placed on the same distance from the specimen surface on its other side. The calculation of viscous losses \(W\) for the motion of vortex kinks driven by a microwave current can be produced as follows:

\[
W = \int_{-\infty}^{0} n_k(z, T) \langle \dot{w}^2 \rangle_{\epsilon,\theta} dz =
\]

\[
= \int_{-\infty}^{0} n_k(z, T) \frac{\langle F^2_r(z, \theta, t) \rangle_{\epsilon,\theta}}{2\eta} dz. \quad (10)
\]

Averaging \(\langle \cdot \rangle_{\epsilon,\theta}\) in (10) means the time and orientation averaging for the dynamics of vortex kinks, \(\theta\) is

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the angle between the local kink and microwave current directions. Equating the losses calculated in (10) to the well-known expression for rf-losses: \( W = \frac{R_i I^2}{2} \) (\( I \) is a sheet current amplitude), we get the surface resistance due to the motion of vortex kinks:

\[
R_s(B, \omega, T) = \frac{B}{\eta \omega} \exp \left(-\frac{(E_k)^2}{kT^2}\right).
\] (11)

It should be noted that, in this case, the frequency and temperature dependences of \( R_s \) caused by vortex oscillations are essentially different from those obtained for point-like pinning sites in (7).

4. Conclusion
In general, HTS films possess a number of unique properties at high frequencies, which make them very attractive for the application in a variety of high-frequency devices [26–28].

The significant contribution both to the understanding of superconductivity in HTS cuprate films at microwave frequencies and to their application in different microwave devices was made by Professor G.A. Melkov with coauthors (see, e.g., their works [29–32]). Nevertheless, the problem of superconducting materials which will be optimal for rf applications remains still open [33].

The obtained results indicate that the implantation of dielectric nanoparticles can significantly eliminate the oscillations of Abrikosov vortices and related microwave energy losses, thus decreasing the contribution of Abrikosov vortices to the \( R_s \) value in the mixed state of HTS films. Such effect of the addition of nanoparticles on the microwave response of HTS films was observed in some recent experiments (see, e.g., [8–10]). In the present work, we have suggested a new model for the rf vortex response in superconductors with nano-sized point-like and columnar strong vortex pinning sites. In the case of point-like pinning sites, this model allows one to specify the Labusch parameter as a function of material properties (which enter through the vortex line tension \( P \)) and the concentration of nano-inclusions \( n_i \), as it is given by Eq. (6). This determines, in turn, the addition to the surface resistance, coming from oscillating vortex segments, \( R_{s,v}(B, \omega, n_i) \), and its dependences on the concentration of nano-inclusions \( n_i \), frequency, and magnetic induction values, as it is given by Eq. (7). In the case where the dominant pinning sites are extended columnar defects with a nano-sized cross-section (e.g., radiation tracks or “nanorods”), we have obtained, for the first time, the expression for the contribution of vortex kinks to the microwave surface resistance \( R_{s,v}(B, \omega, n_i) \), which is given by Eq. (11). There should be a significant difference in losses caused by vortex oscillations in superconductors with strong point-like and extended columnar vortex pinning sites, as it follows from the obtained relations (4) and (8), respectively.

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