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ACTION-AT-A-DISTANCE AND RADIATION REACTION OF POINT-LIKE PARTICLES IN DE SITTER SPACE[1]

The two-particle system with the time-asymmetric retarded-advanced electromagnetic interaction known as the Staruszkiewicz–Rudd–Hill model is considered in the de Sitter space-time. The manifestly covariant descriptions of the model within the Lagrangian and Hamiltonian formalisms with constraints are proposed. It is shown that the model is de Sitter-invariant and integrable. An explicit solution of the equations of motion is derived. We use the covariant electromagnetic Green function in the de Sitter space in order to derive the equation of motion of a point charge in an external electromagnetic field, where the radiation reaction is taken into account.

Keywords: Staruszkiewicz–Rudd–Hill model, de Sitter space, electromagnetic self-force.

1. The model invented by Staruszkiewicz [1, 2], Rudd and Hill [3] describes the following time-asymmetric interaction of two relativistic pointlike charged particles: the advanced field of the first particle acts on the second particle, the retarded field of the second particle acts on the first particle, and a radiation reaction is neglected. This model is built of the time-nonlocal Tetrode-Fokker action functional [4, 5] via replacing its integrand, the symmetric Green function of the d'Alembert equation, by the retarded (or advanced) one. In this way, the model was reformulated to the Lagrangian and then the Hamiltonian [2] form which turned out integrable [6] due to its exact Poincaré-invariance. The Staruszkiewicz–Rudd–Hill model was generalized to various interactions (scalar, gravitational, confining, *etc.*) [7, 8], and the corresponding quantum versions [9, 10] revealed their physical adequacy, despite the artificiality of time-asymmetric interactions.

Here, the Staruszkiewicz–Rudd–Hill model is considered in the de Sitter space-time. Using the representation of the de Sitter space-time as a hyperboloid in the 5-dimensional Minkowski space $\mathbb{M}_5$, we construct the covariant action principle for the time-asymmetric particle dynamics with a constraint and the Hamiltonian description. It is invariant with respect to the de Sitter group SO(1,4) and integrable. A formal solution for this dynamics is built.

2. Tetrode–Fokker variational functional [4, 5], which is a base of the action-at-a-distance Wheeler–Feynman electrodynamics, was generalized for a curved space-time by Hoyle and Narlikar [11] and others [12]. For the system of two charged particles of masses m_a and charges e_a $(a = 1, 2)$, it has a form

$$I = I_{\rm free} + I_{\rm int}, \quad \text{where} \quad I_{\rm free} = -\sum_{a=1}^{2} m_a \int {\rm d}s_a\,, \quad (1)$$

$$I_{\rm int} = -4\pi e_1 e_2 \iint {\rm d}x_1^\mu\, {\rm d}x_2^\nu\, G_{\mu\nu}(x_1, x_2), \quad (2)$$

where $x_a^\mu(\tau_a)$ $(\mu = 0, ..., 3)$ are the space-time coordinates of particle world lines parametrized by the evolution parameters τ_a $(a = 1,2)$. The measures ${\rm d}s_a$ in free-motion terms $I_{\rm free}$ of action (1) are the elementary intervals along particle world lines. The integrand of the interaction term (2) is a symmetric Green function $G_{\mu\nu'}(x, x')$ of the d'Alembert equation.

3. The de Sitter space-time can be presented as a 4-dimensional hyperboloid $\mathbb{H}$ [13, 14]

$$y^2 \equiv y \cdot y \equiv \eta_{MN} y^M y^N = -R^2$$

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in the 5-dimensional Minkowski space $\mathbb{M}_5$ with coordinates y^M $(M = 0, 1, ..., 4)$ and metrics $||\eta_{MN}|| = = \mathrm{diag}(+, -, ..., -)$. The constant R determines the scalar curvature $\mathcal{R}$ of the de Sitter space, and it is related to the cosmological Λ-constant: $\mathcal{R} = -12/R^2 = = 4\Lambda$ (the speed of light is put $c = 1$).

The hyperboloid $\mathbb{H}$ is invariant with respect to the de Sitter group SO(1,4) represented in $\mathbb{M}_5$ by standard pseudoorthogonal transformations. The metrics on $\mathbb{H}$ is induced by the metrics in $\mathbb{M}_5$: $\mathrm{d}s^2 = = \eta_{MN}\mathrm{d}y^M\,\mathrm{d}y^N\,|_{\mathbb{H}}$.

Thus, the configuration space of a two-particle system is $\mathbb{H}^2$. It can be parametrized by either independent variables $x^\mu_a(\tau_a)$ or 5-dimensional variables $y^M_a(\tau_a)$ $(M = 0, 1, ..., 4)$ constrained on a hyperboloid (for each particle, $a = 1, 2$).

For the de Sitter space-time, the symmetric Green function is known from work [15] [2]:

$$G_{\mu\nu'}(x, x') = G^\delta_{\mu\nu'}(x, x') + G^\theta_{\mu\nu'}(x, x') \tag{3}$$

with

$$G^\delta_{\mu\nu'}(x, x') = \frac{1}{16\pi}\bar{g}_{\mu\nu'}(x, x')\delta(\rho),$$

$$G^\theta_{\mu\nu'}(x, x') = \frac{1}{12\pi}\left\{\partial_\mu\partial_{\nu'}\left(\ln Z - \frac{1}{2Z}\right)\right\}\Theta(\rho);$$

$$\bar{g}_{\mu\nu'}(x, x') = -2R^2\left\{\partial_\mu\partial_{\nu'}Z - \frac{(\partial_\mu Z)(\partial_{\nu'} Z)}{Z}\right\},$$

$$Z(x, x') = 1 + \frac{1}{4}\rho(x, x')/R^2,$$

$$\rho \equiv (y - y')^2 = 2R^2\left(\cosh\frac{s}{R} - 1\right) > 0,$$

where s is the interval between points y and y' along the time-like geodesics (for the space-like geodesics, s is imaginary, $\cosh(s/R) = \cos|s/R|$, and $\rho < 0$).

4. Following Staruszkiewicz [1] and Rudd and Hill [3], we replace the symmetric Green function in (2) by

$$G^{(\eta)}_{\mu\nu}(x_1, x_2) = 2\Theta[\eta(x^0_1 - x^0_2)]G_{\mu\nu}(x_1, x_2), \tag{4}$$

which is the retarded (for $\eta = +1$) or advanced ($\eta = = -1$) Green function.

We note that the Green function $G_{\mu\nu}$ consists of two parts: the local part $G^\delta_{\mu\nu'}(x, x')$ which contributes on the light cone surface $\rho = 0$ and the non-local part $G^\theta_{\mu\nu'}(x, x')$ which contributes in the light cone interior $\rho > 0$. The same is true for $G^{(\eta)}_{\mu\nu}$. This is a common feature of curved space-times, contrary to the flat Minkowski space, where the Green functions of massless fields have local part only. But in the present case of the de Sitter space, the contribution of the Green function $G_{\mu\nu}$ or $G^{(\eta)}_{\mu\nu}$ to integral (1) can be effectively reduced to a local one via the integration of I_{int} by parts:

$$I_{\text{int}} = -4\pi e_1 e_2 \iint \mathrm{d}x^\mu_1\,\mathrm{d}x^\nu_2\,G^{(\eta)}_{\mu\nu}(x_1, x_2) \simeq$$

$$\simeq -e_1 e_2 \iint \mathrm{d}\tau_1\,\mathrm{d}\tau_2\;2\Theta(\eta y^0)\,\dot{y}_1\cdot\dot{y}_2\,\delta(y^2)|_{\mathrm{T}\mathbb{H}^2}, \tag{5}$$

where $\dot{y}_a \equiv \mathrm{d}y_a/\mathrm{d}\tau_a$ $(a = 1, 2)$ are 5-vector particle velocities, $y \equiv y_1 - y_2$ is a relative position 5-vector, and the symbols "$\simeq$" denote the equality up to off-integral terms which do not contribute to the variational problem. The integrand on r.-h.s. of (5) is constrained on $\mathrm{T}\mathbb{H}^2$, the tangle bundle over the configuration space $\mathbb{H}^2$.

5. At this point, integral (5) can be integrated out once, similarly to the flat-space case of the Staruszkiewicz–Rudd–Hill model [2]. Consequently, action (1), (5) reduces to a single-time Lagrangian one presented here in the following manifestly covariant 5-dimensional form:

$$I = \int \mathrm{d}\tau\,\left\{L + \lambda_0 y^2 + \sum_{a=1}^{2}\lambda_a(y^2_a + R^2)\right\}. \tag{6}$$

Here, the Lagrangian defined on $\mathrm{T}\mathbb{M}^2_5$ is:

$$L \;\equiv\; -\sum_{a=1}^{2} m_a\sqrt{\dot{y}^2_a} \;-\; e_1 e_2\frac{\dot{y}_1\cdot\dot{y}_2}{|\dot{Y}\cdot y|}, \tag{7}$$

where $Y \equiv \frac{1}{2}(y_1 + y_2)$, and the Lagrangian multipliers λ_a and λ_0 take holonomic constraints into account: the hyperboloid constraints:

$$y^2_a + R^2 = 0, \quad a = 1, 2, \tag{8}$$

and the light-cone constraint generated by the δ-function on r.-h.s. of (5):

$$y^2 \equiv (y_1 - y_2)^2 = 0, \quad \eta y^0 \equiv \eta(y^0_1 - y^0_2) > 0. \tag{9}$$

These constraints define a 7D configuration space of the system $\mathbb{K} \subset \mathbb{H}^2 \subset \mathbb{M}^2_5$.

6. The de-Sitter-invariance of Lagrangian (7) and constraints (8) and (9) provide the existence of ten

[2] The earlier proposal [16] seems to be wrong, since it does not meet demands of the de Sitter-invariance.

Noether integrals of motion collected in the angular momentum tensor:

$$J_{MN} = \sum_{a=1}^{2} (y_{aM}\pi_{aN} - y_{aN}\pi_{aM}) =$$
$$= Y_M\Pi_N - Y_N\Pi_M + y_M\pi_N - y_N\pi_M, \qquad (10)$$

where

$$\pi_{aM} = \partial L/\partial \dot{y}_a^M \quad (a = 1, 2), \qquad (11)$$
$$\Pi_M = \pi_{1M} + \pi_{2M}, \quad \pi_M = \frac{1}{2}(\pi_{1M} - \pi_{2M}).$$

In addition, Lagrangian (7) is invariant under an arbitrary change of the evolution parameter: $\tau \to \to \tau' = f(\tau)$. Thus, the Legendre mapping (11) onto the 20-dimensional phase space $\mathrm{T}^*\mathbb{M}_5^2$ endowed with standard Poisson brackets $\{y_a^M, \pi_{aN}\} = \delta_{ab}\delta_N^M, \ldots$ is degenerated and leads to the manifestly covariant Hamiltonian description with constraints [17].

Ten Noether integrals of motion (10) become, within the Hamiltonian description, the generators of the canonical realization of the de Sitter group, i.e., they satisfy the canonical relations of SO(1,4) algebra:

$$\{J_{MN}, J_{LK}\} = \eta_{ML}J_{NK} + \eta_{NL}J_{ML} -$$
$$- \eta_{MK}J_{NL} - \eta_{NL}J_{MK}.$$

By virtue of the parametric invariance of Lagrangian (7), the canonical Hamiltonian vanishes, while the dynamics of the system is generated by the *dynamical* constraint $\Phi(y_a, \pi_b) = 0$ which together with the holonomic constraints (8), (9) constitutes a set of primary constraints. A form of the *dynamical* constraint (i.e., of the function $\Phi(y_a, \pi_b)$) is determined by Lagrangian (7), but not unique. It is possible to construct $\Phi(y_a, \pi_b)$ of the first class with respect to the holonomic constraints (8), (9). Then the dynamics is self-consistent, and no secondary constraints arise. For this purpose, the function $\Phi(y_a, \pi_b)$ must satisfy the condition $\{\Phi, y_a^2\} = \{\Phi, y^2\} = 0$. It implies that Φ is a function of the following four 5-scalar arguments: $\Pi\cdot y$, $\pi\cdot y$, $J^2 \equiv J_{MN}J^{MN}$, and $V^2 \equiv V_M V^M$, where $V_M \equiv \frac{1}{8}\epsilon_{MABCD}J^{AB}J^{CD}$. The arguments J^2 and V^2 are the Casimir functions of SO(1,4) group, i.e., conserved quantities, both negative for physically meaningful systems.

For Lagrangian (7), the dynamical constraint is

$$\Phi = \pi_\perp^2 + \tfrac{1}{4}\Pi_\perp^2 - \frac{m_1^2\pi_2\cdot y + m_2^2\pi_1\cdot y}{\Pi\cdot y} -$$
$$- \alpha\frac{\Pi_\perp^2 - m_1^2 - m_2^2}{\eta\Pi\cdot y} + \alpha^2\left(\frac{m_1^2}{\eta\pi_1\cdot y - \alpha} + \frac{m_2^2}{\eta\pi_2\cdot y - \alpha}\right) +$$
$$+ \frac{1}{4R^2}\left(\frac{(\pi_1\cdot y)(\pi_2\cdot y)}{\eta\Pi\cdot y} - \alpha\right)(\eta\Pi\cdot y - 4\alpha) = 0, \quad (12)$$

where

$$\alpha = e_1e_2, \quad \Pi_\perp^2 = -\frac{1}{R^2}\left(\frac{1}{2}J^2 + (\pi\cdot y)^2\right),$$

and

$$\pi_\perp^2 = -\frac{1}{R^2(\Pi\cdot y)^2}\left(V^2 - \frac{1}{2}(\pi\cdot y)^2J^2 - (\pi\cdot y)^4\right).$$

7. In order to show that the system under consideration is integrable, let us start from the Hamiltonian equation of motion for the relative position 5-vector y:

$$\dot{y} = \lambda\left\{y, \Phi(\Pi\cdot y, \pi\cdot y;\, J^2, V^2)\right\} =$$
$$= \lambda\left(\frac{\partial\Phi}{\partial\,\pi\cdot y} - 4\frac{\partial\Phi}{\partial J^2}\mathcal{J} - \frac{\partial\Phi}{\partial V^2}\mathcal{K}\right)y; \qquad (13)$$

here, $\mathcal{J} = ||J^M{}_N||$ and $\mathcal{K} = ||K^M{}_N|| \equiv \equiv ||\epsilon^M{}_{NABC}V^AJ^{BC}||$ are conserved matrices, while the Lagrangian multiplier $\lambda(\tau)$ is an arbitrary gauge-fixing function.

If $\Pi\cdot y = \Psi(\tau)$ is known as a function of τ, then $\pi\cdot y = \psi(\tau) \equiv \psi(\Psi(\tau); J^2, V^2)$ would be known too from the dynamical constraint $\Phi(\Psi(\tau), \psi(\tau);\ J^2, V^2) = 0$ (since J^2 and V^2 are conserved).

In turn, the Hamiltonian equation for $\Pi\cdot y = \Psi(\tau)$ is self-consistent and reduces to quadratures:

$$\dot{\Psi} = \lambda\{\Psi, \Phi\} = \lambda\frac{\partial\Phi(\Psi, \psi(\Psi; J^2, V^2); J^2, V^2)}{\partial\psi}\Psi,$$

where the Lagrangian multiplier $\lambda(\tau)$ is meant a desirable fixed function of τ.

8. At this point, Eq. (13) becomes linear in the 5-vector y with known τ-dependent matrix coefficients. In order to split and to solve this equation, the projection operator techniques will be applied.

Following the Hamilton–Cayley theorem, the matrix $\mathcal{J}$ possesses 5 eigenvalues: $\pm\Sigma$, $\pm\,\mathrm{i}\,S$, and 0, where $\Sigma^2 = \sqrt{J^4/16 - V^2} - \frac{1}{4}J^2 > 0$ and $S^2 = = \sqrt{J^4/16 - V^2} + \frac{1}{4}J^2 > 0$. Correspondently, one can construct 5 projection operators: $\mathcal{P}^{(\pm\Sigma)}$, $\mathcal{P}^{(\pm\,\mathrm{i}S)}$ and $\mathcal{P}^{(0)}$ [18], whose explicit form is omitted here.

Next, we perform the substitution $y(\tau) = \frac{\Psi(\tau)}{\Psi(0)}\, r(\tau)$ and decompose r onto eigen-subspaces:

$$r^{(\Sigma)} = (\mathcal{P}^{(+\Sigma)} + \mathcal{P}^{(-\Sigma)})r,$$
$$r^{(S)} = (\mathcal{P}^{(+\,\mathrm{i}\,S)} + \mathcal{P}^{(-\,\mathrm{i}\,S)})r, \quad r^{(0)} = \mathcal{P}^{(0)}r.$$

We arrive at linear equations of motion for the projections $r^{(\Sigma)}$, $r^{(S)}$, and $r^{(0)}$:

$$\dot{r}^{(i)}(\tau) = f^{(i)}(\tau)\,\mathcal{J}\,r^{(i)}(\tau), \quad i = \Sigma, S, 0,$$

where

$$f^{(0)}(\tau) = 0,$$
$$f^{(\Sigma)}(\tau) = -\lambda\left(4\frac{\partial\Phi}{\partial J^2} + 2S^2\frac{\partial\Phi}{\partial V^2}\right),$$
$$f^{(S)}(\tau) = -\lambda\left(4\frac{\partial\Phi}{\partial J^2} - 2\Sigma^2\frac{\partial\Phi}{\partial V^2}\right).$$

The formal solution of these equations is:

$$r^{(i)}(\tau) = \exp\{F^{(i)}(\tau)\mathcal{J}\}y^{(i)}(0),$$

where $F^{(i)}(\tau) = \int_0^\tau \mathrm{d}\tau\, f^{(i)}(\tau)$. Unraveling matrix exponents yield

$$r^{(\Sigma)}(\tau) = \left(\cosh\left(\Sigma F^{(\Sigma)}(\tau)\right) + \right.$$
$$\left. + \frac{\mathcal{J}}{\Sigma}\sinh\left(\Sigma F^{(\Sigma)}(\tau)\right)\right)y^{(\Sigma)}(0),$$
$$r^{(S)}(\tau) = \left(\cos\left(SF^{(S)}(\tau)\right) + \right.$$
$$\left. + \frac{\mathcal{J}}{S}\sin\left(SF^{(S)}(\tau)\right)\right)y^{(S)}(0),$$
$$r^{(0)}(\tau) = y^{(0)}(0).$$

Thus, we arrive at the solution

$$y_a^{(i)}(\tau) = Y^{(i)}(\tau) - \frac{1}{2}(-)^a\, y^{(i)}(\tau) =$$
$$= \frac{\mathcal{J} - \psi(\tau) - \frac{1}{2}(-)^a\,\Psi(\tau)}{\Psi(0)}\, r^{(i)}(\tau),$$

where $a = 1{,}2$ and $i = \Sigma, S, 0$.

Finally, by means of the Legendre transform (11), one can express the constants $\Psi(0)$ and $\mathcal{J}$ (10) in terms of the initial position 5-vectors $y_a(0)$ and $\dot{y}_a(0)$ $(a = 1, 2)$. If $\{y_a(0), \dot{y}_a(0)\} \in \mathrm{T}\mathbb{K}$, then $\{y_a(\tau), \dot{y}_a(\tau)\} \in \mathrm{T}\mathbb{K}$, in particular, $y_a(\tau) \in \mathbb{K}$ by construction.

9. Following works [15,19], the electromagnetic potential of a pointlike charge q in the de Sitter space is

$$A_\mu(x) = q\int\limits_{-\infty}^{+\infty} \mathrm{d}\tau\sqrt{-g[z(\tau)]}G_{\mu\nu'}(x, z(\tau))u^{\nu'}(\tau), \quad (14)$$

where $G_{\mu\nu'}$ is one of the Green functions (3), (4); four functions $z^{\nu'}(\tau)$ parametrize the world line of the charge q, and $u^{\nu'}(\tau) = \mathrm{d}z^{\nu'}(\tau)/\mathrm{d}\tau$. To obtain the electromagnetic field $F_{\alpha\beta} = \partial_\alpha A_\beta(x) - \partial_\beta A_\alpha(x)$, we use the 4-dimensional stereographic coordinates [20]

$$y^\alpha = \Omega(x)x^\alpha, \quad y^4 = -R\Omega(x)\left(1 + \frac{\sigma^2}{4R^2}\right), \quad (15)$$

where $\Omega(x) = \left(1 - \frac{1}{4R^2}\sigma^2\right)^{-1}$ and $\sigma^2 = \eta_{\alpha\beta}x^\alpha x^\beta$. Routine scrupulous calculations yield the retarded electromagnetic field $F^{\mathrm{ret}}_{\alpha\beta}(x) = \partial_\alpha A^{\mathrm{ret}}_\beta(x) - \partial_\beta A^{\mathrm{ret}}_\alpha(x)$,

$$F^{\mathrm{ret}}_{\alpha\beta}(x) = \frac{q}{4\pi}\left\{\Omega^2[z(\tau)]\frac{u_\alpha k_\beta - u_\beta k_\alpha}{r^2} + \Omega^4[z(\tau)]\times\right.$$
$$\times\frac{a_\alpha k_\beta - a_\beta k_\alpha - a_k\,(u_\alpha k_\beta - u_\beta k_\alpha)}{r} -$$
$$- \frac{2}{R^2}\Omega^5[z(\tau)](z\cdot u)\left[\frac{k_\alpha u_\beta - k_\beta u_\alpha}{r} + \right.$$
$$\left.\left. + \frac{1}{2R^2}\Omega(x)\,(k_\alpha z_\beta - k_\beta z_\alpha)\right]\right\}, \quad (16)$$

where all the terms on the right-hand side are referred to the retarded instant $\tau^{\mathrm{ret}}(x)$. Here, $k_\alpha = [x^\alpha - z^\alpha(\tau^{\mathrm{ret}})]/r$, and the scalar r is a retarded distance $r = \eta_{\alpha\beta}\,(x^\alpha - z^\alpha(\tau^{\mathrm{ret}}))\,u^\beta(\tau^{\mathrm{ret}})$; $a_k = (a\cdot k)$ and $\dot{a}_k = (\dot{a}\cdot k)$.

We assume [21] that the radiation reaction part of the electromagnetic field is given by

$$F^{\mathrm{rr}}_{\alpha\beta} = \frac{1}{2}\left(F^{\mathrm{ret}}_{\alpha\beta} - F^{\mathrm{adv}}_{\alpha\beta}\right), \quad (17)$$

where the first term is the retarded field, and the second one is the advanced field. The advanced field is handled via the Green function (4) supported on the future light cone of x. We develop kinematic variables in the advanced field tensor in series in the parameter $\triangle\tau(r) = \tau^{\mathrm{adv}}(x) - \tau^{\mathrm{ret}}(x)$:

$$\triangle\tau = 2\Omega^2(z)r + 2\Omega^4(z)\left[a_k + \frac{1}{R^2}\Omega(z)(z\cdot u)\right]r^2 +$$
$$+ \frac{2}{3}\Omega^5(z)\left\{\Omega(z)\left(3a_k^2 + 2\dot{a}_k\right) + \Omega^3(z)a^2 + \right.$$

$$+\frac{1}{R^2}\left[\Omega^2(z)\left(2(z\cdot a)+9(z\cdot u)a_k\right)+2\right]+$$
$$\left.+\frac{5}{R^4}\Omega^3(z)(z\cdot u)^2\right\}r^3. \quad (18)$$

The analog of the Lorentz–Abraham–Dirac equation in the de Sitter space-time is

$$m\left\{a^\mu+\frac{1}{2R^2}\Omega(z)\left[2(z\cdot u)u^\mu-\Omega^{-2}(z)z^\mu\right]\right\}=$$
$$=q\Omega^{-2}(z)\eta^{\mu\alpha}(F^{\rm ext}_{\alpha\beta}+F^{\rm rr}_{\alpha\beta})u^\beta, \quad (19)$$

where $F^{\rm ext}_{\alpha\beta}$ is the external field. To obtain the radiation reaction field (17), we pass to the limit $r\to 0$

$$F^{\rm rr}_{\alpha\beta}[z(\tau)]=\frac{2q}{3}\Omega^8[z(\tau)]\left(u_\alpha\dot a_\beta-u_\beta\dot a_\alpha\right)+$$
$$+\frac{3q}{R^2}\Omega^9[z(\tau)]\left(z\cdot u\right)\left(u_\alpha a_\beta-u_\beta a_\alpha\right)-$$
$$-\frac{q}{R^4}\Omega^8[z(\tau)]\left(z\cdot u\right)\left(u_\alpha z_\beta-u_\beta z_\alpha\right), \quad (20)$$

where the kinematic characteristics are evaluated at instant τ which specifies point $z(\tau)$ on the world line.

10. The electromagnetic field in a curved space-time spreads not only along the light-cone surface, as it is in the flat space-time, but also over its interior. Thus, the radiation self-action and the interaction between charged particles are time-nonlocal generally, i.e., they depend on a whole particle history [19].

We have shown that, in the case of the maximally symmetric de Sitter space, the time-nonlocal term of the radiation reaction force vanishes due to a specific structure of the covariant electromagnetic Green function (3), (4). In addition, this peculiarity admits an integrable de Sitter generalization of the Staruzskiewicz–Rudd–Hill model.

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ДІЯ НА ВІДСТАНІ ТА РЕАКЦІЯ ВИПРОМІНЮВАННЯ ТОЧКОВИХ ЧАСТИНОК У ПРОСТОРІ ДЕ СІТТЕРА

Р е з ю м е

Двочастинкова система з часо-асиметричною спізнено-випередною взаємодією, відома як модель Старушкевича–Рудда–Гілла, розглядається у часопросторі де Сіттера. Запропоновано явно коваріантні описи моделі в рамках лаґранжевого та гамільтонового формалізмів з в'язями. Показано, що модель є де-сіттер-інваріантною та інтеґровною. Отримано явний розв'язок рівнянь руху. За допомогою коваріантної електромагнітної функції Ґріна отримано рівняння руху точкового заряду в зовнішньому електромагнетному полі, де врахована реакція випромінювання.