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REVISITING TO THE GEIGER–NUTTAL RELATION TO BE EMPLOYED IN THE ESTIMATION OF THE HALF-LIVES OF SUPERHEAVY NUCLEI

The half-lives for the even–even (e–e), even–odd (e–o), odd–even (o–e) and odd–odd (o–o) nuclei in the range $100 \leqslant Z \leqslant 120$ *have been tested within the Viola–Seaborg formula (VSF) and within the analytical formula of Royer (RF). We proposed another formula (Present Work Formula or PWF) with regard for the effect of angular momentum of the alpha decay particle and with the use of the relative neutron excess* $\left(\frac{N-Z}{A}\right)$. *Our formula includes a new set of parameters found by the least square fitting method of alpha decays of 128 nuclei. We obtained the standard deviations for each of the formulas for comparison. The results show an acceptable agreement with available data. The values of the suggested theoretical coefficient (K) for the PWF show a similar behavior of half-lives with* α*-decay, which can be used to predict the new superheavy nuclei.*

Keywords: Geiger–Nuttal, superheavy nuclei, alpha decay, half-lives, neutron excess ratio.

1. Introduction

One of the landmarks in modern physics, shaping the development leading to quantum mechanics, was the formulation of the empirical Geiger–Nuttal (GN) law in 1911 [1] concerning the partial half-life ($T_{1/2}$) with alpha decay.

Recently, the amount of α-decay data for heavy and superheavy nuclei has greatly increased [2, 3, 4]. The universal formula for the alpha and cluster processes reproduces well the practical values for the half-lives for even and odd nuclei with $84 \leqslant Z \leqslant 100$ [5, 6]. Within this procedure, the tunneling probability through the potential barrier was determined, by using the Wentzel–Kramers–Brillouin approximation. An acceptable accuracy was found in this field in comparison with other formulas [7, 8]. This idea was adopted by some authors to investigate the half-lives of superheavy elements with alpha decay within the range of $100 \leqslant Z \leqslant 122$ [5]. Firas and Mayan [9] derived a semiempirical formula based on the Geiger–Nuttal rule and introduced some parameter for a single-body model with suitable constants obtained through the trial and error method. Their model involved the relative neutron excess $\left(\frac{N-Z}{A}\right)$ which is extremely important in calculations of Q_α and the half-live logarithm for even–even heavy nuclei. It is worth to note that the alpha decay model is usually used for the guessing of heavy and superheavy elements (SHE) [10, 11, 12]. Viola and Seaborg suggested an analytical relation for the guessing of half-lives, which is based on a Gamow-type formula [13]. A semiempirical relation was suggested by Poenaru and Ivascu [14] for the alpha decay fission theory for all groups of nuclei.

Moreover, the half-lives for e–e, e–o, o–e, and o–o nuclei with alpha decay in the range $52 \leqslant Z \leqslant 118$ have been tested within the Royer modified formula including new Royer coefficients obtained by the fitting of 356 isotopes [15]. Superheavy isotopes probe the extremes of the structure of nuclei with respect to the mass number of nuclei for a bound system. Their existence and the properties of the decay are one of the most essential challenges in nuclear physics [16, 17]. Sayed & ALmadar have estimated the alpha-decay half-lives for all types of nuclei in the interval $Z = 104$–118 according to the quantum mechanics theory (tunnel effect). Before the emission, the alpha particle moves inside the mother nucleus supposedly in a spherical field determined by the daughter nucleus. The lifetime of a nucleus with alpha-decay may give a rough measure of the extent to which the nuclear structure is capable of guessing the amount of nuclear density [18]. A very important results were achieved by Wang *et al.* [19], who studied many types of formulae in the field of superheavy elements. They found that the semi FFS2 formula is the best one for the prediction of the alpha-decay half-lives. In addition, the formulas UNIV2, VSS, and NRDX with their fewer coefficients have a well-done guessing of SHEs with alpha decay [7, 13, 20, 21, 22, 23]. There is a research which explains that different technical coefficients do not alter significantly the transfer structure of fractional yields of medium heavy isotopes with regard for cluster half-lives [29]. A description of even–even Pd isotopes from $A = 102$ to 106 in the framework of the interacting boson model was carried out in [30].

In this work, we intend to describe the α-decay half-lives with different proposed formulas. First, we apply the Viola–Seaborg–Sobiczewski approach which reveals the relationship between the alpha-decay Q-value and $T_\alpha(1/2\ )$ [13]. Second, we will use the analytical formula for the α-decay half-lives constructed in [24]. We used the formula by Firas and Mayan [9] in a modified form to calculate the alpha-decay half-lives with regard for the relation [9] that can be valid to all types of nuclei (e–e, e–o, o–e, and o–o) in the interval $100 \leqslant Z \leqslant 120$. Eventually, the theoretical coefficient has been proposed for the prediction of new elements. This is done by solving the partial differential equation (8).

2. VSF Tests for the Nuclei under Study

Geiger and Nuttal proposed the following relation between the alpha-particle decay energy (Q) and the alpha-decay half-lives (T_α):

$$\log_{10} T_\alpha = a + bQ_\alpha^{-1/2}, \tag{1}$$

where the parameters a and b depend on the atomic number of a parent nucleus. In 1966, Viola and Seaborg utilized the Geiger–Nuttal formula and proposed the well-known Viola–Seaborg relation [13]:

$$\log_{10} T_\alpha = a + bQ_\alpha^{-1/2} + (cZ + d + h_{\log}), \tag{2}$$

where Z is the atomic number of the parent nucleus. a, b, c, d are the coefficients that can be achieved by fitting the data given in [13]:

$$a = 1.66175, \quad b = -8.5166, \quad c = -0.20228,$$

$$d = -33.9069.$$

The quantity $h_{\log}$ is the hindrance factor for odd–A or odd–odd nuclei calculated by VSF:

$h_{\log} = 0$ even–even nuclei,

$h_{\log} = 0.772$ for odd–even nuclei,

$h_{\log} = 1.066$ for even–odd nuclei,

$h_{\log} = 1.144$ for odd–odd nuclei.

Equation (2) was applied to all nuclei in the range $100 \leqslant Z \leqslant 120$.

3. RF Tests for the Nuclei Under Study

The alpha-decay half-life can be evaluated suggesting that the incoming point is the contact point, and the outgoing point fits the value of the Coulomb energy with the practical Q_α. The inertia coefficient is a miniature mass. Through this model of unified fission, the decay constant is simply the product of the number of collisions and the potential of penetration. There is no pre-modulation parameter [25, 26]. The relation between the Q value of the alpha decay and half-lives suggested by G. Royer [24], by analyzing the process of alpha emission by a nucleus. This relation was applied to all nuclei (e–e, e–o, o–e, and o–o) in the interval $100 \leqslant Z \leqslant 120$.

For even–even nuclei

$$\log_{10}\left[T_{1/2}(S)\right] = -25.31 - 1.1629A^{1/6}\sqrt{Z} + \frac{1.5864Z}{\sqrt{Q_\alpha}}. \tag{3}$$

For even–odd nuclei

$$\log_{10}\left[T_{1/2}(S)\right] = -26.6 - 1.0859A^{1/6}\sqrt{Z} + \frac{1.592Z}{\sqrt{Q_\alpha}}. \tag{4}$$

For odd–even nuclei

$$\log_{10}\left[T_{1/2}(S)\right] = -25.68 - 1.1423A^{1/6}\sqrt{Z} + \frac{1.592Z}{\sqrt{Q_\alpha}}. \tag{5}$$

For odd–odd nuclei

$$\log_{10}\left[T_{1/2}(S)\right] = -29.48 - 1.113A^{1/6}\sqrt{Z} + \frac{1.6971Z}{\sqrt{Q_\alpha}}. \tag{6}$$

Here, Q_α is the experimental value.

4. Present Work Formula (A New Approach)

In our previous work [9], we proposed a semiempirical formula for even-even nuclei in the interval of $82 \leqslant Z \leqslant 102$, in the following form:

$$\log T = \frac{1.65(Z_p-2)}{\sqrt{Q_\alpha}} - 26.6 - \\ - \sqrt{[(1.08)(A-4)^{1/3}+2](Z_p-2)} + \left(\frac{N-Z_p}{A}\right). \quad (7)$$

This formula (semiempirical relation) is based on the Geiger–Nuttal rule and involves some parameters of the single-particle model such as the radius of nucleus represented by $(1.08A_d^{1/3}+2)$ and the atomic number of the daughter nucleus with their suitable constants that were obtained by the trial and error method. Moreover, the model contains the term representing the relative neutron excess $\left(\frac{N-Z}{A}\right)$, which is extremely important for the suitability of calculations of the half-life logarithm and its matching the experimental value. In this work, relation (7) becomes no longer valid to all types of nuclei under study. So, we introduce a more accurate general formula. This is done by adding two additional terms to the l-dependent formula in order to determine the alpha-decay half-lives of the even–even, even–odd, odd–even, and odd–odd nuclei. The formula involves also A, Z, N of the mother nucleus, experimental decay energy Q_α, and angular momentum l. The alpha-particle carries the angular momentum $l \neq 0$ for odd–odd and odd–A nuclei in the ground-state transition which depends on the spin and parity of the parent and daughter nuclei. The minimum angular momentum mainly carried by the alpha particle with regard for the selection rules is zero ($l = 0$) for the ground state of even–even nuclei in view of their spin and parity [26]. As a result, the modified formula reads

$$\log T = a\frac{(Z_p-2)}{\sqrt{Q_\alpha}} - b - \\ - \sqrt{[(1.08)(A-4)^{1/3}+2](Z_p-2)} + \left(\frac{N-Z_p}{A}\right) + \\ + c\,\frac{ANZ(l(l+1))^{1/4}}{Q} + dA[1-(-1)^l], \quad (8)$$

where Q is the alpha-decay energy given in MeV units, and A, Z, and N are the mass, charge and the number of neutrons of the mother nucleus, respectively. The parameters a, b, c, and d are obtained using the least square fitting of α-decay data of the studied nuclei. Taking the last two terms from [27], we get the following.

For even–even nuclei

$$a = 1.65, \quad b = -27.8, \quad l = 0, \\ c = 1.4948\times10^{-6}, \quad d = 10\times10^{-4}.$$

For even–oddb nuclei

$$a = 1.662, \quad b = -28.8, \quad l = 3, \\ c = 8.3678\times10^{-4}, \quad d = 2.343\times10^{-6}.$$

For odd–even nuclei

$$a = 1.64, \quad b = -27.4, \quad c = 1.9003\times10^{-6}, \\ d = 12\times10^{-6}, \quad l = 3.$$

For odd–odd nuclei

$$a = 1.66, \quad b = -26.6, \quad c = \text{zero}, \quad d = \text{zero}, \quad l = 3.$$

Equation (8) was applied to all nuclei in the interval $100 \leqslant Z \leqslant 120$.

5. Proposing a Theoretical Coefficient for the Prediction of New Elements

It is known that Q_α is of importance for calculating $T_{1/2}$ of the alpha decay. Up to now, there was no theoretical formula that could describe accurately the alpha-decay energy with a deviation less than 0.5 MeV and reach the guessing of half-lives with an acceptable accuracy. To avoid this difficulty, we introduce the quantity [28]

$$K = \left|\frac{\partial \log_{10} T_{\alpha(S)}}{\partial Q_\alpha}\right|. \quad (9)$$

After the straightforward transformations, relation (9) becomes

$$K = \left|\frac{\partial \log_{10} T_{\alpha(S)}}{\partial Q_\alpha}\right| = \left| -\frac{1}{2}(a)(Z_p-2)(Q_\alpha)^{3/2} - \\ - cAZN[l(l+1)^{1/4}\,Q_\alpha^{-2}\right|, \quad (10)$$

for e–e, e–o, o–e, and o–o nuclei, where a, l, and c are the same as above. Formula (10) helps us to explain the energy dependence of the alpha-decay half-life. To reveal the behavior of K values more obviously, we determine the K data for all types of superheavy elements ranging from $Z = 100$ to $Z = 120$. Moreover, we will calculate the difference between the experimental $T_{1/2}^{\rm exp}$ and theoretical $T_{1/2}^{\rm theo}$ values for α-decays:

$$\Delta T = (\log_{10} T_{1/2}^{\rm exp}) - (\log_{10} T_{1/2}^{\rm theo}). \qquad (11)$$

In order to measure the deviation of the obtained data, we go in the standard way. The RMS deviations are determined by [31]:

$$\sigma = \left\{ \frac{1}{N} \sum_{i=1}^{N} \left[\left((\log_{10} T_{1/2}^{\rm exp}) - (\log_{10} T_{1/2}^{\rm theo}) \right)^2 \right] \right\}^{1/2}, \qquad (12)$$

where $T_{1/2}$ theo is the theoretical value of the alpha-decay half-life and $N_{\rm tot}$ is the total number of all nuclei under study that decay with the emission of an alpha particle ($N_{\rm tot} = 128$). The determined values of RMS deviations for the three models (Viola–Seaborg, G. Royer, and Present Work Model) for all types of nuclei are shown in Table 5.

6. Results and Discussion

The properties of the alpha decay of 128 superheavy nuclei within the interval $100 \leqslant Z \leqslant 120$ have been studied by evaluating the alpha-decay half-lives using the Viola–Seaborg formula (VSF), Royer formula (RF), and Present Work Formula (PWF). The latter involves the effect of a relative neutron excess [see relation (8)] and the angular momentum (l) of the ejected alpha particle. The alpha emission obeys the spin–parity selection rule:

$$|I_p - I_d \leq l \leq I_p + I_d| \quad \text{and} \quad \pi_p = (-1)^l \pi_d, \qquad (13)$$

where I_p, I_d, and π_p, π_d are the spins and parities of the mother and daughter nucleus, respectively. Tables 1–4 show the evaluated the alpha-decay half-lives for the three models.

A comparative calculation for the standard deviations of e–e, e–o, o–e, and o–o nuclei in the alpha transitions for (VSF), (RF), and (PWF) are listed in Table 5 that shows the most accurate and best alpha-decay half-lives. The first column represents the types of nuclei. The second, third, and fourth columns identify the standard deviations for the (VSF), (RF), and (PWF) models. The last column represents the number of mother nuclei.

Table 1. **The predicted $\log_{10} T_{1/2}^{\rm theo}$ for (VSF), (RF), (PWF) and $\log_{10} T_{1/2}^{\rm exp}$ of even–even nuclei in the interval $100 \leq Z \leq 120$**

Nucleus	Z, N, A	$\log_{10} T_{1/2}^{\rm theo}$ (VSF)	$\log_{10} T_{1/2}^{\rm theo}$ (RF)	$\log_{10} T_{1/2}^{\rm theo}$ (PWF)	$\log_{10} T_{1/2}^{\rm exp}$
Sg	106, 154, 260	–2.3582	–2.3132	–2.1703	–2.444
Sg	106, 156, 271	1.4438	1.2918	1.6133	2.05
Sg	106, 160, 266	1.1399	1.0810	1.36	1.531
Hs	108, 156, 264	–2.6485	–3.2969	-3.1449	–3.585
Hs	108, 162, 270	–0.0952	0.1061	0.3952	0.556
Hs	108, 158, 266	–2.6152	–2.6144	–2.4287	–1.91
Hs	108, 160, 268	–1.4483	–1.4831	–1.2518	–0.69
Hs	108, 164, 272	–1.9881	–2.0998	–1.8554	–1.33
Hs	108, 166, 274	–0.318	–0.4634	–0.1602	0.44
Hs	108, 168, 276	1.8749	1.6971	2.0732	2.77
114	114, 172, 286	–0.792	–0.937	–0.5413	–0.886
114	114, 174, 288	–0.1195	–0.3016	0.1261	–0.097
116	116, 174, 290	–1.9248	–2.0617	–1.6632	–2.167
116	116, 176, 292	–1.4569	–1.6312	–1.2064	–1.745
118	118, 176, 294	–3.3151	–3.4384	–3.0457	–3.046
118	118, 166, 284	–5.2459	–5.1789	–4.908	–4.62
118	118, 168, 286	–4.3803	–4.3528	–4.0436	–3.72
118	118, 170, 288	–3.425	–3.4369	–3.0869	–2.72
118	118, 172, 290	–2.8218	–2.8723	–2.4916	–2.11
120	120, 178, 298	–4.4471	–4.5532	–4.159	–4.523
120	120, 154, 274	–11.8684	–11.498	–11.4832	–11.52
120	120, 156, 276	–11.143	–10.8138	–10.7636	–10.28
120	120, 160, 280	–8.9085	–8.6632	–8.5199	–8.4
120	120, 162, 282	–7.6293	–8.4262	–7.2318	–7.05
120	120, 164, 284	–4.5723	–4.4159	–4.1193	–3.78
120	120, 166, 286	–6.1101	–5.9875	–5.7215	–5.5
Fm	100, 150, 250	3.2163	3.2941	3.4869	3.38
Fm	100, 146, 246	0.3338	0.4718	0.5612	0.17
Fm	100, 148, 248	1.5988	1.7055	1.8422	1.66
Fm	100, 154, 254	4.1851	4.1919	4.4385	4.14
Fm	100, 156, 256	4.418	4.3881	4.6547	4.405
No	102, 156, 258	1.85	1.8222	2.0462	2.08
No	102, 150, 252	0.5152	0.596	0.7422	0.74
No	102, 152, 254	1.5889	1.6365	1.8252	1.82
Rf	104, 152, 256	–0.0228	0.0406	0.2095	–0.52
Rf	104, 154, 258	–0.9207	–0.8997	–0.7401	–1.04
Rf	104, 156, 260	0.1281	0.1151	0.3167	0.00
Ds	110, 152, 262	–5.6376	–5.4879	–5.4072	–5.05
Ds	110, 154, 264	–5.3594	–5.2483	–5.1454	–4.79
Ds	110, 156, 266	–4.9444	–4.8714	–4.7428	–4.37
Ds	110, 158, 268	–5.68	–5.6466	–5.5239	–5.18
Ds	110, 160, 270	–4.0828	–4.0855	–3.905	–3.49
112	112, 152, 264	–6.7267	–6.5345	–6.4641	–6.18
112	112, 154, 266	–6.5888	–6.4356	–6.3468	–6.07
112	112, 156, 268	–6.0867	–5.9722	–5.8551	–5.56
112	112, 158, 270	–5.5902	–5.5141	–5.369	–5.06
112	112, 160, 272	–5.122	–5.0841	–4.9118	–4.56
112	112, 162, 274	–4.5962	–4.5962	–4.3954	–4.02
112	112, 164, 276	–3.8229	–3.8605	–3.6244	–3.22
112	112, 166, 278	–3.3277	–3.4047	–3.1392	–2.72

Table 2. **The predicted $\log_{10}T_{1/2}^{\rm theo}$ for (VSF), (RF), (PWF) and $\log_{10}T_{1/2}^{\rm exp}$ of even–odd nuclei in the interval $100 \le Z \le 120$**

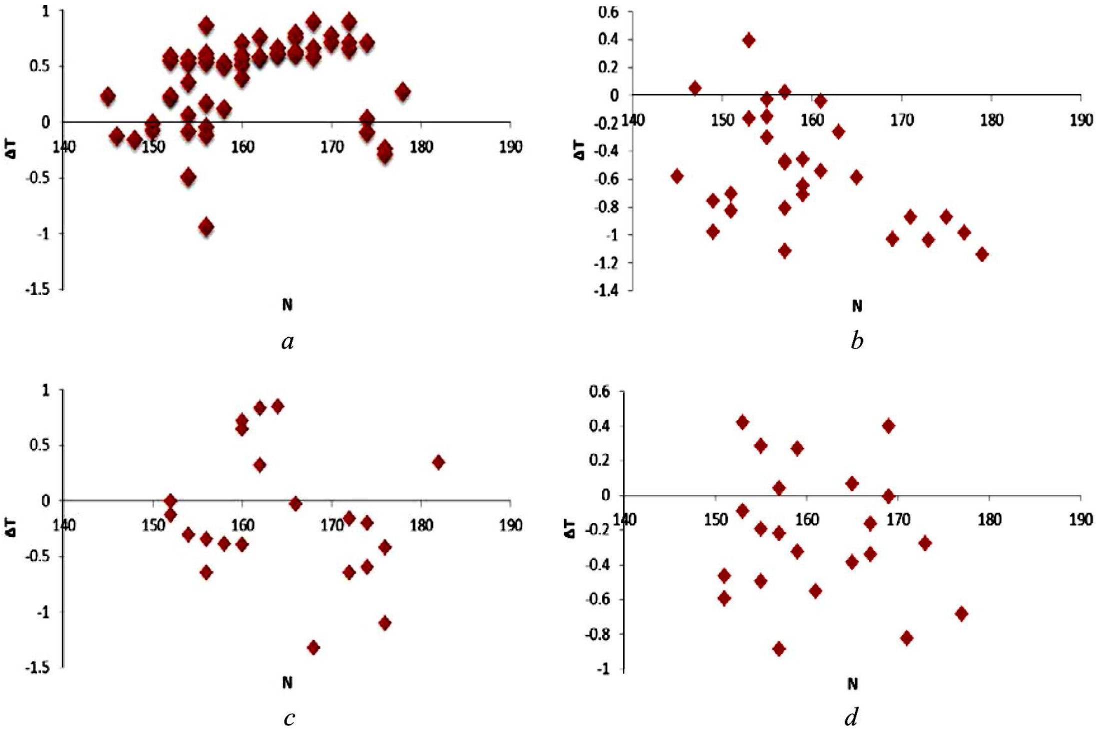
Nucleus	Z, N, A	$\log_{10}T_{1/2}^{\rm theo}$ (VSF)	$\log_{10}T_{1/2}^{\rm theo}$ (RF)	$\log_{10}T_{1/2}^{\rm theo}$ (PWF)	$\log_{10}T_{1/2}^{\rm exp}$
Rf	$104, 151, 255$	1.034	0.8576	0.4576	0.204
Rf	$104, 155, 259$	0.6727	0.4212	0.0994	0.519
Rf	$104, 157, 261$	1.8395	1.5611	1.4128	1.86
Sg	$106, 155, 261$	–0.6066	–0.8168	–1.2555	–0.638
Sg	$106, 157, 263$	0.2847	0.0445	–0.2522	–0.523
Sg	$106, 159, 265$	1.5627	1.2954	1.1841	0.851
Sg	$106, 165, 271$	2.6472	2.2813	2.463	–3.097
Hs	$108, 157, 265$	–2.6125	–2.8158	–3.3849	-1.481
Hs	$108, 159, 267$	–0.8315	–1.0604	–1.4042	0.987
Hs	$108, 161, 269$	1.0269	0.7732	0.6724	–5.523
Ds	$110, 157, 267$	–4.4074	-44.5599	–5.2888	–3.77
Ds	$110, 159, 269$	–3.3119	–3.495	–4.0709	–2.959
Ds	$110, 161, 271$	–2.4185	–2.633	–3.0708	–3.77
Ds	$110, 163, 273$	–3.5050	–3.7603	–4.2522	–3.77
Ds	$110, 169, 279$	0.3304	–0.0115	0.0536	–0.699
Cn	$112, 171, 283$	1.4541	1.1412	1.3961	0.58
114	$114, 173, 287$	0.7178	0.4291	0.6706	–0.319
114	$114, 175, 289$	1.292	0.9707	1.3363	0.415
118	$118, 177, 295$	–2.0134	–2.2432	–2.1695	–3.000
120	$120, 179, 299$	–3.1627	–3.3555	–3.3424	–4.301
No	$102, 151, 253$	2.8091	2.6019	2.3667	1.982
No	$102, 153, 255$	2.4345	2.1879	1.9751	2.27
No	$102, 155, 257$	1.6975	1.4085	1.1814	1.398
No	$102, 157, 259$	4.0127	3.7082	3.7764	3.542
No	$102, 149, 251$	0.9795	0.7928	0.3249	0.00
Fm	$100, 145, 245$	1.1994	1.0455	0.4796	0.62
Fm	$100, 147, 247$	2.0214	1.8387	1.4056	2.07
Fm	$100, 149, 249$	3.3457	3.1394	2.8911	2.59
Fm	$100, 151, 251$	6.7774	6.5684	6.7255	6.07
Fm	$100, 153, 253$	6.3087	6.059	6.2333	6.70

According to the value of σ in Table 5, (PWF) and (RF) can be considered the best models for o–e and e–o nuclei compared with (VSF); (VSF) is the best for o–o nuclei; while (PWF) shows more acceptable results, than (VSF) and (RF). The results of evaluation of the overall effect of proton and neutron shells on the alpha-decay half-life, which allows us to judge the nucleus stability, are shown in Figs. 1, *a, b, c, d*, 2 *a, b, c, d* , and 3, *a, b, c,d.* The ΔT logarithm versus the neutron number (N) curves were calculated with the use of (VSF), (RF), and (PWF) for the even–even, even–odd, odd–even, and odd–odd nuclei emitting alpha particles. These figures show the acceptable results.

Table 3. **The predicted $\log_{10}T_{1/2}^{\rm theo}$ for (VSF), (RF), (PWF) and $\log_{10}T_{1/2}^{\rm exp}$ of odd–even nuclei in the interval $100 \le Z \le 120$**

Nucleus	Z, N, A	$\log_{10}T_{1/2}^{\rm theo}$ (VSF)	$\log_{10}T_{1/2}^{\rm theo}$ (RF)	$\log_{10}T_{1/2}^{\rm theo}$ (PWF)	$\log_{10}T_{1/2}^{\rm exp}$
Db	$105, 152, 257$	–0.0914	–0.5102	–0.4342	–0.097
Db	$105, 156, 260$	0.604	0.1331	0.3186	0.255
Bh	$107, 160, 267$	1.6232	1.1001	1.4842	1.230
Mt	$109, 166, 275$	–1.9844	–2.6002	–2.3065	–2.013
Mt	$109, 158, 267$	–2.94	–3.41	–3.34	–3.33
Mt	$109, 160, 269$	–2.1103	–2.61	–2.3	–1.47
Mt	$109, 162, 271$	–0.48	–1.0168	–0.07	–0.16
113	$113, 172, 258$	0.9029	0.7905	0.8527	0.739
113	$113, 174, 287$	1.8	1.2	1.9	1.6
115	$115, 172, 287$	–0.8100	–1.3915	–0.8268	–1.456
115	$115, 174, 289$	0.0610	–0.6768	–0.0138	–0.658
115	$115, 176, 291$	0.29	–0.3	0.38	–0.18
115	$115, 182, 297$	3.14	2.3	3.4	3.48
117	$117, 176, 293$	–1.3326	–1.936	–1.3005	–1.824
Lr	$103, 152, 255$	1.4687	0297	1.1838	1.338
Lr	$103, 154, 257$	0.1158	–0.3723	–0.2590	–0.187
Lr	$103, 156, 259$	1.4430	0.9280	1.1706	0.792
111	$111, 160, 271$	–3.86	–4.3321	–4	–3.08
111	$111, 168, 279$	–1.3441	–1.94	–1.74	–2.67
111	$111, 164, 275$	–4.340	–4.89	–4	–3.49
111	$111, 162, 273$	–4.5	–4.1	–3.4	–3.87

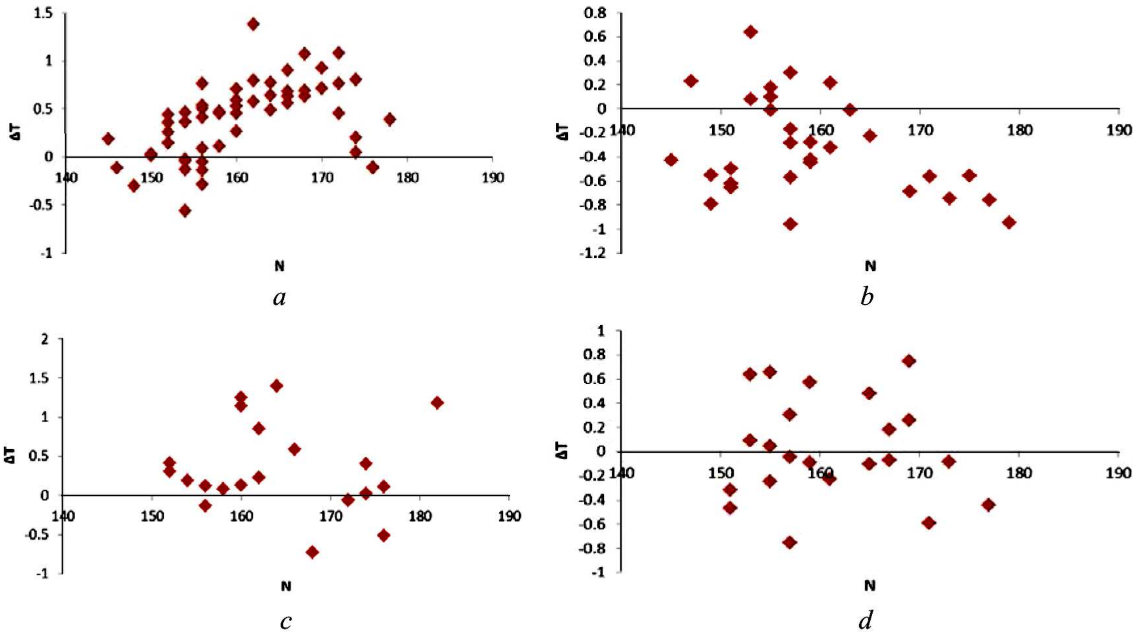
We can note a decrease in the alpha-decay half-lives with an increase in Z, on the whole. But they increase with the neutron number for isotopes of a specific element. The local maximum appears for even-Z nuclei with $N = 156$ and $N = 151$. It is connected with an increasing of the stability in the vicibity of the so-called distorted magic number of neutrons, this result being in agreement with [8]. The alpha-decay half-lives increase with N beyond 146 and up to 158 for all elements. In these nuclei (especially for superheavy ones), the alpha decay would be irrelevant decay channel under the spontaneous fission. it may have a low half-live. In deed, the curves plotted for lower Zs in Fig. 4, *a, b, c, d* show that the peak at $N = 156$ and $Z = 100$ is higher than the peak appeared for $N = 172$ and $Z = 112$, while the discontinuities can be interpreted as the effect which appears due to the closed shell structure. The distance between the peaks for the atomic numbers equal to 108 and 110 are of special interest. The same behavior can be observed for the half-lifes of other isotopic chains described by (VSF) and (RF) for all types of



a *b*

c *d*

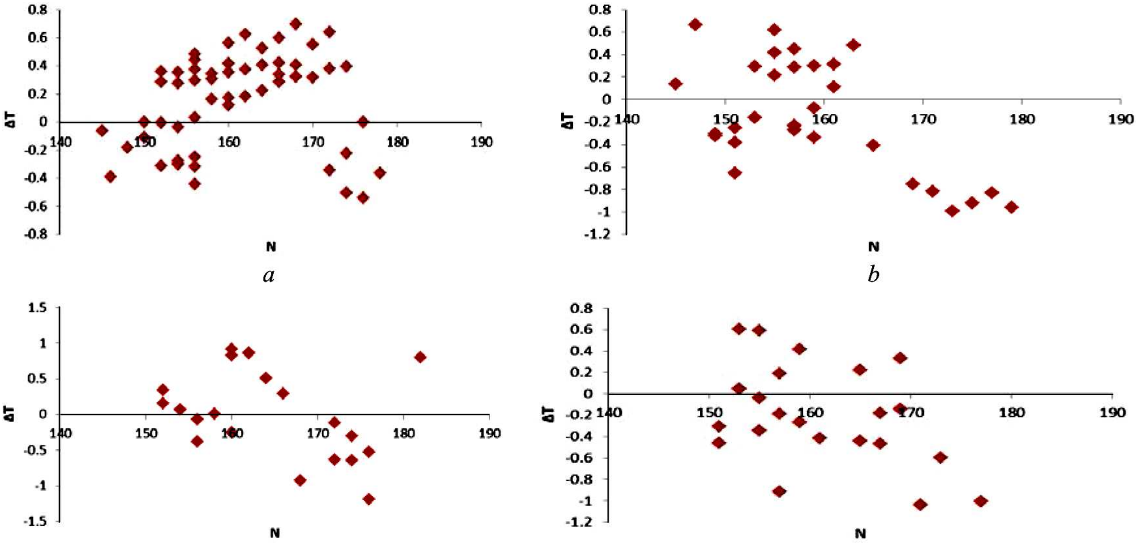
Fig. 1. The disparities between experimental and calculated alpha-decay half-lives for e–e, e–o, o–e, and o–o nuclei versus N for (VSF)



a *b*

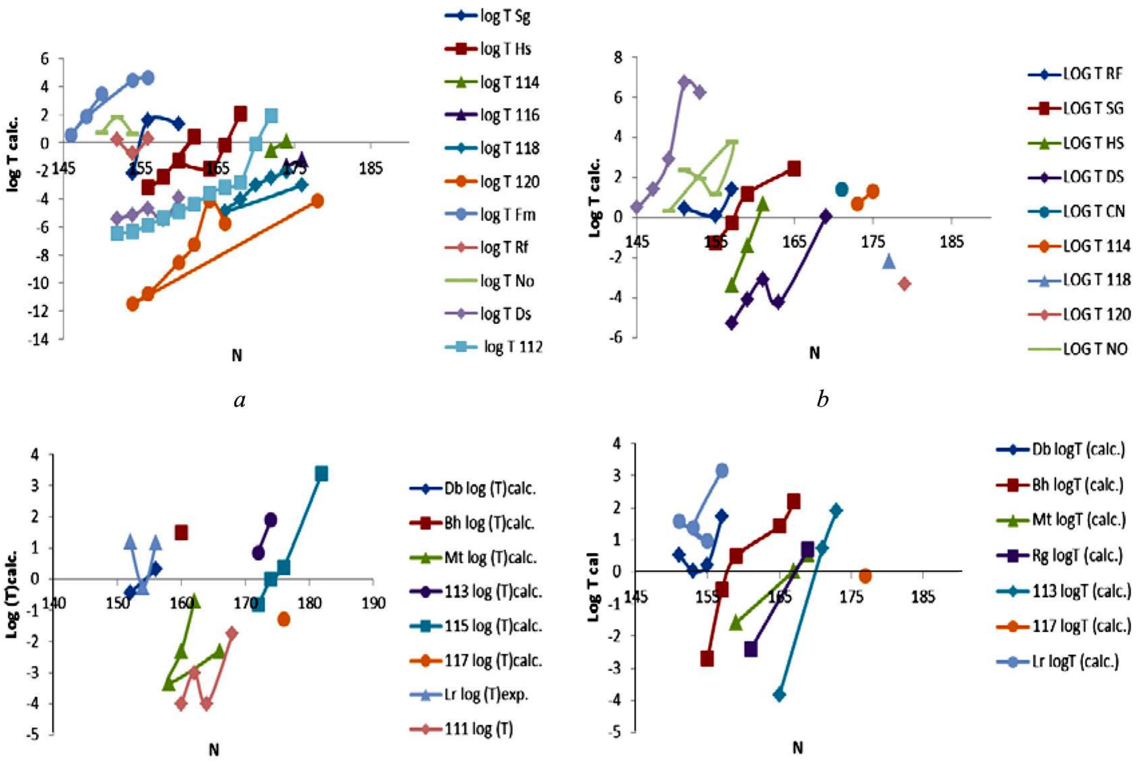
c *d*

Fig. 2. The disparities between experimental and calculated alpha-decay half-lives for e–e, e–o, o–e, and o–o nuclei versus N for (RF)



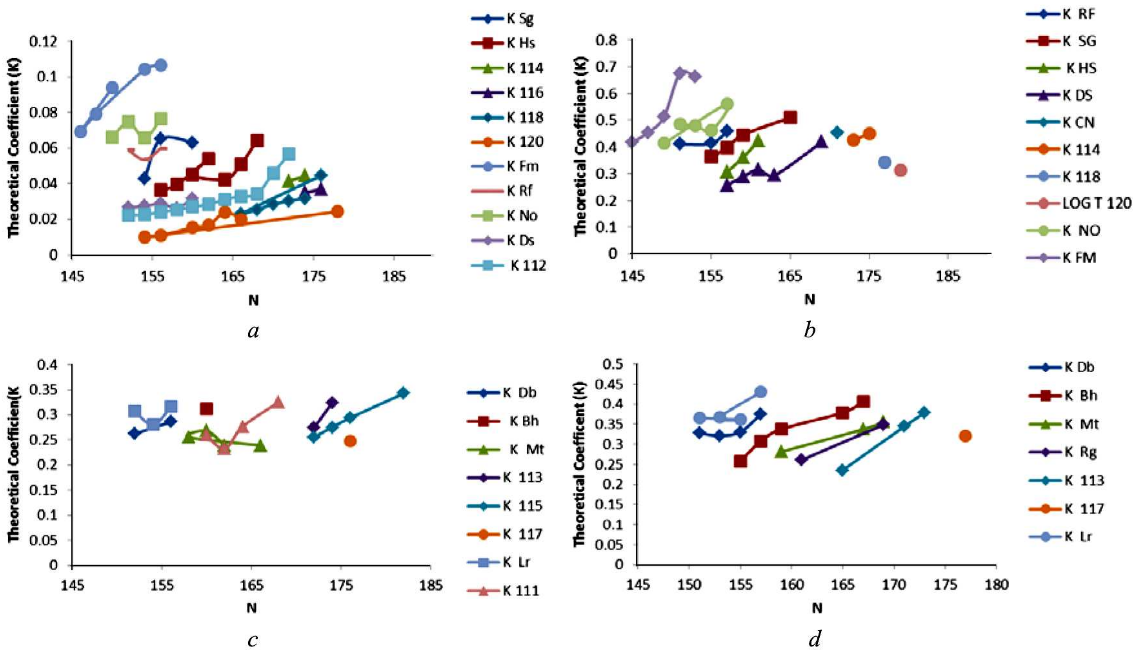
a *b* *c* *d*

Fig. 3. The disparities between practical and calculated half-lives of alpha decay for e–e, e–o, o–e and o–o nuclei versus N for (PWF)



a *b* *c* *d*

Fig. 4. The theoretical alpha-decay half-lives of e–e, e–o, o–e, and o–o nuclei, respectively, as a function of the neutron number N for (PWF) only



a *b*

c *d*

Fig. 5. K for e–e, e–o, o–e, and o–o nuclei versus the neutron number N

Table 4. **The predicted $\log_{10}T_{1/2}^{\rm theo}$ for (VSF), (RF), (PWF) and $\log_{10}T_{1/2}^{\rm exp}$ of odd–odd nuclei in the interval $100 \le Z \le 120$**

Nuc-leus	Z, N, A	$\log_{10}T_{1/2}^{\rm theo}$ (VSF)	$\log_{10}T_{1/2}^{\rm theo}$ (RF)	$\log_{10}T_{1/2}^{\rm theo}$ (PWF)	$\log_{10}T_{1/2}^{\rm exp}$
Db	105, 151, 125	0.6944	0.5511	0.5333	0.230
Db	105, 153, 258	0.2216	0.0062	0.0382	0.643
Db	105, 155, 260	0.3729	0.1317	0.2145	0.176
Db	105, 157, 262	1.7633	1.5876	1.7285	1.544
Bh	107, 155, 262	–2.3807	–2.7542	–2.6895	–2.097
Bh	107, 159, 266	0.5549	0.3211	0.4936	–0.356
Bh	107, 157, 264	–0.3948	–0.6609	–0.5435	0.230
Bh	107, 165, 272	1.3796	1.0971	0.4326	0.991
Bh	107, 167, 274	2.0719	1.8039	2.2009	1.733
Mt	109, 159, 268	–1.4212	–1.7258	–1.5758	–1.5758
Mt	109, 167, 276	0.0217	–0.3243	0.0361	0.0361
Mt	109, 169, 278	0.4819	0.1332	0.5505	0.5505
Rg	111, 161, 272	–2.2686	–2.5936	–2.4113	–2.4113
Rg	111, 169, 280	0.5637	0.2944	0.7003	0.7003
113	113, 165, 278	–3.6831	–4.0999	–3.8418	–3.8418
113	113, 171, 284	0.5031	0.2708	0.7159	0.7159
113	113, 173, 286	1.5686	1.3752	1.8892	1.8892
117	117, 177, 294	–0.4317	–0.6712	–0.1703	–0.1073
Lr	103.151, 254	1.7112	1.5797	1.5769	1.5769
Lr	103, 155, 258	1.1093	0.8588	0.9577	0.9577
Lr	103, 153, 256	1.52	1.3371	1.3857	1.3857
Lr	103, 157, 260	3.1422	3.007	3.1705	3.1705

nuclei. Tables 1–4 show the acceptable agreement between the analytical determinations and the experimental values, which is a good indicator for the guessing for the alpha-decay half-lives of 121 nuclei within all models that are used. The PWF can be verified in a wide region of nuclear structures, as well as can be applied to the comparison between applied and theoretical nuclear models. To show the meaning of the theoretical coefficient K more clearly, we determine it for superheavy elements ranging from $Z = 10$ to $Z = 120$, as shown in Fig. 5.

It has been found that the K value decreases, as Z increases, and increases with N for isotopes of a particular element. This means that it has the same behavior as $\log_{10}T_{1/2}^{\rm theo}$ and hints that the half-lives becomes more and more insensitive to the alpha-decay

Table 5. **The standard deviation for (VSF), (RF), and (PWF) within the new approach with new parameters**

Set	σ (VSF)	σ (RF)	(PWF)	No of nuclei
even–even	0.541	0.55	0.38	55
even–odd	0.68	0.51	0.51	30
odd–even	0.53	0.64	0.52	21
odd–odd	0.43	0.43	0.58	22

energy. For instance, the decrease in the Q_α value by 2 MeV corresponds to an increase of the half-life by ≈8 orders for isotopes of $110 D_S$ (Table 1). It is a good advantage to predict the alpha-decay half-lives for superheavy nuclei, because they are not so sensitive to energy decay value as for medium-heavy nuclei [28]. For nuclei near the closed shell, the K values are low, because they are affected by the closed shell structure, especially for a heavy element with low $\log_{10} T_{1/2}$. This fact proposes that, for a given massive element, the isotopes at the beginning of the closed shell are more insensitive to Q_α-value.

7. Conclusion

We have investigated the alpha-decay half-lives of e–e, e–o, o–e, and o–o superheavy nuclei with use of the Viola–Seaborg–Sobiczewoski formula, Royer formula, and present-work formula with the account for the angular momentum of an alpha particle and the relative neutron excess $\left(\frac{N-Z}{A}\right)$, which is extremely important for the evaluation of the half-live logarithm. The formula we have proposed has a new set of parameters determined by the least square fitting method for the alpha decay of 128 nuclei. We have obtained the standard deviations for each formula and the disparity between the experimental and calculated alpha-decay half-lives for all types of nuclei. The obtained results are compared with the corresponding experimental values, and it is revealed that they show a good matching. The proposed theoretical coefficient K shows a similar behavior of $\log_{10} T_{1/2}$, which can be used to predict new superheavy nuclei. Moreover, the presented formula can be applied to a wide field of physical verifications. But the half-lives of emitters of an alpha particle are insufficient to obtain more information about the nuclear properties and other aspects of superheavy nuclei (such as vibration bands, nuclear isospin, and various nuclear structures).

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АНАЛІЗ СПІВВІДНОШЕННЯ ГЕЙГЕРА–НУТТАЛА ДЛЯ ЗАСТОСУВАННЯ ОЦІНКИ ПЕРІОДІВ НАПІВРОЗПАДУ НАДВАЖКИХ ЯДЕР

Резюме

Періоди напіврозпаду парно-парних, парно-непарних, непарно-парних і непарно-непарних ядер в інтервалі $100 \leqslant Z \leqslant 120$ оцінювалися за формулою Віола–Сіборга (VSF) і за аналітичною формулою Роєра (RF). Ми запропонували іншу формулу (PWF), яка враховує кутовий момент частинки в альфа-розпаді і відносний нейтронний надлишок $\left(\frac{N-Z}{A}\right)$. Наша формула містить новий набір параметрів, які визначаються за методом найменших квадратів для альфа-розпадів 128 ядер. Ми отримали і порівняли стандартні відхилення для кожної з формул. Результати показують прийнятне узгодження з наявними даними. Величини запропонованого теоретичного коефіцієнта K в нашій моделі (PWF) показують схожу поведінку періодів напіврозпаду ядер з альфа-розпадом, що може бути використано для передбачення нових надважких ядер.