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VACUUM BIREFRINGENCE IN SUPERCRITICAL MAGNETIC FIELDS

The birefringence effect in vacuum in strong magnetic fields has been considered. The polarization tensor in a constant external magnetic field is analyzed in the framework of quantum field theory and on the basis of the electron Green's function calculated as the sum over the Landau levels. The case of the lowest Landau levels for photons with the energies below the electron-positron pair creation threshold is considered, and the corresponding refractive indices of the physical vacuum for anomalous and normal waves are determined.

Keywords: vacuum birefringence, quantum electrodynamics, strong magnetic field.

1. Introduction

As follows from the classical field theory, electromagnetic waves propagate independently of one another and do not change their polarization, when propagating in vacuum. This is so because Maxwell's equations are linear (i.e. the Lagrangian of an electromagnetic field is a quadratic function of the electric and magnetic fields). However, according to quantum electrodynamics (QED), photons can create virtual electron-positron pairs in an external field, and those pairs can interact with this field. Thus, nonlinear QED effects may take place; in particular, this is the effect of vacuum birefringence. The latter consists in a variation of the photon polarization due to the creation and annihilation of virtual electron-positron pairs in a strong external electromagnetic field. The external field transforms the physical vacuum into an anisotropic medium with different refractive indices along the field direction and perpendicularly to it.

Despite that this effect was predicted rather long ago, its direct experimental confirmation is still absent. Nevertheless, with the appearance of powerful laser facilities, highly sensitive devices for measuring the ellipticity and the rotation angle of the polarization plane of electromagnetic radiation, and extremely sensitive telescopes that can analyze the polarization of radiation emitted by neutron stars, this task becomes extremely challenging.

In particular, the transformation of a linearly polarized electromagnetic wave passing through a magnetic field into an elliptically polarized one was intensively studied on the PVLAS (polarization of the vacuum with laser) installation [1]. Recent results demonstrate that the available sensitivity of the installation still remains insufficient to observe this effect [2, 3].

The appearance of powerful laser facilities – such as PHELIX (Facility for Antiproton and Ion Research, Germany) [4], Vulcan (Central Laser Facility, UK) [4], and Apollon (Orme des Merisiers, Saclay, France) [5] – brought the scientists to the idea of verifying non-linear QED effects in laser fields. The idea consists in colliding two laser beams, one of which plays the role of a background electromagnetic field with a high intensity, whereas the other is a low intensive and linearly polarized X-ray radiation, which is analyzed for the appearance of the ellipticity. The polarization properties of physical vacuum will be researched in the framework of this scenario in Germany at the Helmholtz International Beamline for Extreme Fields (HIBEF) on the European X-ray Free Electron Laser (XFEL) facility [6].

At present, the Extreme Light Infrastructure (ELI) laser installation (the Czech Republic, Romania, and Hungary) [7] is also under development. It will open new possibilities in studying the interaction of laser radiation with matter. In the framework of this project, experiments aimed at the verification of nonlinear effects in strong electromagnetic fields predicted by quantum electrodynamics are also planned. In particular, this is the research of the birefringence effect in a laser field. It should be noted that an increase of the laser field intensity results in the appearance of new scientific problems concerning the influence of laser fields of various configurations on physical phenomena. Those problems cover diverse domains in both fundamental and applied sciences. In this case, the application of intense fields makes it possible to experimentally verify some of nonlinear QED effects induced by strong fields.

It should be noted that intense experimental researches of a change of the photon polarization in strong magnetic fields of neutron stars have been carried out in the recent years. For instance, in 2016, the polarization variation of optical photons at their passage through the magnetosphere of the isolated neutron star RX J1856.5-3754 was determined for the first time, and the polarization degree of those photons was found [8]. According to the conclusion of the cited authors, this result confirms the polarization properties of physical vacuum.

The results of first theoretical studies were published in works [9, 10], where the fluctuations of the electron-positron field were described in the approximation of weak external fields and for the photon energies much lower than the electron mass, $E/E_c \ll 1$ and $H/H_c \ll 1$. Here, $E_c \approx 1.32 \times 10^{16}$ V/cm is the critical value of the electric field strength, at which electron-positron pairs can be generated spontaneously from the vacuum, and $H_c \approx 4.41 \times 10^{13}$ Gs is the critical value of the magnetic field strength. According to this theory, the account for nonlinear effects gives rise to the appearance of an additional term in the Lagrangian of an electromagnetic field, which has to be relativistically invariant. Among the consequences of the approach on the basis of the Heisenberg–Euler Lagrangian, we obtain the birefringence effect. In other words, the physical vacuum becomes an anisotropic medium in strong electromagnetic fields, and just this anisotropy becomes responsible for the appearance of the vacuum birefringence.

Later, the theoretical study of this effect was continued. Namely, the polarization tensor, which describes the polarization of vacuum in external fields, was determined. In particular, in work [11], the polarization tensor was derived for the first time in the case of a photon in a constant electromagnetic field with an arbitrary configuration. The Schwinger proper-time method [12] was applied at that. In works [13–15], similar calculations were performed in the case of a constant uniform magnetic field, and some limiting cases were analyzed. Later, in work [16], the refractive indices of physical vacuum were calculated in the cases where the magnetic fields are weak or strong in comparison with the critical value, and the photon energies are lower than the electron-positron pair generation threshold. It should be noted that supercritical magnetic fields are observed in magnetars, which have been discovered, when observing radiation in the X- and gamma-ray spectral intervals. In work [17], scalar functions of the polarization tensor were found numerically for arbitrary magnetic field values and for photons with $\omega < 2m$. Finally, works [18,19] should be mentioned where the resonance case was considered; namely, virtual intermediate particles reach the mass shell and become real.

It should be noted that the consideration in works [11–18] was based on the application of the electron Green's function that was obtained in the framework of the Schwinger proper-time method. In this case, the polarization tensor does not explicitely depend on the Landau level numbers. Recently, there appeared works [20, 21], in which the required expressions were obtained. But it was a result of mathematical transformations of the formulas that were derived using Schwinger's Green's function. At present, there is no consecutive study of the polarization tensor using Green's function for the electron in a magnetic field that is determined as the sum over the Landau levels in the basis of exact solutions of the Dirac equation. Note also that the general form of the polarization operator in the approximation of the lowest Landau levels was obtained in work [22] and used in work [23], while solving the problem of magnetic catalysis.

Unlike the previous researches, the consideration of the polarization tensor in a magnetic field is based

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in this paper on the electron Green's function calculated as the sum over the Landau levels. In Section 2, the explicit form of the electron propagator in a magnetic field is considered. Section 3 contains consecutive mathematical calculations of the polarization tensor in the general case. In Section 4, the case of the lowest Landau levels for photons with energies below the electron-positron pair generation threshold is analyzed. In Section 5, this approximation is used to calculate the refractive indices of physical vacuum, which characterize the vacuum birefringence effect.

2. Electron Green's Function via the Sum over the Landau levels

In what follows, the calculations are performed in the relativistic system of units ($\hbar = c = 1$). The Landau gauge for the electromagnetic 4-potential is used: $A = (0, 0, xH, 0)$.

It should be noted that the effect of vacuum birefringence in a magnetic field is associated with the process of virtual electron-positron pair generation by a photon, and the annihilation of this pair into a single photon. In this case, the polarization properties of physical vacuum are described by the polarization tensor. In the one-loop approximation, it looks like

$$\Pi^{\mu\nu}(x, x') = -ie^2 Sp\,\{\gamma^\mu G(x, x')\gamma^\nu G(x', x)\}. \tag{1}$$

The Feynman diagram for the polarization tensor is depicted in Fig. 1.

In expression (1), $G(x, x')$ is the electron Green's function in a magnetic field, which was obtained for the first time in work [24]. Later, it was found in works [25, 26] using a slightly different method and in work [27] using precise solutions of the Dirac equation. It should be noted that this propagator was used in works [28–31] to calculate the amplitudes of second-order processes with the electron as an intermediate particle. The propagator has the form

$$G(x, x') = -\frac{m\sqrt{h}}{(2\pi)^3}\int d^3g e^{-i\Phi}\sum_n \frac{G_H(x, x')}{g_0^2 - E_n^2}, \tag{2}$$

where

$$G_H(x, x') = (\gamma P + m)\left[\tau U_n U_n' + \tau^* U_{n-1}U_{n-1}'\right] + \\ + im\sqrt{2nh}\gamma^1\left[\tau U_{n-1}U_n' - \tau^* U_n U_{n-1}'\right], \tag{3}$$

$$\Phi = g_0(x_0' - x_0) - g_y(x_2' - x_2) - g_z(x_3' - x_3),$$

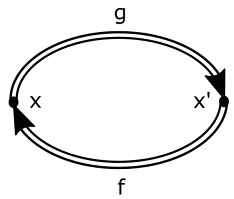

Fig. 1. Feynman diagram for the polarization tensor in a magnetic field

$$U_n = \frac{1}{\sqrt{\sqrt{\pi}2^n n!}}e^{-\frac{\rho^2}{2}}H_n(\rho),$$

$$\tau = \frac{1}{2}(1 + i\gamma^2\gamma^1), \quad P = (g_0, 0, 0, g_z),$$

$h = H/H_\mathrm{c}$, $H_\mathrm{c} = m^2/e = 4.41 \times 10^{13}$ Gs is the Schwinger critical magnetic field, $d^3g = dg_0 dg_y dg_z$, Φ is the phase, γ are the Dirac matrices , $H_n(x)$ is the Hermite polynomial, and $\rho(x) = m\sqrt{h}x + g_y/m\sqrt{h}$ is the argument of the quantity U_n. The primed functions in Eq. (3) depend on x'. The parameter E_n in expression (2) is the electron energy, which has discrete energy levels in a uniform magnetic field,

$$E_n = \sqrt{g_z^2 + \tilde{m}^2}, \tag{4}$$

where g_z is the momentum component directed along the field, $\tilde{m} = m\sqrt{1 + 2nh}$, and n is the number of a Landau level.

After integrating expression (2) over g_y, we obtain

$$G(x, x') = -e^{-i\Phi'(x,x')}G(z), \tag{5}$$

where

$$\Phi'(x, x') = (x_1 + x_1')(x_2 - x_2')\frac{m^2 h}{2},$$

$$z = x - x',$$

$$G(z) = \frac{m^2 h}{(2\pi)^3}\int dg_0 dg_z e^{i(zP)}\sum_n \frac{G_H(z_1, z_2)}{g_0^2 - E_n^2}, \tag{6}$$

$$G_H(z_1, z_2) = e^{-\frac{\xi}{2}}\Big\{(\gamma P + m)(\tau L_n + \tau^* L_{n-1}) - \\ - 2in\gamma^1\left(\frac{\tau}{z_1 + iz_2} + \frac{\tau^*}{z_1 - iz_2}\right)(L_n - L_{n-1})\Big\}, \tag{7}$$

and the argument of Laguerre polynomials L_n is

$$\xi = \left(z_1^2 + z_2^2\right)\frac{m^2 h}{2}.$$

In view of expression (6) for the electron Green's function, the polarization tensor in the magnetic field, Eq. (1), can be rewritten in the form

$$\Pi^{\mu\nu}(z) = -ie^2 \mathrm{Sp}\,\{\gamma^\mu G(z)\gamma^\nu G(-z)\}. \tag{8}$$

3. Polarization Tensor in a Magnetic Field

In this work, while analyzing the polarization properties of vacuum, we will use the Fourier transform of the polarization tensor,

$$\Pi^{\mu\nu}(k) = \int d^4 z e^{-ikz} \Pi^{\mu\nu}(z). \tag{9}$$

Taking expression (6) into account and integrating over the variables z_0 and z_3, we rewrite formula (9) in the form

$$\Pi^{\mu\nu}(k) = -ie^2 \frac{m^4 h^2}{(2\pi)^4} \int d^2 z_\perp d^2 g_\parallel e^{i(k_1 z_1 + k_2 z_2)} \mathrm{Sp} A^{\mu\nu}, \tag{10}$$

where $d^2 z_\perp = dz_1 dz_2$, $d^2 g_\parallel = dg_0 dg_3$, and

$$A^{\mu\nu} = e^{-\xi} \sum_{n,n'=0}^{\infty} \frac{\sum_{j=1}^{16} B_j^{\mu\nu}}{(g_0^2 - E_n^2)(f_0^2 - E_{n'}^2)}. \tag{11}$$

Expression (11) also contains the quantities $B_j^{\mu\nu}$, whose explicit forms are given in Appendix A.

Without loss of generality, we choose a reference system, in which the longitudinal component of a wave vector along the magnetic field direction vanishes:

$$k_z = 0, \tag{12}$$

because the Lorentz transformation along the magnetic field does not change the field itself.

To carry out the integration in expression (10) over the variables g_0 and g_z, we use the α-representation [32], which is analogous to the Schwinger proper-time method [12]:

$$\frac{1}{g_0^2 - E_n^2 + i\epsilon} = -i \int_0^\infty d\alpha e^{i\alpha\left(g_0^2 - E_n^2 + i\epsilon\right)}. \tag{13}$$

We also use the change of variables

$$\alpha = \lambda \frac{1-\xi}{2}, \quad \beta = \lambda \frac{1+\xi}{2}. \tag{14}$$

It is well known that the polarization tensor has a singularity. Therefore, let us apply the regularization or renormalization procedure. For this purpose, we use the regularization method of Bogolyubov [32]. According to this method, the denominator in the expression for Green's function can be rewritten as

$$\frac{1}{g_0^2 - E_n^2 + i\epsilon} = \frac{1}{g_0^2 - E_n^2 + i\epsilon} - \frac{1}{g_0^2 - E_n^2 - M^2 + m^2 + i\epsilon}, \tag{15}$$

where M is an extra mass that tends to infinity, if the regularization is cancelled, and ϵ is a small positive parameter. After applying the regularization procedure (15) and using α-representation (13), the general expression for the polarization tensor reads

$$\Pi^{\mu\nu}(k) = i \frac{e^2 m^4}{8\pi^2} h \sum_{n,n'=0}^{\infty} \int_{-1}^{1} d\xi \int_0^\infty d\lambda E(\lambda, \xi) \sum_{j=1}^{15} \Pi_j^{\mu\nu}. \tag{16}$$

Here, the following notations are introduced:

$$E(\lambda, \xi) = e^{-\epsilon\lambda} \left(e^{-i\lambda A} + e^{-i\lambda B} - e^{-i\lambda C} - e^{-i\lambda D}\right),$$

$$A = m^2 (1 + \Phi), B = m^2 \left(\Phi + \frac{M^2}{m^2}\right),$$

$$C = m^2 \left\{\Phi + \frac{1}{2}(1+\xi) + \frac{M^2}{2m^2}(1-\xi)\right\},$$

$$D = m^2 \left\{\Phi + \frac{1}{2}(1-\xi) + \frac{M^2}{2m^2}(1+\xi)\right\},$$

$$\Phi = \tilde{\omega}^2 \xi^2 - h(n-n')\xi + h(n+n') - \tilde{\omega}^2,$$

where $\tilde{\omega} = \omega/2m$. Explicit expressions for the quantities $\Pi_j^{\mu\nu}$ are given in Appendix B.

4. Soft Photons and a Strong Magnetic Field

Let us consider the case where (i) the photon energy is lower than the energy threshold required for the generation of an electron-positron pair (soft photons) and (ii) the magnetic field exceeds the critical value:

$$\tilde{\omega} < 1, \quad h \gg 1. \tag{17}$$

Then the approximation of the lowest Landau levels is valid, and the polarization tensor (16) can be rewritten as follows:

$$\Pi_{0,0}^{\mu\nu}(k) = i \frac{e^2 m^4}{8\pi^2} h e^{-\eta} \int_{-1}^{1} d\xi \Pi_0^{\mu\nu} \int_0^\infty d\lambda E(\lambda, \xi), \tag{18}$$

where

$$\Pi_0^{\mu\nu} = \eta_\parallel^{\mu\nu} - \tilde{\omega}^2 \left(1 - \xi^2\right) U_\parallel^{\mu\nu}, \tag{19}$$

$$\eta_\parallel^{\mu\nu} = \mathrm{diag}(1, 0, 0, -1), \quad U_\parallel^{\mu\nu} = \mathrm{diag}(1, 0, 0, 1).$$

After the regularization procedure, the integral over the variable λ can be written in the form

$$\int\limits_0^\infty d\lambda E\left(\lambda,\xi\right) = -\frac{i}{m^2}\frac{1}{1+\Phi-i\tilde{\epsilon}}, \tag{20}$$

where $\tilde{\epsilon}=\epsilon/m^2$. Then the polarization tensor (18) takes the form

$$\Pi^{\mu\nu}_{0,0}(k) = e^2\frac{m^2h}{8\pi^2}e^{-\eta}\left[\left(\eta^{\mu\nu}_{\parallel}-U^{\mu\nu}_{\parallel}\right)I_{0,0}+2U^{\mu\nu}_{\parallel}\right], \tag{21}$$

where

$$I_{n-n',n'} = \int\limits_{-1}^{1}\frac{d\xi}{1+\Phi-i\tilde{\epsilon}}. \tag{22}$$

Figures 2 and 3 demonstrate the dependences of the real and imaginary, respectively, parts of integral (22) on $\tilde{\omega}^2$ for some Landau levels at $h=0.1$ and $\tilde{\epsilon}=0.01$.

In the case of the lowest Landau levels and an arbitrary photon energy (as $\tilde{\epsilon}\to 0$), integral (22) can be written in the form

$$I_{0,0} = \begin{cases}\dfrac{2}{\tilde{\omega}\sqrt{1-\tilde{\omega}^2}}\arctan\left(\dfrac{\tilde{\omega}}{\sqrt{1-\tilde{\omega}^2}}\right), & \tilde{\omega}<1;\\ \dfrac{1}{\tilde{\omega}\sqrt{\tilde{\omega}^2-1}}\left[\ln\left|\dfrac{\tilde{\omega}-\sqrt{\tilde{\omega}^2-1}}{\tilde{\omega}+\sqrt{\tilde{\omega}^2-1}}\right|+i\pi\right], & \tilde{\omega}>1.\end{cases} \tag{23}$$

From whence, one can see that if the photon energy does not exceed the generation threshold for an electron-positron pair, the polarization tensor has only a real part (just this case will be considered below). If the photon energy is larger than this value, there appears an imaginary part, which characterizes the process of electron-positron pair generation. It should be noted that, in the case where the photon energy is equal to the e^-e^+-pair generation threshold, the intermediate particles go to the mass shell. In other words, the resonance condition is obeyed. In this case, the process width, which is usually associated with the total probability of the intermediate state decay [29–31], has to be taken into account. From expression (22), it is evident that the parameter $\tilde{\epsilon}$ plays the role of the resonance process width.

It is also known that the polarization tensor must be equal to zero at $\tilde{\omega}=0$ [20, 33]. However, one can see from expression (21) that

$$\Pi^{\mu\nu}_{0,0}(0) = e^2\frac{m^2h}{4\pi^2}e^{-\eta}\eta^{\mu\nu}_{\parallel}. \tag{24}$$

Therefore, the final form of the polarization tensor for the lowest Landau levels can be written as follows:

$$\tilde{\Pi}^{\mu\nu}_{0,0}(k) = \Pi^{\mu\nu}_{0,0}(k)-\Pi^{\mu\nu}_{0,0}(0) = \\ = e^2\frac{m^2h}{8\pi^2}e^{-\eta}\left[\left(\eta^{\mu\nu}_{\parallel}-U^{\mu\nu}_{\parallel}\right)\left(I_{0,0}-2\right)\right]. \tag{25}$$

Expression (25) is completely identical to the results of work [33], where another method of research was applied.

It should be noted that, after the procedure has been carried out, the expression obtained for the polarization tensor in the approximation of the lowest Landau levels demonstrates an explicit transverse structure, which is shown in Eq. (25). At the same

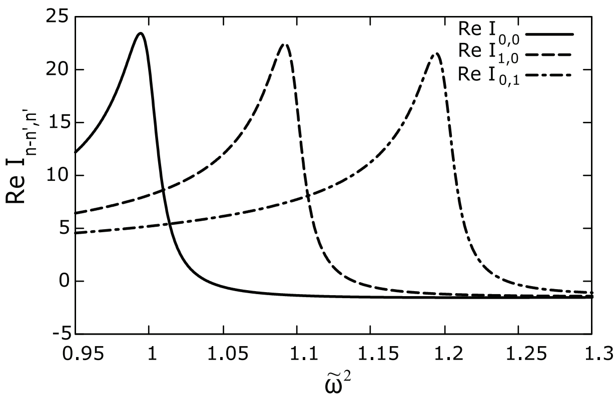

Fig. 2. Dependences of the real part of integral (22) on $\tilde{\omega}^2$ at $h=0.1$

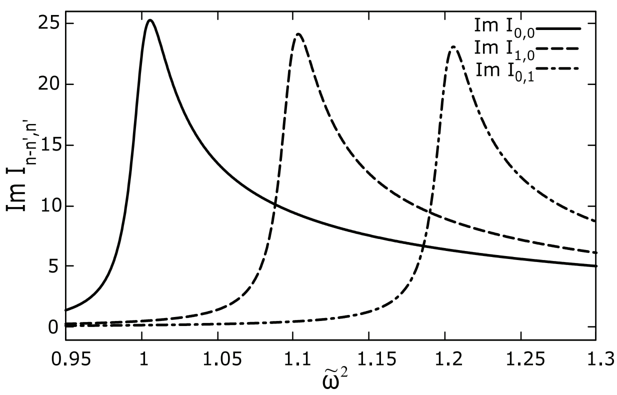

Fig. 3. Dependences of the imaginary part of integral (22) on $\tilde{\omega}^2$ at $h=0.1$

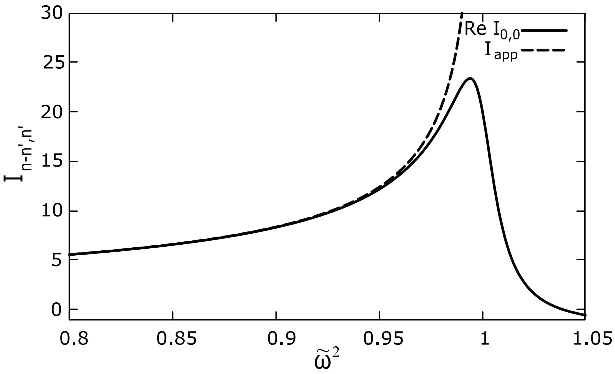

Fig. 4. Dependences of the real part of integral (22) (taking the width into account) and the approximate function (26) for the lowest Landau levels on $\tilde{\omega}^2$

time, the general formulas (10) and (11) for the polarization tensor in a magnetic field, as well as expression (16), are not transverse.

5. Refractive Index in Supercritical Magnetic Fields

With the help of polarization tensor (25), let us determine the refractive indices of physical vacuum along the magnetic field direction and perpendicularly to it. For this aim, let us consider the case where a photon propagates along the Ox axis and possesses an energy that is lower than the pair generation threshold, and let the magnetic field be directed along the Oz axis. Then

$$I_{0,0} = \frac{2}{\tilde{\omega}\sqrt{1-\tilde{\omega}^2}} \arctan\left(\frac{\tilde{\omega}}{\sqrt{1-\tilde{\omega}^2}}\right). \tag{26}$$

Figure 4 illustrates the dependence of the real part of integral (22) on $\tilde{\omega}^2$, in which the width was taken into account, and the approximate function (26) for the lowest Landau levels. The corresponding refractive indices can be found as follows:

$$n_{\parallel,\perp} = 1 - \frac{1}{2\omega^2}\mathrm{Re}\,\Pi_{\parallel,\perp}, \tag{27}$$

where

$$\Pi_{\parallel,\perp} = e_\mu^{\parallel,\perp}\Pi^{\mu\nu}e_\nu^{\parallel,\perp}, \tag{28}$$

$$e_\mu^{\parallel} = (0,0,0,1), \quad e_\mu^{\perp} = (0,0,1,0). \tag{29}$$

Then, on the basis of Eq. (27), the refractive indices along the magnetic field, $n_\parallel$, and perpendicularly to it, $n_\perp$, look like

$$n_\parallel = 1 + \frac{e^2 h}{32\pi^2\tilde{\omega}^2}\,(I_{0,0} - 2), \tag{30}$$

$$n_\perp = 1. \tag{31}$$

In the partial case $\tilde{\omega} \ll 1$ and $h \gg 1$, we obtain

$$n_\parallel - n_\perp = \frac{\alpha}{6\pi}h. \tag{32}$$

6. Conclusions

To summarize, the birefringence effect emerging at the photon propagation in vacuum through a region with a strong magnetic field has been studied. The methods of quantum field theory in the presence of external fields are used (the Furry picture, when the external field is considered as a classical one and is taken into account precisely, whereas the interaction of other particles is considered in the framework of quantum mechanics).

Using this approach, the polarization tensor is studied in a constant external magnetic field on the basis of the electron Green's function calculated as the sum over the Landau levels. The case of the lowest Landau levels (a supercritical magnetic field) is analyzed for photons with energies below the e^-e^+-pair generation threshold. The refractive indices of physical vacuum are determined in this approximation, and it is shown that, as one can see from expression (32), the difference between the refractive indices for the anomalous and normal waves linearly depends on the magnetic field strength. This is in contrast to the case of weak magnetic fields, in which the difference between the indicated refractive indices is proportional to the square of the magnetic field strength. The result obtained is in agreement with works [16, 34], where a different approach, namely, a method based on the Schwinger proper time, was applied.

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APPENDIX A

This Appendix contains explicit expressions for the tensors $B_j^{\mu\nu}$ in formula (10):

$$B_1^{\mu\nu} = \gamma^\mu\,(\gamma g + m)\,\tau\gamma^\nu\,(-\gamma f + m)\,\tau L_n L_{n'},$$

$$B_2^{\mu\nu} = \gamma^\mu\,(\gamma g + m)\,\tau\gamma^\nu\,(-\gamma f + m)\,\tau^* L_n L_{n'-1},$$

$B_3^{\mu\nu} = 2in'\gamma^\mu(\gamma g + m)\tau\gamma^\nu\gamma^1\tau(z_1 + iz_2)^{-1}\times$
$\times L_n(L_{n'} - L_{n'-1})$,
$B_4^{\mu\nu} = 2in'\gamma^\mu(\gamma g + m)\tau\gamma^\nu\gamma^1\tau^*(z_1 - iz_2)^{-1}\times$
$\times L_n(L_{n'} - L_{n'-1})$,
$B_5^{\mu\nu} = \gamma^\mu(\gamma g + m)\tau^*\gamma^\nu(-\gamma f + m)\tau L_{n-1}L_{n'}$,
$B_6^{\mu\nu} = \gamma^\mu(\gamma g + m)\tau^*\gamma^\nu(-\gamma f + m)\tau^* L_{n-1}L_{n'-1}$,
$B_7^{\mu\nu} = 2in'\gamma^\mu(\gamma g + m)\tau^*\gamma^\nu\gamma^1\tau(z_1 + iz_2)^{-1}\times$
$\times L_{n-1}(L_{n'} - L_{n'-1})$,
$B_8^{\mu\nu} = 2in'\gamma^\mu(\gamma g + m)\tau^*\gamma^\nu\gamma^1\tau^*(z_1 - iz_2)^{-1}\times$
$\times L_{n'}(L_n - L_{n-1})$,
$B_9^{\mu\nu} = -2in\gamma^\mu\gamma^1\tau\gamma^\nu(-\gamma f + m)\tau(z_1 + iz_2)^{-1}\times$
$\times L_{n'}(L_n - L_{n-1})$,
$B_{10}^{\mu\nu} = -2in\gamma^\mu\gamma^1\tau\gamma^\nu(-\gamma f + m)\tau^*(z_1 + iz_2)^{-1}\times$
$\times L_{n'-1}(L_n - L_{n-1})$,
$B_{11}^{\mu\nu} = 4nn'\gamma^\mu\gamma^1\tau\gamma^\nu\gamma^1\tau(z_1 + iz_2)^{-2}\times$
$\times(L_n - L_{n-1})(L_{n'} - L_{n'-1})$,
$B_{12}^{\mu\nu} = 4nn'\gamma^\mu\gamma^1\tau\gamma^\nu\gamma^1\tau^*(z_1^2 + z_2^2)^{-1}\times$
$\times(L_n - L_{n-1})(L_{n'} - L_{n'-1})$,
$B_{13}^{\mu\nu} = -2in\gamma^\mu\gamma^1\tau^*\gamma^\nu(-\gamma f + m)\tau(z_1 - iz_2)^{-1}\times$
$\times L_{n'}(L_n - L_{n-1})$,
$B_{14}^{\mu\nu} = -2in\gamma^\mu\gamma^1\tau^*\gamma^\nu(-\gamma f + m)\tau^*(z_1 - iz_2)^{-1}\times$
$\times L_{n'-1}(L_n - L_{n-1})$,
$B_{15}^{\mu\nu} = 4nn'\gamma^\mu\gamma^1\tau^*\gamma^\nu\gamma^1\tau(z_1^2 + z_2^2)^{-1}\times$
$\times(L_n - L_{n-1})(L_{n'} - L_{n'-1})$,
$B_{16}^{\mu\nu} = 4nn'\gamma^\mu\gamma^1\tau^*\gamma^\nu\gamma^1\tau^*(z_1 - iz_2)^{-2}\times$
$\times(L_n - L_{n-1})(L_{n'} - L_{n'-1})$.

APPENDIX B

This Appendix contains explicit expressions for the quantities $\Pi_j^{\mu\nu}$ in formula (16) for the polarization tensor:

$$\Pi_1^{\mu\nu} = J_{n,n'}^2\left\{\eta_\parallel^{\mu\nu} - \tilde{\omega}^2(1-\xi^2)U_\parallel^{\mu\nu}\right\},$$
$$\Pi_2^{\mu\nu} = -J_{n,n'-1}^2\left\{1 + \tilde{\omega}^2(1-\xi^2) - \frac{i}{\lambda m^2}\right\}l^{\mu*}l^\nu,$$
$$\Pi_3^{\mu\nu} = \frac{\sqrt{2n'h}}{2m}e^{-i\phi}J_{n,n'}J_{n,n'-1}(1+\xi)k_\parallel^\mu l^\nu,$$
$$\Pi_4^{\mu\nu} = \frac{\sqrt{2n'h}}{2m}e^{i\phi}J_{n,n'}J_{n,n'-1}(1+\xi)l^{\mu*}k_\parallel^\nu,$$
$$\Pi_5^{\mu\nu} = -J_{n-1,n'}^2\left\{1 + \tilde{\omega}^2(1-\xi^2) - \frac{i}{\lambda m^2}\right\}l^\mu l^{\nu*},$$
$$\Pi_6^{\mu\nu} = J_{n-1,n'-1}^2\left\{\eta_\parallel^{\mu\nu} - \tilde{\omega}^2(1-\xi^2)U_\parallel^{\mu\nu}\right\},$$
$$\Pi_7^{\mu\nu} = \frac{\sqrt{2n'h}}{2m}e^{-i\phi}J_{n-1,n'}J_{n-1,n'-1}(1+\xi)l^\mu k_\parallel^\nu,$$
$$\Pi_8^{\mu\nu} = \frac{\sqrt{2n'h}}{2m}e^{i\phi}J_{n-1,n'}J_{n-1,n'-1}(1+\xi)k_\parallel^\mu l^{\nu*},$$
$$\Pi_9^{\mu\nu} = \frac{\sqrt{2nh}}{2m}e^{-i\phi}J_{n,n'}J_{n-1,n'}(1-\xi)l^\mu k_\parallel^\nu,$$
$$\Pi_{10}^{\mu\nu} = \frac{\sqrt{2nh}}{2m}e^{-i\phi}J_{n,n'-1}J_{n-1,n'-1}(1-\xi)k_\parallel^\mu l^\nu,$$
$$\Pi_{11}^{\mu\nu} = 2\sqrt{nn'}he^{-2i\phi}J_{n,n'-1}J_{n-1,n'}l^\mu l^\nu,$$
$$\Pi_{12}^{\mu\nu} = 4\sqrt{nn'}hJ_{n,n'}J_{n-1,n'-1}\eta_\parallel^{\mu\nu},$$
$$\Pi_{13}^{\mu\nu} = \frac{n}{2m}\sqrt{\frac{2h}{n'}}e^{i\phi}J_{n,n'}J_{n-1,n'}(1-\xi)k_\parallel^\mu l^{\nu*},$$
$$\Pi_{14}^{\mu\nu} = \frac{n}{2m}\sqrt{\frac{2h}{n'}}e^{i\phi}J_{n,n'-1}J_{n-1,n'-1}(1-\xi)l^{\mu*}k_\parallel^\nu,$$
$$\Pi_{15}^{\mu\nu} = 2\sqrt{nn'}he^{2i\phi}J_{n,n'-1}J_{n-1,n'}l^{\mu*}l^{\nu*},$$

where $k_x = |\mathbf{k}|\cos\phi$. In the expressions above, a special function $J_{n,n'}$ was introduced. It was obtained in work [35]. A large number of its properties was considered in work [27]. In particular, in the case $n > n'$,

$$J_{n,n'} = e^{-\frac{\eta}{2}}\eta^{\frac{|n-n'|}{2}}\sqrt{\frac{n!}{n'!}}\frac{1}{(n-n')!}\times$$
$$\times F(-n', n-n'+1, \eta).$$

In the case $n < n'$,

$$J_{n,n'} = e^{-\frac{\eta}{2}}\eta^{\frac{|n-n'|}{2}}(-1)^{n-n'}\sqrt{\frac{n'!}{n!}}\frac{1}{(n'-n)!}\times$$
$$\times F(-n, n'-n+1, \eta),$$

where $\eta = (k_x^2 + k_y^2)/2hm^2$, $l^\mu = (0,1,i,0)$, $k_\parallel^\mu = (\omega,0,0,0)$, and F is the degenerate hypergeometric function.

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ВАКУУМНЕ ПОДВІЙНЕ ПРОМЕНЕЗАЛОМЛЕННЯ В НАДКРИТИЧНОМУ МАГНІТНОМУ ПОЛІ

Р е з ю м е

В роботі розглянуто ефект подвійного променезаломлення фізичного вакууму в сильному магнітному полі. В рамках квантової теорії поля досліджено поляризаційний тензор у постійному зовнішньому магнітному полі на основі функції Гріна електрона через суму по рівнях Ландау. Розглянуто випадок найнижчих рівнів Ландау для фотонів з енергією нижче порога народження електрон-позитронної пари. Знайдено, в даному наближенні, показники заломлення фізичного вакууму для аномальної та нормальної хвилі.