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DYNAMICS OF CHARGED BUNCHES IN THE WAKEFIELD EXCITED BY THEM IN PLASMA

The results of computer simulation concerning the dynamics of charged bunches in the wakefield created by them in homogeneous and inhomogeneous plasmas are reported. The proton and electron bunches in an electron-proton plasma are simulated, by using the particle-in-cell method. The simulation results are compared with those of analytical calculations. It is shown that the inverse influence of excited wakefields on ion bunches can be neglected at a distance of several tens of wakefield wavelengths, and such fields are excited only by the bunch fronts. For the electron bunches, the charge density profile becomes considerably distorted at distances of about the wake wavelength. In this case, some additional mechanisms of wakefield excitation emerge owing to the decay of the initial bunch into microbunches: associated with the Cherenkov resonance (for long bunches) and with the microbunch focusing.

Keywords: charged bunches, wakefield, plasma.

1. Introduction

The problem concerning the excitation of wakefields by electron bunches and the inverse influence of those fields on the bunch dynamics is of interest first of all from the viewpoint of an opportunity to create compact electron wakefield accelerators. In the literature, the possibilities of the wakefield excitation in plasmas (see, e.g., [1, 2]) and insulators [3, 4] is discussed. Electron and ion bunches or sequences of such bunches [5–10], as well as short ultra-powerful laser pulses [11], can be used to excite wakefields. The possibility of constructing the wakefield accelerators was confirmed experimentally (see, e.g., [12]). The problem of the wakefield excitation is also of interest owing to a possibility to diagnose an inhomogeneous plasma, by analyzing the transition radiation emitted by charged particles and bunches [13].

This work is devoted to studying the dynamics of the ion and electron bunches associated with the excitation of a wake wave in homogeneous and inhomogeneous plasmas in the absence of a magnetic field. The most convenient method to solve this problem is a computer simulation using the particle-in-cell (PIC) method. To simulate the interaction of ion and electron bunches with a plasma, a one-dimensional electrostatic code and a 2.5-dimensional electromagnetic code with the axially symmetric geometry [14] were applied. The simulation parameters were selected according to the conditions of typical laboratory experiments [15,18], with the density of bunches being much lower than the density of the background plasma. The results of analytical calculations are also given.

2. One-Dimensional Simulation of the Dynamics of Electron Bunches with Initially Rectangular and Triangular Density Profiles in Homogeneous Plasma

The one-dimensional model corresponds to the simplest possible geometry of the problem concerning the dynamics of charged bunches in the wakefields ex-

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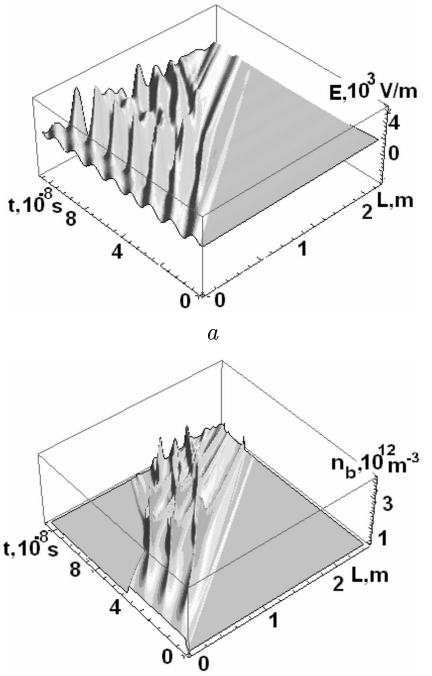


a

b

Fig. 1. Space-time distributions of (*a*) wakefield and (*b*) bunch density for a bunch with the initially rectangular density profile (one-dimensional model): $v_b = 3 \times 10^9$ cm/s, $\tau_b = 3T_{\rm Langm} = 3.3 \times 10^{-8}$ s, $n_b = 1.1 \times 10^6$ cm^{-3}, $n_p = 10^8$ cm^{-3}, and $T_e = 2 \times 10^5$ K

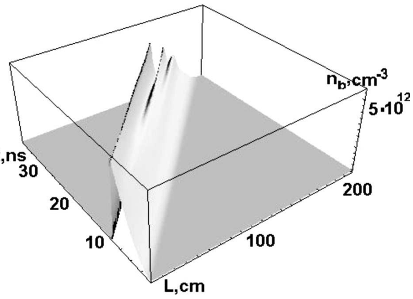


Fig. 2. Space-time density distribution for a bunch with the initially triangular density profile (one-dimensional model). The simulation parameters are the same as in Fig. 1

cited by them. An analytical calculation for a charged bunch with the rectangular density profile was carried out in [17] in the given-current approximation, when the wakefield is excited only by the leading and trailing fronts of the bunch.

The simulation was carried out for electron bunches with the initially rectangular [18] and triangular [19] charge density profiles. The results obtained for the rectangular profile are depicted in Fig. 1. First, the electrons in the bunch move in the wakefield excited by the leading front of the bunch. If the bunch length considerably exceeds the wake wavelength, there emerges a sequence of microbunches. This effect was observed for the first time in [20] (see also [21]). The sequence of microbunches, in turn, excites a wake wave according to the mechanism of Cherenkov resonance. In other words, a beam-plasma instability develops in the system starting from the wake wave excited by the leading front of the bunch. Later, the bunches decay because of different velocities of electrons that form them.

Figure 2 illustrates the space-time evolution of a bunch with the initially triangular density profile characterized by a prolonged leading front and a jump-like trailing one. In this case, the wakefield excited by the leading front turns out much lower than the wakefield excited by a bunch with the rectangular profile with the same parameters [22]. As a result, the wakefield-induced bunch distortion develops considerably slower.

The perturbation of a bunch density profile can be quantitatively characterized by the distortion index

$$\sigma = \frac{\int\limits_{-\infty}^{0} n^2(x)\,dx + \int\limits_{0}^{L} [n_0(x) - n(x)]^2 dx + \int\limits_{L}^{+\infty} n^2(x)\,dx}{\int\limits_{0}^{L} n_0^2(x)\,dx}, \tag{1}$$

where $n_0(x)$ and L are the initial density distribution and the length, respectively, of the bunch, and $n(x)$ is the immediate density distribution in the bunch. The spatial dependences of the distortion indices for bunches with the initially rectangular (curve *1*) and triangular (curve *2*) density profiles are exhibited in Fig. 3. Note that the maximum in curve *1* corresponds to the maximum grouping of microbunches emerged from the initial bunch.

3. Dynamics of Cylindrical Charged Bunches in Homogeneous Plasma

3.1. Results of analytical calculations

The analytical calculation was carried out for the model of cold plasma in the potential approximation and supposing the bunch current to be given [23]; in other words, the influence of a wakefield on the bunch motion was not taken into account. The bunch shape was assumed to be cylindrical, the velocity to be directed along the bunch axis, and the density n_{b0} to be uniform:

$$n_b(r,\xi) = \begin{cases} 0, & |\xi| > L/2; \\ n_b(r), & |\xi| \leqslant L/2, \end{cases}$$

$$n_b(r) = \begin{cases} 0, & r > a; \\ n_{b0}, & r \leqslant a, \end{cases} \quad \xi = z - V_0 t, \tag{2}$$

where r and z are the cylindrical coordinates, and a and V_0 the radius and the velocity of the bunch, respectively. In this case, the perturbation of the background plasma density, δn, induced by the wake wave is given by the formula

$$\delta n(r,\xi) = -\frac{n_b(r)}{2\pi} \exp\left(\frac{\nu\xi}{2V_0}\right) \times$$
$$\times \left\{ \exp\left(\frac{\nu L}{4V_0}\right) \cos\left[\frac{\omega_p}{V_0}(\xi + L/2) + \varphi\right] \theta\left(-\xi - \frac{L}{2}\right) - \right.$$
$$- \exp\left(-\frac{\nu L}{4V_0}\right) \cos\left[\frac{\omega_p(r)}{V_0}(\xi - L/2) + \varphi\right] \times$$
$$\left. \times\, \theta\left(-\xi + \frac{L}{2}\right) \right\}, \quad \varphi = \operatorname{arctg} \frac{\nu}{2\omega_p}, \tag{3}$$

where ω_p is the Langmuir frequency of the background plasma, and ν the frequency of electron collisions with heavy particles (Fig. 4). For the model of cold plasma in the potential approximation, the wakefield is excited only in the region of the background plasma, which is passed through by the electron bunch, because, in this model, the equation describing plasma oscillations contains no coordinate derivations. The wakefields excited by the leading and trailing beam fronts interfere in the region behind the trailing front ($\xi < -L/2$). In Fig. 4, those fields are almost in the anti-phase.

3.2. Simulation results

The motion of a proton bunch in a homogeneous plasma was simulated with the help of the 2.5-dimensional code on the basis of the PIC method. The simulation results (Fig. 5) qualitatively agree with the result of analytical calculation in the given bunch current approximation, because the charge distribution in the bunch is not changed significantly during the bunch motion.

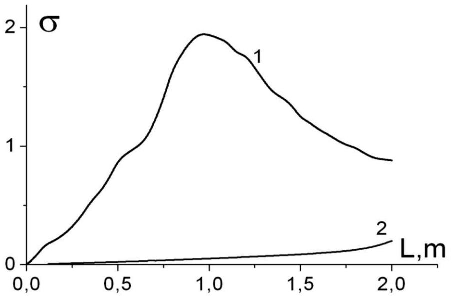


Fig. 3. Spatial dependences of the distortion index for bunches with the initially rectangular (*1*) and triangular (*2*) density profiles (one-dimensional model)

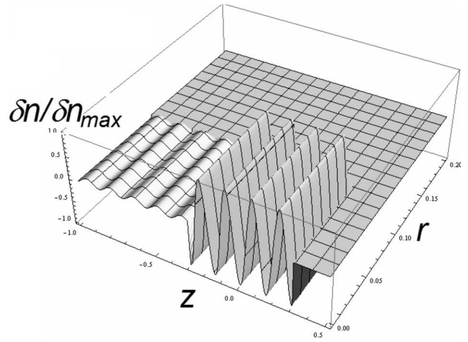


Fig. 4. Density perturbation in a homogeneous plasma by a cylindrical charged bunch (the analytical solution in the potential approximation for the model of cold plasma)

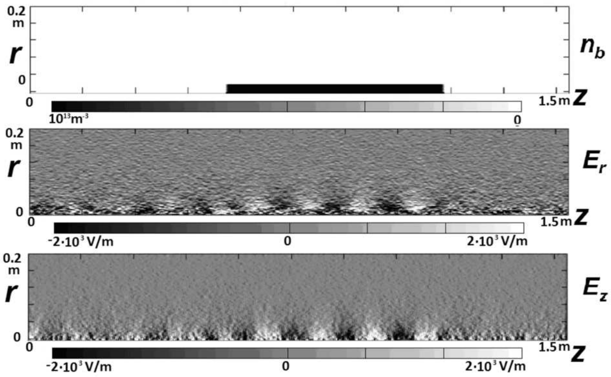


Fig. 5. Spatial distributions of n_b, E_r, and E_z for a cylindrical ion bunch in a homogeneous plasma: $n_p = 5 \times 10^8$ cm^{-3}, $n_b = 8 \times 10^6$ cm^{-3}, $v_b = 3 \times 10^9$ cm/s, $\tau_b = 2 \times 10^{-8}$ s $= 4T_{\text{Langm}}$, $T_e = 1$ eV, and $T_i = 0.1$ eV

Making allowance for a non-zero plasma temperature and going beyond the scope of the potential approximation result in that the wakefield appears outside the region, through which the charged bunch

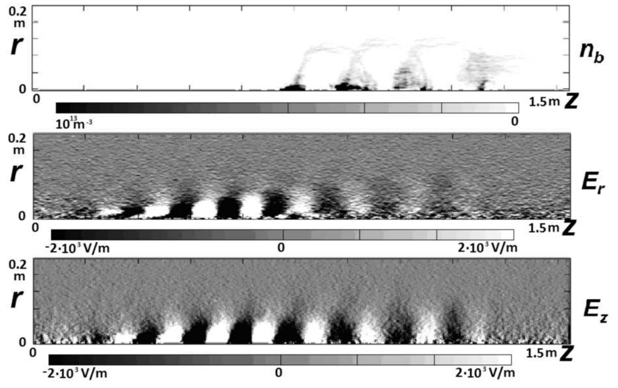


Fig. 6. The same as in Fig. 5, but for a cylindrical electron bunch

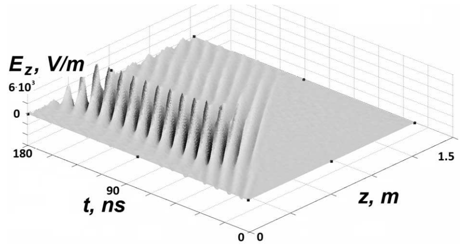


Fig. 7. Space-time distribution of the longitudinal electric field excited by a cylindrical electron beam in a homogeneous plasma near the axis of the system. The simulation parameters are the same as in Fig. 5

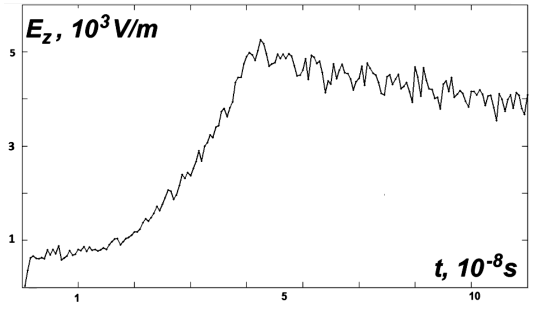


Fig. 8. Time-dependence of the amplitude maximum for a wake wave excited by a cylindrical electron beam in a homogeneous plasma. The simulation parameters are the same as in Fig. 5

passes. In addition, there appear radial components of the electric field and the velocity of electrons in the plasma, which were absent in the potential approximation.

The major difference between the simulation results for electron and ion bunches consists in a substantial redistribution of the electron density in a bunch under the influence of the initial wakefield excited by its sharp leading front [20, 23]. The 2.5-dimensional simulation reveals both the axial and radial focusing and defocusing of the electron bunch (Fig. 6). The arrangement of the region with the most intense wakefield along the first half of the bunch trajectory remains almost invariable owing to a low group velocity of wake waves (Fig. 7).

The maximum of the wake wave amplitude becomes by an order of magnitude larger in comparison with its initial value generated by the leading bunch front (Fig. 8). This effect can be explained by the Cherenkov mechanism exciting a wake wave (cf. Section 2), which develops in the mutual consistency with the dynamics of microbunches. On the other hand, this result can be interpreted as the development of a beam-plasma instability (for long bunches).

4. Dynamics of Cylindrical Charged Bunches in Longitudinally Inhomogeneous Plasma

4.1. Results of analytical calculations

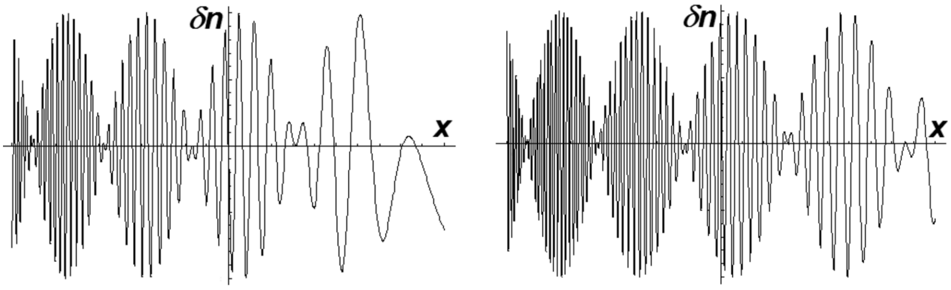
In this section, we discuss the excitation of wakefields and the dynamics of charged bunches in a longitudinally inhomogeneous plasma with a linear density profile [24]

$$n(z) = n_0 (1 + z/D),$$

where D is the characteristic inhomogeneity size. The analytical calculation was carried out, as before, in the potential approximation for the model of cold plasma and assuming the current of a charged bunch to be given. The corresponding spatial distributions of the plasma density perturbation by a long (on the scale of excited wavelengths) bunch at various time points are plotted in Fig. 9.

In the framework of the model of cold plasma in the potential approximation, the oscillations at neighbor plasma points occur independently. Therefore, in an inhomogeneous plasma, the phase shift between oscillations occurring at neighbor points with different frequencies monotonically grows (Fig. 9). Certainly, this effect cannot be observed experimentally.

On the other hand, the inhomogeneity of the background plasma along a bunch trajectory gives rise to a monotonic variation of the phase difference between the wake waves excited by the leading and trailing



a *b*

Fig. 9. Spatial distributions of plasma density perturbations induced by a long cylindrical charged bunch in a longitudinally inhomogeneous plasma at $\omega_{p0}t = 60$ (*a*) and 80 (*b*) (the analytical solution in the potential approximation for the model of cold plasma and the given current approximation)

bunch fronts. As a result, spatial beats of the wakefield are observed (Fig. 9).

4.2. Simulation results

In the simulation, the plasma density along the charged bunch trajectory varied from $2\times\ \times 10^8$ to 8×10^8 cm^{-3}. The other parameters were the same as in the previous section.

The simulation results obtained for the proton bunch testify that, in this case (inhomogeneous plasma), the approximation of given current for such a bunch is well satisfied (cf. Section 3 3.2). Figure 10 illustrates the spatial wakefield beats predicted by the analytical calculation (cf. Fig. 9).

The simulation results for a long electron bunch show that it decays into microbunches, the number of which is determined by the wake wavelength at the initial trajectory section.

In a longitudinally inhomogeneous plasma, the wakefield periodicity near the axis of the system and on the periphery can be different (Fig. 11). This difference is associated with the difference between the behavior of the first microbunch and that of the next ones. The first microbunch is better focused in the longitudinal direction, whereas the next ones in the radial direction. As a result, the wakefield on the periphery is mainly excited by the first microbunch, and the field near the axis of the system by the next microbunches [24].

Figure 12 demonstrates that the maximum amplitude of the electric field in a longitudinally inhomogeneous plasma is also larger in comparison with the value provided by the leading bunch front; however, it turns out much smaller than in a homogeneous plasma (see Fig. 8). Now, the condition of Cherenkov resonance is not satisfied; therefore, the amplitude growth can be associated with the microbunch focusing. Hence, it follows that the energy taken away from

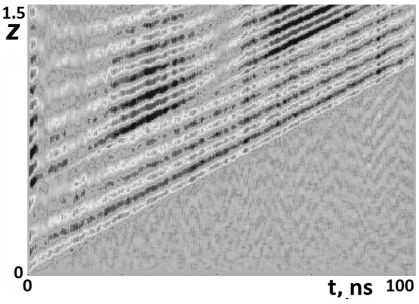


Fig. 10. Space-time distribution of the longitudinal electric field near the axis of the system (cylindrical ion bunch in an inhomogeneous plasma)

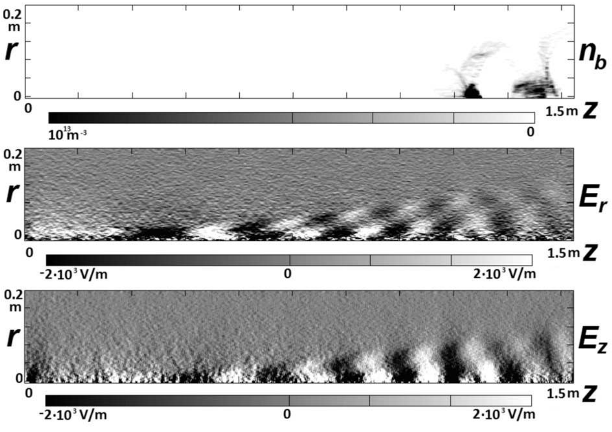


Fig. 11. Spatial distributions of n_b, E_r, and E_z for a cylindrical ion bunch in a longitudinally inhomogeneous plasma

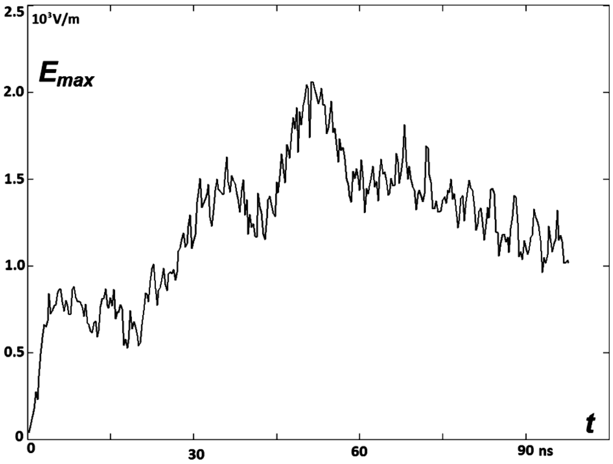


Fig. 12. The same as in Fig. 8, but in the case of a longitudinally inhomogeneous plasma

the bunch in order to excite the wakefield in an inhomogeneous plasma is lower than in the case of a homogeneous plasma.

5. Influence of the Dynamics of Charged Bunches on the Possibility of Their Application to Diagnostics of Inhomogeneous Plasma

While determining whether the reconstruction of a density profile in an inhomogeneous plasma with the use of the transition radiation emitted by modulated beams and bunches of charged particles is possible, the fixed current approximation was used (see, e.g., work [13]); i.e. the dynamics of those particles under the action of excited wakefields was neglected. On the basis of the calculation and simulation results discussed above, some speculations can be made concerning the influence of such a dynamics on the probable diagnostics.

The most general conclusion consists in that a distortion of charged bunches under the wakefield action confines the distance from the injector, at which the diagnostics of an inhomogeneous plasma by analyzing the emitted transition radiation is possible. For the long (on the scale of wake wavelengths) electron bunches with initially rectangular axial concentration profiles, the corresponding distance is of the same order of magnitude as the bunch length; therefore, it is impossible to use them for the diagnostics. The indicated distance can be increased for short (much shorter than the wake wavelength) bunches and for bunches with a smeared leading front and a sharp trailing one. It can also be done for ultra-relativistic electron bunches. At last, ion bunches can be used.

6. Conclusions

1. The approximation of given bunch current is well satisfied for the ion bunches with densities much lower than that of a plasma at distances of the order of several tens of wake wavelengths.

2. The dynamics of a single electron bunch at such distances is governed by the wakefields excited by the bunch itself. For this bunch, the approximation of fixed bunch current becomes wrong at a distance of an order of the wake wavelength.

3. The initial wakefield is excited by the leading front of a charged bunch. In the case of long electron bunches in a homogeneous plasma, this field promotes the formation of a sequence of microbunches and a further growth of the wakefield owing to the Cherenkov resonance. This effect can also be interpreted as the development of a beam-plasma instability.

4. An additional mechanism of wake wave growth is related to the focusing of microbunches and the corresponding growth of their charge density. Unlike the Cherenkov resonance, this mechanism is realized in an inhomogeneous plasma as well.

1. P. Chen, J.M. Dawson, R.W. Huff, and T. Katsouleas, Phys. Rev. Lett. **54**, 693 (1985).
2. M.J. Hogan, T.O. Raubenheimer, A. Seryi, P. Muggli, T. Katsouleas, C. Huang, W. Lu, W. An, K.A. Marsh, W.B. Mori, C.E. Clayton, and C. Joshi, New J. Phys. **12**, 055030 (2010).
3. A. Tremaine, J. Rosenzweig, and P. Schoessow, Phys. Rev. E **56**, 7204 (1997).
4. V.I. Maslov and I.N. Onishchenko. Probl. At. Sci. Technol. Series: Plasma Electronics and New Acceleration Methods **4**, 69 (2013).
5. A. Bazzania, M. Giovannozzic, P. Londrillob, S. Sinigardia, and G. Turchettia, C. R. Mecanique **342**, 647 (2014).
6. A. Caldwell and K.V. Lotov, Phys. Plasmas **18**, 103101 (2011).
7. K.V. Lotov, Phys. Plasmas **20**, 083119 (2013).
8. L. Yi, B. Shen, K. Lotov, L. Ji, X. Zhang, W. Wang, X. Zhao, Y. Yu, J. Xu, X. Wang, Y. Shi, L. Zhang, T. Xu, and Zh. Xu, Phys. Rev. ST Accel. Beams **16**, 071301 (2013).

9. K.V. Lotov, V.I. Maslov, I.N. Onishchenko, and I.P. Yarovaya, Probl. At. Sci. Technol. Series: Plasma Electronics and New Acceleration Methods **4**, 73 (2013).
10. J. Vieira, Y. Fang, W.B. Mori, L.O. Silva, and P. Muggli, Phys. Plasmas **19**, 063105 (2012).
11. T. Tajima and J.M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).
12. I. Blumenfeld, C.E. Clayton, F.J. Decker, M.J. Hogan, C. Huang *et al.,* Nature **445**, 741 (2007).
13. I.O. Anisimov and K.I. Lyubich, J. Plasma Phys. **66**, 157 (2001).
14. Yu.M. Tolochkevych, T.E. Litoshenko, and I.O. Anisimov, Probl. At. Sci. Technol., No. 4, 47 (2010).
15. A.K. Berezin, G.P. Berezina, N.S. Erokhin, S.S. Moiseev, and Ya.B. Fainberg, Pis'ma Zh. Eksp. Teor. Fiz. **14**, 149 (1971).
16. M. Starodubtsev, C. Krafft, P. Thevenet, and A. Kostrov, Phys. Plasmas **6**, 1427 (1999).
17. I.O. Anisimov and K.I. Lyubych, Zh. Fiz. Dosl. **4**, 61 (2000).
18. I.O. Anisimov and Yu.M. Tolochkevich, Ukr. J. Phys. **54**, 454 (2009).
19. I.O. Anisimov, P.V. Parashchenko, and Yu.M. Tolochkevych, Ukr. J. Phys. **55**, 885 (2010).
20. V.A. Balakirev, V.I. Karas', and I.V. Karas', Fiz. Plasmy **28**, 144 (2002).
21. R. Fedele, D. Jovanović, F. Tanjia, and S. DeNicola, Nucl. Instrum. Meth. A **740**, 180 (2014).
22. T. Katsouleas, Phys. Rev. A **3**, 2056 (1986).
23. I.O. Anisimov, T.E. Litoshenko, and Yu.M. Tolochkevich, Probl. At. Sci. Technol., No. 6, 126 (2010).
24. Yu.M. Tolochkevich, T.E. Litoshenko, and I.O. Anisimov, Probl. At. Sci. Technol., No. 4, 34 (2013).

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ДИНАМІКА ЗАРЯДЖЕНИХ ЗГУСТКІВ У ЗБУДЖЕНОМУ НИМИ В ПЛАЗМІ КІЛЬВАТЕРНОМУ ПОЛІ

Резюме

Подано результати комп'ютерного моделювання динаміки заряджених згустків у кільватерному полі в однорідній та неоднорідній плазмі. Моделювання методом крупних частинок у комірках виконувалося для електронних та протонних згустків в електронно-протонній плазмі. Результати моделювання порівнюються з аналітичними розрахунками. Показано, що для іонних згустків зворотним впливом збуджених ними кільватерних полів можна знехтувати на відстанях у декілька десятків довжин кільватерних хвиль, тому механізм збудження таких полів пов'язаний лише з фронтами згустку. Для електронних згустків помітна деформація профілю густини заряду відбувається вже на відстані порядку довжини кільватерної хвилі. В цьому випадку за рахунок розбиття первісного згустку на мікрозгустки з'являються додаткові механізми збудження кільватерних полів: черенковський (для довгих згустків) та пов'язаний із фокусуванням мікрозгустків.