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HYDRODYNAMIC, KINETIC MODES OF PLASMA AND RELAXATION DAMPING OF PLASMA OSCILLATIONS

The hydrodynamics of a completely ionized two-component electron-ion plasma is investigated at the end of the component temperature and velocity relaxation. The problem of accounting for the peculiarities of the Coulomb interaction in the plasma kinetics is discussed. The investigation is based on the Landau kinetic equation and the Chapman–Enskog method generalized on the basis of the Bogolyubov idea of the functional hypothesis. Nonlinear hydrodynamic equations are obtained. Linearized hydrodynamic equations are built, and the hydrodynamic and kinetic modes of the Landau kinetic equation are investigated in the hydrodynamic approximation. The effect of the relaxation processes on the evolution of the system is investigated. On the basis of the Vlasov–Landau equation, the plasma modes are investigated in the main hydrodynamic approximation. Some of them describe the relaxation damping of plasma oscillations, which is much more important than the Landau damping at small wave vectors $k \to 0$.

Keywords: completely ionized plasma, Landau kinetic equation, Vlasov–Landau kinetic equation, Chapman–Enskog method, functional hypothesis, hydrodynamic modes, kinetic modes, relaxation damping of plasma oscillations.

1. Introduction

In his known paper [1], L.D. Landau obtained a kinetic equation for a completely ionized two-component electron-ion plasma. This equation is widely used in the investigation of the plasma kinetics (see, e.g., [2–4]). Of course, it describes the situation in plasma approximately. The Landau equation involves only the short-range part of the Coulomb interaction, because the Coulomb potential is artificially cut in the collision integral at the Debye radius. This can be done exactly by the Balescu–Lenard equation. In the Landau collision integral, the Coulomb potential is also cut-off at small distances, where this potential is big, and the situation needs a special attention. This was done in the exact consideration by A.A. Rukhadze and V.P. Silin. A comparison of the mentioned theories shows that the Landau kinetic equation describes effects of the short-range part of the Coulomb interaction in plasma with a logarithmic accuracy (see the discussion of the mentioned results in [2]).

The present paper is devoted to the investigation of hydrodynamic states of a two-component completely ionized electron-ion plasma on the basis of the Landau kinetic equation and the consistent application of the Chapman–Enskog method [5,6]. Special attention is paid to accounting for relaxation processes in the system. *The term "relaxation process" is understood here in a narrow sense as a process that is possible in spatially homogeneous states.* The obtained equations are applied to the investigation of the hydrodynamic and kinetic modes of the Landau kinetic equation in the hydrodynamic approximation. The corresponding results cannot be found in the literature.

In this connection, we note that, in the hydrodynamic approximation, the complex frequencies $\lambda_i(k)$ of modes are calculated only for small wave vectors k ($\lambda_i(k = 0) = 0$ for hydrodynamic modes, and $\lambda_i(k = 0) \neq 0$ for kinetic modes). The Landau kinetic equation has six hydrodynamic modes and an infinite number of kinetic ones (see, e.g., [7]). *The number of hydrodynamic modes is equal to the number of parameters defining the equilibrium distribution functions* (DFs) (in our case, they are: the component particle densities n_e and n_i, temperature T, and velocity v_n).

However, the Landau collision integral does not involve the long-range part of the Coulomb interaction. Therefore, these effects should be described by some corrections to the Landau kinetic equation. An important idea in this direction is connected with *the Vlasov term, which describes the one-particle effects of a self-consistent field*. However, the investigation of the Vlasov–Landau kinetic equation on the basis of the Chapman–Enskog method meets difficulties. Probably, one could overcome these difficulties, by considering *the intrinsic degrees of freedom of the electromagnetic field in plasma*. This idea is suggested by the results of D. Bohm and D. Pines [8], which were developed further in our paper [9]. It is better to investigate the interacting systems (in our case: an electromagnetic field and a system of charged particles with an effective short-range interaction) in terms of their modes to avoid the high dimensionality problem. In such a theory, *one needs the modes of the Landau kinetic equation*. Therefore, *the problem considered in the present paper is an important part of this program*, which will be elaborated elsewhere.

Hydrodynamic states of plasma are investigated in the present paper with regard for the relaxation processes. The pioneering investigation of the relaxation of the component temperature $T_a(t)$ $(a = e, i)$ for a spatially uniform quasiequilibrium plasma was conducted by L.D. Landau [1] on the basis of his kinetic equation and gave the corresponding relaxation time τ_T. According to his assumption, the plasma components are described in this situation by the Maxwell DF with time-dependent component temperatures $w_{ap}(T_a(t))$

$$w_{ap}(T) \equiv \frac{n_a}{(2\pi m_a T)^{3/2}} e^{-\varepsilon_{ap}/T} \qquad (1)$$

($\varepsilon_{ap} \equiv p^2/2m_a$ is the energy of a particle, n_a is the component particle density). In this approach, the component velocity $v_a(t)$ relaxation can be investigated too, and the corresponding relaxation time τ_u can also be obtained [10], by using $w_{a,p-m_a v_a(t)}(T)$ as the nonequilibrium DF.

A hydrodynamic theory for a completely ionized plasma on the basis of the Landau kinetic equation was built by S.I. Braginsky [11, 12]. In his theory, the plasma is completely described by the component variables $n_a(x,t), T_a(x,t), v_{an}(x,t)$. In [11, 12], as usually in hydrodynamics, the gradients of these reduced description parameters (RDPs) are assumed to be small (let g be the corresponding small parameter). The difference of the component velocities $v_{\text{en}}(x,t) - v_{in}(x,t)$ was considered as a small quantity of the same order g. From the beginning, the Landau kinetic equation was simplified by S.I. Braginsky, by using the smallness of the electron-to-ion mass ratio $m_e/m_i \equiv \sigma^2$ with some discussion of the significance of contributions to the nonequilibrium DFs. However, this was done without a systematic perturbation theory in the small parameter σ and corresponding estimates of the calculation accuracy.

Braginsky's investigation of the plasma hydrodynamics is based on a modification of the Chapman–Enskog method. He assumed that the main contributions to the nonequilibrium DFs of the plasma components $f^{(0)}_{ap}(x,t)$ are given by the Maxwell distributions $w^L_{ap}(x,t) \equiv w_{a,p-m_a v_a(x,t)}(T_a(x,t))$ (*this statement can be called the Landau assumption or the local equilibrium assumption*). However, *these DFs are not an exact solution of the kinetic equation in the zero-order approximation in gradients* (even for a small difference $v_{en}(x,t) - v_{in}(x,t)$). So, this is a deviation from the standard formulation of the Chapman–Enskog method (see, e.g., [5, 6]), and the DF $w^L_{ap}(x,t)$ cannot describe the system adequately. The Braginsky modification of the Chapman–Enskog method can be formulated as follows: to find a solution of the Landau kinetic equation near the states described by the DF $w^L_{ap}(x,t)$. It is also necessary to note that his investigation of the transport phenomena omits a discussion of the diffusion processes.

In the present paper, plasma hydrodynamics is investigated near the end of the component temperature and velocity relaxation processes. In this situation, the DFs *of the plasma components* $f^{(0)}_{ap}(x,t)$ *can be calculated in an additional perturbation theory* in small differences of the component temperatures and velocities $v_{en}(x,t) - v_{in}(x,t)$, $T_e(x,t) - T_i(x,t)$ (the exact definition of the corresponding small parameter μ is given below).

The theory proposed in the present paper is a generalization of the Chapman–Enskog method based on *the Bogolyubov idea of the functional hypothesis* (see about this hypothesis in [6]). This idea is a basis of the Bogolyubov formulation of the standard Chapman–Enskog method [5, 6]. However, it allows us to use a new RDP in the theory and new small parameters for the construction of a perturbation theory.

The generalization is made *to account for the temperature and velocity relaxation processes in plasma hydrodynamics consistently and to understand them in terms of the kinetic modes of the system.* In the solution of the obtained general integral equations, we restrict ourselves to the study of relaxation processes in the one-polynomial approximation, which is equivalent to the linear formulation of the Landau assumption. The two-polynomial approximation will be discussed in a subsequent paper. Note that a general nonlinear theory which describes relaxation processes in a vicinity of the standard hydrodynamic states was discussed in our paper [13].

Our previous results devoted to accounting for relaxation phenomena in plasma hydrodynamics were published in [14–17]. The present paper gives a corrected formulation of these results and proposes the investigation of plasma modes in the hydrodynamic approximation with regard for relaxation phenomena. The method, by which modes are obtained in the hydrodynamic approximation, is rather known [7, 18], but the investigation of plasma modes on the basis of the Landau equation taking the relaxation phenomena into account cannot be found in the literature. In this paper, not only the eigenvalues $\lambda_i(k)$ of the generalized hydrodynamic matrix (the complex frequencies of the system) are investigated, but also the corresponding eigenfunctions are calculated in order to show the coherent movement of the system related to each mode.

As noted above, the investigation of the Vlasov–Landau kinetic equation on the basis of the Chapman–Enskog method meets difficulties. However, in the last section of this paper, it will be shown that the Vlasov–Landau kinetic equation can be applied to the analysis of plasma modes taking the DF of the system in the approximation of zero order in gradients (i.e., neglecting the hydrodynamic dissipative processes: heat conductivity, viscosity, and diffusion). The necessary DF can be found from the Landau equation, because a self-consistent field in uniform states is absent.

The plan of this paper is as follows: in Section 2, the basic equations and definitions of the theory are given; in Section 3, the hydrodynamic equations for the parameters that describe the system are obtained; in Section 4, the plasma component DFs are derived; in Section 5, a linearized theory is built, and the dispersion laws for the modes of the Landau equation and the corresponding coherent movements are investigated; and, in Section 6, the modes of the Vlasov–Landau kinetic equation are discussed, by neglecting the hydrodynamic dissipative processes.

2. Basic Equations of the Theory

The Landau kinetic equation can be written in the standard form

$$\frac{\partial f_{ap}(x,t)}{\partial t} = -\frac{p_{an}}{m_a}\frac{\partial f_{ap}(x,t)}{\partial x_n} + I_{ap}(f(x,t)), \qquad (2)$$

where $f_{ap}(x,t)$ is the component DF $(a,b,c,... = e,i)$. The Landau collision integral I_{ap} is given by the formulas [1]

$$I_{ap}(f) = \sum_c 2\pi(e_a e_c)^2 L J_{ac}(p),$$
$$J_{ac}(p) = -\frac{\partial}{\partial p_n}\int d^3p'\left\{f_{ap}\frac{\partial f_{cp'}}{\partial p'_l} - f_{cp'}\frac{\partial f_{ap}}{\partial p_l}\right\}\times$$
$$\times D_{nl}\left(\frac{p}{m_a} - \frac{p'}{m_c}\right). \qquad (3)$$

Here, e_a is the component charge ($e_e = -e$, $e_i = ze$; e is the elementary electric charge, z is the ion charge number), L is the Coulomb logarithm, and

$$D_{nl}(q) \equiv (q^2\delta_{nl} - q_n q_l)/q^3. \qquad (4)$$

The component temperature T_a, velocity υ_{an}, and particle density n_a are introduced by the standard definition [3, 6] in terms of the DF

$$n_a = \int d^3p f_{ap}, \quad \pi_{an} = m_a n_a \upsilon_{an} = \int d^3p f_{ap} p_n,$$
$$\varepsilon_a = \frac{3}{2}n_a T_a + \frac{m_a n_a {\upsilon_a}^2}{2} = \int d^3p f_{ap}\varepsilon_{ap}, \qquad (5)$$

where π_{an} and ε_a are the component momentum and energy densities. The total particle and mass densities of the system are given by the formulas

$$n = \sum_a n_a, \quad \rho = \sum_a m_a n_a. \qquad (6)$$

From (2) and (5), the following equations can be obtained:

$$\frac{\partial n_a}{\partial t} = -\frac{1}{m_a}\frac{\partial \pi_{an}}{\partial x_n}, \quad \frac{\partial \pi_{an}}{\partial t} = -\frac{\partial t_{anl}}{\partial x_l} + R_{an}, \\ \frac{\partial \varepsilon_a}{\partial t} = -\frac{\partial q_{an}}{\partial x_n} + Q_a, \tag{7}$$

which express the conservation laws. Here,

$$q_{an} \equiv \int d^3p\ \varepsilon_{ap}\frac{p_n}{m_a}f_{ap}, \quad t_{aln} \equiv \int d^3p\ p_l\frac{p_n}{m_a}f_{ap} \tag{8}$$

are the energy and momentum component fluxes, and

$$Q_a \equiv \int d^3p\ \varepsilon_{ap}I_{ap}(f), \quad R_{an} \equiv \int d^3p\ p_n I_{ap}(f) \tag{9}$$

are the energy and momentum component sources. The total fluxes are given by the relations

$$t_{nl} = \sum_a t_{anl}, \quad q_n = \sum_a q_{an}. \tag{10}$$

It can be shown from the expression for the Landau collision integral that the total sources are equal to zero:

$$Q \equiv \sum_a Q_a = 0, \quad R_n \equiv \sum_a R_{an} = 0. \tag{11}$$

The system is considered here at the end of the temperature and velocity relaxation. It is evident that, after the end of those processes, the temperature T and the velocity υ_n of the system are given by the relations

$$\pi_n = \sum_a \pi_{an} = \rho\upsilon_n,\ \varepsilon = \sum_a \varepsilon_a = \frac{3}{2}Tn + \frac{1}{2}\rho\upsilon^2, \tag{12}$$

where π_n and ε are the total momentum and energy densities of the system. Therefore, in what follows, we use the deviations of the electron temperature and velocity from their standard hydrodynamic values,

$$\tau \equiv T_e - T, \quad u_n \equiv \upsilon_{en} - \upsilon^h_{en}, \tag{13}$$

which are estimated by the expressions

$$u_n \sim \mu\sqrt{T/m_e}, \quad \tau \sim \mu T; \quad \mu \ll 1. \tag{14}$$

In the standard hydrodynamics of two-component fluids [6], the component particle densities n_a, temperature T, and velocity υ_n are used as the RDPs, and it is assumed that the relaxation is finished. Note that υ^h_{en} is expressed in terms of the diffusive flux, which, in turn, is expressed in terms of n_a, T, υ_n. For the hydrodynamic processes in the presence of relaxation, the component particle densities n_a, temperatures T_a, and velocities υ_{an} play the role of the RDPs [11, 12]. Relations (5), (12), and (13) show that the deviations of the ion temperature T_i and the velocity v_{in} from their standard hydrodynamic values are expressed in terms of the variables τ and u_n. Thus, the RDPs of the theory with account for the relaxation $\xi_\alpha(x,t)$ can be chosen as

$$n_e(x,t) \equiv \xi_1(x,t), \quad n_i(x,t) \equiv \xi_2(x,t), \\ \upsilon_n(x,t) \equiv \xi_{2+n}(x,t), \quad T(x,t) \equiv \xi_6(x,t), \\ u_n(x,t) \equiv \xi_{6+n}(x,t), \quad \tau(x,t) \equiv \xi_{10}(x,t). \tag{15}$$

The construction of a reduced description of the plasma by the parameters (15) is based here on a functional hypothesis [6] of the form

$$f_{ap}(x,t) \xrightarrow[t \gg \tau_0]{} f_{ap}(x, \xi(t)), \tag{16}$$

where $f_{ap}(x,\xi)$ is a functional of the variables $\xi_\alpha(x')$ as functions of x', and τ_0 is a time which is much shorter than the subsystem velocity and temperature relaxation times τ_u, τ_T. The dependence of the RDP on the coordinates is supposed to be weak. Thus, besides the small parameter of our theory μ, the gradients of the RDPs are small as well:

$$\frac{\partial^s \xi_\alpha}{\partial x_{n_1}...\partial x_{n_s}} \sim g^s, \ \ \alpha = 1, ..., 6, \\ \frac{\partial^s \xi_\alpha}{\partial x_{n_1}...\partial x_{n_s}} \sim \mu g^s, \ \ \alpha = 7, ..., 10, \ \ g \ll 1. \tag{17}$$

The parameter g is estimated as the ratio of the mean free path to the characteristic length of inhomogeneities in the system.

All the results of the theory are calculated finally in an additional perturbation theory in the small electron-to-ion mass ratio

$$\sigma = (m_e/m_i)^{1/2}. \tag{18}$$

3. Hydrodynamic Equations

According to the functional hypothesis (16), the hydrodynamic equations have the structure

$$\frac{\partial \xi_\alpha(x,t)}{\partial t} \equiv L_\alpha(x, f(\xi(t))). \tag{19}$$

Here, the functionals $L_\alpha(x, f)$ can be calculated from (5)–(11) and (15), which gives the relations

$$\frac{\partial n_a}{\partial t} = -\frac{1}{m_a}\frac{\partial \pi_{an}}{\partial x_n}, \qquad \frac{\partial \upsilon_n}{\partial t} = \frac{1}{\rho}\left[\upsilon_n \frac{\partial \pi_l}{\partial x_l} - \frac{\partial t_{nl}}{\partial x_l}\right],$$
$$\frac{\partial T}{\partial t} = \frac{T}{n}\sum_a \frac{1}{m_a}\frac{\partial \pi_{an}}{\partial x_n} - \frac{1}{3n}\upsilon^2 \frac{\partial \pi_n}{\partial x_n} + \frac{2}{3n}\upsilon_n \frac{\partial t_{nl}}{\partial x_l} - \frac{2}{3n}\frac{\partial q_n}{\partial x_n},$$
$$\frac{\partial u_n}{\partial t} = \frac{1}{m_e n_e}\upsilon_{en}\frac{\partial \pi_{el}}{\partial x_l} + \frac{1}{m_e n_e} R_{en} - \frac{1}{m_e n_e}\frac{\partial t_{enl}}{\partial x_l} - \frac{\partial \upsilon^h_{en}}{\partial t}, \tag{20}$$
$$\frac{\partial \tau}{\partial t} = \frac{T_e}{n_e m_e}\frac{\partial \pi_{\mathrm{en}}}{\partial x_n} - \frac{1}{3n_e}\frac{\partial \pi_{en}}{\partial x_n}\upsilon_e^2 + \frac{2}{3n_e}\upsilon_{en}\frac{\partial t_{enl}}{\partial x_l} - \frac{2}{3n_e}\frac{\partial q_{\mathrm{en}}}{\partial x_n} + \frac{2}{3n_e}(Q_e - \upsilon_{\mathrm{en}} R_{en}) - \frac{\partial T}{\partial t}.$$

To obtain the hydrodynamic equations with regard for relaxation processes, one has to calculate fluxes (8) and sources (9) appearing in (20). These hydrodynamic equations are general and are nonlinear.

The necessary DFs $f_{ap}(x, \xi)$ can be found from the kinetic equation (2), which can be rewritten in view of the functional hypothesis (16) and formulas (19) in the form

$$\sum_\alpha \int d^3x' \frac{\delta f_{ap}(x, \xi)}{\delta \xi_\alpha(x')} L_\alpha(x', f(\xi)) = -\frac{p_n}{m_a}\frac{\partial f_{ap}(x, \xi)}{\partial x_n} + I_{ap}(f(x, \xi)). \tag{21}$$

This equation is solved in a double perturbation theory in the above-introduced small parameters μ and g

$$f_{ap} = \sum_{m=0}^{2}\sum_{n=0}^{1} f_{ap}^{(n,m)} + O\left(\mu^2 g^0, \mu^2 g^1, \mu^2 g^2, g^3\right), \tag{22}$$

where $f_{ap}^{(n,m)} \sim \mu^n g^m$. The contributions $f_{ap}^{(0,m)}$ ($m = 0, 1$) are necessary to obtain fluxes (20), and $f_{ap}^{(1,m)}$ ($m = 0, 1$) are necessary to obtain both the fluxes and the sources in (20), and $f_{ap}^{(1,2)}$ is necessary to obtain only the sources.

4. Component Distribution Functions and Fluxes

The generalization of the Chapman–Enskog method presented here is based on the functional hypothesis (16), the corresponding definition of the RDP, and a special perturbation theory. In this section, the solution of the kinetic equation (21) with additional conditions following from the definition of RDP is discussed.

In *the leading approximation*, the nonequilibrium DFs are obviously the Maxwellian ones (1)

$$f_{ap}^{(0,0)} = w_{a,p-m_a\upsilon}, \qquad w_{ap} \equiv w_{ap}(T). \tag{23}$$

The DFs $f_{ap}^{(0,1)}$ *describe the dissipative terms of standard hydrodynamics* and are found in the form

$$f_{ap}^{(0,1)} = w_{ap}\left\{\sum_b \frac{\partial n_b}{\partial x_n} A_{an}^{N_b}(p) + \frac{\partial T}{\partial x_n} A_{an}^{T}(p) + \frac{\partial \upsilon_n}{\partial x_l} A_{anl}^{\upsilon}(p)\right\}_{p \to p - m\upsilon}, \tag{24}$$

which leads, according to Eq. (21) to the following integral equations for the functions $A_{an}^{N_b}(p)$, $A_{an}^{T}(p)$, and $A_{anl}^{\upsilon}(p)$:

$$\hat{K} A_{an}^{N_b}(p) = p_n\left[\frac{1}{\rho} - \frac{1}{n_a m_a}\delta_{ab}\right],$$
$$\hat{K} A_{an}^{T}(p) = p_n \frac{\beta}{m_a}\left[\frac{3}{2} + \frac{n m_a}{\rho} - \beta\varepsilon_{ap}\right], \tag{25}$$
$$\hat{K} A_{anl}^{\upsilon}(p) = -\frac{\beta}{m_a} h_{nl}(p)$$

($\beta \equiv T^{-1}$, $h_{nl}(p) = p_n p_l - \delta_{nl} p^2/3$). Here, the integral operator $\hat{K}$ is defined for an arbitrary function $h_a(p)$ by the relations

$$\hat{K} h_a(p) = \sum_b \int d^3p' K_{ab}(p, p') h_b(p'),$$
$$w_{ap} K_{ab}(p, p') = -w_{bp'} M_{ab}(p, p'),$$
$$M_{ab}(p, p') = \left.\frac{\delta I_{ap}(f)}{\delta f_{bp'}}\right|_{p \to p + m\upsilon,\, p' \to p' + m\upsilon,\, f_c \to w_c}. \tag{26}$$

Equations (25) show that the functions $A_{an}^{T}(p)$, $A_{an}^{N_b}(p)$, and $A_{anl}^{\upsilon}(p)$ have the structure

$$A_{an}^{T}(p) = p_n A_a^{T}(\beta\varepsilon_{ap}),$$
$$A_{an}^{N_b}(p) = p_n A_a^{N_b}(\beta\varepsilon_{ap}), \tag{27}$$
$$A_{anl}^{\upsilon}(p) = h_{nl}(p) A_a^{\upsilon}(\beta\varepsilon_{ap}).$$

The definitions of υ_n and T (12) in terms of the DFs $f_{ap}^{(0,1)}$ give the following additional conditions for the solutions of the integral equations (25):

$$\sum_a \langle p^2 A_a^T(\beta\varepsilon_{ap})\rangle_a = 0, \ \sum_a \langle p^2 A_a^{N_b}(\beta\varepsilon_{ap})\rangle_a = 0. \quad (28)$$

Here, for an arbitrary function $h(p)$,

$$\langle h(p)\rangle_a \equiv \int d^3p w_{ap} h(p).$$

The functions A_a^T, $A_a^{N_b}$, and A_a^υ are sought in the form of Sonine polynomial series

$$\begin{aligned} A_a^{N_b}(\beta\varepsilon_{ap}) &= \sum_{n=0}^{\infty} g_{an}^{N_b} S_n^{3/2}(\beta\varepsilon_{ap}), \\ A_a^T(\beta\varepsilon_{ap}) &= \sum_{n=0}^{\infty} g_{an}^T S_n^{3/2}(\beta\varepsilon_{ap}), \\ A_a^\upsilon(\beta\varepsilon_{ap}) &= \sum_{n=0}^{\infty} g_{an}^\upsilon S_n^{5/2}(\beta\varepsilon_{ap}). \end{aligned} \quad (29)$$

The Sonine polynomials are defined by the formula

$$S_n^\alpha(x) \equiv \frac{1}{n!} e^x x^{-\alpha} \frac{d^n}{dx^n}\left(e^{-x} x^{\alpha+n}\right), \quad (30)$$

and they are orthogonal

$$\int_0^\infty e^{-x} x^\alpha S_n^\alpha(x) S_{n'}^\alpha(x)\, dx = \frac{\Gamma(n+\alpha+1)}{n!}\delta_{nn'}. \quad (31)$$

With the help of (31), the additional conditions (28) can be rewritten in terms of the coefficients g_{an}^T and $g_{an}^{N_b}$ as

$$\sum_a m_a n_a g_{a0}^T = 0, \quad \sum_a m_a n_a g_{a0}^{N_b} = 0. \quad (32)$$

The integral equations (25) can be reduced to a linear set of equations for the coefficients g_{an}^T, $g_{an}^{N_b}$, and g_{an}^υ. It contains the infinite number of equations, and the number of polynomials in the expansions (29) should be artificially truncated in order to solve them. The DFs $f_{ap}^{(0,1)}$ are necessary for obtaining the fluxes of the order $(\mu^0 g^1)$ in (20). According to (24) and (31), the expressions for fluxes (8) *in the accompanying reference frame* in terms of the coefficients g_{an}^T, $g_{an}^{N_b}$, and g_{an}^υ are given by the formulas

$$\begin{aligned} \pi_{an}^{(0,1)} &= n_a m_a T\left[\sum_b \frac{\partial n_b}{\partial x_n} g_{a0}^{N_b} + \frac{\partial T}{\partial x_n} g_{a0}^T\right], \\ q_{an}^{(0,1)} &= \frac{5}{2} n_a T^2 \sum_b [g_{a0}^{N_b} - g_{a1}^{N_b}]\frac{\partial n_b}{\partial x_n} + \\ &+ \frac{5}{2} n_a T^2 [g_{a0}^T - g_{a1}^T]\frac{\partial T}{\partial x_n}, \\ t_{anl}^{(0,1)} &= n_a g_{a0}^\upsilon m_a T^2\left[\frac{\partial \upsilon_n}{\partial x_l} + \frac{\partial \upsilon_l}{\partial x_n} - \frac{2}{3}\frac{\partial \upsilon_m}{\partial x_m}\delta_{nl}\right]. \end{aligned} \quad (33)$$

As is seen, the fluxes are expressed in terms of only two coefficients from A_a^T and $A_a^{N_b}$ and only one coefficient from A_a^υ. That is why these functions are sought in the two- and one-polynomial approximations, respectively.

By multiplying two first equations in (25) by $w_{ap} p_n S_k^{3/2}$ $(k = 0, 1)$, summing over the subscript n, and integrating over d^3p, we obtain the following linear set of equations for the coefficients $g_{ak}^{N_b}$ and g_{ak}^T:

$$\begin{aligned} \sum_{n=0,1}\sum_b g_{bn}^T G_{ak,bn} &= -Y_{ak}, \\ \sum_{n=0,1}\sum_b g_{bn}^{N_e} G_{ek,bn} &= -\frac{3n_i m_i T}{\rho}\delta_{k0}, \\ \sum_{n=0,1}\sum_b g_{bn}^{N_e} G_{ik,bn} &= \frac{3n_i m_i T}{\rho}\delta_{k0}, \\ \sum_{n=0,1}\sum_b g_{bn}^{N_i} G_{ek,bn} &= \frac{3n_e m_e T}{\rho}\delta_{k0}, \\ \sum_{n=0,1}\sum_b g_{bn}^{N_i} G_{ik,bn} &= -\frac{3n_e m_e T}{\rho}\delta_{k0}. \end{aligned} \quad (34)$$

Here,

$$Y_{a0} = 3r_a n_e n_i \frac{m_i - m_e}{\rho}, \quad Y_{a1} = -\frac{15}{2} n_a \quad (35)$$

$(r_e \equiv 1, r_i \equiv -1)$, and $G_{ak,bn}$ are the integral brackets

$$G_{ak,bn} \equiv \{p_l S_k^{3/2}(\beta\varepsilon_{ap}), p_l S_n^{3/2}(\beta\varepsilon_{bp})\}_{ab}, \quad (36)$$

where

$$\{g, h\}_{ab} \equiv \int d^3p d^3p' g(p) w_{ap} K_{ab}(p, p') h(p'). \quad (37)$$

The coefficients $g_{ak}^{N_b}$ and g_{ak}^T are calculated from (32) and (34) in a σ perturbation theory.

By multiplying the third equation in (25) by $w_{ap}\, h_{nl}(p)$, summing over the subscripts n, l, and integrating over d^3p, we obtain the following linear set of equations for the coefficients g^{v}_{a0}:

$$\sum_b g^{v}_{b0} H_{a0,b0} = -10 n_a m_a T, \tag{38}$$

where

$$H_{ak,bn} =$$
$$= \{h_{lm}(p) S^{5/2}_n(\beta\varepsilon_{ap}), h_{lm}(p) S^{5/2}_k(\beta\varepsilon_{bp})\}_{ab}. \tag{39}$$

The coefficients g^{v}_{a0} are calculated from this set of equations in a σ perturbation theory with account for (32). The expressions for g^{v}_{a0}, g^{T}_{ak}, and $g^{N_b}_{ak}$ $(k = 0, 1)$ are rather lengthy. That is why they are not given here and can be found in [16, 17].

The DFs $f^{(1,0)}_{ap}$ *describe relaxation processes in a spatially homogeneous system* and are sought in the form

$$f^{(1,0)}_{ap} = w_{ap} \left\{A_a\left(\beta\varepsilon_{ap}\right)\tau + B_{an}(p) u_n\right\}_{p\to p-mv}, \tag{40}$$

where

$$B_{an}(p) = p_n B_a\left(\beta\varepsilon_{ap}\right). \tag{41}$$

The integral equations for the functions A_a and B_{an} can be obtained from Eq. (21):

$$\begin{aligned} &\hat{K} A_a(\beta\varepsilon_{ap}) = \lambda_T A_a\left(\beta\varepsilon_{ap}\right), \\ &\hat{K} B_{an}(p) = \lambda_u B_{an}(p), \end{aligned} \tag{42}$$

where λ_u and λ_T are the relaxation rates for the variables u_n and τ, respectively (see Eqs. (61)). *Equations* (42) are eigenvalue problems for the operator $\hat{K}$ of the linearized collision integral and *describe the kinetic modes of the system.* These equations follow from our generalization of the Chapman–Enskog method.

The definitions of u_n and τ (13) in terms of the DFs $f^{(1,0)}_{ap}$ give the following additional conditions for the solutions of the integral equations (42):

$$\begin{aligned} &\langle A_a(\beta\varepsilon_{ap})\rangle_a = 0, \quad \langle A_a(\beta\varepsilon_{ap})\varepsilon_{ap}\rangle_a = \frac{3}{2} r_a n_e, \\ &\langle B_a(\beta\varepsilon_{ap})\varepsilon_{ap}\rangle_a = \frac{3}{2} s_a n_e \\ &(s_e \equiv 1, s_i \equiv -\sigma^2). \end{aligned} \tag{43}$$

The functions A_a and B_a are sought in the form of Sonine polynomial series:

$$\begin{aligned} &A_a\left(\beta\varepsilon_{ap}\right) = \sum_{n=0}^{\infty} g_{an} S^{1/2}_n\left(\beta\varepsilon_{ap}\right), \\ &B_a\left(\beta\varepsilon_{ap}\right) = \sum_{n=0}^{\infty} h_{an} S^{3/2}_n\left(\beta\varepsilon_{ap}\right). \end{aligned} \tag{44}$$

By multiplying the first equation in (42) by $S^{1/2}_k$ and integrating over d^3p, we obtain the following set of equations for g_{ak} and λ_T:

$$\sum_{n=0}^{\infty}\sum_b g_{bn} V_{ak,bn} = \lambda_T g_{ak} \frac{2n_a}{k!\sqrt{\pi}} \Gamma\left(k + 3/2\right), \tag{45}$$

where

$$V_{ak,bn} = \{S^{1/2}_k\left(\beta\varepsilon_{ap}\right), S^{1/2}_n\left(\beta\varepsilon_{bp}\right)\}_{ab}. \tag{46}$$

By multiplying the second equation in (42) by $p_n S^{3/2}_k$, summing over the subscript n, and integrating over d^3p, we obtain the following set of equations for h_{ak} and λ_u:

$$\sum_{n=0}^{\infty}\sum_b h_{bn} G_{ak,bn} = \lambda_u h_{ak} \frac{4T m_a n_a}{k!\sqrt{\pi}} \Gamma\left(k + 5/2\right) \tag{47}$$

with the coefficients $G_{ak,bn}$ defined in (36).

The additional conditions (43) lead to the following expressions for the coefficients g_{a0}, g_{a1}, and h_{a0}:

$$\begin{aligned} &g_{a0} = 0, \quad h_{e0} = \beta, \quad h_{i0} = -\frac{n_e}{n_i}\beta\sigma^2, \\ &g_{e1} = -\beta, \quad g_{e1} = -\frac{n_e}{n_i}\beta. \end{aligned} \tag{48}$$

These coefficients define the DFs $f^{(1,0)}_{ap}$ in the one-polynomial approximation, which coincide with the DFs given by the above-mentioned Landau assumption [1]. In this approximation, the relaxation rates calculated from (45) and (47) are given by the expressions

$$\begin{aligned} &\lambda_T = \frac{2^{7/2}\pi^{1/2}}{3}(n_i + n_e)\lambda\sigma^2 + O(\sigma^4), \\ &\lambda_u = \frac{2^{5/2}\pi^{1/2}}{3} n_i \lambda + O\left(\sigma^2\right), \end{aligned} \tag{49}$$

where

$$\lambda \equiv \frac{z^2 e^4 L}{\left(m_e T^3\right)^{1/2}}. \tag{50}$$

These results coincide with those obtained in [1, 10, 11, 19]. It can be shown that the expressions for the Landau distribution functions and for the relaxation rates (49) are the principal-order solutions of (42) in a σ perturbation theory (see [15]). That is why *we restrict ourselves to the DFs $f_{ap}^{(1,0)}$, and the rates λ_T, λ_u are taken in the one-polynomial approximation.* The two-polynomial approximation for these quantities will be discussed elsewhere.

The DFs $f_{ap}^{(1,1)}$ *describe the relaxation processes in spatially non-uniform states* and are sought in the form:

$$f_{ap}^{(1,1)} = w_{ap}\Bigg\{A_{an}^{\tau}(p)\frac{\partial\tau}{\partial x_n} + A_{anl}^{u}(p)\frac{\partial u_n}{\partial x_l} + A_{an}^{\tau T}(p)\,\tau\frac{\partial T}{\partial x_n} + A_{anl}^{uT}(p)\,u_l\frac{\partial T}{\partial x_n} + A_{anl}^{\tau\upsilon}(p)\,\tau\frac{\partial \upsilon_n}{\partial x_l} + A_{amnl}^{u\upsilon}(p)\,u_m\frac{\partial \upsilon_n}{\partial x_l} + \sum_b A_{an}^{\tau N_b}(p)\,\tau\frac{\partial n_b}{\partial x_n} + \sum_b A_{anl}^{uN_b}(p)\,u_l\frac{\partial n_b}{\partial x_n}\Bigg\}_{p\to p-m_a\upsilon}. \tag{51}$$

The integral equations for the functions A_{an}^{τ}, A_{anl}^{u}, $A_{an}^{\tau T}$, $A_{an}^{\tau N_b}$, $A_{anl}^{\tau\upsilon}$, A_{anl}^{uT}, $A_{anl}^{uN_b}$, and $A_{amnl}^{u\upsilon}$ are obtained from the kinetic equation (21) with regard for the results for the DFs $f_{ap}^{(1,0)}$ and the relaxation rates λ_u, λ_T.

The integral equations for A_{an}^{τ} and A_{anl}^{u} have the form

$$\begin{aligned}\hat{K}A_{am}^{\tau}(p) &= \frac{n_e s_a}{m_e n_a T}p_m - \frac{s_a}{n_a m_e T}\,p_n G_{nm}^{\tau} + \frac{n_e r_a}{m_a n_a T}p_m\left(\frac{3}{2} - \frac{\varepsilon_{ap}}{T}\right) + A_{am}^{\tau}(p)\,\lambda_T,\\ \hat{K}A_{anl}^{u}(p) &= \delta_{nl}\left(\frac{\varepsilon_{ap}}{T} - \frac{3}{2}\right)\times\left[\frac{2\left(1-\sigma^2\right)n_e}{3\left(n_e+n_i\right)}\left(1 - \frac{n_e r_a}{n_a}\right) + \frac{2}{3}\frac{n_e r_a}{n_a}\right] + \delta_{nl}\frac{n_e s_a}{n_a} + \frac{2}{3}\frac{r_a}{n_a T}\left(\frac{3}{2} - \frac{\varepsilon_{ap}}{T}\right)G_{nl}^{u} - p_n p_l\frac{n_e s_a}{m_a n_a T} + A_{anl}^{u}(p)\,\lambda_u,\end{aligned} \tag{52}$$

where

$$G_{nm}^{u} \equiv \sum_b \{\varepsilon_{ep}, A_{bnm}^{u}\}_{eb},\qquad G_{nm}^{\tau} = \sum_b \{p_n, A_{bm}^{\tau}\}_{eb}. \tag{53}$$

Other integral equations of the order $\mu^1 g^1$ are not given here because of their complexity. However, the corresponding terms in the DF $f_{ap}^{(1,1)}$ do not contribute to the linearized theory, which is discussed in the next section.

Equations (52) show that the functions A_{an}^{τ} and A_{anl}^{u} have the structure

$$A_{an}^{\tau}(p) = p_n A_a^{\tau}(\beta\varepsilon_{ap}),\qquad A_{anl}^{u}(p) = h_{nl}(p) A_a^{u}(\beta\varepsilon_{ap}) \tag{54}$$

(the contraction of the second equation in (52) with respect to the subscripts n and l vanishes in the terms with known functions). The functions A_a^{τ} and A_a^{u} are sought in the form of Sonine polynomial series

$$A_a^{\tau}(\beta\varepsilon_{ap}) = \sum_{n=0}^{\infty} g_{an}^{\tau} S_n^{3/2}(\beta\varepsilon_{ap}),\qquad A_a^{u}(\beta\varepsilon_{ap}) = \sum_{n=0}^{\infty} g_{an}^{u} S_n^{5/2}(\beta\varepsilon_{ap}). \tag{55}$$

The additional conditions for the solutions of Eqs. (52), which follow from definition (5) of the RDPs u_n, τ, give the restriction only on A_a^{τ}:

$$\langle A_a^{\tau}(\beta\varepsilon_{ap})\varepsilon_{ap}\rangle_a = 0. \tag{56}$$

This condition in terms of the coefficients g_{an}^{τ} takes the form

$$g_{a0}^{\tau} = 0. \tag{57}$$

The contributions of two first terms from (51) to the fluxes in the accompanying reference frame are given by the formulas

$$\begin{aligned}\tilde{\pi}_{an}^{(1,1)} &= n_a m_a T g_{a0}^{\tau}\frac{\partial\tau}{\partial x_n},\\ \tilde{q}_{an}^{(1,1)} &= \frac{5}{2}n_a T^2\left[g_{a0}^{\tau} - g_{a1}^{\tau}\right]\frac{\partial\tau}{\partial x_n},\\ \tilde{t}_{anl}^{(1,1)} &= m_a n_a T^2 g_{a0}^{u}\left[\frac{\partial u_n}{\partial x_l} + \frac{\partial u_l}{\partial x_n} - \frac{2}{3}\frac{\partial u_m}{\partial x_m}\delta_{nl}\right]\end{aligned} \tag{58}$$

(a tilde in these expressions indicates that they give only the parts of the fluxes that contribute to the linearized hydrodynamic theory). Condition (57) shows that one can restrict oneself to the one-polynomial approximation in the calculation of these fluxes.

By multiplying the first equation in (52) by $w_{ap}p_m S_1^{3/2}$, summing over the subscript m, and integrating over d^3p, we obtain a linear set of equations for the coefficients g_{a1}^{τ}. By multiplying the second equation in (52) by $w_{ap}\,h_{nl}(p)$, summing over the subscripts n,l, and integrating over d^3p, we obtain a linear set of equations for the coefficients g_{a0}^{υ}. The coefficients from these sets of equations are calculated in a σ perturbation theory (see the results in [16]).

The DFs $f_{ap}^{(1,2)}$ *describe the effect of the Burnett terms on the relaxation processes* and are necessary only for the calculation of the sources in the hydrodynamic equations. The method of obtaining the DF $f_{ap}^{(1,2)}$ is the same as for obtaining the DFs $f_{ap}^{(0,1)}$ and $f_{ap}^{(1,1)}$, but the corresponding equations are very lengthy, and that is why their derivation is omitted here. We restrict ourselves in the calculations of the DF $f_{ap}^{(1,2)}$ to the one-polynomial approximation.

5. Linearized Hydrodynamic Equations and Modes of the System

The component DF and the fluxes are calculated above in the general nonlinear case. In this section, the linearized hydrodynamic equations are investigated in order to obtain the dispersion laws for the plasma modes (see an example of such investigation in [7]).

In the linearized theory, the RDPs $\xi_\alpha(x,t)$ are taken in a vicinity of their equilibrium values

$$\begin{aligned} &n_a = n_a^{\rm eq} + \delta n_a\,(x,t), \quad \upsilon_n = \delta\upsilon_n\,(x,t),\\ &T = T^{\rm eq} + \delta T\,(x,t), \quad u_n = \delta u_n\,(\mathbf{x},t),\\ &\tau = \delta\tau\,(x,t), \end{aligned} \tag{59}$$

and the deviations $\delta\xi_\alpha(x,t)$ from their equilibrium values $\xi_\alpha^{\rm eq}$ are small (in the reference frame under consideration, $\upsilon_n^{\rm eq} = 0$). The equilibrium values of the RDPs $\xi_\alpha^{\rm eq}$ are constants, and the condition of electroneutrality

$$n_e^{\rm eq} = z n_i^{\rm eq} \tag{60}$$

is satisfied.

The linearized hydrodynamic equations must be rotationally invariant and, therefore, have the structure

$$\begin{aligned} &\partial_t\delta n_e = \gamma_{ee}\Delta\delta n_e + \gamma_{ei}\Delta\delta n_i + \gamma_{e\upsilon}{\rm div}\delta\upsilon + \gamma_{eT}\Delta\delta T +\\ &+ \gamma_{eu}{\rm div}\delta u + \gamma_{e\tau}\Delta\delta\tau,\\ &\partial_t\delta n_i = \gamma_{ie}\Delta\delta n_e + \gamma_{ii}\Delta\delta n_i + \gamma_{i\upsilon}{\rm div}\delta\upsilon + \gamma_{iT}\Delta\delta T +\\ &+ \gamma_{iu}{\rm div}\delta u + \gamma_{i\tau}\Delta\delta\tau,\\ &\partial_t\delta\upsilon_l = \beta_e{\rm grad}_l\delta n_e + \beta_i{\rm grad}_l\delta n_i + \eta_\upsilon\Delta\delta\upsilon_l +\\ &+ \tilde\eta_\upsilon{\rm grad}_l{\rm div}\delta\upsilon + \beta_T{\rm grad}_l\delta T +\\ &+ \eta_u\Delta\delta u_l + \tilde\eta_u{\rm grad}_l{\rm div}\delta u + \beta_\tau{\rm grad}_l\delta\tau,\\ &\partial_t\delta T = \alpha_e\Delta\delta n_e + \alpha_i\Delta\delta n_i + \alpha_\upsilon{\rm div}\delta\upsilon + \alpha_T\Delta\delta T +\\ &+ \alpha_u{\rm div}\delta u + \alpha_\tau\Delta\delta\tau,\\ &\partial_t\delta u_l = -\lambda_u\delta u_l + \chi_u\Delta\delta u_l + \tilde\chi_u{\rm grad}_l{\rm div}\delta u +\\ &+ \chi_\tau{\rm grad}_l\delta\tau,\\ &\partial_t\delta\tau = -\lambda_T\delta\tau + \theta_u{\rm div}\delta u + \theta_\tau\Delta\delta\tau. \end{aligned} \tag{61}$$

Four first equations here (three scalar ones and a vector one) describe the evolution of the standard hydrodynamic variables, and the last two equations describe the evolution of the relaxation RDPs u_n and τ. The coefficients appearing in (61) are constants and should be calculated. Comparing (61) with (20), using (60), and calculating the fluxes and the sources in the linearized theory with the help of the obtained DFs, we get these coefficients in a σ perturbation theory:

$$\begin{aligned} &\alpha_T = \frac{\sqrt{2}+17z}{2z(1+z)(\sqrt{2}+z)}\Lambda + O\,(\sigma),\\ &\alpha_e = \frac{\sqrt{2}+7z}{2(1+z)(\sqrt{2}+z)}\frac{T}{n_i}\Lambda + O\,(\sigma),\\ &\alpha_\upsilon = -\frac{2T}{3},\\ &\alpha_\tau = \frac{25}{2(1+z)(4\sqrt{2}+13z)}\Lambda + O(\sigma),\\ &\alpha_u = -\frac{2Tz}{3(1+z)} + O(\sigma^2), \quad \alpha_i = O(\sigma^2);\\ &\beta_i = \beta_e \equiv \beta, \quad \beta = -\frac{T}{n_i m_e} + O(\sigma^2),\\ &\beta_T = -\frac{1+z}{m_e} + O(\sigma^2);\\ &\gamma_{ie} = O(\sigma^2), \quad \gamma_{ii} = O(\sigma^4), \quad \gamma_{i\upsilon} = -n_i,\\ &\gamma_{iu} = n_i z\sigma^2, \quad \gamma_{iT} = O(\sigma^2), \quad \gamma_{i\tau} = 0, \end{aligned}$$

$$\gamma_{ei} = O(\sigma^2), \quad \gamma_{ev} = -n_i z, \quad \gamma_{eu} = -n_i z,$$

$$\gamma_{e\tau} = 0, \quad \gamma_{ee} = \frac{3}{16}\frac{4\sqrt{2}+13z}{z^2(\sqrt{2}+z)}\Lambda + O(\sigma),$$

$$\gamma_{eT} = \frac{3}{4}\frac{\sqrt{2}+7z}{z(\sqrt{2}+z)}\frac{n_i}{T}\Lambda + O(\sigma);$$

$$\eta_u = O(\sigma^2), \quad \tilde{\eta}_u = \eta_u/3,$$

$$\eta_v = \frac{5}{4\sqrt{2}z^4}\Lambda\sigma + O(\sigma^2), \quad \tilde{\eta}_v = \eta_v/3;$$

$$\theta_\tau = \frac{25}{2z(1+z)\left(4\sqrt{2}+13z\right)}\Lambda + O(\sigma),$$

$$\theta_u = -\frac{2T}{3}\frac{1}{z+1} + O(\sigma^2);$$

$$\chi_u = \frac{15}{4z\left(3\sqrt{2}+z\right)}\Lambda - 4z^2T^3g_A^{(0)}\frac{1}{\Lambda} + O(\sigma),$$

$$\tilde{\chi}_u = \frac{5}{4z\left(3\sqrt{2}+z\right)}\Lambda - 8z^2T^3(g_A^{(0)} + g_B^{(0)})\frac{1}{\Lambda} + O(\sigma),$$

$$\chi_\tau = -\frac{4(\sqrt{2}+7z)}{4\sqrt{2}+13z}\frac{1}{m_e} + O(\sigma), \tag{62}$$

where the notations

$$g_B = \frac{5n_i z}{4G_{e1,e1} - 30\lambda_u n_i z m_e T}\left[\frac{T(zg_{e1}^T + g_{e1}^\tau)}{z+1} + \right.$$
$$\left. + \frac{3}{2}g_{e1}^{N_e} n_i z - g_{e0}^u T\right],$$

$$g_A = \frac{42 g_{e0}^u n_i z m_e^2 T^3}{2G - 1155\lambda_u n_i z m_e^3 T^3}; \tag{63}$$

$$G = \{S_1^{5/2}(\beta\varepsilon_{ep})p_n p_m p_l; S_1^{5/2}(\beta\varepsilon_{ep})p_n p_m p_l\}_{ee};$$

$$\Lambda = \frac{T^{5/2}}{n_i e^4 L\sqrt{2\pi m_e}}$$

are introduced. In (62) and in what follows, the superscript “eq” is omitted in $T^{\rm eq}$ and $n_a^{\rm eq}$; and $g_A^{(0)}$ and $g_B^{(0)}$ are the contributions of the order σ^0 to g_A and g_B. The explicit expressions for χ_u and $\tilde{\chi}_u$ as functions of z are not given here, because they are too lengthy. The second terms on the right-hand sides of expressions (62) for them are given by the source $R_{en}^{(1,2)}$ (but $Q_e^{(1,2)} = O(\sigma)$).

Performing the Fourier transformation in Eqs. (61),

$$\delta\xi_a(k,t) = \int e^{-ik_n x_n}\delta\xi_a(x,t)d^3x, \tag{64}$$

and choosing the x-axis of the coordinates along the wave vector k, we obtain the following linearized hydrodynamic equations:

$$\begin{aligned}
&\partial_t\delta n_e = -k^2(\gamma_{ee}\delta n_e + \gamma_{ei}\delta n_i) + i\gamma_{ev}k\delta v_x - \\
&- \gamma_{eT}k^2\delta T + i\gamma_{eu}k\delta u_x - \gamma_{e\tau}k^2\delta\tau, \\
&\partial_t\delta n_i = -k^2(\gamma_{ie}\delta n_e + \gamma_{ii}\delta n_i) + i\gamma_{iv}k\delta v_x - \\
&- \gamma_{iT}k^2\delta T + i\gamma_{iu}k\delta u_x - \gamma_{i\tau}k^2\delta\tau, \\
&\partial_t\delta v_x = ik(\beta_e\delta n_e + \beta_i\delta n_i) - (\eta_v + \tilde{\eta}_v)k^2\delta v_x + \\
&+ ik\beta_T\delta T - (\eta_u + \tilde{\eta}_u)k^2\delta u_x + ik\beta_\tau\delta\tau, \\
&\partial_t\delta v_{y,z} = -k^2\eta_v\delta v_{y,z} - k^2\eta_u\delta u_{y,z}, \\
&\partial_t\delta T = -k^2(\alpha_e\delta n_e + \alpha_i\delta n_i) + i\alpha_v k\delta v_x - k^2\alpha_T\delta T + \\
&\partial_t\delta\tau = -(\lambda_T + \theta_\tau k^2)\delta\tau + ik\theta_u\delta u_x, \\
&+ ik\alpha_u\delta u_x - k^2\alpha_\tau\delta\tau, \\
&\partial_t\delta u_x = -(\lambda_u + k^2\chi_u + k^2\tilde{\chi}_u)\delta u_x + ik\chi_\tau\delta\tau, \\
&\partial_t\delta u_{y,z} = -(\lambda_u + \chi_u k^2)\delta u_{y,z}.
\end{aligned} \tag{65}$$

Equations (65) can be written in the form

$$\partial_t\delta\xi_\alpha(k,t) = \sum_{\alpha'} M_{\alpha\alpha'}(k)\delta\xi_{\alpha'}(k,t) \tag{66}$$

that defines the generalized hydrodynamic matrix $M_{\alpha\alpha'}(k)$. In order to solve them, let us consider the eigenvalues $\lambda_i(k)$, left and right eigenvectors $\psi_{i\alpha}(k)$ and $\varphi_{i\alpha}(k)$ of the matrix $M_{\alpha\alpha'}(k)$ given by the relations

$$\begin{aligned}
&\sum_{\alpha'} M_{\alpha\alpha'}(k)\varphi_{i\alpha'}(k) = \lambda_i(k)\varphi_{i\alpha}(k), \\
&\sum_{\alpha} \psi_{i\alpha}(k)M_{\alpha\alpha'}(k) = \lambda_i(k)\psi_{i\alpha'}(k)
\end{aligned} \tag{67}$$

with the normalization condition

$$\sum_{\alpha} \psi_{i\alpha}(k)\varphi_{i'\alpha}(k) = \delta_{ii'}. \tag{68}$$

Now, the solution of Eq. (66) is given by the formulas

$$\begin{aligned}
&\xi_\alpha(k,t) = \sum_i c_i(k,t)\varphi_{i\alpha}(k), \\
&c_i(k,t) \equiv c_i(k,0)e^{\lambda_i(k)t}.
\end{aligned} \tag{69}$$

Eigenvalues $\lambda_i(k)$ are complex frequencies for modes of the system, $\delta\theta_i(k,t)$, that are given by the expression

$$\delta\theta_i(k,t) \equiv \sum_\alpha \psi_{i\alpha}(k)\delta\xi_\alpha(k,t) = \delta\theta_i(k,0)e^{\lambda_i(k)t}. \quad (70)$$

The eigenvalues $\lambda_i(k)$ of the matrix $M_{\alpha\alpha'}(k)$ should be calculated from the equation

$$\det|M(k) - \lambda I| = 0 \quad (71)$$

in the k-perturbation theory, because the magnitude of the wave vector according to (17) is small (I is the identity matrix). Then the eigenfunctions $\varphi_{i\alpha}(k)$ and $\psi_{i\alpha}(k)$ can be calculated from definitions (67) and (68) in the same perturbation theory. Further, the obtained results should be investigated in a σ-perturbation theory.

As a result, we obtain the eigenvalues (complex frequencies)

$$\begin{aligned}
&\lambda_{1,2} = \pm ikc_s - D_s k^2 + O(k^3),\\
&c_s = \sigma\sqrt{\frac{5T}{3m_e}(z+1)} + O(\sigma^2),\\
&D_s = \Lambda\,\frac{5(29z+4\sqrt{2})}{2^5 z(z+\sqrt{2})(z+1)} + O(\sigma);\\
&\lambda_3 = -D_3k^2 + O(k^3),\\
&D_3 = \Lambda\,\frac{3\left[a(z)+\sqrt{a(z)^2-b(z)}\right]}{10(z+\sqrt{2})(z+1)} + O(\sigma);\\
&\lambda_4 = -D_4k^2 + O(k^3),\\
&D_4 = \Lambda\,\frac{3\left[a(z)-\sqrt{a(z)^2-b(z)}\right]}{10(z+\sqrt{2})(z+1)} + O(\sigma);\\
&\lambda_{5,6} = -\eta_\upsilon k^2 + O(k^4),\\
&\lambda_7 = -\lambda_T - \left[\theta_\tau + \frac{\theta_u\chi_\tau}{\lambda_u-\lambda_T}\right]k^2 + O\left(k^3\right),\\
&\lambda_8 = -\lambda_u + \left[\frac{\theta_u\chi_\tau}{\lambda_u-\lambda_T} - \chi_u - \tilde{\chi}_u\right]k^2 + O\left(k^3\right),\\
&\lambda_{9,10} = -\lambda_u - \chi_u k^2 + O(k^4),
\end{aligned} \quad (72)$$

where the functions $a(z)$ and $b(z)$ are

$$a(z) = \frac{25}{8} + \frac{5}{16z^2}(4\sqrt{2}+13z), \quad b(z) = \frac{125}{8z^2}(\sqrt{2}+z).$$

The corresponding modes are given in the zero order in k by the relations

$$\begin{aligned}
&\delta\theta_{1,2} \sim \beta_e(\delta n_e + \delta n_i) \pm c_s\delta\upsilon_x + \beta_T\delta T,\\
&\delta\theta_{3,4} \sim \delta T + a_{3,4}\delta n_e + b_{3,4}\delta n_i,\\
&\delta\theta_5 \sim \delta\upsilon_y, \quad \delta\theta_6 \sim \delta\upsilon_z, \quad \delta\theta_7 \sim \delta\tau,\\
&\delta\theta_8 \sim \delta u_x, \quad \delta\theta_9 \sim \delta u_y, \quad \delta\theta_{10} \sim \delta u_z,
\end{aligned} \quad (73)$$

where a_3, a_4, b_3, and b_4 are some lengthy coefficients not given here.

Here, $\delta\theta_1, ..., \delta\theta_6$ are the hydrodynamic modes of the system; $\delta\theta_7, ..., \delta\theta_{10}$ are the kinetic modes of the system, which are related to the relaxation; $\delta\theta_{1,2}$ are the sound modes, $\delta\theta_{3,4}$ are the heat and diffusion modes; $\delta\theta_{5,6}$ are the velocity υ shear modes. The mode $\delta\theta_7$ describes the evolution of the component temperature difference τ, and the modes $\delta\theta_8, ..., \delta\theta_{10}$ describe the evolution of the component velocity difference u_n.

6. Relaxation Damping of Plasma Oscillations

In Introduction, it was noted that *the problem of investigation of the modes of the Landau equation can be considered as an important part of the investigation of plasma modes*. In the mentioned approach suggested by D. Bohm and D. Pines in [8], the long-range part of the Coulomb interaction is described by the degrees of freedom of the plasma electromagnetic field and the short-range part is accounted for by the Landau kinetic equation.

In another approach, the effects of the long-range part of the Coulomb interaction can be investigated by adding the Vlasov term to the Landau kinetic equation, which gives the Vlasov–Landau kinetic equation [6, 20]

$$\frac{\partial f_{ap}}{\partial t} + \frac{p_n}{m_a}\frac{\partial f_{ap}}{\partial x_n} + e_a E_n(f)\frac{\partial f_{ap}}{\partial p_n} = I_{ap}(f). \quad (74)$$

Here, $E_n(f)$ is the self-consistent electric field that satisfies the Poisson equation

$$\frac{\partial E_n}{\partial x_n} = 4\pi\sum_a e_a n_a. \quad (75)$$

Unfortunately, the generalized Chapman–Enskog method developed in the present paper, which is based on the RDPs $\xi_\alpha(x,t)$ (15) and a perturbation theory in the parameters μ and g (17), cannot be rigorously applied to Eq. (74). We note that there are no attempts of a similar investigation in the literature (see, e.g., [6, 20]).

However, we can propose to study the problem in an approximation based on the DF taken in the zero order in the gradients of the RDPs $\xi_\alpha(x)$. The Vlasov–Landau equation (74) in the spatially homogeneous case does not contain the self-consistent field, and, thus, the necessary component DF is given by relations (40)–(42). For simplicity, we restrict ourselves here to the one-polynomial approximation for the solution of Eqs. (42). In this approximation, the DFs are given by formulas (23) and (40), in which the functions A_a and B_a have the form

$$A_e = -\beta\left[\frac{3}{2} - \beta\varepsilon_{ep}\right], \quad A_i = \beta\frac{n_e}{n_i}\left[\frac{3}{2} - \beta\varepsilon_{ip}\right];$$
$$B_e = \beta, \quad B_i = -\beta\sigma^2\frac{n_e}{n_i} \tag{76}$$

(see (44), (48)). As was noted above, such DFs $f^{(1,0)}_{ap}$ coincide with the DFs given by the Landau assumption discussed in Introduction.

The time equations for parameters (15) with regard for the self-consistent field are obtained from definitions (5), (7), (12), and the kinetic equation (74). They coincide with (20), except for the equations for υ_n and u_n, which take the form

$$\frac{\partial\upsilon_n}{\partial t} = \frac{1}{\rho}\left[\upsilon_n\frac{\partial\pi_l}{\partial x_l} - \frac{\partial t_{nl}}{\partial x_l} + E_n\sum_a e_a n_a\right],$$
$$\frac{\partial u_n}{\partial t} = \frac{1}{m_e n_e}\upsilon_{en}\frac{\partial\pi_{el}}{\partial x_l} + \frac{1}{m_e n_e}R_{en} - \tag{77}$$
$$-\frac{e}{m_e}E_n - \frac{1}{m_e n_e}\frac{\partial t_{enl}}{\partial x_l} - \frac{\partial\upsilon_n}{\partial t}.$$

Substituting DFs (23) and (40) with (76) into the fluxes and the sources in the time equations (20) and (77) gives hydrodynamic equations in the nondissipative approximation (they do not contain terms describing hydrodynamic dissipative processes: viscosity, heat conductivity, and diffusion) with account for the self-consistent field and relaxation.

Applying the method developed in Section 5 to the obtained hydrodynamic equations and (75) gives the modes of the Vlasov–Landau kinetic equation calculated in a k, σ-perturbation theory. It is necessary to emphasize that this gives the complex frequencies of modes (72) without hydrodynamic damping rates of the form $\gamma_\alpha = D_\alpha k^2$, but with some modification related to the Vlasov self-consistent field.

As a result, we obtain the sound modes

$$\lambda_{1,2} = \pm ic_s k + O(k\sigma^2, k^2) \tag{78}$$

with c_s from (72); the heat mode

$$\lambda_3 = O(k^2); \tag{79}$$

the plasma (Langmuir) modes

$$\lambda_{4,8} = -\frac{\lambda_u}{2} \pm i\sqrt{\omega_p^2 - \frac{\lambda_u^2}{4}} + O(k^0\sigma^2, k^2); \tag{80}$$

the shear modes

$$\lambda_{5,6} = O(k^2); \tag{81}$$

the mode related to τ

$$\lambda_7 = -\lambda_T + O(k^2); \tag{82}$$

the transversal modes related to u_n

$$\lambda_{9,10} = -\lambda_u + O(k^2). \tag{83}$$

Formula (80) contains the electron plasma frequency defined by $\omega_p = (4\pi e^2 nz/m_e)^{1/2}$.

The estimates $O(k^2)$ are written in (79) and (81), because, as usual, the hydrodynamic dissipation processes give such estimates. The estimates in the expressions for the complex frequencies (78), (80), (82), and (83) are obtained directly from Eq. (71) taken in the approximation considered in this section.

In the framework of the used accuracy, all obtained frequencies except for modes 4 and 8 coincide with expressions (72). Neglecting the self-consistent field can be performed formally by the transition $\omega_p \to 0$. After that, the plasma frequencies (80) coincide with the expressions for modes 4 and 8 from (72) obtained on the basis of the Landau kinetic equation. In the opposite case where the collision integral is omitted, i.e. the Vlasov equation is investigated, the substitution $\lambda_T = \lambda_u = 0$ into (80) give the known plasma oscillations $\lambda = \pm i\omega_p$. Thus, the obtained results are true in the borderline cases of the Landau and Vlasov kinetic equations, which justifies results (78)–(83).

The modes corresponding to frequencies (78), (80), (82), and (83) are given in the zero order in k by the relations

$$\delta\theta_{1,2} \sim \beta(\delta n_e + \delta n_i) \pm c_s \delta \upsilon_x + \beta_T \delta T,$$
$$\delta\theta_{4,8} \sim e(z\delta n_i - \delta n_e), \quad \delta\theta_7 \sim \delta\tau, \tag{84}$$
$$\delta\theta_9 \sim \delta u_y, \quad \delta\theta_{10} \sim \delta u_z.$$

The results for $\delta\theta_{5,6}$ and $\delta\theta_3$ can be obtained only after obtaining the explicit expressions for $\lambda_{5,6}$ and λ_3, so they cannot be found in the nondissipative hydrodynamic approximation. Expressions (84) except for the expressions for the plasma modes coincide with the analogous ones from (73). The expression for the plasma modes $\delta\theta_{4,8}$ corresponds to their common physical understanding as related to the deviation of the charge density from zero.

The modes $\delta\theta_{4,8}$ with the frequencies $\lambda_{4,8}$ describe the plasma oscillations with damping related to the relative velocity relaxation rate λ_u. The frequency of these oscillations is shifted similarly to the influence of the friction on oscillations

$$\omega = \sqrt{\omega_p^2 - \lambda_u^2/4}, \tag{85}$$

and the damping rate

$$\gamma_R = \lambda_u/2 \tag{86}$$

does not vanish at $k = 0$. In a different approach, which is based on the calculation of the dielectric permittivity taking the collisions into account, the damping rate of plasma oscillations was found, for example, in [2]. The above result (86) coincides with the mentioned one if the relaxation rate λ_u is taken from (49). However, our theory of the collisional plasma oscillation damping is more general than the theory developed in [2]. We note that the theory developed in [2] is devoted only to the "jelly" model, where the ion subsystem is an equilibrium one.

As is known, not only the relaxation, but also the Landau damping take place in plasma. If $kr_\mathrm{D} \ll 1$, the rate of the Landau damping is given by the formula [2]

$$\gamma_\mathrm{L} = \sqrt{\frac{\pi}{8}} \frac{\omega_p}{(kr_\mathrm{D})^3} \exp\left(-\frac{1}{2k^2 r_\mathrm{D}^2} - \frac{3}{2}\right), \tag{87}$$

where $r_\mathrm{D} \equiv (T/4\pi n_e e^2)^{1/2}$ is the Debye length. As is seen, γ_L vanishes if $k = 0$, that is why the obtained relaxation damping is much more important than the Landau damping, if k is small.

To illustrate this fact, let us give numerical data for some completely ionized plasmas, considering the case $z = 1$. As is known [10], the Coulomb logarithm is estimated as $L \sim 10 \div 15$. The approximate values of the quantities n_e and T for some plasmas are taken from [10] and given in Table 1. The results which are related to the relaxation damping and the Landau damping coefficients are given in Table 2.

As is seen from the tables, for the widely known cases of a completely ionized plasma, $\lambda_u \ll \omega_p$, and the frequency shift of the plasma oscillation from ω_p in (85) due to the relaxation damping is negligible. However, the relaxation damping rate γ_R is higher than the Landau damping rate γ_L if $k \to 0$. The range of the kr_D values, where the relaxation damping is more important than the Landau one, is given for the considered cases in Table 2 and relations (49), (86), and (87) (the quantity δ is defined as follows: if $kr_\mathrm{D} < \delta$, then $\gamma_\mathrm{L} < \gamma_R$).

Table 1. **Density and temperature for some plasmas**

Plasma	n_e, cm^{-3}	T, K
Tokamak	$10^{14} \div 10^{15}$	10^8
Interplanetary plasma	$10^{-2} \div 10^1$	10^4
Solar corona	$10^4 \div 10^8$	$10^6 \div 10^8$

Table 2. **Damping coefficients for some plasmas**

Plasma	γ_R/ω_p	δ
Tokamak	$(0.3 \div 1.5) \times 10^{-8}$	$0.146 \div 0.152$
Interplanetary plasma	$(0.03 \div 1.5) \times 10^{-9}$	$0.133 \div 0.144$
Solar corona	$3 \times 10^{-14} \div 5 \times 10^{-9}$	$0.119 \div 0.148$

7. Conclusion

The hydrodynamics of a completely ionized two-component plasma is studied on the basis of the Landau kinetic equation. The Landau equation is solved by a generalized Chapman–Enskog method, which involves the component temperature and velocity relaxation at their end and is based on the Bogolyubov idea of the functional hypothesis.

The obtained component fluxes of the particle number, energy, and momentum in the first order in gradients (33) describe the heat conductivity, viscosity, and diffusion in the system. These fluxes can be written in the form that introduces the kinetic coefficients of the system. In the literature, there are several definitions of kinetic coefficients for many-component systems, but in the absence of relaxation (see, for example, [5,6,22]). It is planned to discuss the transport phenomena in the system in detail in a subsequent paper, considering the definitions of kinetic coefficients and the results of their calculation.

The hydrodynamic and kinetic modes (72) of the Landau kinetic equation are obtained in the hydrodynamic approximation with additional account for the small electron-to-ion mass ratio. It is shown that six modes of the system are the standard hydrodynamic modes of a two-component plasma. Four other modes of the system are relaxation ones, and they are due to the component temperature and the velocity relaxation. The calculation is restricted to the solution of the integral equations that describe kinetic modes in the one-polynomial approximation. The other integral equations of the theory are solved in the one- or two-polynomial approximation.

Since the Landau kinetic equation describes only the short-range part of the Coulomb interaction with a logarithmic accuracy, we discuss two approaches taking the long-range part of the Coulomb interaction into account.

The first approach proposes a description of the long-range effects by the intrinsic degrees of freedom of the electromagnetic field in plasma following the ideas of D. Bohm and D. Pines. To overcome the high dimensionality of the system, it is proposed to describe the subsystems of the electromagnetic field and the charged particles by their modes. In doing so, one needs *the modes of the Landau kinetic equation investigated in the present paper*. An example of the use of subsystem modes in an investigation of the modes of a complex system is given in our paper [21].

The second approach describes the long-range effects by adding the Vlasov term to the Landau kinetic equation. This gives the Vlasov–Landau kinetic equation. At the moment, we cannot construct a complete theory of hydrodynamic states of this equation based on the generalized Chapman–Enskog method. Probably, a physically adequate consideration of the problem should justify an understanding of plasma quasineutrality, assuming that the charge density of a nonequilibrium plasma is small. Here, we have investigated the modes of the Vlasov–Landau kinetic equation only in the *nondissipative hydrodynamic approximation* (i.e., without consideration of dissipative hydrodynamic processes: heat conductivity, viscosity, and diffusion). The possibility to solve this problem is related to the following statement: in spatially uniform states, a self-consistent field is absent, and one can calculate the plasma distribution functions only on the basis of the Landau kinetic equation. The distribution functions in the zero order in the gradients of the hydrodynamic variables are calculated above and *the plasma modes in the nondissipative hydrodynamic approximation are investigated with regard for the long-range properties of the Coulomb interaction.* In this process, the relaxation rate, related to the component relative velocity, plays the role of a friction constant, which leads to *the relaxation dumping of plasma oscillations and a shift of the plasma frequency.* These phenomena are related to the short-range part of the Coulomb interaction. The famous Landau damping of plasma oscillations is related to the long-range part of the Coulomb interaction. This follows from the possibility to obtain the Landau damping by considering the attenuation of long longitudinal electromagnetic waves in plasma. Our investigation shows that, *at small wave vectors, the attenuation of plasma oscillations is governed by the relaxation damping, while the Landau damping is negligibly small.*

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ГІДРОДИНАМІЧНІ, КІНЕТИЧНІ МОДИ ПЛАЗМИ І РЕЛАКСАЦІЙНЕ ЗГАСАННЯ ПЛАЗМОВИХ КОЛИВАНЬ

Р е з ю м е

Гідродинаміку повністю іонізованої двокомпонентної електрон-іонної плазми досліджено у випадку, коли релаксація температури та швидкості компонент близька до завершення. Обговорено проблему врахування в кінетиці плазми особливостей кулонівської взаємодії. Дослідження ґрунтується на кінетичному рівнянні Ландау та методі Чемпена–Енскога, узагальненому на основі ідеї функціональної гіпотези Боголюбова. Отримано нелінійні гідродинамічні рівняння. Побудовано лінеаризовані гідродинамічні рівняння і досліджено гідродинамічні та кінетичні моди кінетичного рівняння Ландау у гідродинамічному наближенні. Вивчено вплив релаксаційних процесів на еволюцію системи. На основі кінетичного рівняння Власова–Ландау досліджено моди плазми в недисипативному гідродинамічному наближенні. Деякі з них описують релаксаційне згасання плазмових коливань, яке при малих хвильових векторах $k \to 0$ є набагато більш вагоме за згасання Ландау.