doi: 10.15407/ujpe60.07.0585

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PACS 25.70.z, 25.70.Jj

MINIMUM BARRIER HEIGHT FOR SYMMETRIC AND ASYMMETRIC NUCLEAR SYSTEMS

The interaction potential in symmetric and asymmetric systems composed of two nuclei has been studied with regard for the quadrupole, octupole, and hexadecapole deformations of nuclei. The influence of dynamic multipole deformations of the nuclear surface on the barrier height and the interaction energy between two nuclei is considered. The deformation parameters corresponding to the minimum values of barrier height are evaluated.

Keywords: interaction potential, total interaction energy of nuclei, deformation energy, quadrupole nuclear deformation, octupole nuclear deformation, hexadecapole nuclear deformation, barrier height.

1. Introduction

The potential of nucleus-nucleus interaction is important while describing the collisions of heavy ions and at the formation of compound nuclei [1, 2]. The corresponding barrier height governs the mechanisms and probabilities of nuclear reactions, reactions of subbarrier fusion and synthesis of superheavy elements, as well as various near-barrier reactions [2–18].

The height of the nucleus-nucleus potential barrier depends on the deformation of nuclei and their relative orientation [2, 10–12, 14, 18]. In work [19], the minimum height of the barrier between nuclei was studied in detail, but taking only their quadrupole deformation into account. In this work, the influence of a surface deformation in nuclei of various types on the interaction potential in symmetric and asymmetric systems consisting of a light and a heavy nucleus will be analyzed. The minimum height of the potential barrier between two nuclei is considered in detail with regard for quadrupole, octupole, and hexadecapole deformations of the surfaces of both nuclei. For this purpose, the interaction energy for systems composed of two nuclei with surface deformations characterized by the multipolarities $l = 2$, 3, and 4 is calculated.

2. Interaction of Nuclei with Dynamically Perturbed Surfaces

In order to calculate the minimum height of the barrier at the collision of two nuclei, the dynamic deformation of the latter has to be taken into account. The total potential energy of interaction between the nuclei will be determined as a sum of the Coulomb and nuclear potentials and the deformation energy of each nuclei,

$$V_{\rm full}(R,\theta_1,\theta_2,\phi,\beta_{i2},\beta_{i3},\beta_{i4}) = \\ = V_{\rm coul}(R,\theta_1,\theta_2,\phi,\beta_{i2},\beta_{i3},\beta_{i4}) + \\ + V_{\rm nucl}(R,\theta_1,\theta_2,\phi,\beta_{i2},\beta_{i3},\beta_{i4}) + \\ + V_{\rm def}(\beta_{i2},\beta_{i3},\beta_{i4}), \qquad (1)$$

where R is the distance between the centers of mass of nuclei; θ_1, θ_2, and ϕ are the angles characterizing the relative orientation of the deformed axially symmetric nuclei; and β_{i2}, β_{i3}, and β_{i4} are the parameters of quadrupole, octupole, and hexadecapole dynamic deformations of the corresponding nucleus surface ($i = 1, 2$), which parametrize the radius of the nucleus surface:

$$R_i(\theta_0) = R_{i0}\,[1 + \beta_{i2} Y_{20}(\theta_0) + \\ + \beta_{i3} Y_{30}(\theta_0) + \beta_{i4} Y_{40}(\theta_0)]. \quad (2)$$

Here, R_{i0} is the radius of the i-th nucleus in the spherical approximation, and $Y_{l0}(\theta)$ are the spherical harmonics of the multipolarity l [20]. Multipole dynamic deformations of the nuclei arise as a result of the interaction between them, when they approach each other. The angle θ_0 is determined in the internal coordinate frame of the i-th nucleus. The angles $\theta_1$and θ_2 describe the orientations of the symmetry axes of the deformed nuclei with respect to the axis that passes through their centers of mass, and the angle ϕ describes the orientation of those axes in the plane that is perpendicular to the latter.

The potential of Coulomb interaction between two axially deformed nuclei depends on the angles that determine the relative orientation of the nuclei in space and on the parameters of a quadrupole deformation of nuclei. It looks like [2, 10, 11, 14, 19]

$$V_{\mathrm{coul}}(R, \theta_1, \theta_2, \phi, \beta_{i2}, \beta_{i3}, \beta_{i4}) = \\ = \frac{Z_1 Z_2 e^2}{R} \Big\{ 1 + \sum_{l=2,3,4} [f_{1l}(R, \theta_1, R_{10})\beta_{1l} + \\ + f_{1l}(R, \theta_2, R_{20})\beta_{2l}] + f_2(R, \theta_1, R_{10})\beta_{12}^2 + \\ + f_2(R, \theta_2, R_{20})\beta_{22}^2 + f_3(R, \theta_1, R_{10}, \theta_2, R_{20})\beta_{12}\beta_{22} + \\ + f_4(R, \theta_1, R_{10}, \phi, \theta_2, R_{20})\beta_{12}\beta_{22} \Big\}, \quad (3)$$

where Z_i is the number of protons in the i-th nucleus, e the proton charge, and

$$f_{1l}(R, \theta_i, R_{i0}) = \frac{3R_{i0}^l}{(2l+1)R^l} Y_{l0}(\theta_i), \quad (4)$$

$$f_2(R, \theta_i, R_{i0}) = \frac{6\sqrt{5}R_{i0}^2}{35\sqrt{\pi}R^2} Y_{20}(\theta_i) + \frac{3R_{i0}^4}{7\sqrt{\pi}R^4} Y_{40}(\theta_i), \quad (5)$$

$$f_3(R, \theta_1, R_{10}, \theta_2, R_{20}) = \frac{27R_{10}^2 R_{20}^2}{80\pi R^4} + \\ + [17\cos^2(\theta_1)\cos^2(\theta_2) - 5\cos^2(\theta_1) - 5\cos^2(\theta_2) + 1], \quad (6)$$

$$f_4(R, \theta_1, R_{10}, \phi, \theta_2, R_{20}) = \\ = \frac{27R_{10}^2 R_{20}^2}{40\pi R^4}\,[\cos^2(\phi)\sin^2(\theta_1)\sin^2(\theta_2) - \\ - 2\cos(\phi)\sin(2\theta_1)\sin(2\theta_2)] \quad (7)$$

are the functions describing the dependence of the potential on the orientation angles. This expression contains all terms that are linear or quadratic in the quadrupole deformation parameter of each nucleus. As a rule, the values of octupole and hexadecapole deformation parameters are smaller than that of the quadrupole deformation, being related by the equality $\beta_{i2}^2 \approx \beta_{l>2}$. Hence, the expression that contains simultaneously linear and quadratic terms with respect to the quadrupole deformation parameter and linear terms with respect to higher-multipolarity deformation parameters has accuracy of the order of β_{i2}^2. The next terms in the expansion will be corrections of the order of β_{i2}^3 or $\beta_2\beta_{l>2}$, i.e. too small to considerably affect the energy of interaction between the deformed nuclei. As a rule, the deformation parameters $\beta_{l>4}$ are less than the hexadecapole one. Therefore, the deformations of those types can be neglected.

Note that the center of mass in nuclei with the quadrupole and octupole deformations is shifted [2]. To avoid this, a dipole deformation, which is connected with quadrupole (hexadecapole) and octupole deformations by the relation $\beta_{il} \sim -\beta_{i2(4)}\beta_{i3}$ [2], is introduced. Since $\beta_{i2}^2 \approx \beta_{l>2}$, the magnitude of dipole deformation is $\beta_{il} \sim -\beta_{i2(4)}\beta_{i3} \sim -\beta_{i2}^3$. The terms of the order of β_{i2}^3 were neglected; therefore, the terms with a dipole deformation and the shift of the center of mass of nuclei should also be neglected.

The nuclear part of the interaction energy of the nuclei is obtained in the approximation of the so-called proximity theorem [21]. It is proportional to the interaction potential between the spherical nuclei $V(R)$,

$$V_{\mathrm{nucl}}(R, \theta_1, \theta_2, \phi, \beta_{12}, \beta_{22}) = \frac{C_{10} + C_{20}}{C_{\mathrm{def}}} V(R), \quad (8)$$

where $C_{\mathrm{def}} = \left[\left(C_1^{\parallel} + C_2^{\parallel}\right)\left(C_1^{\perp} + C_2^{\perp}\right)\right]^{1/2}$ is the generalized curvature, $C_{i0} = 1/R_{i0}$ is the curvature of the i-th spherical nucleus, and $V(R)$ is the nuclear part of the interaction potential between the spherical nuclei, when the distance d between their surfaces is equal to that between the deformed nuclei, and the distance between the centers of mass of interacting spherical nuclei is equal to $R = R_1 + R_2 + d$.

The interaction potential between the spherical nuclei is taken from work [22]:

$$V(R) = -1.989843CF(s)\,[1 + 0.003525139 \times \times (A_1/A_2 + A_2/A_1)^{3/2} - 0.4113263(I_1 + I_2)], \quad (9)$$

where $C = R_1R_2/R_{12}, s = R - R_{12} - 2.65$ fm, $R_{12} = R_1 + R_2$, and

$$R_{i0} = R_{ip}(1 - 3.413817/R_{ip}^2) + 1.284589 \times \times (I_i - 0.4A_i/(A_i + 200)). \quad (10)$$

The proton surface radius, R_{ip}, in Eq. (10) equals

$$R_{ip} = 1.24\, A_i^{1/3}\,(1 + 1.646/A_i - 0.191\, I_i), \quad (11)$$

where $I_i = (N_i - Z_i)\,/A_i$, and A_i and N_i are the numbers of nucleons and neutrons, respectively, in the i-th nucleus. The function $F(s)$ is approximated by the exponential dependence

$$F(s) = \left\{1 - s^2\left[0.0541026\,C\exp\left(-\frac{s}{1.760580}\right) - 0.5395420\,(I_1 + I_2)\exp\left(-\frac{s}{2.424408}\right)\right]\right\} \times \exp\left(\frac{-s}{0.7881663}\right), \quad (12)$$

in the case $s \geqslant 0$ (when the nuclei are located at a large distance from each other), and is parametrized by the polynomial

$$F(s) = 1 - \frac{s}{0.7881663} + 1.229218\,s^2 - 0.2234277\,s^3 - 0.1038769\,s^4 - C\,(0.1844935\,s^2 + 0.07570101\,s^3) + (I_1 + I_2)\,(0.04470645\,s^2 + 0.03346870\,s^3) \quad (13)$$

in the case of short distances between the nuclei $(5.65 \leqslant s \leqslant 0)$. This potential describes well the empirical barriers between spherical nuclei [22].

The generalized curvature C_{def} in Eq. (8) is connected with the curvatures $C_i^{\parallel}$ and $C_i^{\perp}$ at the nearest points of nuclei's surfaces. The expressions for those parameters look like [10]

$$C_1^{\parallel} = k_1 + k_1', \quad C_1^{\perp} = k_1 - k_1', \quad (14)$$

$$C_2^{\parallel} = k_2 + k_2'\cos(2\phi), \quad C_2^{\perp} = k_2 - k_2'\cos(2\phi). \quad (15)$$

The curvature parameters k_i and k_i' depend on the deformation parameters and the angles that describe the orientations of nuclei in space:

$$k_i(R_{i0}, \beta_{i2}, \eta',) \approx C_{i0}\left[1 + \sum_{l=2,3,4}\frac{l(l+1) - 2}{2} \times \beta_{il}Y_{l0}(\eta') - 5\,\beta_{i2}^2(Y_{20}(\eta'))^2 + \frac{\beta_{i2}^2}{4\pi}\right], \quad (16)$$

$$k_i'(R_{i0}, \beta_{i2}, \eta') \approx -C_{i0}\frac{3}{8\pi}\cos^2(\eta') \times \left[2\sqrt{5\pi}\beta_{20} + 5\beta_{20}^2 - 30\,\cos^2(\eta')\beta_{20}^2 + 15\sqrt{\pi}\beta_{40}(7\cos^2(\eta') - 1)\right], \quad (17)$$

where η' is the angle in the own coordinate frame that determines a point on nucleus' surface, which is the closest to the surface of the other nucleus.

In the liquid drop model, the deformation energy in the case of axial multipole deformation of the nuclear surface equals [23, 24]

$$V_{\text{def}}(\beta_{1l}, \beta_{2l}) = \frac{1}{2}\sum_{l=2,3,4}\left[\chi_{1l}^2(\beta_{1l}) + \chi_{2l}^2(\beta_{2l})\right], \quad (18)$$

where χ_{il} is the stiffness coefficient for the surface of the i-th nucleus at its deformation. This coefficient depends on the coefficient of surface tension σ and the Coulomb energy. In the case of l-multipole deformation, it looks like [23]

$$\chi_{il} = \frac{1}{4\pi}\,(l-1)\,(l+2)\,b_{\text{surf}}\,A_i^{2/3} - \frac{3}{2\pi}\frac{l-1}{2l+1}\frac{e^2}{r_0}\,Z_i^2\,A_i^{-1/3}, \quad (19)$$

where $b_{\text{surf}} = 4\pi\sigma r_0^2 = a_s(1 - k_sI^2)$, with the values of a_s, k_s, and r_0^2 taken from work [24].

3. Results of Calculations

The barrier between two nuclei has a minimum height, if they are prolate and oriented along the axis of their axial symmetry that connects their centers of mass at the Euler angles $\theta_1 = \phi = 0$ and $\theta_2 = \pi$ [2, 10, 11, 14, 21]. Therefore, the potential was calculated for this orientation of nuclei. Moreover, if $\theta_1 = 0$ and $\theta_2 = \pi$, both the Coulomb [Eq. (3)] and nuclear [Eq. (8)] contributions to the total interaction energy do not depend on ϕ.

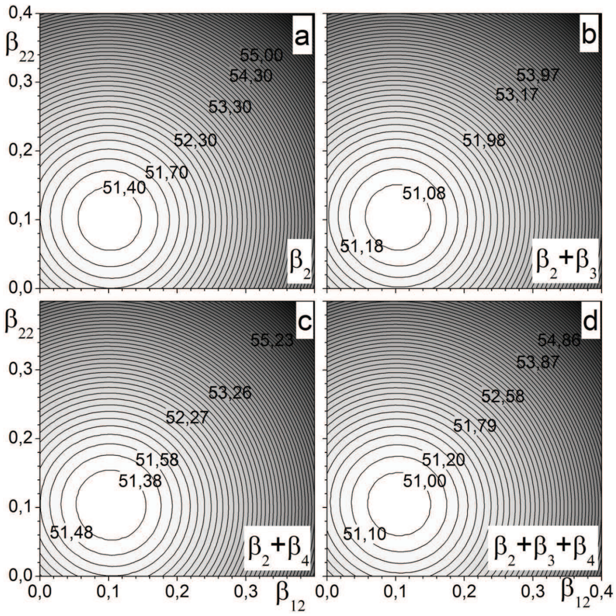

Fig. 1. Barrier dependences on the quadrupole deformation parameters for the ^{48}Ca + ^{48}Ca nuclear system calculated with regard for only the quadrupole deformation (*a*), the quadrupole and octupole deformations (*b*), the quadrupole and hexadecapole deformations (*c*), and the quadrupole, octupole, and hexadecapole deformations of nuclei (*d*). The barrier heights are indicated in MeV

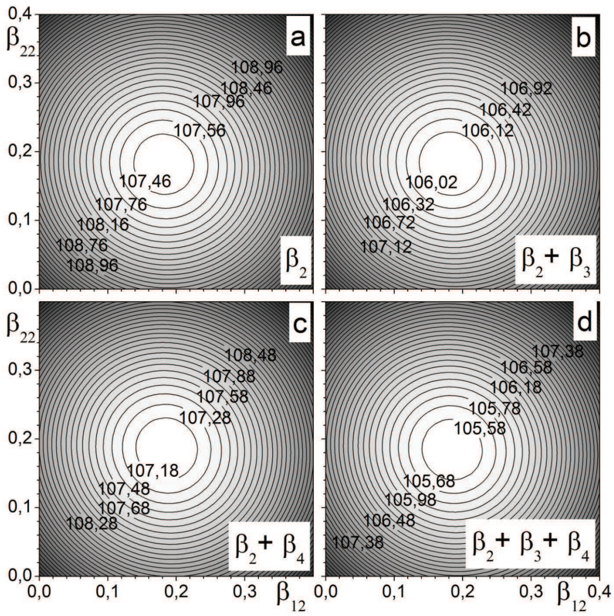

Fig. 2. The same as in Fig. 1, but for the symmetric nuclear system ^{68}Zn + ^{68}Zn

Let us consider the interaction potential in the following two-nuclear symmetric systems in detail: ^{48}Ca + ^{48}Ca, ^{68}Zn + ^{68}Zn, and ^{96}Zr + ^{96}Zr. In the ground state, those nuclei have a spherical shape or a shape close to it. While analyzing the potential, the assumption is made that the shapes of nuclei change, and the features of the interaction potential between the nuclei – in particular, the dependence of the minimum barrier height on the surface deformation parameters β_2, β_3, and β_4 – are studied.

In Fig. 1, the dependences of the barrier height on the quadrupole deformation parameters β_{12} and β_{22} of the nuclei are shown for various multipole deformations of nuclei's surfaces in the system ^{48}Ca + ^{48}Ca. Panel *a* demonstrates this dependence in the case where only the quadrupole deformation of both nuclei is taken into account. One can see that the account of this deformation diminishes the barrier height. The minimum barrier height value and the magnitudes of quadrupole deformation are quoted in Table 1.

In order to determine the influence of the octupole deformation of nuclear surfaces, the results obtained for the barrier minimum with regard for both the quadrupole and octupole deformations are demonstrated in Fig. 1, *b*. For every value of quadrupole deformation parameter, the selected value of octupole deformation corresponded to the barrier height min-

Table 1. **Minimum barrier heights and nuclear deformation parameters at the barrier minimum for symmetric systems ^{48}Ca + ^{48}Ca, ^{96}Zr + ^{96}Zr, and ^{68}Zn + ^{68}Zn**

System	R, fm	β_2	β_3	β_4	$V_{\min}$, MeV
^{48}Ca + ^{48}Ca	10.20	0	0	0	52.10
	10.63	0.100	0	0	51.32
	10.80	0.105	0.043	0	50,99
	10.67	0.105	0	0.005	51.28
	10.93	0.105	0.045	0.016	50.92
^{68}Zn + ^{68}Zn	10.81	0	0	0	110.53
	11.67	0.180	0	0	107.37
	11.98	0.185	0.078	0	105.92
	11.68	0.185	0	0.028	107.09
	12.00	0.185	0.079	0.032	105.49
^{96}Zr + ^{96}Zr	11.60	0	0	0	182.12
	12.94	0.265	0	0	174.38
	12.97	0.260	0.010	0	171.04
	12.75	0.265	0	0.029	173.85
	12.87	0.260	0.090	0.004	170.97

imum. One can see that the account for a octupole deformation slightly reduces the minimum barrier height (see Table 1).

The influence of a hexadecapole deformation on the barrier height is shown in Fig. 1, *c*. In this case, only the quadrupole and hexadecapole deformations are considered. It it evident that the influence of the latter on the barrier height is weaker than that of the octupole deformation.

The simultaneous account for the quadrupole, octupole, and hexadecapole deformations brings about the smallest value of barrier height (see Fig. 2, *d* and Table 1). The values of parameters β_3 and β_4 in Fig. 2 are selected to provide a barrier height minimum for a fixed β_2 value. The minimum barrier heights and the multipole deformations of surfaces with regard for various combinations of multipole deformations are also quoted in Table 1.

In Figs. 2 and 3, the analogous results obtained for the heavier symmetric systems ^{68}Zn + ^{68}Zn and ^{96}Zr + ^{96}Zr, respectively, are exhibited. The barrier height minima and the corresponding values of deformation parameters for the symmetric systems of nuclei are listed in Table 1. By comparing Figs. 1 to 3 and the data in Table 1, one can see that the minimum value of barrier height for the heavier system of nuclei corresponds to the larger value of quadrupole deformation.

The corresponding plots for the asymmetric system ^{62}Zn + ^{74}Zn are shown in Fig. 4. Comparing Figs. 2 and 4, we note that the nuclei interacting in the former (symmetric) system (Fig. 2) have identical atomic masses and identical numbers of protons and neutrons, whereas the nuclei in the asymmetric system (Fig. 4) have different numbers of neutrons and, accordingly, different atomic masses. The Coulomb energy of interaction is the same in both systems. Therefore, the difference between the barriers can follow only from different nuclear interactions in those systems owing to different numbers of neutrons in the interacting nuclei. The asymmetric system has a larger deformation and a higher minimum barrier height between the nuclei in comparison with the symmetric system (the deformation parameters and the minimum heights of interaction barriers between the nuclei are quoted for both systems in Tables 1 and 2). A higher barrier in the asymmetric system is associated with a reduction of the nuclear potential in comparison with the symmetric system. This

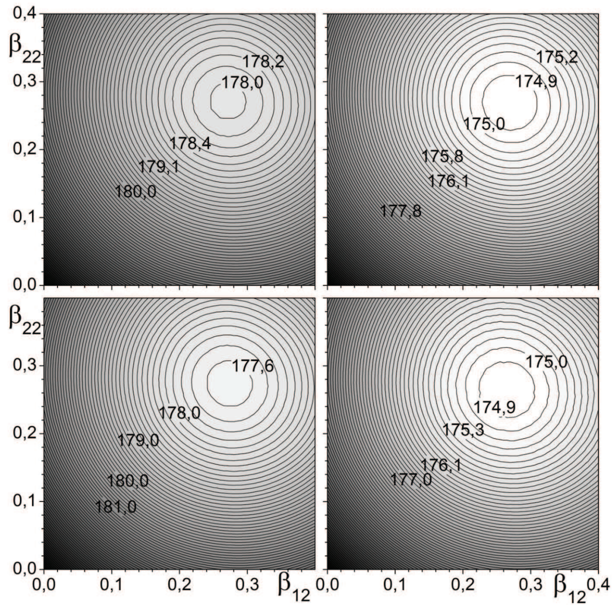

Fig. 3. The same as in Fig. 1, but for the symmetric nuclear system ^{96}Zr + ^{96}Zr

Table 2. **Minimum barrier heights and nuclear deformation parameters at the barrier minimum for the asymmetric systems ^{62}Zn + ^{74}Zn, ^{58}Fe + ^{78}Se, ^{40}Ar + ^{96}Mo, ^{20}Ne + ^{116}Sn**

System	R, fm	$\beta_1 2$	$\beta_2 2$	$\beta_1 3$	$\beta_2 3$	$\beta_1 4$	$\beta_2 4$	$V_{\min}$, MeV
^{62}Zn + ^{74}Zn	10.79	0	0	0	0	0	0	110.67
	11.66	0.185	0.185	0	0	0	0	107.46
	11.97	0.185	0.185	0.078	0.083	0	0	105.98
	11.69	0.190	0.190	0	0	0.025	0.031	107.16
	12.03	0.185	0.190	0.086	0.080	0.030	0.036	105.53
^{58}Fe + ^{78}Se	10.81	0	0	0	0	0	0	108.72
	11.65	0.180	0.180	0	0	0	0	105.65
	11.96	0.180	0.180	0.077	0.078	0	0	104.25
	11.69	0.185	0.185	0	0	0.025	0.030	105.37
	11.69	0.200	0.185	0.084	0.088	0.030	0.036	112.08
^{40}Ar + ^{96}Mo	10.74	0	0	0	0	0	0	93.73
	11.49	0.165	0.160	0	0	0	0	91.38
	11.76	0.165	0.160	0.068	0.070	0	0	90.33
	11.53	0.170	0.165	0	0	0.013	0.030	91.16
	11.90	0.170	0.165	0.070	0.072	0.023	0.033	89.99
^{20}Ne + ^{116}Sn	10.46	0	0	0	0	0	0	63.73
	11.00	0.125	0.120	0	0	0	0	62.61
	11.16	0.130	0.120	0.042	0.051	0	0	62.15
	11.00	0.125	0.120	0	0	0	0.022	62.49
	11.27	0.130	0.125	0.041	0.053	0	0.025	61.98

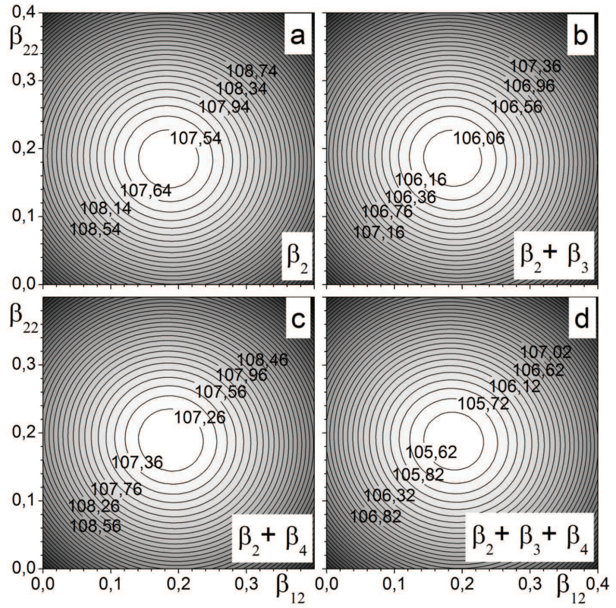

Fig. 4. The same as in Fig. 1, but for the asymmetric nuclear system ^{62}Zn + ^{74}Zn

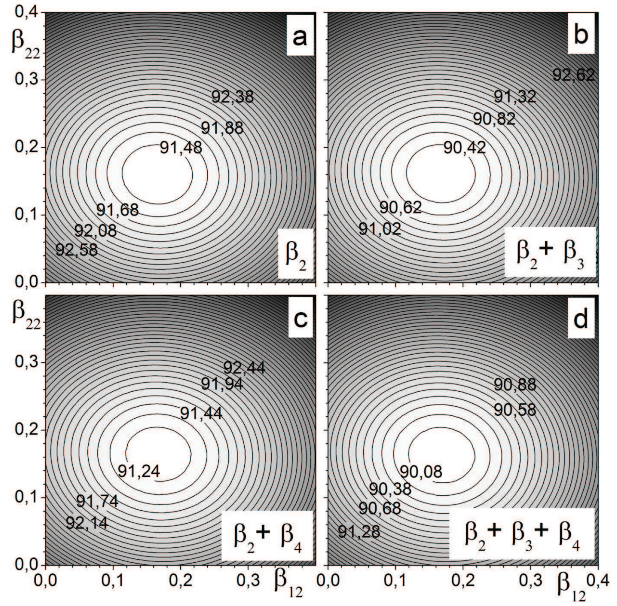

Fig. 6. The same as in Fig. 1, but for the asymmetric nuclear system ^{40}Ar + ^{96}Mo

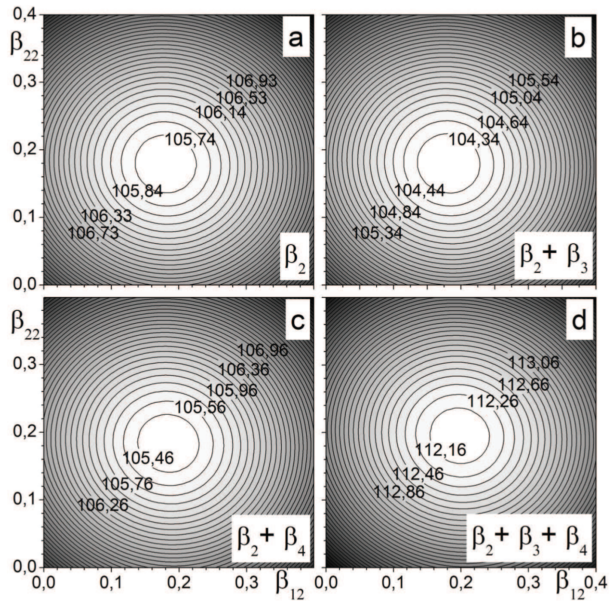

Fig. 5. The same as in Fig. 1, but for the asymmetric nuclear system ^{58}Fe + ^{78}Se

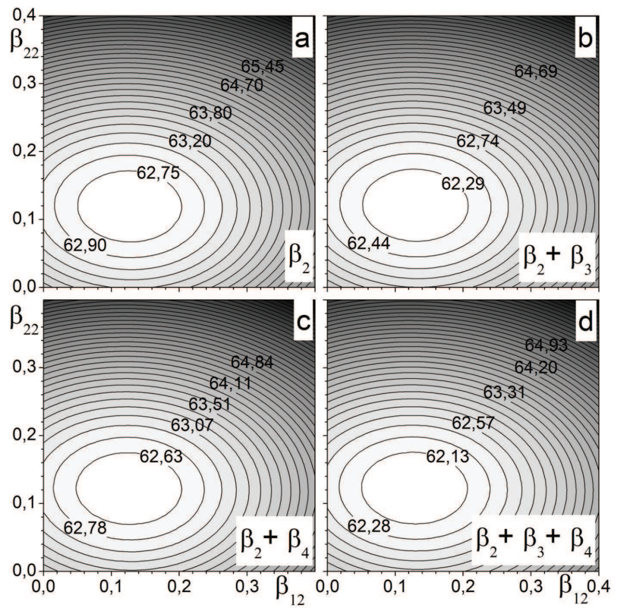

Fig. 7. The same as in Fig. 1, but for the asymmetric nuclear system ^{20}Ne + ^{116}Sn

means that the number of neutrons in the nuclei affects the magnitude of nucleus-nucleus interaction. In addition, the increase in the neutron asymmetry in nuclei gives rise to the growth of a nuclear deformation and the barrier height minimum magnitude.

The interaction potentials for the asymmetric systems ^{62}Zn + ^{74}Zn, ^{58}Fe + ^{78}Se, ^{40}Ar + ^{96}Mo, and ^{20}Ne + ^{116}Sn at various dynamic deformations of nuclear surfaces are depicted in Figs. 4 to 7, respectively. For the sake of comparison, the data on the minimum barrier heights and the corresponding parameters of quadrupole, octupole, and hexadecapole surface deformations of each nucleus are quoted in Table 2. The interacting nuclei have different numbers of nucleons. Each system consists of a light and a heavy nucleus, being a result of the division of ^{136}Nd nucleus, which can be formed, e.g., at the fusion of ^{40}Ca and ^{96}Zr nuclei.

The interaction potentials of nuclei in the case where only the quadrupole deformation is taken into account are shown for each asymmetric system in panels *a* of Figs. 4 to 7. In order to better estimate the influence of an octupole deformation, the dependences of the nuclear interaction potential on only the quadrupole and octupole deformations are shown in panels *b* for each asymmetric system under consideration. For every value of quadrupole deformation, we selected such an octupole deformation that provided the appearance of the minimum barrier height. The dependences of the minimum barrier height on the quadrupole deformation with regard for a hexadecapole deformation of the nuclear surfaces are exhibited in panels *c*. The barrier height minimum (see panels *d* and Table 2) is the smallest, when the quadrupole, octupole, and hexadecapole deformations are taken into account simultaneously. The values of β_3 and β_4 in panels *d* correspond to the minimum barrier height at a fixed value of β_2.

From the analysis of Figs. 4–7 and the data in Table 2, one can see that the account of deformation parameters is important, while determining the interaction potential in symmetric and asymmetric systems of nuclei. For all examined symmetric and asymmetric systems, the minimum barrier height is observed at dynamic deformations of nuclei. The obtained values of those deformation parameters are shown in Figs. 1 to 7 and in Tables 1 and 2. The corresponding analysis demonstrates that, for the symmetric and asymmetric systems of nuclei, the difference between the barrier heights for the spherical and deformed nuclei grows with the nuclear mass and charge.

The quadrupole deformation of the surface is shown to have the largest influence on the barrier height minimum for all examined systems, whereas the octupole deformation gave a smaller effect. The influence of a hexadecapole deformation is insignificant. The effect of octupole and hexadecapole deformations of the nuclear surfaces decreases, as the masses and charges of nuclei increase, provided the minimum potential value, which is responsible for the increase of the stiffness χ_{il} for larger l and A (see Eq. (18)).

Note that the values of quadrupole nuclear deformation parameter corresponding to the barrier height minimum weakly depend on whether the deformations of higher multipolarities are taken into account or not, both for symmetric and asymmetric systems. The parameters of quadrupole deformation that correspond to the minimum of a barrier height for deformed nuclei grow with the mass and charge of interacting nuclei. In the case of asymmetric nuclear systems, the magnitude of quadrupole deformation for heavy nuclei exceeds that for light nuclei (see Tables 1 and 2).

The values of octupole and hexadecapole deformation parameters corresponding to the minimum barrier height for deformed nuclei also grow with the mass and charge of interacting nuclei. For the asymmetric systems of nuclei, the octupole and hexadecapole deformation parameters for a heavy nucleus exceed the corresponding value for a light nucleus (see Tables 1 and 2).

The number of neutrons in nuclei with identical numbers of protons affects the magnitudes of nuclear surface deformation parameters and the barrier height minimum. The barrier height is the smallest for the symmetric systems.

Hence, a nuclear deformation substantially reduces the height of the barrier between the nuclei in comparison with the barrier between the spherical nuclei. This circumstance gives rise to a considerable enhancement of a subbarrier fusion, which was observed experimentally [2, 4–6, 12, 14]. Therefore, while researching the process of fusion in various dinuclear systems and analyzing various near-barrier binary reactions, the account for dynamic deformations of nuclei is mandatory.

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Received 23.03.15.
Translated from Ukrainian by O.I. Voitenko

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МІНІМАЛЬНА ВИСОТА
БАР'ЄРУ ДЛЯ СИМЕТРИЧНИХ
ТА НЕСИМЕТРИЧНИХ СИСТЕМ ЯДЕР

Р е з ю м е

Досліджено потенціал взаємодії симетричних та несиметричних систем двох ядер з урахуванням параметрів квадрупольної, октупольної та гексадекапольної динамічних деформацій ядер. Розглянуто вплив динамічних деформацій поверхні ядра з різною мультипольністю на висоту бар'єра та енергію взаємодії двох ядер, а також знайдено значення параметрів таких деформацій для бар'єрів з найменшою висотою.