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CORRELATION FUNCTION AND SUSCEPTIBILITY OF ISING MAGNET IN A VICINITY OF THE PHASE TRANSITION POINT

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The application of the method of collective variables to study the behavior of non-universal characteristics of a three-dimensional Ising-like system in the critical region has been illustrated by an example of the correlation function and the susceptibility. An analytic procedure for the calculation of those characteristics has been developed in the quartic-distribution approximation for order parameter fluctuations. The asymptotics of the correlation function at large distances obtained for the critical temperature ($T = T_c$) is shown to differ qualitatively from that in the $T \neq T_c$ case because of the presence of the critical regime region for all fluctuation modes.

K e y w o r d s: three-dimensional Ising-like system, phase transition point, non-Gaussian distribution, correlation function, susceptibility.

1. Introduction

The paper is devoted to the calculation of some structural characteristics of a three-dimensional Ising-like system in a vicinity of the phase transition point. The mathematical description is carried out in the framework of the method of collective variables (CV) [1–3] with the use of a non-Gaussian distribution for the order parameter fluctuations. The calculations were performed both with regard for a correction for the averaging of the Fourier transform of the interaction potential $\tilde{\Phi}(k)$, i.e. making allowance for the wave-vector dependence of the Fourier transform of the potential, and without it.

The dependence of $\tilde{\Phi}(k)$ on the wave vector at the calculation of the partition function of the system considerably affects the behavior of the pair correlation function G. It is known (see, e.g., work [4]) that, at the critical temperature $T = T_c$, this function is characterized by a critical exponent η at large distances r, i.e. $G \sim r^{-(d-2+\eta)}$, where d is the space dimensionality. Substituting the quantity $\tilde{\Phi}(k)$ in every layer of the CV phase space by its average value, we obtain that $\eta = 0$. A correction connected with the dependence of the Fourier transform of the potential on the wave vector results in a non-zero η-value. Expressions for the pair correlation function and the susceptibility are also changed both at $T = T_c$ and at temperatures different from T_c.

The Fourier transform of the correlation function $G(T, k)$ in the limit $k \to 0$ is connected with the susceptibility χ by the relation $\lim_{k\to 0} G(T, k) = \beta\chi$. Here, $\beta = 1/(kT)$ is the inverse temperature.

The expressions for the correlation function and the susceptibility of the system were obtained for the temperatures $T > T_c$, $T < T_c$, and $T = T_c$. This work logically supplements our previous researches [5, 6], where a method of determination of the small critical exponent η of the correlation function was developed and used. This supplement makes the general picture of researches comprehensive and the character of the researches more complete, because the developed approach is suitable for the calculation of not only the correlation function exponent, but also the correlation function itself.

2. Correlation Function and Susceptibility of the System Above and Below T_c

The calculation of the pair correlation function near the second-order phase transition point presumes a research of its dependence on the distance between particles, when the latter tends to infinity. In the CV method, it is convenient to work with the Fourier transform of the correlation function $G(T, k)$. It is re-

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lated to the partition function of the system by the formula

$$G(T,k) = -\frac{1}{Z}\frac{\partial Z}{\partial d(k)}. \quad (1)$$

In what follows, we use an expression for the partition function in the quartic measure density approximation (the ρ^4 model), which was presented in works [1, 3, 5]:

$$Z = C\int \exp\left\{-\frac{1}{2}\sum_{\mathbf{k}} d(k)\rho_{\mathbf{k}}\rho_{-\mathbf{k}} - \right.$$

$$\left. - \frac{1}{4!}\frac{a_4}{N}\sum_{\mathbf{k}_1,\ldots,\mathbf{k}_4}\rho_{\mathbf{k}_1}\ldots\rho_{\mathbf{k}_4}\delta_{\mathbf{k}_1+\ldots+\mathbf{k}_4}\right\}(d\rho)^N. \quad (2)$$

Correlation functions near the phase transition point were calculated by many researchers [1, 7, 8]. As a rule, the calculations were carried out, by using the Gaussian measure as a basis one in the framework of the Green's function or other methods [9–12]. The specific feature of the approach to the calculation of correlation functions, which is described below, consists in the application of a non-Gaussian fluctuation distribution as a basis one. The behavior of $G(T,k)$ at $k \to 0$, which corresponds to large distances, is the most important if the critical properties of the system are analyzed. When calculating $G(T,k)$ in the case $T > T_{\rm c}$, it is convenient to present the partition function in the form

$$Z = Z_0 Z_{CR} Z_{TR} Z'. \quad (3)$$

Here, Z_0 is the partition function for the system of noninteracting spins, and Z_{CR} corresponds to the contribution of non-Gaussian fluctuations. The multipliers Z_{TR} and Z' describe long-wave fluctuations, with the component Z' corresponding to extremely large distances, $k \to 0$ (its expression can be found in work [5]). Hence, while calculating the Fourier transform of the pair correlation function (1) at large distances, $r \to \infty$, it is convenient to use the formula

$$G(T,k)\big|_{k\to 0} = -\frac{1}{Z'}\frac{\partial Z'}{\partial \tilde{d}_{p_\tau+1}(k)}, \quad (4)$$

in which the functional derivative with respect to $\tilde{d}_{p_\tau+1}(k)$ is related to the long-wave part of the partition function (3). The notation $p_\tau = m_\tau + m'' + 1$ (see work [3]) was introduced to shorten the expression.

Let us write the Fourier transforms of the pair correlation function and the susceptibility with the use of the quartic basis distribution of fluctuations. At the first stage of calculations, the correction for the averaging of the Fourier transform of the interaction potential will be neglected, i.e. we suppose that $\Delta\tilde{\Phi}(k) = 0$ (see work [5]). Then, the quantity $\alpha^{(0)}$ (see formula (45) in work [5]), which was used to define the critical exponent η of the correlation function, vanishes. As a result, we obtain the condition $\eta = 0$. At $T > T_{\rm c}$, expression (4) takes the form

$$G(T,k) = \left[\tilde{d}_{p_\tau+1}(k)\right]^{-1}, \quad (5)$$

where the quantity $\tilde{d}_{p_\tau+1}(k)$ is defined in works [3, 5]. Using an explicit expression for $\tilde{d}_{p_\tau+1}(k)$ as a function of the temperature and the wave vector, we obtain that, as $k \to 0$,

$$G(T,k) = \frac{1}{D\tau^{2\nu} + D_1 k^2}. \quad (6)$$

Here,

$$D = \left(\frac{c_{1k}}{f_0}\right)^{2\nu} s^{-2m''}\beta\tilde{\Phi}(0)g_0[1 + (g_1 - 2\nu m_2)\tau^{\Delta}],$$
$$D_1 = 2\beta\tilde{\Phi}(0)b^2, \quad (7)$$

ν is the critical exponent of the correlation length, b is the range of the exponentially decreasing interaction potential, s is the parameter of the CV phase space division into layers, and $\tau = (T - T_{\rm c})/T_{\rm c}$ is the relative temperature. The other quantities in Eqs. (7), in particular, $m_2 = -c_{2k}(c_{1k}/f_0)^{\Delta}\Phi_0$, were defined in work [3]. Let us rewrite Eq. (6) in the form

$$G(T,k) = \frac{D_1^{-1}}{\varkappa_+^2 + k^2} \quad (8)$$

and identify the quantity $\varkappa_+$ with the inverse correlation radius ξ_+. For the latter, we find the expression

$$\xi_+ = \xi_+^{(0)}\tau^{-\nu}(1 + a_\xi^+\tau^{\Delta}). \quad (9)$$

Here, the quantity

$$\xi_+^{(0)} = \left(\frac{f_0}{c_{1k}}\right)^{\nu} b s^{m''}\left(\frac{2}{g_0}\right)^{1/2} \quad (10)$$

is the leading critical amplitude, and

$$a_\xi^+ = \nu m_2 - \frac{g_1}{2} \quad (11)$$

is the correlation length amplitude, which determines the correction to scaling (the confluent correction). The exponent of the scaling correction Δ

depends on eigenvalues of the renormalization-group linear-transformation matrix [3]. The susceptibility at $T > T_c$ can be obtained from the relation

$$\chi_+ = kT \lim_{k\to 0} G(T,k) \tag{12}$$

and takes the form

$$\chi_+ = \chi_+^{(0)} \tau^{-2\nu}(1 + a_\chi^+ \tau^\Delta), \tag{13}$$

where

$$\chi_+^{(0)} = kT \left(\frac{f_0}{c_{1k}}\right)^{2\nu} \frac{s^{2m''}}{\beta\tilde{\Phi}(0) g_0}, \qquad a_\chi^+ = 2\nu m_2 - g_1. \tag{14}$$

Note that our estimate

$$a_\xi^+ / a_\chi^+ = 0.5 \tag{15}$$

for the ratio between the amplitudes of confluent corrections for the correlation radius, a_ξ^+, and susceptibility, a_χ^+, is in agreement with the data obtained by other authors. For instance, in the framework of field theory, the value $a_\xi^+ / a_\chi^+ = 0.65 \pm 0.05$ was obtained at $d = 3$ [14]. The application of the high-temperature expansion brings about a result 0.70 ± 0.03 [14], whereas the method of ϵ-expansion (to the second order in ϵ) gives values in the interval of $0.43 - 0.71$ depending on the Padé approximants [13, 15].

Let us consider a case where the temperature is lower the than critical one. Then, the partition function of the model can be written in the form

$$Z = Z_0 Z_{CR} 2^{(N_{\mu_\tau+1}-1)/2} [Q(P_{\mu_\tau})]^{N_{\mu_\tau+1}} Z_{\mu_\tau+1}. \tag{16}$$

In order to calculate the Fourier transform of the correlation function $G'(T,k)$, it is important to know $Z_{\mu_\tau+1}$, since

$$G'(T,k)\big|_{k\to 0} = -\frac{1}{Z_{\mu_\tau+1}} \frac{\partial Z_{\mu_\tau+1}}{\partial \bar{d}_{\mu_\tau+1}(k)}. \tag{17}$$

An expression for $Z_{\mu_\tau+1}$ was given in work [3]. Using Eq. (17), we find

$$G'(T,k) = \left[\bar{d}_{\mu_\tau+1}(k)\right]^{-1}. \tag{18}$$

Then, using the result of calculations for $\bar{d}_{\mu_\tau+1}(k)$ (see work [3]), we obtain

$$G'(T,k) = \frac{1}{D'|\tau|^{2\nu} + D_1 k^2}, \tag{19}$$

where

$$D' = 4f_0 \left(\frac{c_{1k}}{f_0}\right)^{2\nu} \beta\tilde{\Phi}(0) \left[1 - c_{2k} \left(\frac{c_{1k}}{f_0}\right)^{\Delta} 2\nu\Phi_0 |\tau|^\Delta\right], \tag{20}$$

$$D_1 = 2\beta\tilde{\Phi}(0) b^2.$$

In view of this expression for $G'(T,k)$, we find the correlation radius at $T < T_c$:

$$\xi_- = \xi_-^{(0)} |\tau|^{-\nu} (1 + a_\xi^- |\tau|^\Delta). \tag{21}$$

Here, the quantities ν and Δ are the same as in Eq. (9), and

$$\xi_-^{(0)} = b \left(\frac{f_0}{c_{1k}}\right)^{\nu} (2f_0)^{-1/2}, \qquad a_\xi^- = c_{2k} \left(\frac{c_{1k}}{f_0}\right)^{\Delta} \nu\Phi_0. \tag{22}$$

For the susceptibility at $T < T_c$, we have

$$\chi_- = \chi_-^{(0)} |\tau|^{-2\nu} (1 + a_\chi^- |\tau|^\Delta), \tag{23}$$

where

$$\chi_-^{(0)} = kT \left(\frac{f_0}{c_{1k}}\right)^{2\nu} \frac{1}{\beta\tilde{\Phi}(0)} (4f_0)^{-1}, \qquad a_\chi^- = 2c_{2k} \left(\frac{c_{1k}}{f_0}\right)^{\Delta} \nu\Phi_0. \tag{24}$$

Similarly to the case $T > T_c$, we come to the relation

$$a_\xi^- / a_\chi^- = 0.5. \tag{25}$$

Unlike the critical exponents ν and Δ, the leading critical amplitudes $\xi_\pm^{(0)}$ and $\chi_\pm^{(0)}$, as well as the correction-to-scaling amplitudes $a_\xi^\pm$ and $a_\chi^\pm$, depend on the microscopic parameters of the Hamiltonian. This dependence is contained in the multipliers c_{1k} and c_{2k}, whose expressions are given in work [3]. However, the ratios of amplitudes at temperatures above and below T_c are universal quantities. Besides equalities (15) and (25), we have

$$\xi_+^{(0)} / \xi_-^{(0)} = 2s^{m''} \left(\frac{f_0}{g_0}\right)^{1/2} \tag{26}$$

and

$$\chi_+^{(0)} / \chi_-^{(0)} = 4s^{2m''} \frac{f_0}{g_0}. \tag{27}$$

It should be noted that the ratios

$$\frac{a_\xi^+}{a_\xi^-}, \quad \frac{a_\chi^+}{a_\chi^-}, \quad \frac{a_\xi^+}{a_c^+}, \quad \frac{a_\xi^-}{a_c^-}, \tag{28}$$

and many other combinations of critical amplitudes are also universal [16–20]. The universal character of the ratios between confluent corrections amplitudes of type (28) in the CV method is provided by a reduction of the non-universal factor $c_{2k}(c_{1k}/f_0)^{\Delta}$.

The relations given above were obtained in the approximation $\eta = 0$. Making allowance for the dependence of the Fourier transform of the potential $\tilde{\Phi}(k)$ on the wave vector at integrating the partition function over the CVs allows similar calculations to be performed at $\eta \neq 0$. In this case, expressions (5) and (18) remain the same, but the quantities $d_n(k)$ are changed. In the case $T > T_c$, neglecting the confluent correction, we find in the limit $k \to 0$ that

$$\tilde{G}(T,k) = \frac{1}{\tilde{D}\tau^{\tilde{\gamma}} + \tilde{D}_1 k^2}, \tag{29}$$

where

$$\begin{aligned} \tilde{D} &= \left(\frac{\tilde{c}_{1k}}{\tilde{f}}\right)^{\tilde{\gamma}} \beta\tilde{\Phi}(0) s^{-2m''} \tilde{g}_0, \\ \tilde{D}_1 &= 2\beta\tilde{\Phi}(0) b^2 \left(\frac{\tilde{c}_{1k}}{\tilde{f}}\right)^{-\eta\tilde{\nu}} \tau^{-\eta\tilde{\nu}}, \end{aligned} \tag{30}$$

and $\tilde{\gamma} = (2-\eta)\tilde{\nu}$. The exponent $\tilde{\nu}$ together with other quantities in Eqs. (30) is defined in works [3, 5]. The correlation radius $\tilde{\xi}_+$ obtained with regard for the potential averaging correction takes the form

$$\tilde{\xi}_+ = \tilde{\xi}_+^{(0)} \tau^{-\tilde{\nu}}, \tag{31}$$

where the critical amplitude $\tilde{\xi}_+^{(0)}$ is defined by the expression

$$\tilde{\xi}_+^{(0)} = \left(\frac{\tilde{f}}{\tilde{c}_{1k}}\right)^{\tilde{\nu}} b s^{m''} \left(\frac{2}{\tilde{g}_0}\right)^{1/2}. \tag{32}$$

The susceptibility calculated from expression (29) in the limit $k \to 0$ can be presented in the form

$$\tilde{\chi}_+ = \tilde{\chi}_+^{(0)} \tau^{-\tilde{\gamma}}. \tag{33}$$

Here, the critical amplitude

$$\tilde{\chi}_+^{(0)} = 2kT\tilde{\gamma}_4^+ \tag{34}$$

is characterized by the coefficient

$$\tilde{\gamma}_4^+ = \left(\frac{\tilde{f}}{\tilde{c}_{1k}}\right)^{\tilde{\gamma}} \frac{s^{2m''}}{2\beta\tilde{\Phi}(0)\tilde{g}_0}. \tag{35}$$

Expressions (32) and (34) transform into Eqs. (10) and (14), respectively, if the correction connected with the wave-vector dependence of the Fourier transform of the potential tends to zero.

In the case where $T < T_c$ and $\eta \neq 0$, the Fourier transform of the correlation function is given by the formula

$$\tilde{G}'(T,k) = \frac{1}{\tilde{D}'|\tau|^{\tilde{\gamma}} + \tilde{D}_1 k^2}, \tag{36}$$

where

$$\begin{aligned} \tilde{D}' &= 4\tilde{f} \left(\frac{\tilde{c}_{1k}}{\tilde{f}}\right)^{\tilde{\gamma}} \beta\tilde{\Phi}(0), \\ \tilde{D}_1 &= 2\beta\tilde{\Phi}(0) b^2 \left(\frac{\tilde{c}_{1k}}{\tilde{f}}\right)^{-\eta\tilde{\nu}} |\tau|^{-\eta\tilde{\nu}}. \end{aligned} \tag{37}$$

On the basis of Eq. (36), the following expressions for the correlation radius $\tilde{\xi}_-$ and the susceptibility $\tilde{\chi}_-$ are obtained:

$$\tilde{\xi}_- = \tilde{\xi}_-^{(0)} |\tau|^{-\tilde{\nu}}, \quad \tilde{\chi}_- = \tilde{\chi}_-^{(0)} |\tau|^{-\tilde{\gamma}}. \tag{38}$$

The expressions for the leading critical amplitudes $\tilde{\xi}_-^{(0)}$ and $\tilde{\chi}_-^{(0)}$ are similar to the formulas for $\xi_-^{(0)}$ and $\chi_-^{(0)}$ in Eqs. (22) and (24), where the quantities f_0, c_{1k}, and ν have to be substituted by the quantities $\tilde{f}$, $\tilde{c}_{1k}$, and $\tilde{\nu}$ renormalized owing to the inequality $\Delta\tilde{\Phi}(k) \neq 0$.

Note that formulas (13) for the susceptibility χ_+ (the case $T > T_c$ with the confluent correction) and (33) for $\tilde{\chi}_+$ (the case $T > T_c$ with the correction for the averaging of the Fourier transform of the interaction potential), which were obtained with the help of the Fourier transform of the correlation function, coincide with the corresponding expressions for the susceptibilities, which found earlier on the basis of the free energy (see works [5, 21]), to within the factor $(kT)^2/\mu_B^2$, where μ_B is the Bohr magneton. Those susceptibilities have different dimensionalities. In particular, the susceptibilities χ_+ and $\tilde{\chi}_+$ were obtained in terms of kT units (or $\tilde{\Phi}(0)$ ones), whereas the susceptibilities determined on the basis of the free energy were calculated in $\mu_B^2/\tilde{\Phi}(0)$ units. Here, $\tilde{\Phi}(0)$ if the Fourier transform of the interaction potential at the zero wave vector.

3. Pair Correlation Function at $T = T_c$

In the calculations carried out for the correlation function above and below T_c, the presence of a re-

gion with either the limiting (at $T > T_c$) or inverse (at $T < T_c$) Gaussian regime was substantially used. Each of those regimes is characterized by a Gaussian basis measure. The specific feature of this measure is a non-analytic temperature dependence of the dispersion.

In the case $T = T_c$, there are strong correlations between spins that are located at arbitrarily large distances in the system. No region of the Gaussian regime arises at that. At $T = T_c$, the region of the critical regime characterized by the renormalization group symmetry exists for all fluctuation modes including $k = 0$. Therefore, expression (4) is inapplicable here for the calculation of $G(T_c, k)$.

The partition function for a system of Ising spins at $T = T_c$ looks like

$$Z = Z_0 Z_{CR}, \tag{39}$$

where

$$Z_{CR} = \prod_{n=0}^{\infty} \tilde{Q}_n, \tag{40}$$

and the expression for $\tilde{Q}_n$ is given in works [3,5]. The Fourier transform of the correlation function will be calculated for every separate n-th block structure. Let G_n be the average value of the correlation function for the n-th block structure. Let us introduce, instead of $G(k) = \langle \rho_{\mathbf{k}} \rho_{-\mathbf{k}} \rangle$, the quantity

$$G_n = \frac{1}{N_n - N_{n+1}} \sum_{B_{n+1} < k \le B_n} \langle \rho_{\mathbf{k}} \rho_{-\mathbf{k}} \rangle, \tag{41}$$

where the symbol $\langle ... \rangle$ means the averaging over the non-Gaussian-type interacting system. The quantity G_n is calculated using the relation

$$G_n = \frac{\beta}{N_n - N_{n+1}} \frac{\partial F_c}{\partial \tilde{d}_n(B_{n+1}, B_n)}. \tag{42}$$

The free energy of the system, F_c, looks like [22]

$$F_c = -kT_c \sum_{n=0}^{\infty} \Big[N_n \ln \tilde{Q}(d_n) + N_{n+1} \ln Q(P_n) \Big]. \tag{43}$$

A specific feature of the expressions given above for G_n and F_c is an unusual behavior of the quantities $\tilde{d}_n(B_{n+1}, B_n)$ and $\tilde{a}_4^{(n)}$ as functions of the block structure number n (see works [5, 22]). In the general case, the quantities α_n, $\tilde{r}_n$ and $\tilde{u}_n$, in terms of which the quantities $\tilde{d}_n(B_{n+1}, B_n)$ and $\tilde{a}_4^{(n)}$ are expressed, depend on the number n. Only at $T = T_c$, the quantity α_n is independent of n. Using the solutions of the renormalization group equations at $T = T_c$ for $\tilde{r}_n$ and $\tilde{u}_n$, we obtain

$$\begin{aligned} &\tilde{d}_n(B_{n+1}, B_n) = s^{-n(2-\eta)}(\tilde{r} + q + \tilde{c}_2 \tilde{R} \tilde{E}_2^n), \\ &\tilde{a}_4^{(n)} = s^{-2n(2-\eta)}(\tilde{u} + \tilde{c}_2 \tilde{E}_2^n). \end{aligned} \tag{44}$$

Here, $\tilde{r}$ and $\tilde{u}$ are the coordinates of a fixed point [5], $q = \bar{q}\beta\tilde{\Phi}(0)$, $\bar{q}$ corresponds to the average value of k^2 in the interval $(1/s, 1]$, $\tilde{R} = \tilde{R}_{12}/(\tilde{E}_2 - \tilde{R}_{11})$, $\tilde{c}_2 = \tilde{c}_{2k}(\beta_c \tilde{\Phi}(0))^2$, and $\tilde{R}_{ij}$ and $\tilde{E}_2$ are the elements and one of the eigenvalues of the renormalization-group linear-transformation matrix, respectively. In accordance with Eqs. (44) and taking into account that $\tilde{E}_2 < 1$, the quantity $x_n = \sqrt{3}\tilde{d}_n(B_{n+1}, B_n)(\tilde{a}_4^{(n)})^{-1/2}$ at large n looks like

$$x_n = \tilde{x} \left[1 + \left(\frac{\tilde{c}_2 \tilde{R}}{\tilde{r} + q} + \frac{\tilde{c}_2}{2\tilde{u}} \right) \tilde{E}_2^n \right], \tag{45}$$

where $\tilde{x} = \sqrt{3}(\tilde{r} + q)(\tilde{u})^{-1/2}$. The quantity $\tilde{E}_2^n$ can be presented in the form

$$\tilde{E}_2^n = s^{-n\tilde{\omega}}, \quad \tilde{\omega} = \tilde{\Delta}/\tilde{\nu}.$$

The exponent $\tilde{\nu} = \ln s / \ln \tilde{E}_1$ is determined by the larger eigenvalue of the renormalization group transformation matrix ($\tilde{E}_1 > 1$). It characterizes the behavior of the correlation length. The exponent $\tilde{\Delta} = -\ln \tilde{E}_2 / \ln \tilde{E}_1$ characterizes the correction to scaling. Therefore, neglecting the term proportional to $\tilde{E}_2^n$ in expression (45) means that the correction to scaling is not taken into consideration.

Let us consider a certain block structure with the number n_0. According to Eq. (42), the expression for G_n at $n = n_0$ should be sought from the relation

$$\begin{aligned} G_{n_0} = \frac{\beta}{N_{n_0} - N_{n_0+1}} \Bigg[&\frac{\partial F_c}{\partial \tilde{d}_{n_0}(B_{n_0+1, B_{n_0}})} + \\ &+ \left(\frac{\partial F_c}{\partial x_{n_0}} + \frac{\partial F_c}{\partial y_{n_0}} \frac{dy_{n_0}}{dx_{n_0}} \right) \frac{\partial x_{n_0}}{\partial \tilde{d}_{n_0}(B_{n_0+1}, B_{n_0})} \Bigg], \end{aligned} \tag{46}$$

taking formulas for $\tilde{Q}(d_n)$ and $Q(P_n)$ into account (see works [3, 22]). When calculating G_{n_0}, the main difficulty is connected with the fact that, besides the

derivatives of the quantities with the subscript n_0, Eq. (46) also contains the derivatives of the quantities with the subscripts n_0+1, n_0+2, and so on.

For the derivative in the first term on the right-hand side of Eq. (46), we can write

$$\frac{\partial F_c}{\partial \tilde{d}_{n_0}} = kT_c \frac{1-s^{-3}}{2} \sum_{n=n_0}^{\infty} N_n \frac{\partial \ln \tilde{d}_n(B_{n+1}, B_n)}{\partial \tilde{d}_{n_0}}. \quad (47)$$

In the limit of large n, the quantity $\tilde{E}_2^n$ quickly decreases. On the basis of Eqs. (44) to within $\tilde{E}_2^{n_0+m}$, we obtain the following simplified relations:

$$\begin{aligned} &\tilde{d}_{n_0+m}(B_{n_0+m+1}, B_{n_0+m}) = \\ &= s^{-m(2-\eta)} \tilde{d}_{n_0}(B_{n_0+1}, B_{n_0}), \\ &\tilde{a}_4^{(n_0+m)} = s^{-2m(2-\eta)} \tilde{a}_4^{(n_0)}. \end{aligned} \quad (48)$$

In the critical temperature case, using Eqs. (47) and (48), we come to the expression

$$\frac{1}{2(1-s^{-3})} \frac{1}{\tilde{d}_{n_0}(B_{n_0+1}, B_{n_0})},$$

which corresponds to the contribution of the first term on the right-hand side of Eq. (46) to G_{n_0}. Two other terms in Eq. (46) give no contribution to G_{n_0}, because $x_n \approx \tilde{x}$ and does not depend on $\tilde{d}_{n_0}(B_{n_0+1}, B_{n_0})$ ($\tilde{E}_2^{n_0+m} \ll 1$).

Hence, at $T = T_c$, we have

$$G_{n_0} = \frac{1}{2}(1-s^{-3})^{-1} [\tilde{d}_{n_0}(B_{n_0+1}, B_{n_0})]^{-1}. \quad (49)$$

Taking the equality

$$\tilde{d}_{n_0}(B_{n_0+1}, B_{n_0}) = (\tilde{r}+q) s^{-n_0(2-\eta)}$$

into account, we find

$$G_{n_0} = G_0 s^{n_0(2-\eta)}, \quad (50)$$

where

$$G_0 = \frac{1}{2} \frac{(1-s^{-3})^{-1}}{\tilde{r}+q}. \quad (51)$$

The quantity G_{n_0} is the Fourier transform of the correlation function for the n_0-th block structure.

In the course of the step-by-step integration of the partition function, every interval of wave-vector values $k \in (B_{n+1}, B_n]$, where $B_n = B' s^{-n}$ and $B' = (b\sqrt{2})^{-1}$, is assigned the average value

$$\langle k^2 \rangle_{B_{n+1}, B_n} = B'^2 \bar{q} s^{-2n}. \quad (52)$$

Earlier, we obtained the quantity G_{n_0} averaged in the n_0-th layer. It depends on the average wave-vector value in the n_0-th block structure. Its asymptotics at large n differs from expression (52), because the potential averaging correction is taken into account. It is this correction that gives rise to the appearance of extra factors

$$(1+\alpha_0)(1+\alpha_1)...(1+\alpha_{n-1})$$

in the term proportional to $k^2 \rho_{\mathbf{k}} \rho_{-\mathbf{k}}$ in the exponent of the distribution function (see work [5]). At $T = T_c$, we have $\alpha_0 = \alpha_1 = ... = \alpha_n = \alpha^{(0)}$. Accordingly, owing to the smallness of α_n, the product $(1+\alpha_0)...(1+\alpha_{n-1})$ can be approximated by $\exp(n\alpha^{(0)})$. Then, Eq. (52) takes the form

$$(1+\alpha^{(0)})^n \langle k^2 \rangle_{B_{n+1}, B_n} = s^{-n(2-\eta)} B'^2 \bar{q}, \quad (53)$$

where the equality $\alpha^{(0)} = \eta \ln s$ was used. Comparing Eqs. (52) and (53), we obtain the expression

$$s^{-n(2-\eta)} B'^2 \bar{q} = \langle k^{2-\eta} \rangle_{B_{n+1}, B_n}. \quad (54)$$

Hence, the correction for the potential averaging leads to the substitution of s^{-2n} by $s^{-n(2-\eta)}$ and corresponds to the renormalization of the exponent: $k^2 \to k^{2-\eta}$. As a result, at $T = T_c$, the Fourier transform of the correlation function for the n_0-th block structure in the limit $n_0 \to \infty$, which corresponds to small wave-vector magnitudes k, reads

$$G_c = \lim_{n_0 \to \infty} G_{n_0} = G^{(0)} \lim_{k \to 0} \langle k^{-2+\eta} \rangle, \quad (55)$$

where

$$G^{(0)} = \frac{\bar{q}}{4b^2} \frac{(1-s^{-3})^{-1}}{\tilde{r}+q}. \quad (56)$$

The quantity η corresponds to the critical exponent of the correlation function.

4. Conclusions

Important information on the behavior of physical systems can be obtained by calculating their correlation functions. Those functions describe main features that arise near the phase transition temperature [1,7,8,10]. This problem is especially challenging for the description of three-dimensional systems, for which an exact solution cannot be obtained, as a rule.

In this work, a new method has been proposed for the analytic calculation of the pair correlation function for a three-dimensional spin system with a one-component order parameter. The method is based on the functional differentiation of the partition function for the Ising magnet. The calculation of the susceptibility in a vicinity of the phase transition point is connected with the limit of the Fourier transform of the correlation function as $k \to 0$. Calculations were carried out in the high-temperature ($T > T_c$) and low-temperature ($T < T_c$) regions, as well as at $T = T_c$. The technique used to calculate the Fourier transform of the correlation function at $T = T_c$ differs from that used in the case of temperatures different from T_c. This difference results from the existence (at the critical temperature) of the critical regime region for all modes of spin moment density oscillations, including modes with the wave vector $k = 0$. This region of the renormalization group symmetry corresponds to the presence of strong correlations between spins located at arbitrarily large distances from one another. At $T = T_c$, only the critical regime region exists, and the region of the Gaussian regime does not arise, in contrast to the case $T \neq T_c$.

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КОРЕЛЯЦІЙНА ФУНКЦІЯ
ТА СПРИЙНЯТЛИВІСТЬ ІЗИНГОВОГО МАГНЕТИКА
В ОКОЛІ ТОЧКИ ФАЗОВОГО ПЕРЕХОДУ

Р е з ю м е

Застосування методу колективних змінних до вивчення поведінки неуніверсальних характеристик тривимірної ізингоподібної системи в критичній області проілюстровано на прикладі кореляційної функції та сприйнятливості. Аналітичну процедуру для розрахунку кореляційної функції та сприйнятливості системи розвинуто в наближенні четвірного розподілу флуктуацій параметра порядку. Показано, що асимптотика кореляційної функції на великих відстанях при критичній температурі ($T = T_c$) якісно відрізняється від випадку $T \neq T_c$ внаслідок наявності ділянки критичного режиму для всіх мод флуктуацій.