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ON PENDRY'S EFFECTIVE ELECTRON MASS

In 1996, J. Pendry, an English theoretical physicist put forward an idea about the dependence of the effective electron mass on the magnetic field, while interpreting the dielectric response of metal wire mesh structures. The idea was based on the well-known relation between the kinematic and canonical momenta of a charged particle moving in the magnetic field. In this paper, proceeding from the universal character of that relation, the applicability of Pendry's effective electron mass m_{eff} to the problem of electrons in metal mesh structures, as well as to a wide class of problems for charges moving in the magnetic field, has been examined. The general properties of m_{eff} following directly from its definition were found, and an analogy between the effective electron mass m_{eff} and m^ known in the solid-state theory was established. A physical interpretation of m_{eff} was proposed. It was demonstrated in several examples that, despite the generality of the defining relation for the effective mass m_{eff} , the use of m_{eff} beyond the problem of the dielectric response of metal wire mesh structures leads to incorrect results.*

Keywords: effective mass, magnetic field, canonical momentum, Hamiltonian.

1. Introduction

The concept of effective mass for charge carriers is widely used to describe various phenomena in metals and semiconductors. In particular, it is used in the theories of electroconductivity, Hall effect, and cyclotron resonance in metals [1–3].

In the solid-state theory, the effective mass of conduction electrons, m^* , is usually determined from their dispersion law $E(\mathbf{k})$, where E and \mathbf{k} are the energy and the wave vector of an electron, respectively. For example, in the the simplest case of isotropic quadratic dispersion law [3], we have

$$m^* \equiv \left(\frac{1}{\hbar^2} \frac{\partial^2 E}{\partial k^2} \right)^{-1}.$$

The difference between m^* and the rest mass of a free electron, m , is known to emerge owing to the interaction of conduction electrons in a solid with the periodic internal crystal field. As a result of this interaction, electrons in the solid are accelerated under

the action of external electromagnetic fields, however, not as free particles with the mass m , but as certain imaginary particles with the mass m^* . According to experimental data, the value of m^* can significantly (by 1–2 orders of magnitude) differ from m [4].

In the case of anisotropic dispersion, the effective mass of the electron depends on the direction of its acceleration. In this case, the dynamical properties of the electron are not characterized by a single scalar m^* , but a collection of three scalars m_i^* ($i = x, y, z$), which are reciprocals of the principal values of the tensor of inverse effective mass of electrons, [3]

$$(m^*)_{ij}^{-1} = \left(\frac{1}{\hbar^2} \frac{\partial^2 E}{\partial k_i \partial k_j} \right).$$

It is worth noting that the effective mass m^* (or m_i^*) is determined by specific features in the electron energy spectrum of that or another solid, being a function of the wave vector \mathbf{k} , temperature, and pressure [5].

In 1996, Pendry *et al.* [6], while considering the frequency dependence of the dielectric function of

metal wire mesh structures, introduced the concept of effective electron mass dependent on the magnetic field. Unlike the effective mass of electrons m^* , which is considered in the solid-state theory, the effective mass m_{eff} introduced in work [6] (below, it will be called Pendry's effective mass) is defined on the basis of the relation, known from the classical mechanics, between the kinematic and canonical momenta of a non-relativistic spinless charged particle moving in the magnetic field (similarly to works [6–8], the SI units are used below):

$$m_{\text{eff}}\mathbf{v} \equiv \mathbf{P}, \quad (1)$$

$$\mathbf{P} = \mathbf{p} + q\mathbf{A}, \quad (2)$$

where $\mathbf{p} \equiv m\mathbf{v}$ and $\mathbf{P} \equiv \partial L/\partial \mathbf{v}$ are the kinematic and canonical, respectively, momenta of the particle; m , \mathbf{v} , q , and L are its mass, velocity, and charge and the Lagrangian, respectively; and \mathbf{A} is the vector potential of the magnetic field ($\mathbf{B} = \text{rot } \mathbf{A}$). Using the idea of m_{eff} , Pendry *et al.* derived an analytical expression for the mesh plasma frequency ω_p , which is in good agreement with experimental data (see, e.g., works [9, 10]).

Pendry's effective mass m_{eff} cardinally differs from the mass m^* considered in the solid-state theory: not only by definition, but also by properties. For instance, in contrast to m^* , the value of m_{eff} can easily reach giant values of the order of $(10^4 \div 10^6)m$ [6, 7]. Moreover, as a result of the gage invariance of the vector potential \mathbf{A} entering Eq. (2), m_{eff} turns out an ambiguously defined quantity [11]. Taking this fact into account, the critical remarks were made concerning both the correctness of the definition of m_{eff} [11] and the reality of predictions on the basis of m_{eff} [12, 13]. Those remarks forced some authors to search for the ways of deriving the major results of work [6] without attracting the m_{eff} concept. The success attained in those searches [14–17] testifies that, despite the productivity of this concept demonstrated by Pendry *et al.*, its introduction is not required for the explanation of the dielectric response of a metal mesh structure. Nevertheless, in last books on metamaterials [7, 8], this response was explained, by engaging m_{eff} .

Can the notion of m_{eff} be used beyond the scope of work [6]? Relation (2) between \mathbf{p} and \mathbf{P} is rather general, being used not only in classical mechanics, but also in quantum one. Proceeding from its uni-

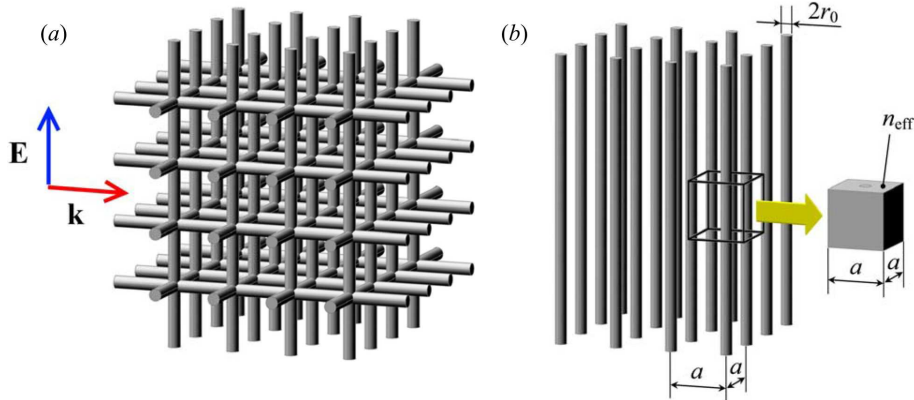
versal character and on the basis of the principle of integrity of physics, one may hope for that the concept of m_{eff} could be applied, while considering a wide range of problems (both classical and quantum-mechanical) concerning the motion of charges in a magnetic field. However, the possibility of its application has not been considered till now.

Earlier, the interpretation of m_{eff} was not considered. The absence of such an interpretation that would be compatible at least with the results of work [6] invokes a number of questions concerning the physical meaning of the effective mass m_{eff} . For example, does m_{eff} characterize the inertial properties of an electron in the magnetic field? Or does the relation $m_{\text{eff}} = 10^4 m$ mean that the inertial and, owing to the Einstein equivalence principle, gravitational masses of an electron increase in the magnetic field by four orders of magnitude? Statements like “Electrons become as heavy as hydrogen atoms” [6, p. 4775] are based on a free interpretation of the relation $m_{\text{eff}} = 10^4 m$ rather than the physical interpretation of m_{eff} itself, and more likely entangle the situation rather than clarify it.

It is worth to recall that analogous questions concerning the “ordinary” effective mass m^* of electrons in a solid arose when the solid-state theory was formulated in the 1930s, in particular, during the discussions concerning the interpretation of the results of classical experiments by Tolman and Stewart [18] and Barnett [19]. Answers to them were given only after the proper interpretation of m^* , which was based on its definition and properties, had appeared (see, e.g., work [20]).

The problem concerning the interpretation of the effective mass m_{eff} proposed by Pendry *et al.* could also be removed after a detailed analysis of the m_{eff} properties that follow from its definition. However, to the knowledge of the author of this work, nobody has carried out such an analysis till now.

This work was aimed at establishing the general properties of the effective mass m_{eff} , which follow directly from its definition; formulating a physical interpretation of m_{eff} , which would be compatible, at least, with the verified results of work [6]; and analyzing the application of m_{eff} beyond the problem of metal wire mesh structures, namely, in a wide range of problems concerning the motion of charges in the external magnetic field, when the relation between \mathbf{p} and \mathbf{P} in the definition of m_{eff} should manifest itself.



Scheme of a metal wire mesh structure excited by an external electromagnetic wave. Electric oscillations arise in the mesh conductors (they are called active), which are parallel to the vector \mathbf{E} of the incident wave. If the orientation of \mathbf{E} with respect to the mesh structure is arbitrary, all the conductors, generally speaking, are active (a). At the homogenization of the sublattice of active conductors in the cubic metal mesh structure, its unit cell, which covers a conductor segment with an arbitrary length, is considered as being filled with a homogeneous electron plasma with concentration n_{eff} (b)

The paper is organized as follows. In Section 2, the main results of work [6] concerning the effective mass m_{eff} of electrons in metal mesh structures and the plasma frequency ω_p of those meshes are presented. It was done not only to demonstrate a way, in which the concept of m_{eff} is used at the solution of a specific problem, but also to give the reader unfamiliar with this concept some initial information on m_{eff} , in particular, its order of magnitude and its dependence on the parameters of the problem. In Section 3, on the basis of definition (1), the general properties of m_{eff} are found, and an interpretation of m_{eff} is proposed. In Section 4, a number of examples illustrating the application of the m_{eff} notion in various situations associated with the motion of classical or quantum-mechanical charged particles in a magnetic field are considered.

2. Effective Mass of an Electron in Metal Wire Mesh Structures

The metal wire mesh structures are, as a rule, three-dimensional periodic structures fabricated from thin metal wire conductors (Figure, a). Such meshes have already been studied for more than half a century: first, as artificial insulators and, at the modern stage, as photonic crystals and metamaterials (see a short historical excursion in work [21]). In the branch of metamaterials, the wire meshes are more often called *mesh metamaterials* and usually applied as media with a negative effective dielectric permittivity ε_{eff} .

Since the dielectric function $\varepsilon_{\text{eff}}(\omega)$ of mesh structures has a Drude form (it was established as early as in the 1960s), negative values of ε_{eff} are attained at frequencies lower than the plasma frequency ω_p of the mesh. The plasma frequency ω_p depends on the mesh structure parameters. The knowledge of the explicit form of this dependence is important both for the mesh design and for the interpretation of experimental data. For the simplest model of a cubic mesh structure composed of infinitely thin ideal conductors, this dependence looks like

$$\omega_p = \sqrt{\frac{2\pi c^2}{a^2 \ln(a/r_0)}}, \quad (3)$$

where a is the mesh period, r_0 the radius of conductor wires ($r_0 \ll a$), and c the speed of light in vacuum. Expression (3) was derived for the first time in work [6] on the basis of the following reasoning.

For continuous media, the plasma frequency is known to be given by the Langmuir formula [22]

$$\omega_p = \sqrt{\frac{ne^2}{\varepsilon_0 m}}, \quad (4)$$

where n is the concentration of electrons in the medium, $m = 9.11 \times 10^{-31}$ kg is the electron mass, $e = 1.60 \times 10^{-19}$ C is the elementary charge, and $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m is the electric constant. Formula (4) can be elementarily derived, while considering the dynamics of electrons at plasma oscillations.

The plasma frequency of a cubic metal mesh structure equals that of the sublattice of its active conductors. This sublattice can be regarded as a certain continuous medium (Figure, *b*) with the effective electron concentration

$$n_{\text{eff}} = \frac{\pi r_0^2}{a^2} n, \quad (5)$$

where n is the electron concentration in the material of mesh conductors. In addition, it was proposed in work [6] to take into account that the dynamics of electrons in the mesh is influenced by the magnetic field of currents I , which arise in the active conductors of the mesh structure at plasma oscillations. This influence can be effectively presented as a change of the electron mass $m \rightarrow m_{\text{eff}}$, where m_{eff} is defined by formulas (1) and (2), in which \mathbf{A} is the vector potential of the magnetic field induced by the currents I .

For an infinite mesh structure, the vector potential \mathbf{A} of currents I is periodic. In every unit cell of the mesh centered at the conductor axis, it can be calculated by the formula [6, 9]

$$A(r) = \frac{\mu_0 I}{2\pi} \ln(a/r), \quad (6)$$

where r is the distance from the conductor axis ($r \in [r_0, a/2]$), and $\mu_0 = 4\pi \times 10^{-7}$ H/m is the magnetic constant. Taking into account that $I = \pi r_0^2 n e v$, where v is the drift velocity of electrons in the conductor, and that electrons move in ideal conductors only on their surface and, therefore, “feel” the field $A(r_0)$, we obtain

$$A(r_0) = \frac{\mu_0 r_0^2 n e v}{2} \ln(a/r_0).$$

For real mesh metamaterials, the term $eA(r_0)$ in Eq. (2) turns out much larger than the kinematic momentum mv of conduction electrons (see the numerical example below). Therefore, neglecting the latter, it is possible to assume that

$$m_{\text{eff}} = \frac{eA(r_0)}{v}$$

or

$$m_{\text{eff}} = \frac{\mu_0 r_0^2 n e^2}{2} \ln(a/r_0). \quad (7)$$

This is a sought expression for Pendry’s effective electron mass in metal mesh structures. Formula (3) for

the mesh plasma frequency can now be easily obtained from the Langmuir formula (4) by substituting Eqs. (5) and (7) for n and m , respectively, into it.

Below, we give a few remarks concerning m_{eff} .

1. An idea about the magnitude of effective electron mass in metal wire mesh structures can be obtained from the following numerical example [6]. For a mesh with the parameters $a = 5$ mm and $r_0 = 1$ μ m (so that $a/r_0 = 5000 \gg 1$) and made from aluminum conductors ($n = 1.81 \times 10^{29}$ m⁻³), we have

$$m_{\text{eff}}/m \approx 2.7 \times 10^4.$$

By the way, the giant value of this ratio ($m_{\text{eff}}/m \gg \gg 1$) means that the relation $eA \gg mv$ used in the derivation of expression (7) for m_{eff} is satisfied.

The electron dilution coefficient for the considered mesh structure equals

$$n_{\text{eff}}/n = a^2 / (\pi r_0^2) \approx 8.0 \times 10^6,$$

and the mesh plasma frequency turns out by five orders of magnitude lower than the plasma frequency $\omega_p(\text{Al})$ for bulk aluminum,

$$\frac{\omega_p(\text{Al})}{\omega_p} = \sqrt{\frac{n}{n_{\text{eff}}} \frac{m_{\text{eff}}}{m}} \approx 4.6 \times 10^5. \quad (8)$$

Estimate (8) is confirmed by experimental data.

2. As one can see from the example given above, the increase of the electron mass, $m \rightarrow m_{\text{eff}}$, plays a substantial role in the formation of the value of ω_p . The account for only one effect – the dilution of charge carriers, $n \rightarrow n_{\text{eff}}$, which is intuitively clear – would not allow one to obtain correct numerical values for ω_p .

3. According to Eq. (7), the effective electron mass depends on the geometrical mesh parameters a and r_0 in such a way that, by increasing the mesh period, i.e. passing to the limiting case of isolated conductor, “one could easily obtain electrons with an effective mass of, say, 1 kg” [12].

4. The authors of work [6] assume the mutual inductance L of mesh’s conductors to be the physical origin of the electron mass increase. However, while deriving formulas for m_{eff} and ω_p , the quantity L has not been used.

3. Properties and Interpretation of m_{eff}

3.1. General properties of m_{eff}

One of the properties of the effective mass m_{eff} that follows immediately from its definition, namely, its

uncertainty owing to the gage invariance of the vector potential \mathbf{A} , which participates in the m_{eff} definition, was already noted in work [11]. Let us consider other properties of m_{eff} that follow from the definition

$$m_{\text{eff}}\mathbf{v} = m\mathbf{v} + q\mathbf{A} \quad (9)$$

but have not been noticed by anybody till now.

1. Generally speaking, the effective mass m_{eff} is a tensor quantity. Indeed, for non-parallel \mathbf{v} and \mathbf{A} , we also obtain that $\mathbf{v} \nparallel \mathbf{P}$. The latter fact means that the quantity m_{eff} in the expression $m_{\text{eff}}\mathbf{v} = \mathbf{P}$ is a tensor rather than a scalar. The quantity m_{eff} is a scalar only in the case $\mathbf{v} \parallel \mathbf{A}$.

An analytical expression for the tensor m_{eff} can be obtained from Eq. (9) by regarding the vector \mathbf{A} as a result of the action of some operator \mathbb{T} on \mathbf{v} :

$$\mathbf{A} = \mathbb{T}\mathbf{v}.$$

The action of this operator on the vector \mathbf{v} consists in the elongation of the latter by the factor A/v and the rotation of the result until the new vector becomes superposed on \mathbf{A} . Proceeding from that, let us express \mathbb{T} in the form

$$\mathbb{T} = \frac{A}{v}\mathbf{R}(\mathbf{n}, \varphi),$$

where $\mathbf{R}(\mathbf{n}, \varphi)$ is the operator of rotation by the angle φ around the axis defined by the unit vector \mathbf{n} , with

$$\varphi = \angle(\mathbf{v}, \mathbf{A}), \quad \mathbf{n} = \frac{\mathbf{v} \times \mathbf{A}}{|\mathbf{v} \times \mathbf{A}|}.$$

In such a manner, we obtain

$$m_{\text{eff}} = m \cdot \mathbf{1} + \frac{qA}{v}\mathbf{R}(\mathbf{n}, \varphi)$$

from Eq. (9) or, in the coordinate representation,

$$(m_{\text{eff}})_{ij} = m\delta_{ij} + \frac{qA}{v}R_{ij}(\mathbf{n}, \varphi),$$

where $\mathbf{1} \equiv \{\delta_{ij}\}$ is the unit operator (δ_{ij} is the Kronecker symbol), the matrix elements R_{ij} of the operator $\mathbf{R}(\mathbf{n}, \varphi)$ are known to have the form (see, e.g., work [23])

$$R_{ij}(\mathbf{n}, \varphi) = \delta_{ij} \cos \varphi - \epsilon_{ijk}n_k \sin \varphi + n_i n_j (1 - \cos \varphi),$$

and ϵ_{ijk} is the Levi-Civita symbol.

2. In the case where the effective mass m_{eff} is a scalar, it can be both larger and smaller than particle's mass m . Moreover, m_{eff} can accept a zero value or even a negative one.

3. In any case, the magnitude and the sign of m_{eff} are determined by the relative orientation of the vectors \mathbf{v} and \mathbf{A} , and by the relation between the absolute values of terms $\mathbf{p} = m\mathbf{v}$ and $q\mathbf{A}$, which define m_{eff} . For instance, for a positively charged particle ($q > 0$), we have $m_{\text{eff}} \leq 0$ if the particle moves in the direction opposite to the field \mathbf{A} ($\mathbf{v} \uparrow\downarrow \mathbf{A}$) and $v \leq qA/m$.

4. Generally speaking, the effective mass depends on the time: $m_{\text{eff}} = m_{\text{eff}}(t)$. This phenomenon is associated with the fact that a moving charged particle passes through various spatial points, where both the field \mathbf{A} (owing to its spatial nonuniformity or time dependence) and the particle velocity can be different.

3.2. Analogy between m_{eff} and m^*

Some of the indicated properties of Pendry's effective mass m_{eff} are similar to the properties of the effective mass m^* of conduction electrons in solids. For example, in the general case, both m_{eff} and m^* can be anisotropic and should be described by the corresponding tensors¹. Those properties, in particular, distinguish both effective masses m_{eff} and m^* from the ordinary rest mass of particles m , which is always a scalar larger than or equal to zero.

3.3. Interpretation of m_{eff}

Which is the physical meaning of the mass m_{eff} ? Is it "real"? Do the giant values of m_{eff} for electrons in metal wire mesh structures (and not only in them, see below) mean that the electron mass m actually grows gigantically, so that the electron becomes extremely heavy?

The assumption that the electron mass increases in the magnetic field looks unphysical, because we cannot specify any real mechanism that could be responsible for the enormous growth of the electron mass under the physical conditions that exist in the meshes. However, this assumption allows quantitatively proper results to be obtained for the plasma frequency in metal mesh structures. A collision between the nonphysical character and the practical value of this assumption can be eliminated by giving a proper interpretation of m_{eff} .

¹ Strictly speaking, as was mentioned in Introduction, it is the inverse mass $1/m^*$ rather than the mass m^* itself that has tensor properties.

As was mentioned in Introduction, the issue concerning the physical content of m_{eff} is similar to those arisen in due time with respect to m^* . Taking this fact into account, as well as a certain similarity (not only a terminological one, see the previous item) between m_{eff} and m^* , we can make attempt to interpret m_{eff} in the spirit of m^* . Namely, we may consider that *a real charged particle with mass m moves under the action of electromagnetic forces in a magnetic field with a potential \mathbf{A} , as a certain particle characterized by the same charge and the mass m_{eff} moves under the action of the same forces but without magnetic field.* Here, as well as in the interpretation of the effective mass m^* in the solid-state theory [5], a key issue is that m_{eff} makes it possible to calculate the particle acceleration (in this case, in the magnetic field) under the action of only electromagnetic forces. The acceleration of the same particle owing to the action of forces of any other origin, e.g., gravitational forces, depends on its rest mass m , which remains the same in the magnetic field, as it was in the absence of a field.

It is easy to see that the proposed interpretation at least agrees with the results obtained for the metal wire mesh structures described in Section 2. Does it contradict other known results? Let us consider some examples.

4. Effective Mass m_{eff} from Different Sides

In this section, we analyze Pendry's effective mass m_{eff} in various situations where charges move in a magnetic field. In so doing, we ignore the problem associated with the ambiguity in the choice of a field potential \mathbf{A} and, accordingly, the ambiguity of a mass m_{eff} . Similarly to works [6–9], it is supposed that the results presented below are valid for a certain calibration of the potential \mathbf{A} , the choice of which is not specified and remains offscreen.

4.1. Electron in the magnetic field of a conductor with current

In Section 2, it was demonstrated that the effective electron mass m_{eff} can easily reach large (of an order of $10^4 m$) values in metal mesh structures at a proper choice of their parameters. Now, let us estimate the effective mass of an electron moving in the magnetic field of an infinitely long conductor with current. For a single straight infinite conductor with radius r_0 and

with current I , the vector potential \mathbf{A} of the magnetic field created by the current is directed in parallel to the conductor and looks like [24]

$$A(r) = -\frac{\mu_0 I}{4\pi} \left(1 + 2 \ln \frac{r}{r_0} + C \right), \quad (10)$$

where r is the distance from the conductor axis, and C is an arbitrary constant. For the choice $C = \mu_0 I / (4\pi)$, potential (10) has the “minimal form”

$$A(r) = \frac{\mu_0 I}{2\pi} \ln \frac{r}{r_0}. \quad (11)$$

This formula almost coincides with expression (6) for the potential of a magnetic field induced by currents in wire mesh structures: the difference consists in the substitution $a \rightarrow r_0$.

Let us consider an electron that moves with the velocity $v = 1$ mm/s (this is a typical order of magnitude for the drift velocity of electrons in metals at a current density of 1 A/mm²) at the distance $r = 1$ m from the axis of a conductor with the radius $r_0 = 1$ mm and the current $I = 1$ A. According to Eq. (11), the magnetic field potential of the current at the electron position equals $A \approx 10^{-6}$ T · m. The corresponding contribution eA to the canonical momentum (2) of the electron has an order of magnitude of 10^{-25} kg · m/s. At the same time, the kinematic momentum of the electron $p = mv$ under the same conditions has an order of magnitude equal to 10^{-33} kg · m/s. Since $eA/p \approx 10^8$, we obtain in this case that $m_{\text{eff}} \approx 10^8 m$.

Let us pay attention to the following issues.

1. In order to achieve the enormous values of $m_{\text{eff}} \approx 10^8 m$, it is not necessary to use superstrong magnetic fields or create some exotic physical conditions. Such values can be easily realized under usual laboratory conditions.

2. Do the large values of m_{eff} mean that electrons become extremely heavy indeed? If it have been so, then, as an example, the mass of the rotor windings of electric motors (and the mass of the electric motors in whole) would have increased by several orders of magnitude at their start. Surely, it was never observed. The interpretation proposed in item 3 (see Subsection 3.3) allows the misleading interpretation of great m_{eff} -values as the effect of an increase in the electron mass m to be avoided.

4.2. Electron in Earth's magnetic field

In the majority of electric engineering problems, the magnetic field of the Earth is neglected, because the corresponding effects are small. However, the effective electron mass m_{eff} in this field turns out even larger than in the example with the magnetic field of the current considered in Subsection 4.1.

Really, the induction of the magnetic field created by a rectilinear current at a distance of 1 m equals $B = \mu_0 I / (2\pi r) = 0.2 \mu\text{T}$. The magnetic field of the Earth near its surface is known to be equal to approximately $50 \mu\text{T}$ [25], i.e., it is by two orders of magnitude stronger. From the linear relation between \mathbf{B} and \mathbf{A} (this is a consequence of the relation $\mathbf{B} = \text{rot}\mathbf{A}$), it follows that the potentials of those two fields are also different by two orders of magnitude. In view of the results of the previous example, we arrive at the conclusion that the effective mass of an electron moving near the Earth surface at the same velocity $v = 1 \text{ mm/s}$ should differ from the rest mass of electron by 10 (!) rather than 8 orders of magnitude.

4.3. Free charges in electric and magnetic fields

The analysis of free charge motions in electric and magnetic fields of various configurations – uniform or non-uniform, parallel or crossed, stationary or alternating – is of interest for a number of physics domains, e.g., the physics of accelerators and the plasma physics. Results of this analysis are well-known (see, e.g., works [26, 27]) and predict that, for a description of the charge motion to be correct, it is necessary that the both components of the Lorentz force acting on the charge – the electric, $q\mathbf{E}$, and magnetic, $q\mathbf{v} \times \mathbf{B}$, ones – be taken into account in the explicit form. In contrast to that, in the framework of the effective mass concept, the magnetic field can be excluded from consideration, so that the motion of a particle with the mass m_{eff} only in the electric field can be analyzed (see item 3 in Section 3.3.1). It is clear that the well-known classical results are not reproducible in the framework of this consideration. For instance, instead of the known $\mathbf{E} \times \mathbf{B}$ -drift of particles in crossed \mathbf{E} and \mathbf{B} fields, which is a complicated motion in the plane perpendicular to \mathbf{E} and \mathbf{B} [27] we obtain a simple uniformly accelerated motion of a particle with the mass m_{eff} along the vector \mathbf{E} in the framework of the effective mass concept.

4.4. Influence of m_{eff} on electric conductivity

The authors of work [13] paid attention to that the giant values of effective electron mass in metal mesh structures should induce not only a reduction of the mesh plasma frequency in comparison with that for the mesh conductor material, but also to a corresponding giant reduction of the mesh conductivity and a growth of Joule losses in the mesh. It can be explained by the relation between the conductivity σ of the medium and the mass of charge carriers in it. In the case of electron conductivity, [2]

$$\sigma = \frac{ne^2\tau}{m}, \quad (12)$$

where τ is the electron relaxation time. For crystalline solids, the effective mass m^* characterizing the dynamic properties of electrons in a crystal is used instead of m . In the case of mesh structures, the substitution $m \rightarrow m_{\text{eff}}$ should be made in Eq. (12). Then, bearing in mind the numerical estimations for m_{eff} , the conductivity σ should decrease, and, accordingly, the Joule losses should increase by several orders of magnitude. As is well-known, neither of those effects is observed in mesh structures.

With regard for the results described in items 1 and 2 in Section 4, we may assert that the influence of m_{eff} on the electric conductivity should be substantial not only in mesh structures, but also in bulk metal specimens. For example, item 2 implies that, under the conditions of terrestrial laboratories, the conductivity and the resistivity of an ordinary metal conductor can differ by 10 orders of magnitude from the values that would be obtained in the absence of Earth's magnetic field. It is clear that this prediction contradicts the whole body of accumulated experimental data.

4.5. Electrons in a solid

Let us demonstrate now that the concept of effective mass m_{eff} is not pertinent to the quantum-mechanical solid-state theory as well. First, it will be recalled that the effective mass m^* of conduction electrons used in the solid-state theory implicitly makes allowance for the effect of electron interaction with a periodic field $U(\mathbf{r}) = U(\mathbf{r} + \mathbf{a})$ in the crystal lattice, so that this field can be excluded from the Schrödinger equation. In this case, the Hamiltonian of the initial Schrödinger equation,

$$\hat{H} = \frac{1}{2m}(\hat{\mathbf{p}} - e\mathbf{A})^2 + U + V,$$

which describes a real electron with mass m that moves in the periodic crystal field U , an external magnetic field with the vector potential \mathbf{A} , and a field V created by external forces of all other types, e.g., electric and gravitational ones, is reduced to a simpler form²

$$\hat{H} = \frac{1}{2m^*}(\hat{\mathbf{p}} - e\mathbf{A})^2 + V, \quad (13)$$

corresponding to a free particle with mass m^* moving in the fields \mathbf{A} and V [3].

Can the Hamiltonian of an electron in the solid be simplified further with the use of the concept of effective electron mass m_{eff} dependent on the magnetic field? According to the interpretation of m_{eff} , Hamiltonian (13) could be simplified by excluding \mathbf{A} from it and simultaneously by making the substitution $m^* \rightarrow m_{\text{eff}}$ (in this case, m_{eff} should be calculated from Eqs. (1) and (2), in which the substitution $m \rightarrow m^*$ was preliminarily made). As a result, we obtain the following Hamiltonian of a free particle with the mass m_{eff} that moves in the field V :

$$\hat{H} = \frac{1}{2m_{\text{eff}}}\hat{\mathbf{p}}^2 + V.$$

Do the solutions of the Schrödinger equation with this Hamiltonian coincide with the solutions corresponding to Hamiltonian (13)? It is clear that this is not the case. This becomes the most evidently if $V = 0$. All the practice of considering various phenomena in solid-state physics, in which the magnetic field plays an essential role, e.g., the cyclotron resonance and the Hall effect, testifies that, for those phenomena to be described correctly, the dependence of \hat{H} on \mathbf{A} must be taken into consideration explicitly. Hence, the well-known results of the solid-state theory cannot be reproduced in the framework of the concept of effective mass m_{eff} .

5. Conclusions

The concept of effective mass of an electron in the magnetic field was introduced in work [6], while solving the problem of the plasma frequency of metal mesh structures. The way of introducing the mass m_{eff} substantially uses the concept of canonical momentum, $\mathbf{P} \equiv \partial L / \partial \mathbf{v}$, of charged particle in an external electromagnetic field and the relation between

the kinematic, $\mathbf{p} \equiv m\mathbf{v}$, and canonical particle's momenta. The concept of canonical momentum is an important element of the Lagrange and Hamilton formalisms of classical mechanics; it is also used in the quantum mechanics. The universal character of this notion and the relationship between \mathbf{P} and \mathbf{p} give a hope for that the concept of effective mass m_{eff} can be used not only in the specific problem, where it was introduced, but also in other problems (both classical and quantum-mechanical ones) dealing with the motion of charges in the magnetic field.

The analysis of the effective mass m_{eff} , which was carried out in this work, testifies to its rather unusual properties. For instance, m_{eff} turns out, generally speaking, to be a tensor quantity. In some cases, it can acquire zero or even negative values. A certain analogy between m_{eff} and the effective mass m^* known from the solid-state theory allows the former to be interpreted in the spirit of the latter, by removing, in such a way to a certain extent, the problem of a non-physical character of m_{eff} marked earlier by various authors. In addition, this interpretation of m_{eff} does not contradict the experimentally confirmed results of work [6].

Although the relationships, on which the definition of m_{eff} is based, have universal character, the application of the m_{eff} concept beyond the problematics of the dielectric response of metal wire mesh structures does not allow one to obtain plausible results or to reproduce the already known ones. For example, the dependence of the effective electron mass on the magnetic field gives rise to a conclusion about an enormous increase of the metal resistance even in weak magnetic fields. Taking this fact into account, this concept has to be recognized as an *ad hoc* hypothesis, which allows the proper results to be obtained only for a specific problem dealing with the frequency of plasma oscillations in thin metal wire mesh structures.

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² Here, for simplification, the simplest case of isotropic quadratic dispersion law is considered, for which the effective mass is a scalar.

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ПРО ЕФЕКТИВНУ МАСУ ЕЛЕКТРОНА ЗА ПЕНДРІ

Р е з ю м е

У 1996 р. англійський фізик-теоретик Дж. Пендрі, пояснюючи діелектричний відгук металевих сіток, висунув ідею про залежність ефективної маси електрона від магнітного поля. Ця ідея ґрунтується на відомому співвідношенні між кінематичним і канонічним імпульсами заряду, що рухається в магнітному полі. В даній статті, виходячи з універсальності зазначеного співвідношення, досліджується можливість застосування поняття ефективної маси електрона за Пендрі m_{eff} не лише до електронів у металевих сітках, а й до більш широкого кола задач про рух зарядів у магнітному полі. Встановлено загальні властивості ефективної маси m_{eff} , які випливають безпосередньо з її означення. Виявлено аналогію між m_{eff} та ефективною масою електронів m^* , що розглядається у теорії твердого тіла. Запропоновано фізичну інтерпретацію m_{eff} . На кількох прикладах показано, що, незважаючи на універсальність використовуваного в означенні m_{eff} співвідношення між кінематичним і канонічним імпульсами, застосування поняття m_{eff} поза межами проблематики діелектричного відгуку металевих сіток не дозволяє одержати правильні результати.