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INTERACTION OF ELASTIC AND SPIN WAVES IN A UNIAXIAL FERROMAGNET

The dispersion laws of coupled magnetoelastic waves have been calculated for all ground states in a uniaxial ferromagnet. The magnetoelastic interaction is shown to take place not for all sound modes in those ground states. The obtained dispersion laws testify that the magnetoelastic interaction coefficient depends on both the magnetization direction and the wave vector direction. It is demonstrated that the magnetoelastic interaction between sound and spin waves in the uniaxial ferromagnet is characterized by the constants B_{44} and B_{66}, whereas the other magnetoelastic constants govern only the formation of a magnetoelastic gap in the spectrum of coupled waves.

Keywords: magnetoelastic interaction, dispersion law, uniaxial ferromagnet, elastic modulus.

1. Introduction

Researches of ferromagnets with uniaxial symmetry is of special interest, because there exist degenerate ground states for them, in which the magnetic moment is not directed along the easy axis [1, 2] and the spectrum of spin waves in those ground states is gapless. This phenomenon is responsible for the emergence of Goldstone spin waves in a crystal and is accompanied by a number of characteristic features [2]. At the same time, it is well known that, in the spectrum of spin waves in magnetically ordered materials, there appears a magnetoelastic gap as a result of the interaction between spin and sound waves. In work [3], the appearance of a magnetoelastic gap was supposed to be associated with the violation of the magnetic Hamiltonian symmetry due to the introduction of a magnetoelastic interaction. The corresponding calculations of spin spectra for this phenomenon were carried out recently [4]; however, no comprehensive study of the dispersion laws for coupled magnetoelastic oscillations has been done.

Magnetoelastic interactions in uniaxial ferromagnets have been considered for rather a long time and under various conditions [1,5]. However, the attention was focused only on the ground state of the "easy axis" type, which is not degenerate, whereas calculations for other magnetization directions were not executed. Modern experimental data [6, 7] point to a dependence of the elastic properties of materials on the direction of an applied external magnetic field and, accordingly, the magnetization direction of the specimen. However, no consistent theoretical calculations for the dependence of the magnetoelastic interaction on the magnetic state have been performed for uniaxial ferromagnets as well. This fact stimulated the author to carry out corresponding theoretical researches.

2. Dispersion Laws for Coupled Magnetoelastic Waves in a Uniaxial Ferromagnet

The phenomenological description of the dynamical properties of a ferromagnetic crystal is based on the expression for the free energy that reflects the corresponding symmetry of a ferromagnet [1]. In order to take the magnetoelastic interaction into account, the total energy of the ferromagnet has to be written in the form

$$F = F_m + F_e + F_{me}. \tag{1}$$

Here, F_m is the magnetic energy of the crystal. In the case of uniaxial ferromagnet, it looks like [11]

$$F_m = \frac{\alpha}{2}\frac{\partial\mu}{\partial x_i}\frac{\partial\mu}{\partial x_k} - \frac{1}{2}K_1\mu_z^2 - \frac{1}{4}K_2\mu_z^4 - \mathbf{MH}, \tag{2}$$

where α is the constant of the inhomogeneous exchange interaction (for simplification, the case $\alpha_{ik} = \alpha$ will be considered), K_1 and K_2 are the constants of uniaxial anisotropy (all constants have a dimensionality of energy), $\mathbf{M}$ and $\mathbf{H}$ are the vectors of magnetization and external magnetic field, respectively, $\mu = \mathbf{M}/M_0$ is the normalized magnetization vector, and M_0 the saturation magnetization. The term F_e in Eq. (1) is the elastic energy density, which looks like [8]

$$F_e = \frac{1}{2}C_{11}(\mathrm{E}_{xx} + \mathrm{E}_{yy})^2 + \frac{1}{2}C_{33}\mathrm{E}_{zz}^2 + C_{13}(\mathrm{E}_{xx} + \mathrm{E}_{yy})\mathrm{E}_{zz} + 2C_{44}(\mathrm{E}_{xz} + \mathrm{E}_{yz})^2 + \frac{1}{2}C_{66}(\mathrm{E}_{xx}^2 + \mathrm{E}_{yy}^2 + 2\mathrm{E}_{xy}^2), \tag{3}$$

where E_{ik} are components of the strain tensor, and C_{ik} the elastic moduli of the second order for the uniaxial crystal. Finally, the term F_{me} in Eq. (1) determines the interaction between the magnetic and elastic subsystems [5, 9],

$$F_{me} = \frac{1}{2}B_{11}(\mu_x^2 + \mu_y^2)(\mathrm{E}_{xx} + \mathrm{E}_{yy}) + \frac{1}{2}B_{13}\mu_z^2(\mathrm{E}_{xx} + \mathrm{E}_{yy}) + \frac{1}{2}B_{31}(\mu_x^2 + \mu_y^2)\mathrm{E}_{zz} + \frac{1}{2}B_{33}\mu_z^2\mathrm{E}_{zz} + \frac{1}{2}B_{44}(\mu_x\mu_z\mathrm{E}_{xz} + \mu_y\mu_z\mathrm{E}_{yz}) + \frac{1}{2}B_{66}(\mu_x^2\mathrm{E}_{xx} + \mu_y^2\mathrm{E}_{yy} + 2\mu_x\mu_y\mathrm{E}_{xy}), \tag{4}$$

where B_{ik} are the constants of magnetoelastic interaction in the case of uniaxial symmetry.

By minimizing the magnetic energy (2), it is possible to demonstrate that there are three ground states for the magnetization vector in the uniaxial ferromagnet without external magnetic field ($\mathbf{H} = 0$):

(i) along the easy magnetization axis, $\mathbf{M}||\langle 001\rangle$; this is the "easy axis" phase; the corresponding condition for its existence is $K_1 + K_2 > 0$;

(ii) in the basis plane, e.g., $\mathbf{M}||\langle 100\rangle$; this is the "easy plane" phase; the corresponding condition of existence is $K_1 < 0$; and

(iii) at a certain angle with respect to the easy magnetization axis, which is determined by the expression $\cos^2\theta = -K_1/K_2$; this is the "angular phase"; the existence conditions are $K_2 < 0$ and $0 < K_1 < -K_2$ [2].

The "easy plane" and "angular phase" ground states are degenerate, and, without external magnetic field, the spin wave spectra for them are gapless [2].

In real experiments aimed at studying the elastic and magnetic properties, the external magnetic field $\mathbf{H}$ is directed, as a rule, along the direction $\langle 001\rangle$ or $\langle 100\rangle$. Therefore, the corresponding ground states "easy axis" and "easy plane" will be considered below.

In accordance with the standard phenomenological description of the magnetic moment dynamics [1, 2], small adiabatic oscillations of the magnetic moment density μ of a ferromagnet are considered. In this case, we may write

$$\mu(\mathbf{r}, t) = \mu_0 + \mathbf{m}(\mathbf{r}, t), \tag{5}$$

where $\mathbf{m}(\mathbf{r}, t)$ are small deviations from the equilibrium value μ_0 owing to fluctuations, and the equilibrium value μ_0 of magnetization vector has the components $\mu_0 = (0, 0, 1)$ in the "easy axis" case and $\mu_0 = (1, 0, 0)$ in the "easy plane" one.

Similarly to the magnetic moment μ, the components of the strain tensor E_{ik} can also be represented as the sums of equilibrium values E_{ik}^0 and corresponding small deviations ε_{ik}:

$$\mathrm{E}_{ik} = \mathrm{E}_{ik}^0 + \varepsilon_{ik}. \tag{6}$$

The equilibrium values E_{ik}^0 of the strain tensor components for the ground states of a uniaxial ferromagnet can be easily determined from the condition $\partial F/\partial\mathrm{E}_{ik} = 0$. Below, they will be presented for each ground state separately. The inhomogeneous part of the elastic strain tensor can be expressed in terms of the particle displacement vector $\mathbf{U}$ using the formula [10]

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right). \quad (7)$$

The dispersion laws for coupled magnetoelastic waves can be calculated with the help of the dynamical equations for the magnetization vector μ (the Landau–Lifshits equation),

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma\mu \times \mathbf{H}_{\text{eff}}, \quad (8)$$

and the particle displacement vector $\mathbf{U}$ [1, 8],

$$\rho\ddot{\mathbf{U}} = -\frac{\delta F}{\delta \mathbf{U}}, \quad (9)$$

where $\mathbf{H}_{\text{eff}} = -\delta F/\delta\mathbf{M}$ is the effective magnetic field, $\gamma = g|\mu_B|/\hbar \approx 2|\mu_B|/\hbar$ is the gyromagnetic ratio, and ρ the material density.

For further calculations, let us expand the total energy density (1) in a power series of small deviations m_i and ε_{ik}, substitute the result into the dynamical equations (8) and (9), linearize them, and change to their Fourier transform components with respect to the time t and the coordinates $\mathbf{r}$ for small deviations $\mathbf{m} = \mathbf{m}_0 \exp\{i(\mathbf{kr} - \omega t)\}$ and $\mathbf{U} = \mathbf{U}_0 \exp\{i(\mathbf{kr} - \omega t)\}$, where ω is the frequency, and $\mathbf{k}$ is the wave vector of collective waves. Then, Eqs. (8) and (9) give rise to a system of six equations for the components of the vectors $\mathbf{m}_0$ and $\mathbf{U}_0$. For two ground states of a uniaxial ferromagnet, the corresponding systems are as follows.

The "easy axis" phase: $\mathbf{H} \parallel \mathbf{M} \parallel \langle 001\rangle$.

In this ground state, there are the following nonzero equilibrium values of components of the strain tensor, which can easily be obtained from the condition $\partial F/\partial \mathrm{E}_{ik} = 0$; namely,

$$\mathrm{E}^0_{xx} = \mathrm{E}^0_{yy} = \frac{B_{13}C_{33} - B_{33}C_{13}}{2(2C_{13}^2 - C_{33}(2C_{11} + C_{66}))},$$
$$\mathrm{E}^0_{zz} = \frac{-B_{13}C_{13} - B_{33}(2C_{11} + C_{66})}{2(2C_{13}^2 - C_{33}(2C_{11} + C_{66}))}. \quad (10)$$

The system of dynamical equations looks like

$$(\rho\omega^2 - (C_{11} + C_{66})k_x^2 - \frac{1}{2}C_{66}k_y^2 - C_{44}k_z^2)U_{0x} -$$
$$-\left(\left(C_{11} + \frac{1}{2}C_{66}\right)k_xk_y + C_{44}k_z^2\right)U_{0y} -$$
$$-((C_{13} + C_{44})k_xk_z + C_{44}k_yk_z)U_{0z} +$$
$$+ i\frac{1}{4}B_{44}k_zm_{0x} + iB_{13}k_xm_{0z} = 0; \quad (11a)$$
$$-\left(\left(C_{11} + \frac{1}{2}C_{66}\right)k_xk_y + C_{44}k_z^2\right)U_{0x} +$$
$$+\left(\rho\omega^2 - \frac{1}{2}C_{66}k_x^2 - (C_{11} + C_{66})k_y^2 - C_{44}k_z^2\right)U_{0y} -$$
$$-((C_{13} + C_{44})k_yk_z + C_{44}k_xk_z)U_{0z} +$$
$$+ i\frac{1}{4}B_{44}k_zm_{0y} + iB_{13}k_ym_{0z} = 0; \quad (11b)$$
$$-((C_{13} + C_{44})k_xk_z + C_{44}k_yk_z)U_{0x} -$$
$$-((C_{13} + C_{44})k_yk_z + C_{44}k_xk_z)U_{0y} +$$
$$+(\rho\omega^2 - C_{44}(k_x + k_y)^2 - C_{33}k_z^2)U_{0z} +$$
$$i\frac{1}{4}B_{44}k_xm_{0x} + i\frac{1}{4}B_{44}k_ym_{0y} + iB_{33}k_ym_{0z} = 0; \quad (11c)$$
$$-i\frac{1}{4M_0}\gamma B_{44}k_xU_{0y} - i\frac{1}{4M_0}\gamma B_{44}k_yU_{0z} +$$
$$+ i\omega m_{0x} - \gamma M_0\omega_{m\parallel}m_{0y} = 0; \quad (11d)$$
$$i\frac{1}{4M_0}\gamma B_{44}k_zU_{0x} + i\frac{1}{4M_0}\gamma B_{44}k_xU_{0z} +$$
$$+ \gamma M_0\omega_{m\parallel}m_{0x} + i\omega m_{0y} = 0; \quad (11e)$$
$$i\omega m_{0z} = 0. \quad (11f)$$

In expressions (11d) and (11e), the following notation was introduced:

$$\omega_{m\parallel} = \frac{\alpha k^2}{M_0^2} + \frac{H}{M_0} + \frac{K_{me}}{M_0^2} + \frac{K_1}{M_0^2} + \frac{K_2}{M_0^2}, \quad (12)$$

where $K_{me} = (B_{11} - B_{13} + B_{66})\mathrm{E}^0_{xx} + (B_{11} - B_{13})\mathrm{E}^0_{yy} + (B_{31} - B_{33})\mathrm{E}^0_{zz}$.

The "easy plane" phase: $\mathbf{H} \parallel \mathbf{M} \parallel \langle 100\rangle$.

The equilibrium values of strain tensor components in this ground state look like

$$\mathrm{E}^0_{xx} = -\frac{B_{66}}{4C_{66}} - \frac{2B_{31}C_{13} - C_{33}(2B_{11} + B_{66})}{4(2C_{13}^2 - C_{33}(2C_{11} + C_{66}))},$$
$$\mathrm{E}^0_{yy} = \frac{B_{66}}{4C_{66}} - \frac{2B_{31}C_{13} - C_{33}(2B_{11} + B_{66})}{4(2C_{13}^2 - C_{33}(2C_{11} + C_{66}))}, \quad (13)$$
$$\mathrm{E}^0_{zz} = \frac{B_{31}(2C_{11} + C_{66}) - C_{13}(2B_{11} + B_{66})}{2(2C_{13}^2 - C_{33}(2C_{11} + C_{66}))}.$$

The system of dynamical equations reads

$$\left(\rho\omega^2 - (C_{11} + C_{66})k_x^2 - \frac{1}{2}C_{66}k_y^2 - C_{44}k_z^2\right) U_{0x} - \\ - \left(\left(C_{11} + \frac{1}{2}C_{66}\right) k_x k_y + C_{44}k_z^2\right) U_{0y} - \\ - ((C_{13} + C_{44})k_x k_z + C_{44}k_y k_z)U_{0z} + \\ + i(B_{11} + B_{66})k_x m_{0x} + i\frac{1}{2}B_{66}k_y m_{0y} + \\ + i\frac{1}{4}B_{44}k_z m_{0z} = 0; \tag{14a}$$

$$- \left(\left(C_{11} + \frac{1}{2}C_{66}\right) k_x k_y + C_{44}k_z^2\right) U_{0x} + \\ + \left(\rho\omega^2 - \frac{1}{2}C_{66}k_x^2 - (C_{11} + C_{66})k_y^2 - C_{44}k_z^2\right) U_{0y} - \\ - ((C_{13} + C_{44})k_y k_z + C_{44}k_x k_z)U_{0z} + \\ + iB_{11}k_y m_{0x} + i\frac{1}{2}B_{66}k_x m_{0y} = 0; \tag{14b}$$

$$- ((C_{13} + C_{44})k_x k_z + C_{44}k_y k_z)U_{0x} - \\ - ((C_{13} + C_{44})k_y k_z + C_{44}k_x k_z)U_{0y} + \\ + (\rho\omega^2 - C_{44}(k_x + k_y)^2 - C_{33}k_z^2)U_{0z} + \\ + iB_{31}k_z m_{0x} + i\frac{1}{4}B_{44}k_x m_{0z} = 0; \tag{14c}$$

$$i\omega m_{0x} = 0; \tag{14d}$$

$$- i\frac{1}{4M_0}\gamma B_{44}k_z U_{0x} - i\frac{1}{4M_0}\gamma B_{44}k_x U_{0z} + \\ + i\omega m_{0y} - \gamma M_0\omega_{m1\perp}m_{0z} = 0; \tag{14e}$$

$$i\frac{1}{4M_0}\gamma B_{66}k_y U_{0x} + i\frac{1}{4M_0}\gamma B_{66}k_x U_{0y} + \\ + \gamma M_0\omega_{m2\perp}m_{0y} + i\omega m_{0z} = 0. \tag{14f}$$

In expressions (14e) and (14f), the following notations were introduced:

$$\omega_{m1\perp} = \frac{\alpha k^2}{M_0^2} + \frac{H}{M_0} - \frac{K_1}{M_0^2} - \frac{K_{me}}{M_0^2}, \\ \omega_{m2\perp} = \frac{\alpha k^2}{M_0^2} + \frac{H}{M_0} + \frac{B_{66}^2}{2M_0^2C_{66}}. \tag{15}$$

In the both cases, using the condition that the determinant of the system of dynamical equations should equal zero, we obtain the dispersion laws for coupled magnetoelastic waves in the ground states of a uniaxial ferromagnet. Let us consider a few directions for the wave vector of elastic waves, which are used in experimental researches of sound waves in ferromagnets with uniaxial symmetry: along the "easy axis" and in the "easy plane".

The "easy axis" phase: $\mathbf{H} \parallel \mathbf{M} \parallel \langle 001\rangle$.

The case $\mathbf{k} \parallel \langle 100\rangle$ or $\mathbf{k} \parallel \langle 010\rangle$:

$$\left(\omega^2 - \frac{(C_{11}+C_{66})}{\rho}k^2\right)\left(\omega^2 - \frac{C_{66}}{2\rho}k^2\right)\left[\left(\omega^2 - \frac{C_{44}}{\rho}k^2\right) \times \\ \times (\omega^2 - \gamma^2M_0^2\omega_{m\parallel}^2) - B_{44}^2\left\{\frac{\omega_{m\parallel}\gamma^2k^2}{16\rho}\right\}\right] = 0. \tag{16}$$

The case $\mathbf{k} \parallel \langle 001\rangle$:

$$\left(\omega^2 - \frac{C_{33}}{\rho}k^2\right)\left[\omega^2\left(\omega^2 - \frac{2C_{44}}{\rho}k^2\right)(\omega^2 - \gamma^2M_0^2\omega_{m\parallel}^2) - \\ - B_{44}^2\left\{\frac{\omega_{m\parallel}\gamma^2k^2}{8\rho}\left(\omega^2 - \frac{C_{44}}{\rho}k^2\right)\right\}\right] = 0. \tag{17}$$

The case $\mathbf{k} \parallel \langle 110\rangle$:

$$\left(\omega^2 - \frac{(C_{11}+C_{66})}{\rho}k^2\right)\left(\omega^2 - \frac{C_{66}}{2\rho}k^2\right)\left[\left(\omega^2 - \frac{2C_{44}}{\rho}k^2\right) \times \\ \times (\omega^2 - \gamma^2M_0^2\omega_{m\parallel}^2) - B_{44}^2\left\{\frac{\omega_{m\parallel}\gamma^2k^2}{16\rho}\right\}\right] = 0. \tag{18}$$

The case $\mathbf{k} \parallel \langle 1\bar{1}0\rangle$ or $\mathbf{k} \parallel \langle \bar{1}10\rangle$:

$$\left(\omega^2 - \frac{(C_{11}+C_{66})}{\rho}k^2\right)\left(\omega^2 - \frac{C_{66}}{2\rho}k^2\right) \times \\ \times \left[\omega^2(\omega^2 - \gamma^2M_0^2\omega_{m\parallel}^2) - B_{44}^2\left\{\frac{\omega_{m\parallel}\gamma^2k^2}{16\rho}\right\}\right] = 0. \tag{19}$$

The "easy plane" phase: $\mathbf{H} \parallel \mathbf{M} \parallel \langle 100\rangle$.

The case $\mathbf{k} \parallel \langle 100\rangle$ or $\mathbf{k} \parallel \langle 010\rangle$:

$$\left(\omega^2 - \frac{(C_{11}+C_{66})}{\rho}k^2\right)\left(\omega^2 - \frac{C_{44}}{\rho}k^2\right)\left[\left(\omega^2 - \frac{C_{66}}{2\rho}k^2\right) \times \\ \times (\omega^2 - \gamma^2M_0^2\omega_{m1\perp}\omega_{m2\perp}) - B_{66}^2\left\{\frac{\omega_{m1\perp}\gamma^2k^2}{4\rho}\right\}\right] = 0. \tag{20}$$

The case $\mathbf{k} \parallel \langle 001\rangle$:

$$\left(\omega^2 - \frac{C_{33}}{\rho}k^2\right)\left[\omega^2\left(\omega^2 - \frac{2C_{44}}{\rho}k^2\right) \times\right.$$

$$\times(\omega^2-\gamma^2M_0^2\omega_{m1\perp}^2\omega_{m2\perp})-$$
$$-B_{44}^2\left\{\frac{\omega_{m2\perp}\gamma^2k^2}{16\rho}\left(\omega^2-\frac{C_{44}}{\rho}k^2\right)\right\}\Bigg]=0. \quad (21)$$

The case $\mathbf{k}\parallel\langle 110\rangle$:

$$\left(\omega^2-\frac{C_{66}}{2\rho}k^2\right)\left(\omega^2-\frac{2C_{44}}{\rho}k^2\right)\left[\left(\omega^2-\frac{(C_{11}+C_{66})}{\rho}k^2\right)\times\right.$$
$$\left.\times(\omega^2-\gamma^2M_0^2\omega_{m1\perp}\omega_{m2\perp})-B_{66}^2\left\{\frac{\omega_{m1\perp}\gamma^2k^2}{4\rho}\right\}\right]=0. \quad (22)$$

The case $\mathbf{k}\parallel\langle 1\bar{1}0\rangle$ or $\mathbf{k}\parallel\langle\bar{1}10\rangle$:

$$\omega^2\left(\omega^2-\frac{C_{66}}{2\rho}k^2\right)\left[\left(\omega^2-\frac{(C_{11}+C_{66})}{\rho}k^2\right)\times\right.$$
$$\left.\times(\omega^2-\gamma^2M_0^2\omega_{m1\perp}\omega_{m2\perp})-B_{66}^2\left\{\frac{\omega_{m1\perp}\gamma^2k^2}{4\rho}\right\}\right]=0. \quad (23)$$

Hence, expressions (16)–(23) are dispersion laws written in the general form for coupled magnetoelastic waves in a ferromagnet with uniaxial symmetry. By their structure, these dispersion equations are standard [1, 10]. If the magnetoelastic interaction is neglected ($B_{ik}\to 0$), they are transformed into the classical dispersion laws for spin waves [1] and elastic waves [8] in uniaxial crystals.

Interaction of sound modes with spin waves in the ferromagnet with uniaxial symmetry

Sound mode and wave vector direction	"Easy axis" phase: $\mathbf{H}\|\|\mathbf{M}\|\|\langle 001\rangle$	"Easy plane" phase: $\mathbf{H}\|\|\mathbf{M}\|\|\langle 100\rangle$
s_1 $\mathbf{k}\|\|\langle 100\rangle$ та $\mathbf{k}\|\|\langle 010\rangle$	B_{44}	No interaction
s_2 $\mathbf{k}\|\|\langle 001\rangle$	B_{44}	B_{44}
s_2 $\mathbf{k}\|\|\langle 110\rangle$	B_{44}	No interaction
s_3 $\mathbf{k}\|\|\langle 100\rangle$ та $\mathbf{k}\|\|\langle 010\rangle$	No interaction	B_{66}
s_4 $\mathbf{k}\|\|\langle 110\rangle$	No interaction	B_{66}
s_4 $\mathbf{k}\|\|\langle 1\bar{1}0\rangle$	No interaction	B_{66}

3. Analysis of the Results Obtained and Conclusions

The calculated dispersion laws for coupled magnetoelastic waves in the ferromagnet with uniaxial symmetry [Eqs. (16)–(23)] make it possible to estimate the influence of the magnetic subsystem on the elastic properties of the crystal, namely, on the corresponding elastic moduli. From those dispersion laws, it follows that, in the uniaxial ferromagnet, the following sound modes interact with spin waves: $s_1^2 = C_{44}/\rho$, $s_2^2 = 2C_{44}/\rho$, $s_3^2 = C_{66}/2\rho$, and $s_4^2 = (C_{11}+C_{66})/\rho$. At the same time, the sound mode $s_5^2 = C_{33}/\rho$ does not interact at all with spin waves in those ground states.

As a rule, the influence of the magnetoelastic interaction on any sound s_i (and the corresponding elastic modulus C_{ii}) can be described by considering a magnetoacoustic resonance at the corresponding frequency $\omega_{ph}=s_ik$ [11,12]. In this case, the dispersion laws of coupled magnetoelastic oscillations are transformed into the following dispersion equation, which has a common form for all ground states and wave vector directions:

$$(\omega^2-\omega_{ph}^2)(\omega^2-\omega_{sw}^2)-B_{ii}^2\xi=0, \quad (24)$$

where ω_{sw} is the spin wave frequency depending on the magnetic state ($\omega_{sw}=\gamma M_0\omega_{m\parallel}$ for the "easy axis" ground state, and $\omega_{sw}=\gamma M_0(\omega_{m1\perp}\omega_{m2\perp})^{1/2}$ for the "easy plane" one), and $\xi\sim\frac{\omega_{mi}\gamma^2k^2}{\rho}$ is the coefficient of magnetoelastic interaction depending on the direction of the magnetic moment in the crystal and the wave vector direction of elastic vibrations.

For the uniaxial ferromagnet, as follows from Eqs. (16)–(23), the dispersion laws are decomposed in most cases into Eq. (24) and the spectra of the sound mode, which, in the case concerned, do not interact with spin waves. Hence, in those cases, the estimation of the magnetoelastic interaction has no frequency restrictions. Only in cases (17) and (21), it is necessary to consider a magnetoacoustic resonance, so that the frequencies should be selected close to $\omega_{ph}=(2C_{44}/\rho)^{1/2}k$.

For the consideration to be more illustrative, let us consider Table reflecting the presence of the magnetoelastic interaction for each sound mode depending on the direction of magnetic moment in the uniaxial ferromagnet. If such an interaction takes place, the corresponding magnetoelastic constants are indicated.

The solution of Eq. (24) looks like

$$\omega_{\pm}^2 = \frac{1}{2}\left\{\omega_{ph}^2 + \omega_{sw}^2 \pm [4\xi B_{ii}^2 + (\omega_{ph}^2 - \omega_{sw}^2)^2]^{1/2}\right\}. \quad (25)$$

This dispersion law consists of two branches: quasimagnon and quasiphonon ones [11, 12]. From expression (25), we can easily see that, when the system approaches the magnetoacoustic resonance, $\omega_{sw} \to \omega_{ph}$, it is the quantities ξ and B_{ii} that govern the "repulsion" between the quasimagnon and quasiphonon branches.

From Table, one can also clearly see that the sound modes s_3 and s_4 do not interact with spin waves in the "easy axis" ground state, and the sound mode s_1 with spin waves in the "easy plane" one. The magnetoelastic interaction between the sound and spin waves is characterized exclusively by the constants B_{44} (the sound modes s_1 and s_2) and B_{66} (the sound modes s_3 and s_4). All other magnetoelastic constants correspond only to the formation of a magnetoelastic gap in the spectrum of coupled oscillations [see expressions (12) and (15)].

From expression (15), it also follows that the magnetoelastic interaction eliminates the degeneration of the "easy plane" ground state [4]. The degeneration disappears both in the absence of an external magnetic field and even in the case of isotropic magnet ($K_1 = 0$), which is in complete agreement with general principles expounded in work [3] and the results obtained in work [4].

From the dispersion laws (16)–(23), it also follows that the coefficient of magnetoelastic interaction ξ can depend not only on the magnetic state, but also on the direction of the wave vector of elastic vibrations (i.e., in cases (17) and (21), the parameter ξ has different values). Nevertheless, it is worth noting that, in the case of uniaxial symmetry, this dependence manifests itself to a less extent than in a cubic ferromagnet [11, 12].

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ВЗАЄМОДІЯ ПРУЖНИХ ТА СПІНОВИХ ХВИЛЬ В ФЕРОМАГНЕТИКУ ОДНООСНОЇ СИМЕТРІЇ

Р е з ю м е

Розраховано закони дисперсії зв'язаних магнітопружних хвиль для основних станів "легка вісь" та "легка площина" феромагнетика одноосної симетрії. Показано, що в даних основних станах не всі звукові моди взаємодіють із спіновим хвилями. Отримані закони дисперсії показують, що коефіцієнт магнітопружної взаємодії залежить як від напрямку магнітного моменту феромагнетика, так і від напрямку хвильового вектора пружних коливань. Показано, що магнітопружна взаємодія між звуковими та спіновими хвилями в одноосному феромагнетику характеризується виключно константами B_{44} та B_{66}, інші магнітопружні константи відповідають тільки за формування магнітопружної щілини у спектрі зв'язаних коливань.