With the help of the Linearized Augmented Plane Wave (LAPW) method, the role of some structural types of CoTiSb alloy in the formation of its energy, spatial, spectral, and spin characteristics has been clarified. The ground state of CoTiSb alloy, which is characterized by the highest cohesive energy, is found to be realized in the case where atoms and vacancies are arranged like in the C1b\(^a\) phase. Transitions to the L2\(^a\)\(\gamma\) and B2\(^c\)\(\gamma\) phases with different arrangements of alloy components in their crystal lattices are accompanied by the emergence of high-energy metastable states. CoTiSb alloy in the ground state is a nonmagnetic insulator. The metastable phases transform into metals with spin-polarized electron states and magnetic moments mainly localized at cobalt atoms.

**Keywords**: band structure calculations, X-ray electron spectra, spintronics.

1. Introduction

CoTiSb alloy was first synthesized in work [1]. At present, it is a classical representative of half-Heusler compounds, which are characterized by a certain combination of magnetic, transport, optical, magneto-optical, superconducting, and other important properties. With the help of those compounds, the topologic insulators and the so-called half-metallic state of a solid can be implemented. The latter has an uncompensated spin density of band electrons at the Fermi level, which is an important property required in technologies aimed at the creation of materials for spintronics. The cubic modification is the most typical of them. This modification is most often imagined in the form of four face-centered crystal lattices penetrating one another. The composition of those alloys is predominantly described by the formula VABC, where V stands for an atomic vacancy, A and B are metals, and C is a nonmetal.

A bright feature of the Heusler phases consists in the capability for their composition components to migrate over all four mentioned fcc sublattices, which substantially manifests itself in the Heusler phase properties [2]. This migration is described by arbitrary values for the occupation numbers of the sites in those sublattices with respect to the components of indicated alloys. Evidently, this circumstance will result in the infinite number of structural types characterized by different combinations in the arrangement of atoms and vacancies. However, if the allowed occupation numbers are confined to the set \(\{0, \frac{1}{2}, 1\}\), forty-two independent structural types can be formed in the cubic modification of half-Heusler alloys. Mathematically, they are defined by the corresponding matrices of population parameters (see below) for the sites in the crystal sublattices under discussion [3]. According to the conclusions of work [3], information about
those matrices can be obtained from diffraction measurements.

Some problems concerning interrelations between atomic disorderings and the properties of half-Heusler alloys were considered in a number of works (see, e.g., works [2,4–7]). However, those publications contain only scarce information on CoTiSb alloy. In particular, some mutual exchanges of atoms transform the alloy from the semimetallic (this is obviously a false statement, at least for temperatures exceeding 113 K [8, 9]) non-magnetic state into the metallic one [4].

The statement about the influence of possible atomic disordering on the resistive and thermoelectric properties of this alloy subjected to various conditions of a thermal treatment [7] also turned out poorly substantiated.

Therefore, in that work, where band structure calculations were mainly used, a separate model problem was considered. It concerns the role of some atomic disorderings (structural types) in CoTiSb alloy in the formation of the energy, spatial, spectral, and spin characteristics of this compound.

2. Calculation Procedure

In this work, we will calculate the electron band structures for CoTiSb alloy in its three structural modifications. The modifications differ from one another by the arrangement of atoms and vacancies in the crystal lattice. Each modification corresponds to a definite structural type. Each type is defined in the general form by means of a matrix of occupation numbers for four fcc sublattices \((\alpha, \beta, \gamma, \delta)\), which mutually penetrate one another (Fig. 1) [3]:

\[
\begin{pmatrix}
\alpha_V & \alpha_A & \alpha_B & \alpha_C \\
\beta_V & \beta_A & \beta_B & \beta_C \\
\gamma_V & \gamma_A & \gamma_B & \gamma_C \\
\delta_V & \delta_A & \delta_B & \delta_C
\end{pmatrix}
\]

Here, for example, the quantity \(\gamma_B\) means the probability of population (the occupation number) of the \(\gamma\)-sublattice by component B, and \(\delta_V\) the site population in the \(\delta\)-sublattice by vacancies V. In our specific case of CoTiSb alloy, the components are vacancies (component V), Co (component A), Ti (component B), and Sb (component C). Three structural types (phases, modifications) defined by the matrices of occupation numbers [3] were chosen for calculations, namely:

<table>
<thead>
<tr>
<th>C1b(^a)</th>
<th>L2(^a)</th>
<th>B2(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0 0 0</td>
<td>(\frac{1}{2} \frac{1}{2} 0)</td>
<td>(\frac{1}{2} 0 \frac{1}{2} 0)</td>
</tr>
<tr>
<td>0 0 1 0</td>
<td>0 0 1 0</td>
<td>(\frac{1}{2} 0 \frac{1}{2} 0)</td>
</tr>
<tr>
<td>0 1 0 0</td>
<td>(\frac{1}{2} 1 0)</td>
<td>(\frac{1}{2} 0 \frac{1}{2} 0)</td>
</tr>
<tr>
<td>0 0 0 1</td>
<td>0 0 0 1</td>
<td>(\frac{1}{2} 0 \frac{1}{2} 0)</td>
</tr>
</tbody>
</table>

The values of structure-amplitude factors, which were calculated on the basis of those matrices, can be compared with their corresponding values that are determined from diffraction data. In such a way, we can establish which of the CoTiSb alloy phases is realized in the synthesized specimens with the highest probability. It turned out [3] that, in the case of CoTiSb alloy, the C1b\(^a\) phase is realized with the highest probability, whereas the formation of the L2\(^a\) structure has a low probability. The highest probability for the structure-amplitude factor of the B2\(^c\) phase is that it does not correspond to experimental observations, which means that the researched alloy cannot exist in the B2\(^c\) modification.

In this work, in order to simplify the calculation procedure for the examined phases, the positions of alloy components were assigned by means of their multiplication, by using the symmetry operations for a simple cubic lattice \(P\). Note that in the framework of the standard group-theory approach to the classification of crystalline solids, the arrangement of atoms in the C1b\(^a\) phase in the symmetry of \(P\) lattice corresponds to the cubic system with the space group F-43m (No. 216) [9]. In this work, this coincidence was used as one of the criteria of that the procedure selected for the calculation of the electron band structure in the analyzed phases with the use of \(P\) lattice is correct.

The band structure calculations were performed, by using the Linearized Augmented Plane Wave (LAPW) method [10] with the generalized gradient...
Fig. 2. Total electron densities of states in CoTiSb alloy in its F-43m and C1b a modifications normalized to the height of the corresponding main maximum. Zero energy values correspond to the Fermi level position approximation (GDA) of the electron density (see work [11]). While calculating the parameters of the electron band structure, a spin-polarized variant of this method was applied [12]. There is no information in the literature concerning the values of the parameter a for the cubic lattices in the L2 1 and B2 c modifications. Therefore, they were calculated by minimizing the total energy of the CoTiSb alloy spatial structure [12]. In so doing, the data obtained experimentally [9] for the F-43m configuration, which was optimized together with the structure of the alloy concerned in the C1b a modification (it was done to implement the same approach in calculations), were taken as the initial values for the parameter a.

The results obtained are quoted in Table. They testify that the optimized value of the parameter a for the F-43m structural type differs from the experimentally measured one only by 0.18%.

As was expected, the optimized values of the parameter a for the F-43m and C1b a modifications practically coincide. The total electron densities of states calculated for those structures are also identical (Fig. 2). Both those facts testify now that the structure modeling with the use of P lattice for the researched phases is a proper procedure.

The muffin-tin (MT) radii for the atomic spheres were chosen to minimize the dimensions of region II between the spheres in the C1b a modification, which has the smallest volume of the unit cell. For all spatial configurations and all atoms, these radii amounted to 2.18a0, where a0 = 5.2918 × 10−11 m is the Bohr radius. While calculating the indices of the electron band structure for all structural modifications of CoTiSb compound, 172 points were used in the irreducible part of their Brillouin zones. The APW+lo bases were used to approximate the wave functions of 3d electrons in all atoms, and the LAPW ones for the wave functions of other valence electrons. The product of the MT sphere radius Rmt and the wave vector maximum for plane waves Kmax was chosen to equal seven. The maximum values of the quantum number were selected to equal ℓ = 10 for the partial waves in the spheres and ℓ = 4 in the calculation of non-muffin-tin matrix elements.

The cohesive energy was calculated as the difference between the total energy of the corresponding CoTiSb phase and the sum of the total energies of the atoms that composed that phase but moved away from each other by an “infinite” distance. The atoms were determined in accordance with the recommendations of work [13].

The polarization degree P of Fermi electrons was determined by the formula [14]

\[ P = \frac{D_\uparrow(E_F) - D_\downarrow(E_F)}{D_\uparrow(E_F) + D_\downarrow(E_F)}, \]

where \( D_\uparrow(E_F) \) and \( D_\downarrow(E_F) \) are the total electron densities of states at the Fermi level \( (E_F) \) with the spins directed upward and downward, respectively.

### Optimized values of the parameter a for a unit cell in various structural types of CoTiSb compound

<table>
<thead>
<tr>
<th>Structural type</th>
<th>F-43m</th>
<th>C1b a</th>
<th>L2 1 a</th>
<th>B2 c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a, Å</td>
<td>5.8934</td>
<td>5.8825*</td>
<td>5.907</td>
<td>6.2123</td>
</tr>
</tbody>
</table>

*Experiment [9].
3. Discussion of the Results Obtained

As one can see from the results depicted in Fig. 3, the states with both close (with a difference of a few thousandths of electronvolt) and considerably different values of cohesive energy can be realized in the system of mentioned spatial configurations. The former expectedly include the F-43m and C1b\(^a\) phases, as well as the compositions of the structural types L2\(_1\)\(^a\) and B2\(_c\). At the same time, the transition from the C1b\(^a\) phase to the L2\(_1\)\(^a\) one is accompanied by a drastic (1.024 eV) increase of the cohesive energy. As a result, the L2\(_1\)\(^a\) and B2\(_c\) phases can be regarded as metastable, energetically highly excited states, which cannot be realized in practice. The transition from the ground state into a highly excited one results in the loosening of interatomic chemical bonds, which gives rise to the growth of the parameter \(a\), especially large at the transition into the B2\(_c\) phase.

The variations in the component arrangement over the crystallographic positions of CoTiSb alloy manifest themselves in the total electron densities of states, which are shown in Fig. 4. In order to find a “true” (corresponding to the experiment) distribution of states among them, the photoelectron spectrum of CoTiSb compound [15] is also exhibited. Its \(c\)-peak is superimposed with the corresponding component in the total electron density of states obtained for the C1b\(^a\) modification. This procedure of matching between the photoelectron spectra and the total electron density of states on the basis of their characteristic peaks is standard (see, e.g., works [16, 17]). A mismatch obtained between the Fermi level positions that were determined either experimentally on the scale of valence electron binding energies (\(E_b\)) or with the help of band structure calculations is caused by the fact that the excited states (a vacancy in the final state of the photoionization process) reveal themselves in the experimental case, whereas the calculation concerns the ground state of the compounds and gives underestimated values for the Fermi level energies. In our case, the indicated mismatch between the Fermi level positions amounted to 1.12 eV. As to the distributions of other electron densities in the L2\(_1\)\(^a\) and B2\(_c\) phases, they are coupled with the previous one by their Fermi levels.

From Fig. 4, it follows that the energy distribution of valence electrons corresponding to the atomic configuration of the C1b\(^a\) structural type maximally correlates with the experiment. In particular, the en-
energy positions of peaks c, d, and e in the photo-
electron spectrum coincide with their counterparts
in the electron state distribution. At the same time,
the positions of bright components a and b in the
experimental spectrum correspond to humps in the
density of states. The calculations also testified to
a non-magnetic character (see below) of the C1bₐ
phase. Furthermore, in this phase, there exists a di-
rect minimum energy gap of 1.087 eV at point Γ of
the Brillouin zone, which separates occupied and free
electron states from one another. According to the
results of other band structure calculations, the mag-
nitude of this gap falls within an interval from 0.95
[8, 18] to 1.06 eV [15].

The revealed correlation between the calculation
and experimental results testifies, first, that the pro-
posed calculation procedure is correct; second, that
the ground state of the researched compound is re-
ally realized in the atomic configuration of the C1bₐ
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The revealed correlation between the calculation
and experimental results testifies, first, that the pro-
posed calculation procedure is correct; second, that
the ground state of the researched compound is re-
ally realized in the atomic configuration of the C1bₐ
structural type; and, finally, that the determination
of atomic configurations in the ground state with the
use of band structure calculations can be useful in
the researches dealing with the spatial structure in
atomically disordered compounds.

It is no surprise that the energy distributions of
the total density of valence electron states for the
L₂¹ₐ and B₂عش phases substantially differ from
the previous one. Mismatches are observed not only
in the energy positions of density components and their
shapes, but also in their presence exactly at the Fermi
level. Really, if the C1bₐ composition is an insulator
with a zero density of valence electrons at the Fermi
level, the other two compositions transform into met-
als with a state density of 9.94 state/eV for the L₂¹ₐ
phase and 15.64 state/eV for the B₂عش one. Note that
the overestimated value of the total density of states
obtained for valence electrons at the Fermi level for
the B₂عش phase testifies again to the extreme instabil-
ity of this phase and the impossibility of its synthesis.

The transformation to metastable modifications of
the examined alloy is accompanied by the spin polar-
ization of their electron states, which is illustrated in
Fig. 5. One can see that the polarization degrees P
for the L₂¹ and B₂عش phases substantially differ from
zero. The zero polarization is typical of the ground
state modifications F-43m and C1bₐ. In turn, transi-
tions between metastable phases change the sign of P.

The polarization of electron states substantially af-
fects the formation of magnetic moments at the atoms
in separate phases of CoTiSb alloy. From Fig. 5, it fol-
lows that the F-43m and C1bₐ phases of the ground
state are really non-magnetic [18]. At the same time,
atoms in metastable formations acquire magnetic mo-
mements. The corresponding values of magnetic mo-
ments at cobalt atoms turn out anomalously large and
change their sign at the transition from a metastable
phase to the other one. The variations of magnetic
moments at titanium and antimony atoms are less con-
siderable, and those moments themselves are close
to zero.

4. Conclusions

The ground state of CoTiSb alloy with the largest
cohesive energy is realized in the C1bₐ phase. Transi-
tions to the L₂¹ and B₂عش phases with different ar-
rangements of alloy components in their crystal lat-
tices are accompanied by the emergence of metastable
high-energy excited states. CoTiSb compound is a
non-magnetic insulator in the ground state, whereas
the metastable phases are transformed into metals with
spin-polarized electrons and the magnetic mo-
mments mainly localized at cobalt atoms.


