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EVALUATION OF ARTERIAL WALL ELASTICITY DURING ULTRASOUND DIAGNOSTICS

A strained biological tissue, namely, arterial walls under the propagation of a pulse wave, has been studied. The artery is considered as a channel in an elastic medium, and blood as an ideal liquid. The problem is reduced to a system of equations including both the equations for elastic equilibrium, which describe the behavior of the arterial environment, and the hydrodynamic Euler equations, which describe the blood flow in the artery. The obtained solution of the system relates the shear modulus of the elastic medium with those parameters of a pulse wave that can be determined, by using the conventional ultrasound method, namely, the velocity of a blood flow and the deformation of an aortic wall. As an example, the shear modulus of the carotid artery wall is found.

K e y w o r d s: arterial pressure, pulse pressure, ultrasound research, shear modulus, vascular wall.

1. Introduction

The subject of this paper concerns medical physics, a branch of science that arose at the interface between physics and medicine. As is indicated in the title, the specific problem that we are interested in consists in the deformation properties of an arterial wall. It is well known (see, e.g., work [1]) that the deformability is an important factor for the functioning of the cardiovascular system.

In the medical literature (see, e.g., work [2]), vascular wall deformations are conventionally regarded to be elastic. In particular, they are characterized by Young's modulus E, which is determined from the formula $c = \sqrt{\frac{Eh}{2R\rho}}$, where c is the propagation velocity of a pulse wave, h the arterial wall thickness, R the internal arterial radius, and ρ the blood density. Of the quantities entering this formula, the parameters c and R are determined experimentally. The value of the parameter R can be found from the M-echogram, whereas c is determined by the formula $c = l/t$, where t is the time required for a pulse to cross the distance l.

In our opinion, this method is not sufficiently correct in view of its following shortcomings. First, we have no standard non-invasive methods for the determination of a wall thickness. From practice, this quantity is known to vary from 0.1 to 1 mm. Therefore, such measurements cannot be done, by using the ultrasound (US) diagnostics method, because its resolution is considerably lower than the indicated values. Second, there are some restrictions associated with sound velocity measurements. Really, the time moment of the pulse wave passage is registered by an US sensor. We have to register two time moments: the time, when the pulse wave enters a spatial element of the length l and the time, when it leaves it. So, two sensors are required. The frequency of pulses generated by the sensor amounts to 50 Hz, so that the time scale of measurements equals 0.02 s. If we set that the pulse propagation velocity equals 5 m/s, the distance between the sensors has to be substantially longer than 10 cm. In the literature, distances of about 1 m are discussed. But both the vascular radius and the elasticity modulus can change substantially over such distances. As a result, this method does not allow one to obtain a local value for Young's modulus.

Unlike the method described above, we propose another one, which makes it possible to determine the local value of elasticity modulus, the shear modulus G. In this work, we consider how the modulus G for a vascular wall can be determined, when the patient is examined.

2. Physical Model of Artery

Let us consider a blood vessel as a cylindrical channel in an elastic medium with the shear modulus G. When flowing through the channel, blood creates a pressure on the channel wall. As a result, the elastic medium around the channel is deformed. As the pressure P, we will consider the pulse arterial pressure, i.e. the maximum difference between the systolic and diastolic pressures, assuming that the vessel environment is in the undeformed state in diastole. Let the channel diameter be designated as $d_0 = 2b_0$ in diastole and as $d = 2b$ in systole, where b_0 and b are the corresponding radii.

Let us introduce cylindrical coordinates with the axis Z directed along the channel axis. The problem is assumed to be axially symmetric. The radial coordinate is r. The longitudinal and radial shifts are denoted by u_z and u_r, respectively. For the radial shift of vessel walls, $u_r(r = b)$, the notation u_b is introduced.

Actually, the solution of the formulated problem consists of two stages. The first stage includes the calculation of stresses in the medium around the artery ($r > b_0$). This is a problem of the theory of elasticity. The second stage includes the calculation of the blood pressure and velocity in the vessel ($r < b_0$). The formulation of this problem belongs to hydrodynamics. A comparison of the results of both indicated stages will allow us to connect the modulus G with the parameters characterizing the blood flow.

From the viewpoint of the theory of elasticity (see, e.g., work [3]), the problem is formulated as follows. The equation of elastic equilibrium

$$\mathrm{div}\sigma = 0, \tag{1}$$

where σ is the strain tensor, has to be solved, provided the boundary condition

$$\sigma_{rr} = -P, \tag{2}$$

where σ_{rr} is the radial strain. The solution of this problem is known (see, e.g., work [3]). In particular, the formula

$$u_r = \frac{B}{r} \tag{3}$$

is obtained for the shift, and the expression

$$\sigma_{rr} = -2G\frac{B}{r^2} \tag{4}$$

for the radial strain. The value of the constant B is obtained by substituting equality (4) into condition (2):

$$B = \frac{Pb^2}{2G}. \tag{5}$$

Accordingly, for the vessel wall shift, we have

$$u_b = \frac{b}{2G}P. \tag{6}$$

This expression can be rewritten in the form

$$P = \alpha u_b, \tag{7}$$

where the notation

$$\alpha = \frac{2G}{b} \tag{8}$$

is used.

As one can see from formula (6), the shear modulus G can be calculated, if the quantities P, u_b, and b are known. The values of two first parameters are governed by the features of a blood flow in the vessel. Therefore, let us consider the propagation of a pulse wave in the artery.

3. Pulse Wave in the Artery

The appearance of the arterial pressure P is known to be associated with the propagation of a pulse wave in the vessel. Therefore, taking the necessity to determine this quantity into account, let us address the theory of pulse waves. We will use the variant of this theory that was expounded in work [4]. Blood will be considered as an ideal fluid, and the blood flow will be described with the help of Euler's equations

$$\rho \frac{\partial \upsilon_z}{\partial t} = \frac{\partial P}{\partial z}, \tag{9}$$

$$\rho \frac{\partial \upsilon_r}{\partial t} = \frac{\partial P}{\partial r}, \tag{10}$$

where t is the time, and υ_z and υ_r are the axial and radial velocity components. Expression (7) continues to be the required boundary condition. The solution of Eq. (9) is sought in the form

$$\upsilon_z = f\left(\frac{t-z}{c}\right). \tag{11}$$

Assuming blood to be an incompressible fluid, we may write

$$\frac{\partial \upsilon_z}{\partial z} + \frac{1}{r}\frac{\partial (r\upsilon_r)}{\partial r} = 0. \tag{12}$$

By definition,

$$\upsilon_r = \frac{\partial u_r}{\partial t}. \tag{13}$$

Introducing the notation $\xi = t - \frac{z}{c}$ and taking formula (2) into accoiunt, we obtain

$$\frac{\partial \upsilon_z}{\partial z} = \frac{\partial t}{\partial \xi}\left(-\frac{1}{c}\right), \tag{14}$$

$$\frac{\partial \upsilon_z}{\partial t} = \frac{\partial t}{\partial \xi}. \tag{15}$$

Comparing those expressions, we have

$$\frac{\partial \upsilon_z}{\partial z} = \frac{\partial \upsilon_z}{\partial t}\left(-\frac{1}{c}\right). \tag{16}$$

Substituting Eqs. (13) and (16) into formula (12), we obtain

$$\frac{\partial \upsilon_z}{\partial t}\left(-\frac{1}{c}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial t}\right) = 0. \tag{17}$$

Integrating Eq. (17) over r and t, we have

$$u_r = \frac{\upsilon_z r}{2c} + \frac{S(t)}{r} + \frac{\psi(r)}{r}, \tag{18}$$

where $S(t)$ and $\psi(r)$ are arbitrary functions of t and r. In the case $\upsilon_z = 0$, the radial shift has to be equal to zero, so that

$$S(t) = \psi(r) = 0. \tag{19}$$

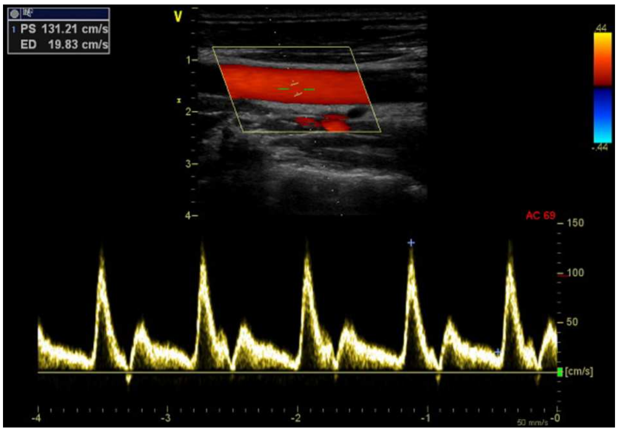

Fig. 1. Doppler echogram for vessel

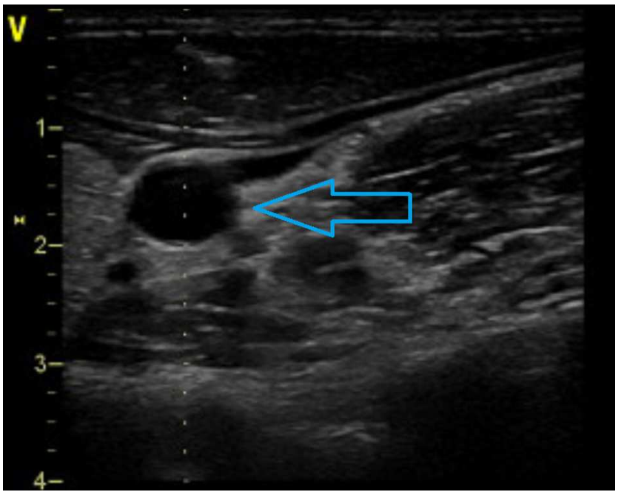

Fig. 2. *B*-echogram for vessel

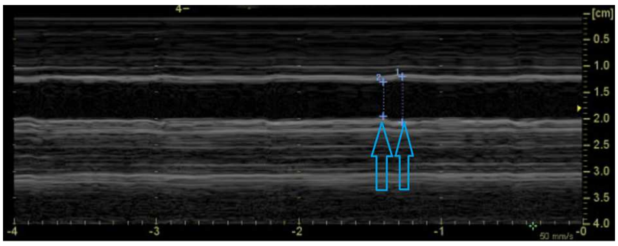

Fig. 3. *M*-echogram for vessel

Accordingly, formula (18) reads

$$u_r = \frac{\upsilon_z r}{2c}. \quad (20)$$

The propagation velocity of a pulse wave is known to be determined by the formula

$$c = \sqrt{\frac{\alpha b}{2\rho}}. \quad (21)$$

Substituting equality (21) into formula (20) and taking expression (8) into account, we obtain

$$G = \frac{\rho \upsilon_z^2 b^2}{4u_b^2}. \quad (22)$$

4. Determination of the Shear Modulus of an Arterial Wall during an Ultrasonic Investigation *in vivo*

As one can see from formula (22), the shear modulus can be calculated, by knowing the blood flow velocity υ_z, vessel radius b, blood density ρ, and vessel wall shift u_b. Except for the blood density, which can be considered constant (1.05×10^3 kg/m^3), all other quantities are determined with the help of modern US devices [5]. In particular, the velocity υ_z is determined with the use of Doppler echograms, whereas the wall shift u_r and the vessel radius can be found from M-echograms.

As an example, we will demonstrate how we measured the shear modulus of a carotid wall. Figure 1 illustrates the Doppler echogram for this vessel, and Figs. 2 and 3 the B- and M-echograms, respectively. The arrow in the B-echogram (Fig. 2) points to a cross-section of the examined artery. The value of z required for calculations corresponds to the size of the arrow in Fig. 1. Two arrows in the M-echogram (Fig. 3) mark the vessel diameters in diastole and systole.

The measurements gave the following numerical data: $d_0 = 6.7$ mm, $d = 8.7$ mm, and $\upsilon_z = 1.35$ m/s. For the blood density, we took the value $\rho = 1.05 \times 10^3$ kg/m^3. Substituting those data into formula (22), we obtain $G \approx 2.5 \times 10^4$ Pa, which characterizes the elasticity of the researched arterial wall.

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ВИЗНАЧЕННЯ ПРУЖНОСТІ СТІНОК АРТЕРІЙ
В ПРОЦЕСІ УЛЬТРАЗВУКОВОГО ДОСЛІДЖЕННЯ

Р е з ю м е

В даній статті вивчається напружений стан біологічної тканини, а саме: стінки артерії при поширенні пульсової хвилі. Артерія розглядається як канал в пружному середовищі. Моделлю крові є ідеальна рідина. Задача зводиться до розв'язання системи рівнянь, що складається з рівнянь пружної рівноваги та ейлерових рівнянь гідродинаміки. Перші описують поведінку оточення артерії, другі – течію крові в артерії. В результаті розв'язання системи рівнянь, отримано формулу, що пов'язує зсувний модуль пружного середовища із характеристиками пульсової хвилі – швидкістю крові та деформацією стінки аорти, які можна визначати за допомогою традиційного ультразвукового дослідження. Як приклад, визначено зсувний модуль стінки сонної артерії.