doi: 10.15407/ujpe62.06.0473

O.I. DAVIDOVSKAYA, V.YU. DENISOV, V.A. NESTEROV

Institute for Nuclear Research, Nat. Acad. of Sci. of Ukraine
(47, Nauky Ave., Kyiv 03028, Ukraine; e-mail: denisov@kinr.kiev.ua)

EFFECTIVE NUCLEUS-NUCLEUS POTENTIAL WITH THE CONTRIBUTION OF THE KINETIC ENERGY OF NUCLEONS, AND THE CROSS-SECTIONS OF ELASTIC SCATTERING AND SUBBARRIER FUSION

PACS 25.70-z, 25.70-Jj

The microscopic double-folding approach to the calculation of the nucleus-nucleus interaction has been discussed in detail. The nucleus-nucleus interaction potentials for the $^{16}O + ^{208}Pb$ system with and without the contribution of the kinetic energy of nucleons in the nuclei are constructed, and the cross-sections of elastic scattering and subbarrier fusion are calculated. The experimental values of those cross-sections are shown to be described well, if the contribution of nucleons' kinetic energy is taken into account.

K e y w o r d s: nucleus, interaction potential, nucleon density distribution, fusion cross-section, kinetic energy, elastic scattering.

1. Introduction

In order to calculate the cross-sections of nuclear reactions, one has to know the potential energy of interaction between nuclei. Therefore, both the magnitude and the radial dependence of the interaction potential between nuclei are very important [1–4]. The energy of interaction between spherical nuclei is associated with the Coulomb interaction of protons and the nuclear interaction of all nucleons composing those nuclei. Therefore, the interaction potential between nuclei can be presented as the sum of the Coulomb, nuclear, and centrifugal interactions. The Coulomb and centrifugal interactions between nuclei are known well enough, whereas the interaction between nuclei associated with nuclear forces has been studied much less. Therefore, a considerable number of various approximations have been proposed at present to calculate this interaction [1–28].

Note that the nuclear part of the interaction potential, which is described either by the Woods–Saxon potential [1–4, 18] or with the use of the double-folding potential [1, 2, 4–12], is attractive at any distance between nuclei. On the contrary, the "proximity" potential [15] or the potentials obtained in the framework of the energy density approach with the help of a modified Thomas–Fermi method [13, 14, 17, 19–28] are attractive at large and medium distances between the interacting nuclei, but demonstrate the repulsion at short distances, when the nuclei are strongly overlapped. This repulsion is due to a large value of the coefficient of incompressibility of the nuclear matter, a contribution of the kinetic energy to the potential, and the account for the Pauli principle [19].

While determining the magnitude of nucleus-nucleus interaction, one should apply the most exact methods developed for the description of the interaction energy between nuclei and the properties of nuclei and the nuclear matter. Therefore, we use the double-folding method [1, 2, 4–12], which is widely applied to the description of nuclear reactions. The double-folding potential has the constant N related to the force of nucleus-nucleus interaction, which is usually fitted to better describe the elastic scattering data [4–7, 9–11]. Note also that, when describing the cross-sections of subbarrier fusion of nuclei, the double-folding potential is summed up with a phenomenological repulsive potential [21] used to describe experimental data.

At short distances between nuclei, the nuclear densities substantially overlap each other, and, due to

the Pauli principle, the internal kinetic energy of nucleons in nuclei is considerably changed. This work is aimed at a detailed consideration of the influence of the contributions made by the kinetic energy to the potential of nucleus-nucleus interaction. With the help of a potential that takes those contributions into account, the cross-sections of subbarrier fusion are calculated, as well as the angular distribution of the elastic scattering in the framework of the optical model. The next section is devoted to the description of the nucleus-nucleus interaction in the framework of this approach. Sections 3 and 4 include the discussion of the results obtained and the conclusions, respectively.

2. Effective Potential of Nucleus-Nucleus Interaction

The potential of nucleus-nucleus interaction $V(R)$ can be presented as the sum of the nuclear, $V_N(R)$, Coulomb, $V_{\rm COUL}(R)$, and centrifugal, $V_L(R)$, components depending only on the distance R between the nuclei:

$$V(R) = V_N(R) + V_{\rm COUL}(R) + V_L(R). \quad (1)$$

Here,

$$V_{\rm COUL}(R) = \begin{cases} \dfrac{Z_1 Z_2 e^2}{R}, & R \geq R_{\rm C}, \\ \dfrac{Z_1 Z_2 e^2}{R_{\rm C}} \left[\frac{3}{2} - \frac{R^2}{2R_{\rm C}^2}\right], & R < R_{\rm C}, \end{cases} \quad (2)$$

$$V_L(R) = \frac{\hbar^2 l(l+1)}{2M[A_1A_2/(A_1+A_2)]R^2}, \quad (3)$$

where Z_1 and Z_2 are the numbers of protons in the nuclei, e is the proton charge, A_1 and A_2 are the numbers of nucleons in the nuclei, $R_{\rm C} = r_{\rm C}(A_1^{1/3} + A_2^{1/3})$ is the radius of the Coulomb interaction, and l is the value of the orbital moment.

In the energy density approximation [13, 14, 16, 17, 19], the potential of nucleus-nucleus interaction is equal to the difference between the energies of the system of two nuclei located at the finite, $E_{12}(R)$, and infinite, $E_1 + E_2$, distances from each other:

$$V_N(R) = E_{12}(R) - E_1 - E_2. \quad (4)$$

The corresponding binding energies can be determined, if we know the nucleon density distribution in the nuclei and the functional of nuclear energy density,

$$E_{12}(R) = \int d\mathbf{r}\, \varepsilon\left[\rho_{1p}(\mathbf{r}) + \rho_{2p}(\mathbf{r}, R), \rho_{1n}(\mathbf{r}) + \rho_{2n}(\mathbf{r}, R)\right], \quad (5)$$

$$E_{1(2)} = \int d\mathbf{r}\, \varepsilon\left[\rho_{1(2)p}(\mathbf{r}), \rho_{1(2)n}(\mathbf{r})\right], \quad (6)$$

where

$$\varepsilon[\rho_p(\mathbf{r}), \rho_n(\mathbf{r})] = \tau[\rho_p(\mathbf{r})] + \tau[\rho_n(\mathbf{r})] + \mathcal{V}_{\rm Sk}[\rho_p(\mathbf{r}), \rho_n(\mathbf{r})] \quad (7)$$

is the energy density [13, 14, 16, 17, 19, 29–31]; ρ_{1n} and ρ_{1p} are the neutron and proton densities, respectively, in the first nucleus; ρ_{2n} and ρ_{2p} are the nucleon densities in the second nucleus; and R is the distance between the centers of nuclear masses. Note that the energy density functional contains terms that are associated with both the kinetic ($\tau[\rho_p(\mathbf{r})] + \tau[\rho_n(\mathbf{r}))$]) and potential ($\mathcal{V}_{\rm Sk}[\rho_p(\mathbf{r}), \rho_n(\mathbf{r})]$) energies of nucleons. In this work, we use the Skyrme forces [32], which enables us to obtain the well-known explicit expression for the energy density [16, 17, 19, 29–31]. The kinetic energy density has the following form to within the second-order terms in $\hbar$: $\tau = \tau_{\rm TF} + \tau_2$, where $\tau = \tau_n + \tau_p$ is the sum of the kinetic energy densities for the protons and neutrons [29, 30],

$$\tau_{{\rm TF}n(p)} = k\,\rho_{n(p)}^{5/3} \quad (8)$$

is the kinetic energy density for the neutrons in the Thomas–Fermi approximation, $k = \frac{5}{3}(3\pi^2)^{2/3}$, τ_2 is the total expression for the gradient correction of the second order in $\hbar$:

$$\tau_{2n(p)} = b_1 \frac{(\nabla\rho_{n(p)})^2}{\rho_{n(p)}} + b_2\nabla^2\rho_{n(p)} + b_3\frac{\nabla f_{n(p)}\nabla\rho_{n(p)}}{f_{n(p)}} + b_4\rho_{n(p)}\frac{\nabla^2 f_{n(p)}}{f_{n(p)}} + b_5\rho_{n(p)}\frac{(\nabla f_{n(p)})^2}{f_{n(p)}^2} + b_6 h_m^2\rho_{n(p)}\left(\frac{\mathbf{W}_{n(p)}}{\rho_{n(p)}}\right)^2, \quad (9)$$

$b_1 = 1/36$, $b_2 = 1/3$, $b_3 = 1/6$, $b_4 = 1/6$, $b_5 = -1/12$, and $b_6 = 1/2$ are numerical coefficients; and $h_m = \hbar^2/2m$. The last term in formula (9) describes

the spin-orbit interaction. The following notations are also introduced:

$$\mathbf{W}_{n(p)} = \frac{\delta\varepsilon(r)}{\delta \mathbf{J}_{n(p)(r)}} = \frac{W_0}{2}\nabla(\rho + \rho_{n(p)}), \tag{10}$$

$$f_{n(p)} = 1 + \frac{2m}{\hbar^2}\left[\frac{1}{4}\left[t_1\left(1+\frac{x_1}{2}\right) + t_2\left(1+\frac{x_2}{2}\right)\right]\rho + \right.$$
$$\left. + \frac{1}{4}\left[t_2\left(x_2+\frac{1}{2}\right) - t_1\left(x_1+\frac{1}{2}\right)\rho_{n(p)}\right]\right], \tag{11}$$

where x_1, x_2, t_1, t_2, and W_0 are the constants of Skyrme forces, which depend on the parametrization choice. The contribution of the Thomas–Fermi term dominates, especially in the nucleus bulk. However, gradient corrections start to play a substantial role at the nucleus surface. The term proportional to b_1 is the so-called Weizsäcker correction. The coefficient b_1 is often used as a fitting parameter in order to compensate the neglect of some terms in the series expansion of the kinetic energy to an accuracy of $\hbar^2$ or $\hbar^4$. However, in this case, the simultaneous description of the binding energies and the nucleon distribution densities becomes impossible.

Note that, in the energy density approximation (4)–(7), the potential has two terms, which are associated with the kinetic energy of nucleons, $V_T(R)$, and the energy of nucleon-nucleon interaction, $V_{nn}(R)$:

$$V_N(R) = V_T(R) + V_{nn}(R), \tag{12}$$

where

$$V_T(R) =$$
$$= \int d\mathbf{r}\,\tau[\rho_{1p}(\mathbf{r}) + \rho_{2p}(\mathbf{r},R), \rho_{1n}(\mathbf{r}) + \rho_{2n}(\mathbf{r},R)] -$$
$$- \int d\mathbf{r}\,\tau[\rho_{1p}(\mathbf{r}), \rho_{1n}(\mathbf{r})] - \int d\mathbf{r}\,\tau[\rho_{2p}(\mathbf{r}), \rho_{2n}(\mathbf{r})], \tag{13}$$
$$V_{nn}(R) =$$
$$= \int d\mathbf{r}\,V_{\rm Sk}[\rho_{1p}(\mathbf{r}) + \rho_{2p}(\mathbf{r},R), \rho_{1n}(\mathbf{r}) + \rho_{2n}(\mathbf{r},R)] -$$
$$- \int d\mathbf{r}\,V_{\rm Sk}[\rho_{1p}(\mathbf{r}), \rho_{1n}(\mathbf{r})] - \int d\mathbf{r}\,V_{\rm Sk}[\rho_{2p}(\mathbf{r}),\, \rho_{2n}(\mathbf{r})]. \tag{14}$$

The contribution of the kinetic energy of nucleons to the potential is very important at short distances between the nuclei, since it takes into account that nucleons are fermions and obey the Pauli principle. Note that the contribution of the kinetic energy of nucleons to the nucleus-nucleus potential is analogous by its origin to the contribution of the kinetic energy of electrons to the potential of intermolecular interaction, which has been taken into account since the 1950s [33, 34], while calculating various parameters of collisions between atoms and molecules.

Double-folding potentials have been widely used recently [2–12]. In order to calculate the potential of nucleus-nucleus interaction in the framework of the double-folding method, the potential of nucleus-nucleus interaction ν and the nucleon density distributions $\rho_{1(2)}$ in the ground states of colliding nuclei have to be given:

$$V_{\rm DF}(R) = N\,G(E)\int d\mathbf{r}_1 d\mathbf{r}_2 \rho_1(\mathbf{r}_1) F(\rho_1(\mathbf{r}_1) +$$
$$+ \rho_2(\mathbf{r}_2)) v(R + \mathbf{r}_2 - \mathbf{r}_1)\rho_2(\mathbf{r}_2). \tag{15}$$

Here, $\mathbf{r}_1$ and $\mathbf{r}_2$ are radius vectors describing the positions of interacting nucleons in the coordinate systems coupled with the centers of mass of the nuclei, and R is the distance between those centers. The functions $F(\rho)$ and $G(E)$ [5, 7, 12] describe the dependence of the nucleon-nucleon interaction on the nucleon densities and the collision energy, and the multiplier N is usually selected to better describe the scattering data [4–7, 9–11].

Note that the nucleon distribution densities in nuclei can be obtained in the framework of different approaches, and the forces acting between nucleons can also be different. In this work, we use modern modifications of the double-folding method and modern interaction parameterizations. In particular, these are nucleon-nucleon forces $DDM3Y$ [5, 7, 12] depending on the nucleon density distribution in nuclei and the collision energy. According to Refs. [5, 7, 12],

$$G(E) = 1 - 0.002\,E, \tag{16}$$

$$F(\rho) = C\,[1 + \alpha\,\exp(-\beta\,\rho)], \tag{17}$$

where E is the collision energy in MeV, ρ the nucleon distribution density, and C, α, and β are parameters determined by fitting the scattering data.

The contribution of the internal kinetic energy of nucleons $V_T(R)$, which was directly taken into account, when considering the nucleus-nucleus potential in the energy density approximation (5)–(11), is ignored in the standard double-folding method [see

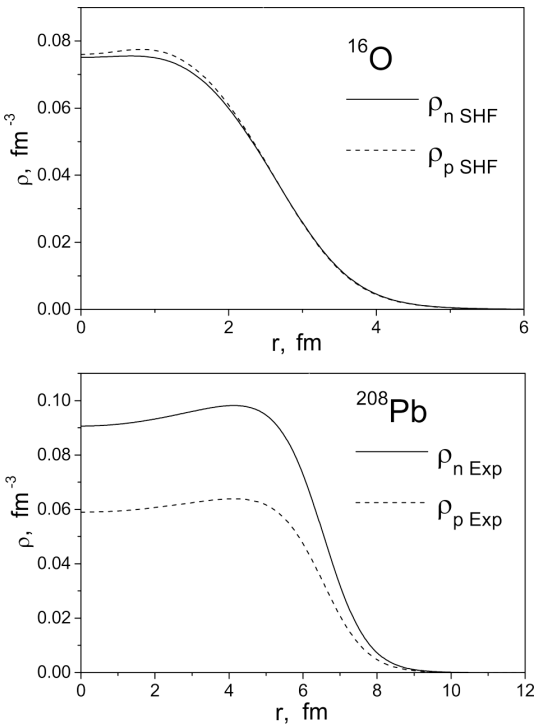

Fig. 1. Nucleon distribution densities in ^{16}O (the shell model) and ^{208}Pb (experimental data) nuclei

Eq. (15)]. Therefore, the potentials obtained in the framework of standard double-folding method for the description of reactions between heavy nuclei are very deep [5–12]. For this reason, for a better description of scattering data for heavy nuclei, the multiplier N in modern parametrizations of the nucleon-nucleon interaction is chosen, as a rule, from the interval $0.7 \leq N \leq 1$ [5, 9–11].

The double-folding potential uses the "frozen" densities of nuclei in the ground state. At small distances between the nuclei, the nuclear densities substantially overlap each other, and, owing to the Pauli principle, the internal kinetic energy of nucleons in the nuclei is considerably changed. The contribution of the internal kinetic energy of nucleons to the double-folding potential for a certain nucleon distribution in the coordinate space can be calculated in the framework of the standard or modified Thomas–Fermi approximation. As a result, the double-folding potential with the account for the contribution from the internal kinetic energy can be determined as follows [4, 25, 26]:

$$V_{\mathrm{DF-kin}}(R) = V_T(R) + V_{\mathrm{DF}}(R). \quad (18)$$

Here, $V_T(R)$ is the contribution of the internal kinetic energy of nucleons to the potential determined in the framework of the energy density approach (4)–(11), and $V_{\mathrm{DF}}(R)$ is the ordinary double-folding potential described by Eq. (15) and associated only with the nucleon-nucleon interaction. The double-folding potential (15) is associated with the "frozen" densities in the nuclear ground states [2, 3, 13, 14]. Therefore, when calculating the contribution of the internal kinetic energy of nucleons in Eq. (18), the corresponding "frozen" densities must also be used.

Nucleon-nucleon forces that are usually selected to calculate the potential in the double-folding approximation do not depend on the velocity. Therefore, in order to calculate the internal kinetic energy of nucleons, the effective mass m^* [29, 30] is usually put to be equal to the ordinary mass m. However, if the nucleon-nucleon forces depending on the density are used in calculations, the corresponding procedure used to determine the contribution of the kinetic energy of nucleons to the potential has to take into account that m^* and m can be different.

In this work, we use local potentials for the nucleus-nucleus interaction. Since we deal with energies in a vicinity of the Coulomb barrier and a little above it, the application of the local approximation looks quite justified, because the nucleon velocities are much higher than those of nuclei. The non-local character of the interaction mainly appears owing to the Pauli principle, the effect of which is known to diminish, as the collision energy increases.

As nucleon densities, both experimental nucleon densities [35] and the densities calculated in the framework of various theoretical approaches (the modified Thomas–Fermi method [29–31], shell model, and Hartree–Fock method [36]) can be used.

3. Discussion of the Results

Figure 1 illustrates the nucleon density distributions in ^{16}O and ^{208}Pb nuclei, which were chosen by us to calculate the nucleus-nucleus interaction potential, as well as the cross-sections of subbarrier fusion and elastic scattering of those nuclei. The best results for ^{16}O nucleus were obtained with the use of the Hartree–Fock model with the Skyrme forces SkM^* for the nucleon density distribution and the BCS approximation for the nucleon pairing [29, 36]. As the distribution of protons in ^{208}Pb, we took an experimen-

tal charge density from Ref. [35]. Since the double-folding method uses the total nucleon density making no distinction between protons and neutrons, the corresponding neutron density was constructed in a proportional way, by using the isotopic symmetry. In this case, the root-mean-square proton radii of ^{16}O and ^{208}Pb nuclei are close to their experimental values [37].

In Fig. 2, the nucleus-nucleus interaction potentials $V(R) = V_N(R) + V_{\rm COUL}(R)$ for the ^{16}O + ^{208}Pb system in a vicinity of the barrier, which were obtained either in the framework of the conventional double-folding method or with the contribution of the internal kinetic energy, are compared. The nucleus-nucleus potential was calculated for the nucleon-nucleon interaction $DDM3Y1$ with the Reid potential [5, 7, 12] and $N = 1$; i.e. we did not fit the value of multiplier N, but used non-modified potential values. At distances close to the summed up radii of nuclei, the double-folding potential calculated with the internal kinetic energy is less attractive than the ordinary one.

As a rule, the calculation of the cross-sections of nuclear reactions with the help of various programs demands a parametrization of the obtained potentials of nuclear interaction in the form of the Woods–Saxon potential. Therefore, we approximated the obtained double-folding interaction potentials $V_{\rm DF}(R)$ and $V_T(R) + V_{\rm DF}(R)$ by the corresponding Woods–Saxon potentials

$$V_{\rm WS}(R) = \frac{-V_0}{1 + \exp[(R - r_0(A_1^{1/3} + A_2^{1/3}))/d_0]} \qquad (19)$$

at distances close to the sum of nuclear radii. The following parameters were determined for the potentials both taking and not taking the contribution of the kinetic energy into account (see Table). The Woods–Saxon potential describes well the nuclear part of interaction at distances where the nuclei are in contact (see Fig. 3). This interval is important for the calculation of cross-sections of fusion and elastic scattering reactions at energies close to the barrier height.

On the basis of the obtained potentials and with the help of the software code CCFULL [38], we calculated the cross-sections of subbarrier nuclear fusion ^{16}O + + ^{208}Pb. The parameters of quadrupole and octupole deformations of low-energy vibrational states in those

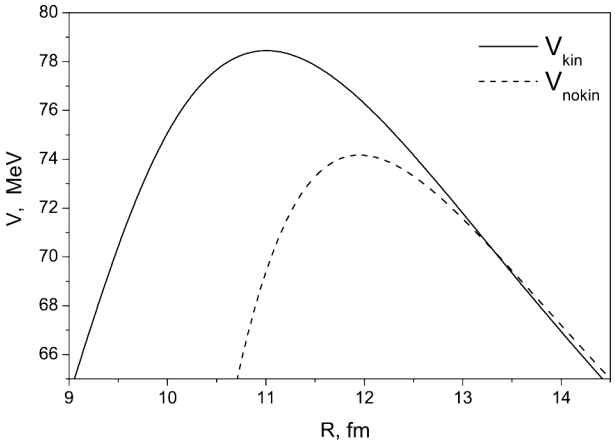

Fig. 2. Nucleus-nucleus interaction potentials for the ^{16}O + ^{208}Pb system obtained in the framework of the double-folding method with the $DDM3Y1$ forces with ($V_{\rm kin}$) and without ($V_{\rm nokin}$) the kinetic energy contribution

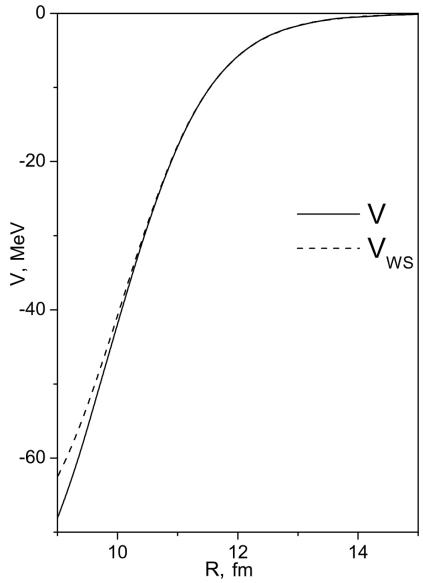

Fig. 3. Approximation of the nuclear part of the nucleus-nucleus interaction potential obtained making no allowance for the contribution of the kinetic energy of nucleons and with the help of the Woods–Saxon potential in a vicinity of about 10 fm near the contact point

Parameters of Woods–Saxon potentials

	V_0, MeV	r_0, fm	d_0, fm
kin	588.12	0.8786	0.8361
nokin	280.03	1.042	0.7465

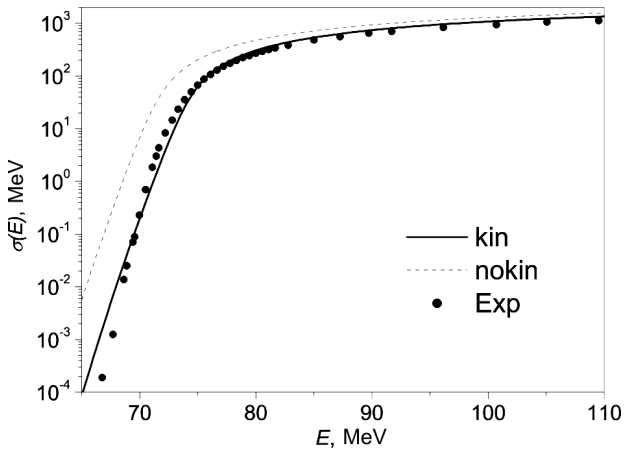

Fig. 4. Cross-sections of sub-barrier fusion $^{16}O + {}^{208}Pb$ calculated taking (kin) and not taking (nokin) the kinetic energy contribution into account, and experimental data (Exp) [41]

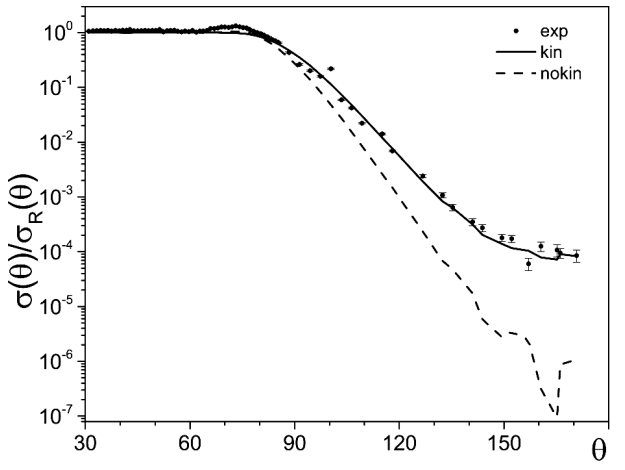

Fig. 5. Experimental data for the $^{16}O + {}^{208}Pb$ elastic scattering at a beam energy of 95 MeV (Exp) [42] and the results of calculations in the framework of the optical model with the use of the nucleus-nucleus potential obtained both taking (kin) and not taking (nokin) the kinetic energy contribution into account

nuclei were taken from tables in Refs. [39, 40]. Those parameters involve the connection with the channels characterized by low-energy vibrational states in the colliding nuclei, when calculating the fusion cross-sections. The cross-sections obtained for the potential that was found with the contribution of the kinetic energy of nucleons agree well with experimental data (see Fig. 4). The potential barrier obtained making no allowance for the contribution of the kinetic energy of nucleons is lower than the barrier obtained with this contribution (see Fig. 2). Therefore, the fusion cross-section calculated without the contribution of the kinetic energy of nucleons turned out considerably overestimated in comparison with experimental values (Fig. 4). Note that, in all our calculations, we used the value of the Coulomb radius $r_C = 1.2$ fm.

We also calculated the cross-sections for the elastic scattering of nuclei $^{16}O + {}^{208}Pb$ at an energy of 95 MeV in the framework of the optical model for the calculated potentials. When calculating the elastic scattering cross-sections, the imaginary part of the nuclear potential was presented as the sum of volume and surface terms:

$$W(R) = -\frac{W_W}{1+\exp\left[R - r_W(A_1^{1/3} + A_2^{1/3})/d_W\right]} - \\ - \frac{W_S \exp\left[R - r_S(A_1^{1/3} + A_2^{1/3})/d_S\right]}{d_S\left\{1+\exp\left[R - r_S(A_1^{1/3} + A_2^{1/3})/d_S\right]\right\}^2}. \qquad (20)$$

Here, the parameters W_W, r_W, d_W, W_S, r_S, and d_S are the force, radius, and diffuseness, respectively, of the volume and surface parts of the imaginary nuclear potential. Such a representation of the imaginary part is general in the theory of nucleus-nucleus collisions [1, 4, 10, 11]. The parameters of the imaginary part of the nuclear potential were found for the double-folding potential, by taking the contribution of the internal kinetic energy into account. The following values were obtained: $W_W = 60.594$ MeV, $r_W = 1.194$ fm, $d_W = 0.3885$ fm, $W_S = 6.035$ MeV, $r_S = 1.315$ fm, and $d_S = 0.67$ fm.

The results of calculations are illustrated in Fig. 5. Here, the magnitudes of the elastic scattering cross-section are exhibited in the form normalized to the Rutherford cross-section. The kinetic energy was and was not taken into account. For comparison, experimental data [42] are also included. From the figure, one can see that the account for the internal kinetic energy of nucleons made it possible to considerably improve the description of experimental data in this case as well.

4. Conclusions

In Figs. 4 and 5, the results of our calculations for the cross-sections of $^{16}O + {}^{208}Pb$ sub-barrier fusion and elastic scattering, respectively, are compared with experimental data. The results of calculations obtained

with the contribution of the internal kinetic energy to the double-folding potential describe experimental data quite well. The results of calculation of the fusion cross-section with the use of the ordinary double-folding potential are considerably overestimated in the subbarrier region. The elastic scattering cross-sections obtained without the contribution of the internal kinetic energy to the double-folding potential also have a worse agreement with the experimental data. Note that the experimental data for the elastic scattering and sub-barrier fusion cross-sections are described without fitting the multiplier N, whose value is usually selected to provide a better description of scattering data.

The authors express their sincere gratitude to Prof. M. Dasgupta for experimental data on $^{16}O + ^{208}Pb$ subbarrier fusion cross-sections [41].

1. G.R. Satchler. *Direct Nuclear Reactions* (Oxford Univ. Press, 1983) [ISBN: 0198512694 9780198512691].
2. P. Frobrich, R. Lipperheide. *Theory of Nuclear Reactions* (Clarendon Press, 1996) [ISBN: 0198537832].
3. R. Bass. *Nuclear Reactions with Heavy Ions* (Springer, 1980) [ISBN: 978-3-540-09611-5].
4. V.Yu. Denisov, V.A. Plyuyko. *Problems of Nuclear Physics and Physics of Nuclear Reactions* (Kyiv University Publ. Center, 2013) (in Russian) [ISBN: 978-966-439-673-5].
5. Dao T. Khoa, W. von Oertzen, H.G. Bohlen. Double-folding model for heavy-ion optical potential: Revised and applied to study ^{12}C and ^{16}O elastic scattering. *Phys. Rev. C* **49**, 1652 (1994).
6. M.E. Brandan, G.R. Satchler. The interaction between light heavy-ions and what it tells us. *Phys. Rep.* **285**, 143 (1997).
7. Dao T. Khoa, W. von Oertzen. A nuclear matter study using the density dependent M3Y interaction. *Phys. Lett.* **304**, 8 (1993).
8. V.B. Soubbotin, W. von Oertzen, X. Viñas, K.A. Gridnev, H.G. Bohlen. Pauli distorted double folded potential. *Phys. Rev. C* **64**, 014601 (2001).
9. A.A. Ogloblin, T. Khoa Dao, Y. Kondō, Yu. A. Glukhov, A.S. Dem'yanova, M.V. Rozhkov, G.R. Satchler, S.A. Goncharov. Pronounced Airy structure in elastic ^{16}O + ^{12}C scattering at $E_{\rm lab}$ = 132 MeV. *Phys. Rev. C* **57**, 1797 (1998).
10. A.A. Ogloblin, Yu.A. Glukhov, W.H. Trzaska, A.S. Dem'yanova, S.A. Goncharov, R. Julin, S.V. Klebnikov, M. Mutterer, M.V. Rozhkov, V.P. Rudakov, G.P. Tiorin, D.T. Khoa, G.R. Satchler. New measurement of the refractive, elastic ^{16}O + ^{12}C scattering at 132, 170, 200, 230, and 260 MeV incident energies. *Phys. Rev. C* **62**, 044601 (2000).
11. Dao T. Khoa, W. von Oertzen, H.G. Bohlen, F. Nuoffer. Study of diffractive and refractive structure in the elastic ^{16}O + ^{16}O scattering at incident energies ranging from 124 to 1120 MeV. *Nucl. Phys. A* **672**, 387 (2000).
12. T. Khoa Dao, W. von Oertzen, H.G. Bohlen, S. Ohkubo. Nuclear rainbow scattering and nucleus–nucleus potential. *J. Phys. G* **34**, R111 (2007).
13. V.Yu. Denisov, V.A. Nesterov. Potential of interaction between nuclei and nucleon-density distribution in nuclei. *Phys. Atom. Nucl.* **69**, 1472 (2006).
14. K.A. Brueckner, J.R. Buchler, M.M. Kelly. New theoretical approach to nuclear heavy-ion scattering. *Phys. Rev. C* **173**, 944 (1968).
15. J. Blocki, J. Randrup, W.J. Swiatecki, C.F. Tsang. Proximity forces. *Ann. Phys. (N.Y.)* **105**, 427 (1977).
16. V.Yu. Denisov. Interaction potential between heavy ions. *Phys. Lett. B* **526**, 315 (2002).
17. V.Yu. Denisov, W. Nörenberg. Entrance channel potentials in the synthesis of the heaviest nuclei. *Eur. Phys. J. A* **15**, 375 (2002).
18. V.Yu. Denisov. Nucleus-nucleus potential with shell-correction contribution. *Phys. Rev. C* **91**, 024603 (2015).
19. V.Yu. Denisov, V.A. Nesterov. Effect of the Pauli exclusion principle on the potential of nucleus-nucleus interaction. *Phys. Atom. Nucl.* **73**, 1142 (2010).
20. Ş. Mişicu, H. Esbensen. Signature of shallow potentials in deep sub-barrier fusion reactions. *Phys. Rev. C* **75**, 034606 (2007).
21. T. Izumoto, S. Krewald, A. Faessler. Nuclear matter approach to the heavy-ion optical potential. *Nucl. Phys. A* **341**, 319 (1980).
22. S. Hossain, M.N.A. Abdullah, K.M. Hasan, M. Asaduzzaman, M.A.R. Akanda, S.K. Das, A.S.B. Tariq, M.A. Uddin, A.K. Basak, S. Ali, F.B. Malik. Shallow folding potential for ^{16}O + ^{12}C elastic scattering. *Phys. Lett. B* **636**, 248 (2006).
23. V.Yu. Denisov, O.I. Davidovskaya. Repulsive core potential and elastic heavy-ion collisions. *Yad. Fiz.* **73**, 429 (2010).
24. V.Yu. Denisov, O.I. Davidovskaya. Repulsive core potential and elastic heavy-ion collisions. *Ukr. J. Phys.* **54**, 669 (2009).
25. O.I. Davidovskaya, V.Yu. Denisov, V.O. Nesterov. Nucleus-nucleus potential with repulsive core and elastic scattering. Part 1. Nucleus-nucleus interaction potential. *Yad. Fiz. Energet.* **11**, No. 1, 25 (2010) (in Ukrainian).
26. O.I. Davidovskaya, V.Yu. Denisov, V.O. Nesterov. Nucleus-nucleus potential with repulsive core and elastic scattering. Part 2. The elastic scattering cross sections with and without core. *Yad. Fiz. Energet.* **11**, No. 1, 33 (2010) (in Ukrainian).
27. V.Yu. Denisov, O.I. Davidovskaya. Elastic scattering of heavy ions and nucleus-nucleus potential with a repulsive core. *Izv. Ross. Akad. Nauk Ser. Fiz.* **74**, 611 (2010) (in Russian).
28. V.Yu. Denisov, O.I. Davidovskaya. Elastic ^{16}O + ^{16}O scattering and nucleus-nucleus potential with a repulsive core. *Ukr. Fiz. Zh.* **55**, 861 (2010).

29. M. Brack, C. Guet, H.B. Håkanson. Self-consistent semiclassical description of average nuclear properties – a link between microscopic and macroscopic models. *Phys. Rep.* **123**, 275 (1985).
30. M. Brack, R.K. Bhaduri. *Semiclassical Physics* (Addison-Wesley, 1997) [ISBN: 0-201-48351-3].
31. V.Yu. Denisov, V.A. Nesterov. The binding energy and the density distribution of atomic nuclei in the framework of modified Thomas–Fermi method. *Yad. Fiz.* **65**, 814 (2002) (in Russian).
32. T.H.R. Skyrme. The effective nuclear potential. *Nucl. Phys.* **9**, 615 (1959).
33. O.B. Firsov. Scattering of ions by atoms. *Zh. Èksp. Teor. Fiz.* **34**, 447 (1958) (in Russian).
34. I.G. Kaplan, *Theory of Molecular Interactions* (Elsevier, 1986) [ISBN: 9780444426963].
35. H. de Vries, C.W. de Jager, C. de Vries. Nuclear charge-density-distribution parameters from elastic electron scattering. *At. Data Nucl. Data Tabl.* **36**, 495 (1987).
36. P. Ring, P. Schuck. *The Nuclear Many-Body Problem* (Springer, 1980) [ISBN: 978-3-540-21206-5].
37. I. Angeli, K.P. Marinova. Table of experimental nuclear ground state charge radii: An update. *At. Data Nucl. Data Tabl.* **99**, 69 (2013).
38. K. Hagino, N. Rowley, A.T. Kruppa. A program for coupled-channel calculations with all order couplings for heavy-ion fusion reactions. *Comput. Phys. Commun.* **1 23**, 143 (1999).
39. B. Pritychenko, M. Birch, B. Singh, M. Horoi. Tables of E2 transition probabilities from the first 2^+ image states in even–even nuclei. *At. Data Nucl. Data Tabl.* **107**, 1 (2016).
40. T. Kibédi, R.H. Spear. Reduced electric-octupole transition probabilities, $B(E3; 0_1^+ \to 3_1^-)$ – an update. *At. Data Nucl. Data Tabl.* **80**, 35 (2002).
41. C.R. Morton, A.C. Berriman, M. Dasgupta, D.J. Hinde, J.O. Newton, K. Hagino, I.J. Thompson. Coupled-channels analysis of the ^{16}O + ^{208}Pb fusion barrier distribution. *Phys. Rev. C* **60**, 044608 (1999).
42. V.P. Rudakov, K.P. Artemov, Yu.A. Glukhov, S.A. Goncharov, A.S. Demyanova, A.A. Ogloblin, V.V. Paramonov, M.V. Rozhkov. Elastic $^{12}C + ^{208}Pb$ and $^{16}O + ^{208}Pb$ scattering and the form of the potential barrier. *Bull. Rus. Acad. Sci. Phys.* **65**, 57 (2001).

Received 00.00.16.
Translated from Ukrainian by O.I. Voitenko

О.І. Давидовська, В.Ю. Денисов, В.О. Нестеров

ЕФЕКТИВНИЙ ЯДЕРНО-ЯДЕРНИЙ ПОТЕНЦІАЛ З УРАХУВАННЯМ ВНЕСКУ КІНЕТИЧНОЇ ЕНЕРГІЇ НУКЛОНІВ ТА ПЕРЕРІЗИ ПРУЖНОГО РОЗСІЯННЯ І ПІДБАР'ЄРНОГО ЗЛИТТЯ

Р е з ю м е

Детально обговорюється мікроскопічній підхід подвійної згортки для розрахунку ядерно-ядерної взаємодії. Побудовано потенціали ядерно-ядерної взаємодії для системи $^{16}O + ^{208}Pb$ з урахуванням та без урахування внеску кінетичної енергії нуклонів у ядрах, розраховано перерізи пружного розсіяння та підбар'єрного злиття. Показано, що експериментальні значення перерізів пружного розсіяння та підбар'єрного злиття добре описуються з урахуванням внеску кінетичної енергії нуклонів.