doi: 10.15407/ujpe62.07.0565

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PACS 21.10.-k, 21.60.-n, 29.85.-c

STATISTICAL ANALYSIS OF THE DISTRIBUTION OF 0^+ STATE ENERGIES IN THE ACTINIDE NUCLEI

The statistical analysis of the distributions of the 0^+ state energies in actinide nuclei is carried out, by using the Brody distribution function. It reveals an intermediate character of the experimental 0^+ spectra between order and chaos. This function is turned out to be different for the available experimental data and the calculated ones in the frame of the quasiparticle-phonon model.

Keywords: 0^+ states, Brody distribution, quasiparticle-phonon model.

Multiple 0^+ states were observed in deformed nuclei, in particular in the actinide region [1–4]. The excitation of 0^+ states has an advantage over excitation of other states. They are easily identified via the angular distributions of the cross-section in the (p, t) reactions in otherwise complicated and dense excitation spectra. The shape of the angular distribution is rather independent of the specific structure of the individual states, as well as of the transfer configurations. The 0^+ states are strongly excited in the one-step process as the natural-parity excitations. Therefore, the measured spectra of 0^+ states have to be complete, allowing a statistical analysis, namely the possibility of the analysis in terms of the ordered or chaotic (mixed) nature of the energy spectra. Information on the degree of mixing of nuclear levels can be obtained by evaluating the distances between the nearest neighbors and the energy difference between the levels of the same spin and parity [5], the 0^+ levels in this case. In the case of strong mixing, we can describe the distribution of distances between nearest neighbors with the Wigner distribution. In the case of a small mixing of configurations, the Poisson distribution more accurately describes the distribution of the nearest neighbor spacing.

Another goal of such an analysis is the problem of the energy location of the 0^+ spectra. All experiments were performed at energies below 3 MeV. Figure 1 shows the 0^+ state spectra in the studied actinide nuclei. As one can see, the 0^+ states in this limited area are observed in the form of bumps. As follow from [1–4], the local groups of 2^+, 4^+, and 6^+ states are shifted relative to 0^+ states in the direction of higher energies. The assumption that the 0^+ states are localized mainly in a limited region, and that the density of 0^+ levels above 3 MeV is, at least, negligible was made in [1]. To this purpose, the triton spectra from the ^{232}Th$(p, t)^{230}$Th reaction were measured in the energy range 3÷4 MeV, but only for the angles 12.5° and 26°. Two lines in the spectra meet the condition for the 0^+ state, but not only; they meet the conditions for 6^+ states as well.

At the same time, the interacting boson model (IBM) [6] and the quasiparticle-phonon model (QPM) [7] predict an increase in the number of 0^+ states with the energy. The impact of the inclusion of these additional levels can be seen from the statistical analysis of the level density for actinide nuclei, experimental data in the energy interval of 0–3 MeV and predicted by the QPM in an energy interval of 0–4 MeV (Fig. 3). The Brody distribution was used for the fitting of the normalized nearest-neighbor energy spac-

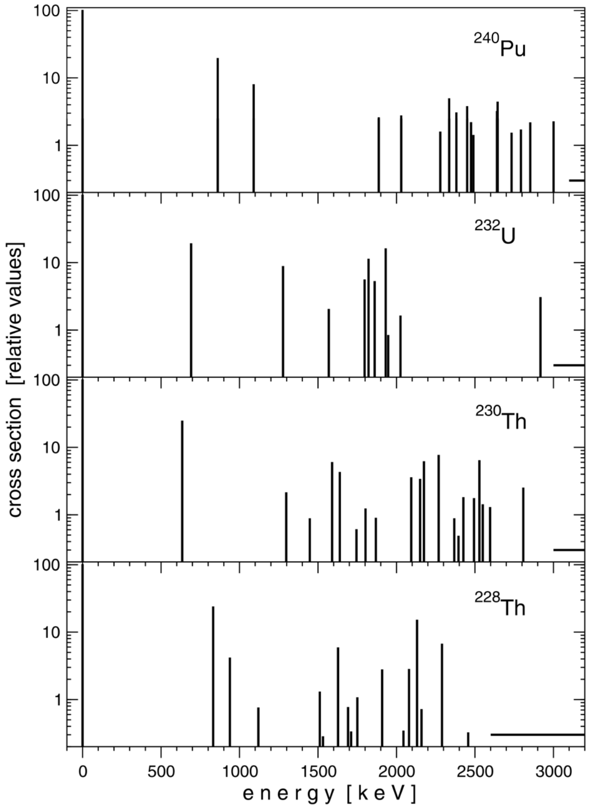

Fig. 1. (Color online) Location and the (p,t) strength of 0^+ states in ^{228}Th, ^{230}Th, ^{232}U, and ^{240}Pu. Horizontal lines indicate limitations in the investigation energy

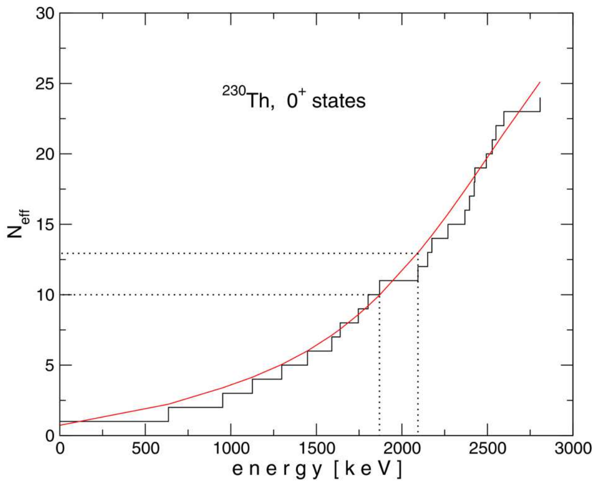

Fig. 2. (Color online) Histogram of 0^+ state energy spacings in ^{230}Th, showing the method of fitting of a quadratic function to the level distribution for the extraction of a normalized effective nearest neighbor spacing distribution, $N_{\rm eff}$

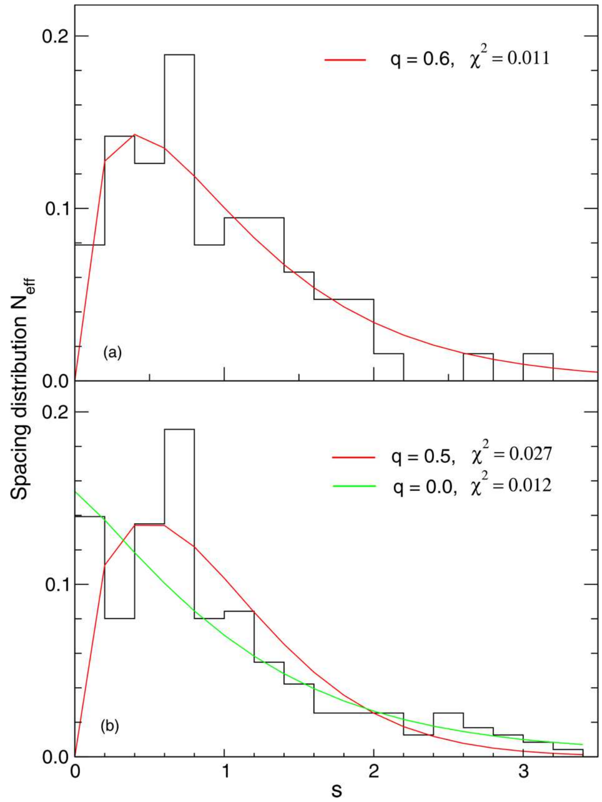

Fig. 3. (Color online) Normalized nearest-neighbor spacing as a function of the dimensionless spacing variable s and the fit with the Brody distribution: experimental data for ^{228,230}Th, ^{232}U, and ^{240}Pu in the energy interval of 0–3 MeV (*a*), calculated by the QPM data for ^{228,230}Th and ^{232}U in an energy interval of 0–4 MeV (*b*)

ing as a function of the dimensionless spacing variable s [8]. The Brody distribution describes systems with intermediate degrees of the level mixing depending on the parameter q in Eq. (1). It ranges from 0 for a Poisson distribution (ordered nature) in the case of small configuration mixing to 1 for the Wigner distribution (chaotic nature) in the case of strong configuration mixing. The used procedure is a standard unfolding method [10]. To transform the energy spacing into the dimensionless variable, a quadratic function $N_{\rm eff}$ was fitted to the histogram plot of the number of assigned 0^+ states for each nucleus individually as a function of the excitation energy. The procedure can be understood from Fig. 2. The difference $s_i = \Delta N_{\rm eff}$ was determined for every energy spacing, thus generating a space distribution. The $\Delta N_{\rm eff}$ distributions for each nucleus were then binned, and the resulting distribution of the energy spacing as a function of the dimensionless spacing variable s was fitted by the Brody distribution

$$N_{\rm eff} = As^q \exp\left(-bs^{q+1}\right), \quad (1)$$

where the parameters b and A are determined by the value of q:

$$b = [\Gamma((2+q)/(1+q))]^{q+1}$$

and $A = b(1+q)$. To get the value of χ^2, the parameter A was left free. In such a way, the experimental data for ^{228,230}Th [1, 2], ^{232}U [3], and ^{240}Pu [4] are fitted by the Brody distribution for $q = 0.6$ with $\chi^2 = 0.011$.

The theoretical data from [9] can be fitted by the Brody distribution for $q = 0.5$, but only with a worse $\chi^2 = 0.027$. In both cases, the obtained values of the parameter A are close to $A = b(1+q)$. A much better fit is obtained for the Poisson distribution with $\chi^2 = 0.012$. This means that the experimental 0^+ spectrum in an energy interval of 0–3 MeV is intermediate between ordered and chaotic ones in structure, while the ordered nature is preferred for the theoretical spectrum in an energy interval of 0–4 MeV. In addition, the mean number of 0^+ states observed in one nucleus in an energy interval of 0–3 MeV is about 18, while the number of theoretically predicted 0^+ states in an energy interval of 0–4 MeV is about 80. Therefore, it is important to investigate at least the region 3–4 MeV for the presence of additional 0^+ excitations. Such an experiment is planned for nuclei ^{158}Gd and ^{184}W, for which the energy calibration is possible in this energy region.

The phenomenologic IBM even in its simplified two-parametric form is known for its capability to study chaos and transitions between order and chaos in the properties of low-lying collective states of even-even nuclei [10, 11]. In the microscopic QPM, the introduction of multiphonon states (three and more) seems to be necessary to move from order toward chaos [12]. This idea is supported by the analysis performed for odd nuclei [13, 14], where the addition of one-quasiparticle plus two-phonon states (i.e. 5-qp states) to the standard one-quasiparticle and one-quasiparticle plus one-phonon states led to a fit of the calculated $17/2^+$ ^{209}Pb spectra to the Brody distribution with the parameter $q = 0.6$, thus corresponding to a transitional region between order and chaos.

In conclusion, the comparison of the experimental nearest-neighbor spacing distribution of the 0^+ states in an interval of 0–3 MeV for four actinide isotopes (^{228,230}Th, ^{232}U, and ^{240}Pu) to the Brody distribution has revealed an intermediate character of the experimental 0^+ spectrum between ordered and chaotic ones in structure. A similar distribution for the data obtained from the QPM calculations in an interval of 0–4 MeV somewhat differs from the experimental one and is closer to the ordered one. Thus, the increased role of multiphonon states in the model at higher energies means the movement to chaos. In any case, these results provide the means for estimating the future microscopic calculations. The (p,t) and $(p,t\gamma)$ experiments for higher energies could provide the additional information on the nature of 0^+ excitations.

1. A.I. Levon, G. Graw, Y. Eisermann, R. Hertenberger, J. Jolie, N.Yu. Shirikova, A.E. Stuchbery, A.V. Sushkov, P.G. Thirolf, H.-F. Wirth, N.V. Zamfir. Spectroscopy of ^{230}Th in the (p,t) reaction. *Phys. Rev. C* **79**, 014318 (2009).
2. A.I. Levon, G. Graw, R. Hertenberger, S. Pascu, P.G. Thirolf, H.-F. Wirth, P. Alexa. 0^+ states and collective bands in ^{228}Th studied by the (p,t) reaction. *Phys. Rev. C* **88**, 014310 (2013).
3. A.I. Levon, P. Alexa, G. Graw, R. Hertenberger, S. Pascu, P.G. Thirolf, and H.-F. Wirth. Spectroscopy of ^{232}U in the (p,t) reaction: More information on 0^+ excitations. *Phys. Rev. C* **92**, 064319 (2015).
4. M. Spieker, D. Bucurescu, J. Endres, T. Faestermann, R. Hertenberger, S. Pascu, S. Skalacki, S. Weber, H.-F. Wirth, N.V. Zamfir, and A. Zilges. Possible experimental signature of octupole correlations in the 0_2^+ states of the actinides. *Phys. Rev. C* **88**, 041303(R) (2013).
5. A. Bohr, B.R. Mottelson. *Nuclear Structure: Nuclear Deformations* (World Scientific, 1998), Vol. 2.
6. F. Iachello, A. Arima. *The Interacting Boson Model* (Cambridge Univ. Press, 1987).
7. V.G. Soloviev. *Theory of Atomic Nuclei: Quasiparticles and phonons* (Institute of Physics, 1992).
8. T.A. Brody. A statistical measure for the repulsion of energy levels. *Lett. Nuovo Cim.* **7**, 482 (1973).
9. N. Lo Iudice, A.V. Sushkov, N.Yu. Shirikova. Microscopic structure of low-lying 0+ states in deformed nuclei. *Phys. Rev. C* **72**, 034303 (2005), and A.V. Sushkov (private communication).
10. Y. Alhassid, A. Novoselsky, N. Whelan. Chaos in the low-lying collective states of even-even nuclei. *Phys. Rev. Lett.* **65**, 2971 (1990).
11. P. Cejnar, J. Jolie. Quantum phase transitions in the interacting boson model. *Progr. Part. Nucl. Phys.* **62**, 210 (2009).

12. R.K. Sheline, P. Alexa. Search for the beginning of chaos in the low-energy region of well deformed even-even nuclei. *Acta Phys. Pol. B* **39**, 711 (2008).
13. Ch. Stoyanov, V. Zelevinsky. High-lying single-particle modes, chaos, correlational entropy, and doubling phase transition. *Phys. Rev. C* **70**, 014302 (2004).
14. Ch. Stoyanov. Quasiparticles, phonons and beyond (nuclear structure calculations in a large domain of excitation energies). *Rom. J. Phys.* **58**, 1096 (2013).

Received 15.06.17

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СТАТИСТИЧНИЙ АНАЛІЗ
ЕНЕРГІЙ 0^+ СТАНІВ В ЯДРАХ АКТИНІДІВ

Р е з ю м е

Для статистичного аналізу розподілу енергій 0^+ станів в ядрах актинідів використана функція розподілу Броді. Аналіз виявив проміжний характер експериментальних спектрів між порядком і хаосом. Цей розподіл виявився різним для експериментальних даних і розрахованих в рамках квазічастинково-фононної моделі.