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GENERALIZED HEISENBERG UNCERTAINTY PRINCIPLE IN QUANTUM GEOMETRODYNAMICS AND GENERAL RELATIVITY

We focus on the energy flows in the Universe as a simple quantum system and are concentrating on the nonlinear Hamilton–Jacobi equation, which appears in the standard quantum formalism based on the Schrödinger equation. The cases of the domination of radiation, barotropic fluid, and the quantum matter-energy are considered too. As a result, the generalized Heisenberg uncertainty principle (GHUP) is formulated for a metric tensor. We also use the Kuzmichev–Kuzmichev geometrodynamics as a way to quantify the interrelationship between the GHUP for a metric tensor and conditions postulated as to a barotropic fluid, i.e. a dust for the early Universe conditions.

Keywords: generalized Heisenberg uncertainty principle, general relativity, Universe, cosmology, quantum geometrodynamics.

1. Introduction

The answer to the question brought up in the title of this paper can be provided after comparative descriptions of the Universe by classical and quantum theories. As is well known, the Universe is subject to classical theories on large space-time scales, whereas, on small space-time scales comparable with Planck's scales and length, it should be described from a quantum-theoretical perspective.

The first goal of our research will be to introduce a framework about the speed of gravitons in "heavy gravity", and this is important eventually, as illustrated by C. Will [1, 2], as it could possibly be observed. Second, it also will involve an upper bound to the rest mass of a graviton. The third aspect of the inquiry of our manuscript will be to come up with a variant of the HUP, involving a metric tensor, as well as the stress energy tensor, which will allow us to establish a lower bound to the mass of a graviton, preferably at the start of a cosmological evolution. The article concludes as answering a statement by Mukhanov, in Marcel Grossman 14 as to his interpretation as to the importance of causal barriers, in place in terms of prior-to-present transitions of the Universe in cosmology. In the Mukhanov view, causal barriers create an averaging effect of contributions from prior Universe conditions to the present Universe initial conditions. In fact, this means that, in the case of a multiverse, the existence of prior Universe contributions from a multiverse would correspond effectively to a single Universe repeating itself. In other words, our view is very similar to the ergodic mixing protocol even in the case of multiverse contributions to a present Universe. This is the basis of much of our analysis. Mukhanov stated that, instead of the ergodic mixing of prior contributions

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from a multiverse, the causal structure would ALWAYS restrict our analysis of information from a prior ensemble to be the same as a repeating single Universe model for cyclic universes. We regard the Mukhanov interpretation as indefensible and state why it is so in the last chapter of this article.

We reference what was done by Will in his living reviews of relativity article as to the "Confrontation between GR and experiment". Specifically, we make use of his experimentally based formula from [1, 2], where v_{graviton} is the speed of a graviton, m_{graviton} is the rest mass of a graviton, and E_{graviton} is defined in the inertial rest frame:

$$\left(\frac{v_{\text{graviton}}}{c}\right)^2 = 1 - \frac{m^2_{\text{graviton}} c^4}{E^2_{\text{graviton}}}. \tag{1}$$

Our take away from formulae 1 is that if a graviton is massive, then the speed of a graviton drops below the value of c, the speed of light, and massless gravitons are travelling with the speed of light. In addition, this puts restrictions upon the energy of a graviton and argues against using overly simplified approximations. Hence, we follow [2] in terms of the following ideas as given in formula (2):

$$\begin{aligned}
&\frac{v_{\text{graviton}}}{c} = 1\text{–}5\times 10^{-17}\left(\frac{200Mpc}{D}\right)\left(\frac{\Delta t}{1\text{ s}}\right) \doteq \\
&\doteq 1\text{–}5\times 10^{-17}\left(\frac{200Mpc}{D}\right)\times \\
&\times\left(\frac{\Delta t = \Delta t_a - (1+z)\Delta t_e}{1\text{ s}}\right) \Leftrightarrow \\
&\Leftrightarrow \frac{2m_{\text{graviton}}c^2}{E_{\text{graviton}}} \approx 5\times 10^{-17}\left(\frac{200Mpc}{D}\right)\times \\
&\times\left(\frac{\Delta t_a - (1+z)\Delta t_e}{1\text{ s}}\right).
\end{aligned} \tag{2}$$

Here, $z = \frac{\Delta U}{c^2}$, ΔU is the difference in the Newtonian potentials between an emitter and a receiver, D is a distance between the emitter (of gravitational waves) and the receiver (of gravitational waves), Δt_a is the difference in the arrival times, and Δt_e is the difference in the emission times in the case of the early Universe, i.e. near the Big Bang. Assuming that there is an average $E_{\text{graviton}} \approx \hbar\omega_{\text{graviton}}$ at the beginning of time, we have

$$\Delta t_a \sim 4.3\times 10^{17}\text{ s},\ \Delta t_e \sim 10^{-33}\text{ s},\ z \sim 10^{50}. \tag{3}$$

Then $\left(\frac{\Delta t_a - (1+z)\,\Delta t_e}{1\text{ s}}\right) \sim 1$. If $D \sim 4.6\times 10^{26}$ m = = radius (universe), one can set

$$\left(\frac{200Mpc}{D}\right) \sim 10^{-2}. \tag{4}$$

If we set the mass of a graviton [3] into Eq. (1) and if we look at primordial time generated gravitons, then we have, in the present era, the small time interval value given below as

$$\Delta t_a \sim 4.3\times 10^{17}\text{ s},\ \Delta t_e \sim 10^{-33}\text{ s},\ z \sim 10^{55}. \tag{5}$$

Note that the above-given frequency for a graviton is for the present era, but it starts, by assuming an initial genesis, from an (initial) inflationary starting point, which is not a space-time singularity.

Note that this comes from a scale factor, if $z \sim \sim 10^{55} \Leftrightarrow a_{\text{scale-factor}} \sim 10^{-55}$, i.e., by 55 orders of magnitude smaller than what would be normally considered. Here, we indicate that the scale factor is not zero, so we do not have a space-time singularity.

We will next discuss the implications of this point in the next section concerning a nonzero smallest scale factor. Second, we are working with a massive graviton, as given will be given some credence as to when we obtain a lower bound, as will come up in our derivation of modifications of the values [3]

$$\begin{aligned}
&\left\langle (\delta g_{uv})^2 \left(\hat{T}_{uv}\right)^2 \right\rangle \geq \frac{\hbar^2}{V^2_{\text{Volume}}} \xrightarrow[uv\to tt]{} \\
&\xrightarrow[uv\to tt]{} \left\langle (\delta g_{tt})^2 \left(\hat{T}_{tt}\right)^2 \right\rangle \geq \frac{\hbar^2}{V^2_{\text{Volume}}}\ \& \\
&\&\ \ \delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+,
\end{aligned} \tag{6}$$

where V_{Volume} is the volume of space-time used by the generalized uncertainty principle for the evaluation of the HUP, $\hat{T}_{uv}$ is an operator version of the stress energy tensor, and δg_{uv} is a fluctuation of the metric tensor g_{uv}. It should be noted that the stress-energy tensor $\hat{T}^{\mu\nu}$ in (6) is for the probe field. It is not the stress-energy tensor $T_{\mu\nu}$ for the matter distribution, which gives $g_{\mu\nu}$ via Einstein's field equations. This is to be expected, as (6) is essentially an uncertainty relation for the field. However, as we are estimating the metric with field measurements, the uncertainty in the field in (6) has been replaced by the uncertainty in the metric.

The reasons for saying so about this set of arguments in favor of a variation of the non g_{tt} metric will be in Section 3, and it is due to the smallness of the square of the scale factor in a vicinity of the Planck time interval, leading to the nonzero initial entropy as stated in Appendix A. We also examine a Ricci scalar value at the boundary between the pre-Planckian to Planckian regime of space-time, setting the magnitude of Ricci scalar k, as approaching the flat space conditions right after the Planck regime. Furthermore, we use the following value as an approximation specifying a starting point of the initial entropy production near the genesis of the Universe: $S_{\text{initial(graviton)}} \sim 10^{37}$. Then we get an initial version of the cosmological "constant," which is linked to the initial value of a graviton mass, as is shown in Appendix D. Appendix E is written for the Riemann–Penrose inequality, which is either a nonzero nonlinear electrodynamics (NLED) scale factor or a quantum bounce of linear quantum gravity (LQG). Finally, Appendix F gives conditions so that a pre-Planckian kinetic energy (inflaton) value greater than the potential energy occurs, which is foundational to the lower bound to the graviton mass. Further, we will add a more elaborate mathematical structure to this calculation to confirm via a precise calculation that the lower bound to the graviton mass is about 10^{-70} g. Our lower bound is a dimensional approximation so far. We will make it exact.

2. Kuzmichevs Quantum Constraint Equations

In this section, we follow Ref. [4] and consider the homogeneous, isotropic, and spatially closed quantum cosmological system (Universe). The geometry of such a Universe is described by the Robertson–Walker metric. This metric has a maximally symmetric three-dimensional subspace of the four-dimensional space-time. Since we consider the spatially closed universe, the geometry of the space-time depends on a single cosmological parameter, namely, the cosmic scale factor a, which describes the overall expansion or contraction of the Universe [5]. The scale factor is a field variable, which determines gravity in the formalism under consideration. We assume that, from the beginning, the Universe is filled with matter in the form of a uniform scalar field ϕ, the state of which is given by some Hermitian Hamiltonian, $H_\phi = H_\phi^\dagger$. This Hamiltonian is defined in a curved space-time and, therefore, depends in the general case on a scale factor a as a parameter, $H_\phi = H_\phi(a)$. In addition, it will be accepted that the Universe is filled with a perfect fluid in the form of relativistic matter (further referred as radiation) with the proper energy $M_\gamma = \frac{E}{2a}$ in the comoving volume $\frac{1}{2}a^3$, where E is a real constant proportional to the number of particles of the perfect fluid. The perfect fluid defines a material reference frame [6, 7].

The restrictions in the form of the first-class constraint equations are imposed on the state vector of the quantum Universe $\Psi = \langle a, \phi | \Psi(T) \rangle$, where T is a time parameter. These constraints can be reduced to two equations [7–9]:

$$\left(-i\partial_T - \frac{2}{3}E\right)\Psi = 0, \tag{7}$$

$$\left(-\partial_a^2 + a^2 - 2aH_\phi - E\right)\Psi = 0, \tag{8}$$

where Eq. (7) describes the time evolution of Ψ, when the number of particles of the perfect fluid conserves, while Eq. (8) determines the quantum states of the Universe at some fixed instant of time $T = T_0$, T_0 being an arbitrary constant taken as a time reference point. The coefficient $\frac{2}{3}$ in Eq. (7) is caused by the choice of the parameter T as the time variable. This time variable is connected with the proper time τ by the differential equation $d\tau = a dT$. Following the ADM formalism [10, 11], one can extract the so-called lapse function N, which specifies the time reference scale, from the total differential dT: $dT = N d\eta$, where η is the "arc time" [12, 13].

The quantum constraints (7) and (8) can be rewritten in the form of the time-dependent Schrödinger-type equation

$$-i\partial_T \Psi = \frac{2}{3}\mathcal{H}\Psi, \tag{9}$$

where

$$\mathcal{H} = -\partial_a^2 + a^2 - 2aH_\phi. \tag{10}$$

The minus sign before the partial derivative ∂_T is stipulated by the specific character of the cosmological problem, namely that the classical momentum conjugate to the variable a is defined with the minus sign [14, 15].

The partial solution of Eqs. (7) and (8) has a form

$$\Psi(T) = e^{i\frac{2}{3}E(T-T_0)}\Psi(T_0), \tag{11}$$

where the vector $\Psi(T_0) \equiv \langle a, \phi|\psi\rangle$ satisfies the stationary equation

$$\mathcal{H}|\psi\rangle = E|\psi\rangle. \tag{12}$$

From the condition

$$0 = \frac{d}{dT}\int D[a,\phi]\,|\Psi|^2 = = -i\frac{2}{3}\int D[a,\phi]\,\Psi^*\left[\mathcal{H}^\dagger - \mathcal{H}\right]\Psi, \tag{13}$$

where $D[a, \phi]$ is the measure of integration with respect to the fields a and ϕ chosen in an appropriate way, it follows that operator (10) is Hermitian: $\mathcal{H} = \mathcal{H}^\dagger$.

3. Initially Nonzero Scale Factor and What is This Telling Us Physically, Starting with a Configuration from Unruh?

Begin with the starting point of [16]:

$$\Delta l\,\Delta p \geq \frac{\hbar}{2}. \tag{14}$$

We will use the approximation given by Unruh [16]. A generalization can be written as

$$(\Delta l)_{ij} = \frac{\delta g_{ij}}{g_{ij}}\,\frac{l}{2}, \quad (\Delta p)_{ij} = \Delta T_{ij}\,\delta t\,\Delta A, \tag{15}$$

where Δl is a change in length, $(\Delta l)_{ij}$ is a change in length due to fluctuations in the metric tensor as given by g_{ij}, $(\Delta p)_{ij}$ is a change in the pressure due to a change in the "surface" area ΔA, and ΔT_{ij} is a change in the stress-energy tensor.

From the Roberson–Walker metric [17], we have

$$g_{tt} = 1, \quad g_{rr} = \frac{-a^2(t)}{1-k\,r^2}, \quad g_{\theta\theta} = -a^2(t)\,r^2, \quad g_{\phi\phi} = -a^2(t)r^2\sin^2\theta. \tag{16}$$

Following Unruh [16], we write an uncertainty of the metric tensor as

$$a^2(t) \sim 10^{-110}, \quad r \equiv l_{\mathrm{P}} \sim 10^{-35}\ \mathrm{m}. \tag{17}$$

If $\Delta T_{tt} \sim \Delta\rho$, the surviving version of Eq. (14) and Eq. (15) is

$$\begin{aligned} &V^{(4)} = \delta t\,\Delta A\,r, \\ &\delta g_{tt}\,\Delta T_{tt}\,\delta t\,\Delta A\,\frac{r}{2} \geq \frac{\hbar}{2} \\ &\Leftrightarrow \delta g_{tt}\,\Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}. \end{aligned} \tag{18}$$

Equation (18) is such that we can extract, up to a point, the HUP principle for uncertainty in time and energy with one very large caveat added, namely, if we use the fluid approximation of space-time [17]:

$$T_{ii} = \mathrm{diag}(\rho, -p, -p, -p). \tag{19}$$

Then

$$\Delta T_{tt} \sim \Delta\rho \sim \frac{\Delta E}{V^{(3)}.} \tag{20}$$

So, Eq. (18), Eq. (19), and Eq. (20) yield

$$\delta t\Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2}, \quad \text{Unless } \delta g_{tt} \sim O(1). \tag{21}$$

How likely is $\delta g_{tt} \sim O(1)$? Not going to happen. Why? The homogeneity of the early Universe will keep with

$$\delta g_{tt} \neq g_{tt} = 1. \tag{22}$$

In fact, we have from Ref. [17], that if ϕ is a scalar function and $a^2(t) \sim 10^{-110}$, then

$$\delta g_{tt} \sim a^2(t)\,\phi \ll 1. \tag{23}$$

There is no way that Eq. (21) is going to come close to $\delta t\Delta E \geq \frac{\hbar}{2}$. Hence, the Mukhanov suggestion is not feasible, as will be discussed toward the end of this article. Finally, we will discuss how we obtain computationally a lower bound to the graviton mass.

4. How can We Justify Very Small $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$ Values

To begin this process, we introduce the following coordinates:

In the rr, $\theta\theta$, and $\phi\phi$ coordinates, we will use the fluid approximation, $T_{ii} = diag(\rho, -p, -p, -p)$ [18] with

$$\begin{aligned} &\delta g_{rr}T_{rr} \geq -\left|\frac{\hbar\,a^2\,(t)\;r^2}{V^{(4)}}\right| \xrightarrow[a\to 0]{} 0, \\ &\delta g_{\theta\theta}T_{\theta\theta} \geq -\left|\frac{\hbar\,a^2\,(t)}{V^{(4)}\,(1-k\,r^2)}\right| \xrightarrow[a\to 0]{} 0, \\ &\delta g_{\phi\phi}T_{\phi\phi} \geq -\left|\frac{\hbar\,a^2\,(t)\,\sin^2\theta\,d\phi^2}{V^{(4)}}\right| \xrightarrow[a\to 0]{} 0. \end{aligned} \tag{24}$$

If, as an example, we have negative pressure, with T_{rr}, $T_{\theta\theta}$, and $T_{\phi\phi} < 0$, and $p = -\rho$, then the only choice we have is to set $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$, because there is no way for $p = -\rho$ to be zero valued. Having said this, the value of δg_{tt} being nonzero will be a part of how we will seek a lower bound to the graviton mass, which is not zero.

In our analysis of the pre-Planckian space-time according to the HUP, which is written here in terms of a reduction of the contributions of all but the time component of the metric tensor, we face the problem of arguing how fluctuations drop off, unless they are directly connected to the time component. This makes sense, since if there is a nonsingular start to the Universe, as given by [19, 20], the pre-Planckian space-time regime is a part and a parcel of the emergent space-time, which would place a premium upon the nonspatial metric tensor fluctuations. Hence, we will delineate reasons for why the metric tensor fluctuations are restricted to the time components only.

5. Lower Bound to the Graviton Mass Using Barbour's Emergent Time

In order to start this approximation, we will use Barbour's value of emergent time [21,22] restricted to the Planck spatial interval and massive gravitons [23]

$$(\delta t)^2_{\text{emergent}} = \frac{\sum_i m_i l_i \, l_i}{2\,(E-V)} \to \frac{m_{\text{graviton}} l_{\text{P}} \, l_{\text{P}}}{2\,(E-V)}. \tag{25}$$

Initially, as postulated by Barbour [21, 22], this set of masses given in the emergent time structure could be, say, the planetary masses of the solar system. Our identification is to have an initial mass value, at the start of creation, for an individual graviton. If $(\delta t)^2_{\text{emergent}} = \delta t^2$ in Eq. (18), we can arrive, by using Eq. (18) and Eq. (25), at the identification

$$m_{\text{graviton}} \geq \frac{2\hbar^2}{(\delta g_{tt})^2 \, l_{\text{P}}^2} \, \frac{(E-V)}{\Delta T_{tt}^2}. \tag{26}$$

The key to Eq. (26) will be the identification of the kinetic energy, which is written as $E-V$. This identification will be the key point raised in our work. Note that it raises the distinct possibility of an initial state, just before the "Big Bang," of a kinetic energy dominating in the "pre-inflationary" universe, i.e., in terms of an inflaton $\dot{\phi}^2 \gg (P.E \sim V)$ [18]. The key finding of [24] is that if the kinetic energy is dominated by the "inflaton," then

$$\text{K.E.} \sim (E-V) \sim \dot{\phi}^2 \propto a^{-6}. \tag{27}$$

This is done with the proviso that $w < -1$, where w = pressure/density [25]. It is worth to mention that w is the equation of state parameter relating the energy density to the pressure. In other words, the convention referred to is of avoiding density = − pressure, which is used frequently. In effect, what we are saying is that, during the "Planckian regime," we can seriously consider the initial density proportional to the kinetic energy and call this k.e. as proportional to [18]

$$\rho_w \propto a^{-3(1-w)}. \tag{28}$$

We are starting our analysis as if we are in a very small Planckian regime of space-time. Then we can write Eq. (28) as proportional to $g^* T^4$ [18], with g^* initial degrees of freedom, and T the initial temperature as being set very low just before the inflation onset. The questions arise: what is the number of the initial degrees of freedom, and what is the temperature, T, at the expansion start? For what it is worth, the starting supposition is that there would be a likelihood for the initial low-temperature regime.

6. Metric Uncertainty Principle as the Interrelationship of General Relativity and Quantum Geometrodynamics

We will use the mathematical inputs from Section 2 extensively as a way to intertwine the predictions as to a HUP connected with the metric tensor of space-time and the resulting initial conditions for space-time according to geometrodynamics. The end result will be that we are supplying the initial conditions, which cannot be obtained by other means. We also will quantify, via a version of the dust dynamics, how this affects the candidate for DM and, possibly, DE contributions to the initial cosmological conditions. To do this, we will review the concepts used in both the Heisenberg uncertainty principle for metric tensors and the geometrodynamics equations used. The conclusion of what we are talking about is the use of the HUP for metric tensors to form bounds on the geometrodynamics equations in the pre-Planckian space-time era.

6.1. Application of the HUP to metric tensors

We will examine a Friedmann equation for the evolution of the scale factor, using explicitly the following cases: one case where the acceleration of expansion of the scale factor is kept in, another one where it is out, and the intermediate case with the acceleration factor where the scale factor is important but not dominant. In doing so, we will be trying it in our discussion with the earlier work done on the HUP, but from the context of how the acceleration term will affect the HUP. We will also make sense of why our generalized uncertainty principle, as given in the beginning of Eq. (29), is from [3, 16, 19] leading to a restriction of the metric tensor fluctuations to being the time component only in the denominator of the modified HUP expression. Ref. [3] gives us the initial generalized HUP, and Refs. [16, 19] express the fluctuation restricted to

$$\left\langle \left(\delta g_{uv}\right)^2 \left(\hat{T}_{uv}\right)^2 \right\rangle \geq \frac{\hbar^2}{V_{\text{volume}}^2} \xrightarrow[uv \to tt]{} \\ \xrightarrow[uv \to tt]{} \left\langle \left(\delta g_{tt}\right)^2 \left(\hat{T}_{tt}\right)^2 \right\rangle \geq \frac{\hbar^2}{V_{\text{volume}}^2} \,\& \\ \& \, \delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+. \tag{29}$$

Namely, we will be working with

$$\delta t \Delta E = \frac{\hbar}{\delta g_{tt}} \equiv \frac{\hbar}{a^2(t)\,\phi} \ll \hbar \Leftrightarrow \\ \Leftrightarrow S_{\text{initial}}(\text{with}[\delta g_{tt}]) = (\delta g_{tt})^{-3} S_{\text{initial}}(\text{without}[\delta g_{tt}]) \gg \\ \gg S_{\text{initial}}(\text{without}[\delta g_{tt}]), \tag{30}$$

i.e., the fluctuation $\delta g_{tt} \ll 1$ dramatically boosts the initial entropy. Not what it would be if $\delta g_{tt} \approx 1$. The next question to ask would be how could one actually have

$$\delta g_{tt} \sim a^2(t)\,\phi \xrightarrow{\text{Very Large}} 1. \tag{31}$$

Furthermore, we have that Eq. (29) has an explicit restriction of the modified HUP: to be influenced by only the time fluctuation of the metric tensor, which is given by δg_{tt}, and it is $\ll 1$ in the denominator of the modified HUP. Equation (30) is highlighted by the term $\ll 1$ in the denominator of the modified HUP, leading to the specific entropy generation. As is expected, in the pre-Planckian to Planckian transition referred to in Eq. (30), the second line delineates, if $\ll 1$ that the entropy generation is very different, than when it approaches 1, which is after the pre-Planckian to Planckian emergent physical regime. In addition, Eq. (31) specifically alludes to physical processes, which are significant if it approaches 1, marking the transition to the Planckian regime and beyond, and this is due to the inflaton growing extremely large.

In short, we would require an enormous "inflaton"-style ϕ-valued scalar function and $a^2(t) \sim 10^{-110}$. How could ϕ be initially quite large? Within the Planck time, the following lower bound for a mass holds:

$$m_{\text{gravitation}} \geq \frac{2\hbar^2}{(\delta g_{tt})^2 l_p^2} \frac{(E-V)}{\Delta T_{tt}^2}. \tag{32}$$

Here, we use the following approximation for the kinetic energy at the beginning of the expansion of the Universe:

$$\text{K.E.} \sim (E-V) \sim \dot{\phi}^2 \infty a^{-6}. \tag{33}$$

Then, up to the first order, we could approximate, with H.O.T. being higher order terms,

$$\dot{\phi} \sim a^{-3} \Leftrightarrow \phi \approx t\,a^{-3} + \text{H.O.T.} \tag{34}$$

Equation (34) will be considerably refined in the subsequent study.

6.2. Metric uncertainty principle and its applications in geometrodynamics

From Eq. (8), we have

$$\langle u_k | H_\phi | u_{k'} \rangle = M_k(a)\, \delta_{k,k'} \xrightarrow{V(\phi)-\lambda_\alpha \phi^\alpha \,\&\; k-k'} \\ \xrightarrow{V(\phi)-\lambda_\alpha \phi^\alpha \,\&\; k-k'} \varepsilon_k \left(\frac{\lambda_\alpha}{2}\right)^{\left(\frac{2}{2+\alpha}\right)} a^{\left(\frac{3(2-\alpha)}{2+\alpha}\right)} \xrightarrow[\alpha - 2]{} \\ \xrightarrow[\alpha - 2]{} \langle u_k | H_\phi | u_{k'} \rangle = M_k(\alpha)\, \delta_{k,k'} \xrightarrow{V(\phi)-\lambda_\alpha \phi^\alpha \,\&\; k-k'} \\ \xrightarrow{V(\phi)-\lambda_\alpha \phi^\alpha \,\&\; k-k'} \varepsilon_k \left(\frac{\lambda_\alpha}{2}\right)^{\left(\frac{2}{2+\alpha}\right)} a^{\left(\frac{3(2-\alpha)}{2+\alpha}\right)} \xrightarrow[\alpha - 2]{} \\ \xrightarrow[\alpha - 2]{} \sqrt{2\lambda_2}\,(k+1/2) = M_2(a). \tag{35}$$

Here, we can assign a density functional and then a change of the energy as given by $\Delta E = 2 \times 10^{-\gamma} \times \times l_p^3 M_2(a)/a^3$. So, then, we have

$$\rho_m = 2M_2(a)/a^3 = \sqrt{2^3\,\lambda_2}\, a^3\,(k+1/2), \\ \Delta E = 2 \times 10^{-\gamma}\, l_p^3 M_2(a)/a^3 = \\ = 10^{-\gamma}\, l_p^3 \sqrt{2^3\,\lambda_2}\, a^{-3}\,(k+1/2). \tag{36}$$

Here, the subscript k, as in Eq. (36), is a "particle count," and we will refer to this repeatedly in the rest of this paper. With Eq. (36) and the emergent field reference, a change in the energy in the pre-Planckian domain is as follows:

$$\delta g_{tt} \approx \frac{\hbar}{\delta t \Delta E} = \\ = (\delta t)^{-1} \frac{\hbar}{10^{-\gamma}\, l_p^3 \sqrt{2^3 \lambda_2}\, a^{-3}(k+1/2)}. \tag{37}$$

If the inequality is strictly adhered to, we have

$$\delta g_{tt} \geq \frac{\hbar}{\delta t \Delta E} = \\ = (\delta t)^{-1} \frac{\hbar}{10^{-\gamma}\, l_p^3 \sqrt{2^3 \lambda_2}\, a^{-3}(k+1/2)}. \tag{38}$$

The smallness of the initial scale factor would be of the order of $a^{-3} \sim 10^{165}$. We have that $k \sim 10^{20}$ initially and $l_p^3 \sim 10^{-105}$. We pick $\hbar = 1$ dimensionally; so, if $\delta t \sim 10^{-44}$ and if we use Eq. (37) as an estimator, the following has to be hold in the pre-Planckian space-time:

$$\lambda_2 \leq 10^{-74+2\gamma} \Leftrightarrow \delta g_{tt} \leq 1 \Leftrightarrow \\ \Leftrightarrow \delta t \Delta E \geq 1 \,\& \\ \& \,\lambda > 10^{-74+2\gamma} \Leftrightarrow \delta g_{tt} > 1 \Leftrightarrow \\ \Leftrightarrow \delta t \delta E < 1, \tag{39}$$

i.e., the violation of the uncertainty principle for commences for any situation, which implies restraints on $\lambda_2 \leq 10^{-74+2\gamma} \Leftrightarrow \delta g_{tt} \leq 1 \Leftrightarrow \delta t \Delta E \geq 1$, when

$$\lambda_2 > 10^{-74+2\gamma} \Leftrightarrow \delta g_{tt} > 1 \Leftrightarrow \delta t \Delta E < 1. \tag{40}$$

For the problem represented by Eq. (40) to hold, it would mean that the following pre-Planckian potential energy would be then small when the potential energy given in Eq. (41) is much smaller than the kinetic energy given in Eq. (30)

$$V(\phi) = \lambda_\alpha \phi^\alpha = \lambda_2 \phi^2. \tag{41}$$

From the inspection, for Eq. (41) to hold for our physical system, we would want Eq. (39) to hold, which would mean an extremely small potential energy, as opposed to the large value of the kinetic energy given by Eq. (32). Hence, the role of geometrodynamics given in Eqs. (35) and (36) will imply, in the case of a quartic potential, that Eq. (41) as the potential energy is much smaller than the kinetic energy as represented for the pre-Planckian space-time physics.

7. Discussion and Conclusions

A way to rewrite the approach given here in terms of the early Universe theory is to refer to Einstein spaces [26], as well as to make certain of the terms and components of the stress energy tensor [27], as we can write it as a modified Einstein field equation. With N as a constant,

$$R_{ij} = \mathrm{N} g_{ij}. \tag{42}$$

Here, the term on the left-hand side of the metric tensor is a constant, so we write with R also to be a constant [27]:

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{ij}} = -\frac{1}{8\pi} \left[\mathrm{N} - R + \Lambda\right] g_{ij}. \tag{43}$$

If we use the fluid approximation given by Eq. (19) and the metric given in Eq. (16), we get a constant energy term on the RHS of Eq. (43), by restricting i, and j, to, correspondingly, t and t.

So, we recover, via the Einstein spaces, the seemingly heuristic argument given above. Furthermore, when we refer to the kinetic energy space as an inflaton $\dot{\phi}^2 \gg (P.E \sim V)$ [18], we can utilize the operator equation for the generation of an "inflaton field" given by the following set of equations:

$$\begin{aligned} &\phi\,(t,\cdot) = \cos(t\sqrt{K}) f + \frac{\sin(t\sqrt{K})}{\sqrt{K}} g, \\ &f(x) = \phi(0,x), \\ &g(x) = \frac{\partial \phi(0,x)}{\partial t}, \\ &-\frac{\partial^2 \phi}{\partial t^2} = K\phi. \end{aligned} \tag{44}$$

Consider the case of the general elliptic operator K. If we use the Fulling reference [28] in the case of the above Roberson–Walker metric, the elliptic operator becomes

$$K = -\nabla^2 + (m^2 + \xi R) = \\ = -\sum_{i,j} \frac{\partial_i \left(g^{ij} \sqrt{|\det g|}\, \partial_j\right)}{\sqrt{|\det g|}} + (m^2 + \xi R) \xrightarrow[i,j \to t,t]{} \\ \xrightarrow[i,j \to t,t]{} -\frac{\partial^2}{\partial t^2} + (m^2 + \xi R). \tag{45}$$

Let R in Eq. (45) be initially a constant and m be the inflation mass. According to [28], we have

$$\begin{aligned} &\phi(t,\cdot) = \cos(t\sqrt{K})f, \\ &-\frac{\partial^2}{\partial t^2} \to \omega^2 \Leftrightarrow \\ &\Leftrightarrow \phi(t,\cdot) = \cos(t\sqrt{\omega^2 + (m^2 + \xi R)}) \end{aligned} \tag{46}$$

Setting the unspecified quantity c_1 to be constant will lead in the first approximation to the kinetic-energy-dominated initial configuration. The details can be be cleaned from [28–30]. To give more details to the following equation, R is linked to the space-time curvature, and m is the inflaton mass related to the field $\phi(t,\cdot) = \cos(t\sqrt{K})f$ with the result that

$$\dot{\phi}^2(t,\cdot) \approx \left[\omega^2 + (m^2 + \xi R)\right] c_1 \gg V(\phi), \tag{47}$$

where c_1 is a proportionality factor, so that the frequency squared times c_1 has the dimension of energy. For Eq. (47) with the use of Planck units, this would mean that c_1 would have the dimension of $V(\phi)$, which is the potential energy.

If the frequency of gravitons is of the order of the Planck frequency, then the corresponding term would likely dominate in Eq. (47). More of the details of this will be worked out, and the candidates for $V(\phi)$ will be ascertained. Most likely, we will look for the Rindler vacuum, as specified in [31], as well as for details of what is relevant to maintain a local covariance in the initial space-time fields as given in [32].

Why is a refinement of Eq. (47) necessary?

The details of the elliptic operator K will be cleaned from [28–30], whereas the details of inflaton $\dot{\phi}^2 \gg (P.E \sim V)$ [18] are important to get a refinement of the lower mass of a graviton. Eq. (45), we consider the mass m of an inflaton, not a graviton, in order to have links to the beginning of the expansion of the Universe. We look to what Corda did in [33] for guidance to pick the values of m relevant to the early Universe conditions.

Finally, as far as Eq. (47) is concerned, there is one serious linkage to classical and quantum mechanics, which should be the bridge between classical and quantum regimes, as far as the space-time applicability. Namely, it follows from [31] that, for all arbitrary operators A and B,

$$(\Delta A)^2 (\Delta B)^2 \geq \left(\frac{1}{2i}\langle [A,B]\rangle\right). \tag{48}$$

As we can anticipate, the pre-Planckian regime may be analyzed within the classical mechanics. Then we pass to the Planckian regime, which would be quantum mechanical. In view of [31], this would lead to a symplectic structure via a modification of the Hamilton equations of motion. Namely, from (26), we get

$$\begin{aligned} &\frac{dq_\mu}{dt} = \frac{\partial H}{\partial p_\mu}, \quad \frac{dp_\mu}{dt} = -\frac{\partial H}{\partial q_\mu}, \\ &H = H(q_1, ..., q_n; p_1, ..., p_n), \\ &y = (q_1, ..., q_n; p_1, ..., p_n), \\ &\Omega^{\mu\nu} = 1, \text{ if } \nu = \mu + n, \\ &\Omega^{\mu\nu} = 0, \text{ otherwise } \frac{dy^\mu}{dt} = \sum_{\nu=1}^{n} \Omega^{\mu\nu} \frac{\partial H}{\partial y^\nu}. \end{aligned} \tag{49}$$

Then there exists a reformulation of the Poisson brackets, as seen by

$$\{f,g\} = \Omega^{\mu\nu} \nabla_\mu f \nabla_\nu g. \tag{50}$$

For the classical observables f and g, we can write, by [31],

$$\begin{aligned} &\wedge : \Theta \to \hat{\Theta}, \\ &\Theta = \text{classical} - \text{observable}, \\ &\hat{\Theta} = \text{quantum} - \text{observable}, \\ &\hbar = 1, \\ &\left[\hat{f}, \hat{g}\right] = i\ (\{f, \hat{g}\}). \end{aligned} \tag{51}$$

Then Eq. (48) and Eq. (51) take the form

$$\begin{aligned} &\left[\hat{f}, \hat{g}\right] = i\ (\{f, \hat{g}\}), \\ &f = \text{classical} - \text{observable}, \\ &\hat{f} = \text{quantum} - \text{observable}, \\ &(\Delta \hat{f})^2 (\Delta \hat{g})^2 \geq \left(\frac{1}{2i}\left(\left\langle \left[\hat{f}, \hat{g}\right] \right\rangle\right)\right) = \left(\frac{1}{2}\left((\{f, \hat{g}\})\right)\right). \end{aligned} \tag{52}$$

If so, then we can set, with regard for the interconnection between the Planck and pre-Planck regimes, the classical variables as follows:

$$\begin{aligned} &f = -\frac{[\aleph - R + \Lambda]\, g_{tt}}{8\pi}, \\ &g = \delta g_{tt}. \end{aligned} \tag{53}$$

Then, by using Eq. (52), we are able to reach a higher precision in our calculations of the early Universe and to understand how to construct the partition function Z, by basing upon the interrelationship between Eq. (52) and Eq. (53). The entropy given in [31] reads

$$S(\text{entropy}) = \ln \mathrm{Z} + \beta \mathrm{E}. \tag{54}$$

If this program were realized with the first-principles construction of a partition function, we may be able to answer if the entropy were zero in the Planck regime or something else, which would give us a more motivation to examine the sort of partition functions as stated in [34, 35]. See Appendix A as to possible scenarios. It is worth to keep in mind that, in the Planck regime, we have a nonstandard physics. Appendix A indicates that, due to the variation we have worked out in the Planckian regime of space-time, the initial entropy is not zero. The consequences of this fact are considered in Appendix B with a specific formulation of the Ricci scalar. The consequences of Appendix A and Appendix B may be relevant for a small cosmological constant and the large "Hubble expansion" with an initially large magnitude of the cosmological pressure, even if negative. This would give credence to a nonzero cosmological entropy. Moreover, the large negative pressure, even in the pre-Planckian regime, will lead to the terms with large ΔT_{tt} which would appear in Eq. (1A), even if we use a partition function based upon lattice Hamiltonians as in [35]. In a lattice gauge arrangement, they would have considerably smaller contributions than ΔT_{tt}. Note that, under conditions of a flat space, Eq. (B9) almost vanishes due to the behavior of the numerator, no matter how a^2_{initial} is small. The supposition is that the numerator becomes far smaller than a^2_{initial}. The initiation of the conditions of flat space is also the regime, for which we think that the nonzero entropy is started. Appendix C gives an initial estimate of what we think the entropy would be in the aftermath of the uncertainty relationship we have outlined, i.e. to the first order, $S_{\text{initial(graviton)}} \sim \sim 10^{37}$. We finalize our treatment of space-time fluctuations and the geometry, by considering the applications of Appendix D to the graviton mass and of Appendix E to the Riemann–Penrose inequality for the conditions of a minimum frequency, as a consequence of the cosmological evolution, and what it portrays as consequences for electromagnetic fields. Appendix D and E give varying initial graviton masses as a starting point, with Appendix D giving a higher initial graviton mass than what is assumed as of today. Finally, Appendix F states a pre-Planckian kinetic energy so the inflaton $\dot{\phi}^2 \gg \gg (P.E \sim V)$ [18]. This last step so important to our development will be considerably refined in a future paper.

What we are doing now is confirming the material given in this paper, as well as giving an explanation for our future research activity. The quartic potential we used above is the simplest version of the potential systems considered here. The cases of nonquartic potential should be examined fully, as part of a comprehensive study. This will be a part of the research project, which the authors will initiate in future publications. We should keep this discussion and the discussion of scalar fields separately from the ideas of inflation, namely of the fluctuations not necessarily having an upper bound of

$$\tilde{\tilde{\phi}} > \sqrt{\frac{60}{2\pi}}\, M_p \approx 3.1 M_p \equiv 3.1. \tag{55}$$

Since our modelling is not predicated upon the inflationary model of cosmology, but is addressing the issue brought up in [36], which is the contribution of the pre-Planckian space time to the cosmological evolution, we wish to adhere to noninflationary treatments as to Eq. (41) and Eq. (55), but will adhere to the questions posed at the beginning of this work. Furthermore, we will adhere to, in future papers, delineating a departure from the standard treatment of the evolution of the scalar field given in conventional inflation cosmology as the follows:

$$\frac{d\phi}{dt} = -\frac{V'(\phi)}{3H(\phi)} + \frac{H^{3/2}(\phi)}{2\pi}\xi(t). \tag{56}$$

This has the term of a "quasiquantum mechanical" effective white noise $\xi(t)$ similar to the term in the first-order differential equation, being a "driving" term for a quasichaotic oscillatory behavior of the scalar field. We argue that Eq. (56) in [37] is wrong, albeit well motivated by the conventional inflationary cosmology. A part of our future discussion will be concern with the pre-Planckian regime of the space-time as partly brought up in [38], by discussing what we are putting in instead as a replacement. Equation (56) contravenes our description of the kinetic energy as the dominant term in the pre-Planckian space-time physics, which deserves future developments for establishing experimental measurements.

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APPENDIX A
Scenarios as to the Value of Entropy at the Beginning of Space-Time Nucleation

We will be looking at inputs from page 290 of [35]. If $E \sim M \sim \Delta T_{tt}\, \delta t_{\text{time}}\, \Delta A\, l_{\text{P}}$

$$S(\text{entropy}) = \ln \mathrm{Z} + \frac{(\mathrm{E} \sim \Delta \mathrm{T_{tt}}\, \delta \mathrm{t}\, \Delta \mathrm{A}\, \mathrm{l_P})}{\mathrm{k_B T_{temperature}}}, \tag{A1}$$

and for Ng's infinite quantum statistics, we have, as the first approximation [39, 40],

$$S(\text{entropy}) \sim \ln \mathrm{Z} + \frac{((\mathrm{E} \sim \Delta \mathrm{T_{tt}})\, \delta \mathrm{t}\, \Delta \mathrm{A}\, \mathrm{l_P})}{\mathrm{k_B T_{temperature}}} \sim$$
$$\sim \ln Z + \left(\frac{\hbar}{k_B T_{\text{temperature}} \delta g_{tt}}\right) \xrightarrow[T_{\text{temperature}} \to \#\text{anything}]{}$$
$$\xrightarrow[T_{\text{temperature}} \to \#\text{anything}]{} [S(\text{entropy}) \sim \mathrm{n_{count}}] \neq 0. \tag{A2}$$

This is due to a very small but non vanishing δg_{tt} with the partition functions covered by [35], and due to [39, 40] with n_{count} to be a nonzero number of initial "particle" or information states, about the Planck regime of space-time, so that the initial entropy is nonzero.

APPENDIX B
Calculation of the Ricci tensor for the Roberson–Walker space-time, with its effect upon the measurement of if or not the space-time, is open, closed, or flat

We begin with Ref. [18] the discussion of the Roberson–Walker metric, if R is the Ricci scalar, and k the measurement of if we have a close, open, or flat universe, that if

$$a = a_{\text{initial}} \exp(H\, t). \tag{B1}$$

Then, by [18],

$$H^2 = -\frac{k}{a^2} + \frac{8\pi G \rho}{3}, \tag{B2}$$

$$3H^2 + \left[\frac{2k}{a^2} + \frac{R}{6}\right] = 0, \tag{B3}$$

leading to

$$a^2 = \frac{1}{k}\left[\frac{R}{6} + 8\pi G \rho\right]. \tag{B4}$$

If $\rho = -p$ [18], then, with a bit of algebra,

$$|p| = \frac{1}{8\pi G}\left[\frac{R}{6} + (a_{\text{initial}})^2 \exp\left[\sqrt{\frac{4\Lambda}{3}}\, t_{\text{time}}\right]\right]. \tag{B5}$$

Next, using [41], we will find at the boundary between the pre-Planckian to Planckian space-time that

$$R = 8\pi\left(T_0^0 + T_1^1 + T_2^2 + T_3^3\right) +$$
$$+ 4\Lambda \xrightarrow[\text{Pre-Planckian-Conditions}]{} 8\pi\left(T_0^0\right) + 4\Lambda. \tag{B6}$$

Then we can obtain right at the start of the Planckian era:

$$|p|_{\text{Planckian}} \sim \frac{1}{8\pi G}\left[\frac{8\pi\left(T_0^0 + T_1^1 + T_2^2 + T_3^3\right) + 4\Lambda}{6}\right]. \tag{B7}$$

The consequences of this would be that, right after the entry into the Planckian space-time, that there would be the following change of the pressure:

$$\begin{aligned} &|p|_{\text{Pre-Planckian}} = \tfrac{1}{8\pi G}\left[\tfrac{8\pi(T_0^0)+4\Lambda}{6} + (a_{\text{initial}})^2\right] \times \\ &\times \left[\exp\left[\sqrt{\tfrac{4\Lambda}{3}}\, t_{\text{time}}\right]\right] \Rightarrow \\ &\Rightarrow |p|_{\text{Pre-Planckian}} \sim \tfrac{1}{8\pi G}\left|\tfrac{8\pi(T_0^0)+4\Lambda}{6} + 0^+\right|, \\ &|p|_{\text{Planckian}} \sim \tfrac{1}{8\pi G}\left|\tfrac{8\pi\left(T_0^0+T_1^1+T_2^2+T_3^3\right)+4\Lambda}{6}\right|, \\ &\Delta P = |p|_{\text{Planckian}} - |p|_{\text{Pre-Planckian}} \sim \\ &\sim \left[\tfrac{\left(T_1^1+T_2^2+T_3^3\right)}{6G}\right]. \end{aligned} \tag{B8}$$

Then the change in the k term would be like to that from the pre-Planckian to Planckian space-time

$$\Delta k = \frac{1}{a_{\text{initial}}^2}\, [8\pi G(\rho - \Delta P)]. \tag{B9}$$

This goes almost to zero, if the numerator shrinks far more than the denominator, even if the initial scale factor is of the order of 10^{-110} or so.

APPENDIX C
Initial entropy from the first principles

We are making use of the Padmanabhan publications [42], where we have

$$\rho_\Lambda \approx \frac{G E_{\text{system}}^6}{c^8 \hbar^4} \Leftrightarrow \Lambda \approx \frac{1}{l_{\text{Planck}}^2}\, (E_{\text{system}}/E_{\text{Planck}})^6. \tag{C1}$$

Then if E_{system} is the energy of the Universe after the initiation of Eq. (18) as a bridge between the pre-Planckian to Planckian physical regimes, we can write

$$E_{\text{system}} \propto n_{\text{gravitons}}\, m_{\text{graviton}},$$

$$\Lambda \approx \frac{1}{l^2_{\text{Radius-Universe-today}}} \Leftrightarrow$$

$$\Leftrightarrow m_{\text{graviton}} \sim 10^{-62}\text{grams} \Rightarrow n_{\text{gravitons}} \sim 10^{37} \Rightarrow$$

$$\Rightarrow S_{\text{initial(graviton)}} \sim 10^{37}(\text{at} - \text{Planck} - \text{time}). \tag{C2}$$

The value of initial entropy, $S_{\text{initial(graviton)}} \sim 10^{37}$ should be contrasted with the entropy for the entire Universe, as given in [43] below.

APPENDIX D
Information flow, gravitons, and upper bounds to the graviton mass

Here, we view the possibility of considering the following, namely [44] is extended by [45], so we can make the identification

$$N = N_{\text{graviton}}\big|_{r_H} = \frac{c^3}{G\hbar}\,\frac{1}{\Lambda} \approx \frac{1}{\Lambda}. \tag{D1}$$

Should the N above be related to the entropy and Eq. (15)? This supposition has to be balanced against the following identification, namely, as given by T. Padmanabhan [42] has got

$$\Lambda_{\text{Einstein-Const.Padmanabhan}} = 1/l^2_{\text{Planck}}\,(E/E_{\text{Planck}})^6. \tag{D2}$$

But should the energy in the numerator in Eq. (D2) be given as say by (C2) in Appendix C, we have de facto quintessence. Then there would have been de facto quintessence, i.e., a variation in the "Einstein constant", which would have a large impact upon the graviton mass with a sharp decrease in g_*, being consistent with an evolution to the ultra light value of a graviton and with initial frequencies corresponding to wavelengths about the size of an atom:

$$\omega_{\text{initial}}\big|_{r_H \sim \text{atomic-size}} \sim 10^{21}\ \text{Hz}. \tag{D3}$$

The final value of the frequency would be of a magnitude smaller than one Hertz, so the graviton mass would be of the order of 10^{-62} g [23], due to Eq. (D2) approaching [44]. Namely,

$$\Lambda_{\text{Einstein-Const.}} = 1/l^2_{\text{Radius-Universe}}, \tag{D4}$$

leading to the upper bound of the graviton mass of about 10^{-62} g [44, 45]. In the present era,

$$m_{\text{graviton}} = \frac{\hbar}{c}\sqrt{\frac{(2\Lambda)}{3}} \approx \sqrt{\frac{(2\Lambda)}{3}}. \tag{D5}$$

Equation (D5) has a different value, if the entropy/particle count is lower, as has been postulated in this note. But Eq. (D5) gives the graviton mass of about 10^{-62} g [23] in the present era, which is in line with the entropy being far larger in this era [43].

APPENDIX E
The Riemann–Penrose Inequality with applications to fluctuations

Let us consider

$$\delta g_{tt} \sim a^2(t)\,\phi \ll 1. \tag{E1}$$

Refining the inputs from Eq. (E1) means to study the possibility of a nonzero minimum scale factor [20], as well as the nature of ϕ, as specified by Giovannini [17]. We hope for that this can be done to give quantifiable estimates and may link the nonzero initial entropy to either loop quantum gravity "quantum bounce" considerations [46] and/or other models, which may presage a modification of initial singularities of the sort given in [1]. Furthermore, if the nonzero scale factor is correct, it may give us opportunities to finely tune the parameters given below [20]:

$$\alpha_0 = \sqrt{\frac{4\pi G}{3\mu_0 c}}\,B_0,$$

$$\lambda(\text{defined}) = \Lambda c^2/3,$$

$$a_{\min} = a_0\left[\frac{\alpha_0}{2\,\widehat{\lambda}\ (\text{defined})}\right]\times$$

$$\times\left[\left(\sqrt{\alpha_0^2 + 32\,\widehat{\lambda}\ (\text{defined})\,\mu_0\omega\,B_0^2} - \alpha_0\right)\right]^{1/4}, \tag{E2}$$

where the following is possibly linkable to minimum frequencies linked to E and M fields [20], and, possibly, to relic gravitons:

$$B > \frac{1}{2\sqrt{10\mu_0\,\omega}}. \tag{E3}$$

So, we now investigate the question of applicability of the Riemann–Penrose inequality, which is presented in [47]. **Riemann–Penrose Inequality**: Let (M, g) be a complete asymptotically flat 3-manifold with nonnegative scalar curvature and total mass m, whose outermost horizon Σ has total surface area A. Then

$$m_{\text{total-mass}} \geq \sqrt{\frac{A_{\text{surface-area}}}{16\pi}}. \tag{E4}$$

The equality holds, if (M, g) is isometric to the spatial isometric Schwartzschield manifold M of mass m outside their respective horizons.

Assume that the frequency is, say, the frequency of Eq. (E3), and $A \approx A_{\min}$ of Eq. (E4) is employed. So, we have, by using the dimensional analysis appropriately, that

$$(v = \text{velocity} \equiv c) = f(\text{frequency}) \times \lambda(\text{wavelength}) \Rightarrow$$

$$\Rightarrow \omega \approx \omega_{\text{initial}} \sim \frac{c}{d_{\min}} \sim \frac{1}{d_{\min}}\bigg|_{c\equiv 1} \& d_{\min} \sim A^{1/3} \propto a_{\min}. \tag{E5}$$

Assume that we also set the input frequency as to Eq. (E3) according to $10 < \zeta \leq 37$, i.e.,

$$\left(\mathrm{m}_{\text{total-mass}} \sim 10^{\zeta}\,\mathrm{m}_{\text{graviton}}\right)^2 \propto a^3_{\min}/16\pi \Leftrightarrow$$

$$\Leftrightarrow \omega \approx \omega_{\text{initial}} \sim \frac{1}{d_{\min}} \sim \left(16\pi \times 10^{\zeta}\,m_{\text{graviton}}\right)^{-2/3}. \tag{E6}$$

Our supposition is that Eq. (E6) should give the same frequency as of Eq. (D3) above. This is a frequency input into Eq. (E3) above, where we safely assumed a graviton mass to be about [23]

$$\mathrm{m}_{\mathrm{total-mass}} \sim 10^{37}\,\mathrm{m}_{\mathrm{graviton}}, \qquad m_{\mathrm{graviton}} \sim 10^{-62}\ \mathrm{g}. \tag{E7}$$

Does the following make sense? When $10 < \zeta \leq 37$, we have

$$\left(m_{\mathrm{total-mass}} \sim 10^{\zeta}\, m_{\mathrm{graviton}}\right)^2 \propto a^3_{\min}/16\pi \Leftrightarrow$$

$$\Leftrightarrow \omega \approx \omega_{\mathrm{initial}} \sim \frac{1}{d_{\min}} \sim \left(16\pi \times 10^{\zeta}\, m_{\mathrm{graviton}}\right)^{-2/3}. \tag{E8}$$

We claim that if this is an initial frequency and if it is connected with the relic graviton production, the minimum frequency would be relevant to Eq. (E3) and may play a role in admissible B fields.

Note that if Appendix D is used, this makes a redo of Eq. (E8) which is a way of saying that the graviton mass given by [23] does not hold.

In either case, Eq. (E8) and Eq. (E3) in some configuration may argue for the implementation of work, it was done in Ref. [48], as to relic cylindrical GW, i.e., their allowed frequency and magnitude, so considered.

APPENDIX F
First-principles treatment of pre- Planckian kinetic energy so the inflaton $\dot{\phi}^2 \gg (P.E \sim V)$

We give this as a plausibility argument, which undoubtedly will be considerably refined, but its importance cannot be overstated, i.e., this is for the pre-inflationary pre-Planckian physics, so as to get a lower bound to the graviton mass. To do this, we look at results in [18] and will be enlisting the new references [49], and [50] as to details to put in, so as to confirm a dominance of the kinetic energy. Let us start with the Friedman equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k_{\mathrm{curvature}}}{a^2} = \frac{4\pi G}{3}\frac{p_\phi^2}{a^6} + \Lambda. \tag{F1}$$

We will treat then the Hubble parameter as

$$\left(\frac{\dot{a}}{a}\right) = H_{\mathrm{initial}} \equiv \frac{2}{t\left(1+\frac{P}{\rho}\right)} \xrightarrow[P=-\rho+\varepsilon^+]{} \xrightarrow[P=-\rho+\varepsilon^+]{} \frac{2}{t\left(\frac{\varepsilon^+}{\rho}\right)} \xrightarrow[t\to t_{\mathrm{P}}]{} \frac{2\rho}{t_{\mathrm{P}}\,\varepsilon^+}. \tag{F2}$$

Now, from Ref. [50], we can write the density in terms of the flux:

$$\frac{d\rho}{dt} = \frac{1}{V^{(3)} = \mathrm{Volume}}\,(A = \mathrm{Area})\,(\Im = \mathrm{Flux}) \sim \sim \frac{(\Im = \mathrm{Flux})}{l_{\mathrm{P}}}. \tag{F3}$$

Let T be the temperature and let N be the particle count in the flux region per unit time (say, the Planck time). Using the “ideal gas law” approximation for superhot conditions, we have

$$\begin{aligned}
&\frac{d\rho}{dt} = \frac{1}{V^{(3)} = \mathrm{Volume}}\,(A = \mathrm{Area})(\Im = \mathrm{Flux}) \sim \\
&\sim \frac{(\Im = \mathrm{Flux})}{l_{\mathrm{P}}}, \\
&\rho \sim \frac{(\Im=\mathrm{Flux})}{c} \Rightarrow H = \frac{N}{\varepsilon^+} \times \\
&\times \frac{1}{V^{(4)} = 4 - \mathrm{DimVolume}}\sqrt{\frac{8}{\pi}}\sqrt{\frac{k_{\mathrm{B}}T}{m_{\mathrm{flux-particle}}}}.
\end{aligned} \tag{F4}$$

Next, according to [49], we can make the following substitution:

$$p_\phi = a^3\,\dot{\phi}. \tag{F5}$$

Therefore,

$$\begin{aligned}
&\dot{\phi}^2 \approx a^{-6}\,(12\pi G)\,V^{(4)}\left(H^2 + |\Lambda|\right) \approx a^{-6}\,(12\pi G)\;V^{(4)} \times \\
&\times \left(\left[\frac{N}{\varepsilon^+}\frac{1}{V^{(4)} = 4 - \mathrm{DimVolume}} \times \right.\right. \\
&\left.\left.\times \sqrt{\frac{8}{\pi}}\sqrt{\frac{k_{\mathrm{B}}T}{m_{\mathrm{flux-particle}}}}\right]^2 + |\Lambda|\right).
\end{aligned} \tag{F6}$$

If the scale factor is very small, say, of the order of $a = a_{\mathrm{initial}} \sim \sim 10^{-55}$, then no matter how fall the initial volume is, in the 4-space (it cancels out in the first part of the brackets), it is easy to see then that $\dot{\phi}^2 \gg (P.E \sim V)$ [18].

In the future, we will add a more structure to this calculation and will confirm via a precise calculation that the lower bound to the graviton mass is about 10^{-70} g.

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УЗАГАЛЬНЕНИЙ ПРИНЦИП НЕВИЗНАЧЕНОСТІ ГАЙЗЕНБЕРГА В КВАНТОВІЙ ГЕОМЕТРОДИНАМІЦІ ТА ЗТВ

Р е з ю м е

В даній роботі розглядається потік енергії у Всесвіті на основі простої квантової системи, що описується нелінійним рівнянням Гамільтона–Якобі, яке виникає в рамках стандартного квантового формалізму рівняння Шредінґера. Розглядаються також випадки домінування випромінювання, баротропної рідини та квантової матерії-енергії. В результаті, сформульовано узагальнений принцип невизначеності Гайзенберга (УПНГ) для метричного тензора та, на основі формалізму Кузьмичових для квантової геометродинаміки, встановлено внутрішній взаємозв'язок між УПНГ для метричного тензора та умовами, які постулюються у випадку стану, в якому домінуючою є баротропна рідина у формі пилу.