Contribution of the processes $B^0_d \rightarrow \bar{K}^{*0} \rightarrow K^- \pi^+$ with vector mesons $V = \rho(770)$, $\omega(782)$, $\phi(1020)$, $J/\psi$, $\psi(2S)$, and others decaying into the $\mu^+\mu^-$ or the CP-asymmetry in the decay $B^0_d \rightarrow \bar{K}^{*0} \rightarrow K^- \pi^+\mu^-\mu^-$ induced by the flavor-changing neutral currents are calculated. For the description of the transition $b \rightarrow s\mu^+\mu^-$, the most general form of the effective weak-interaction Hamiltonian is applied. Predictions are made for the CP-asymmetry of the decay $B^0_d \rightarrow \bar{K}^{*0} \rightarrow K^- \pi^+\mu^-\mu^-$ in the framework of the standard model, as well for two scenarios of new physics. The obtained results are compared with experimental data of the LHCb Collaboration.

Keywords: B-meson, CP symmetry, vector mesons, decay, vector-meson dominance.

1. Introduction

Researches of flavor-changing neutral currents (FCNCs) of quarks in the $b \rightarrow s$ and $b \rightarrow d$ transitions are an important test for the Standard Model (SM) and its probable extensions (see, e.g., work [1]). Among the decay processes associated with FCNCs, the $b \rightarrow s\ell^+\ell^-$ ($\ell = e, \mu$) transition attracts keen interest. In the SM framework, this transition stems from one-loop and box electroweak diagrams with regard for quantum-chromodynamic (QCD) contributions, when a virtual photon, a $Z$-boson, and a $W^+W^-$-pair transform into a lepton-antilepton pair. The structure of the amplitude of the $b \rightarrow s\ell^+\ell^-$ process is sensitive to SM extensions in the gauge theory sector, as well as in the fermionic and Higgs ones. For that reason, a considerable attention is paid to the $B^0_d \rightarrow \mu^+\mu^-$, $B \rightarrow K\ell^+\ell^-$, and $B \rightarrow K^*\ell^+\ell^-$ processes. Their experimental and theoretical studies can result in the observation of new physics (NP) signals beyond the SM [1]. In particular, the manifestations of NP effects in the $B \rightarrow K^*(\rightarrow K\pi)\ell^+\ell^-$ decay were discussed by many authors (see, e.g., works [2–35]).

In this work, the CP-asymmetry of the decay process $B^0_d \rightarrow \bar{K}^{*0}\mu^+\mu^-$ is studied for the most general form of the effective weak-interaction Hamiltonian of the $b \rightarrow s\mu^+\mu^-$ transition. We recall that, in the framework of SM, the CP-asymmetry of this decay induced by the $b \rightarrow s\mu^+\mu^-$ process is predicted at a level of $10^{-3}$ [12, 13]. Therefore, the observation of a considerable CP-asymmetry in this decay will testify to a detection of NP signals.

In general, the amplitude of the decay $B^0_d \rightarrow \bar{K}^{*0}\mu^+\mu^-$ is composed of the contributions from short (SD) and long (LD) distances. The former are expressed in terms of the Wilson coefficients $C_i$ calculated in the framework of the QCD perturbation theory up to the corresponding order in $\alpha_s(\mu)$, the constant of effective QCD interaction at the $\mu$-scale. They contain information about the processes running on energy scales of the order of $m_W$ and $m_t$. On scales of the order of the $b$-quark mass, $m_b$, their values are determined by using the renormalization group methods.

The SD effects, which describe hadronization processes, are expressed in terms of matrix elements of the operators of $b \rightarrow s$ transitions between the initial, $B$, and final, $K^*$, states. As a rule, they are written in the form of transition form factors [36, 37]. In addition, there also exist other LD contributions arising owing to the decays $B^0_d \rightarrow \bar{K}^{*0}V \rightarrow K^{*0}\mu^+\mu^-$, where $V = \rho(770)$, $\omega(782)$, $\phi(1020)$, $J/\psi$, $\psi(2S)$, and so on are vector mesons [38, 39].
In this work, we calculated the CP-asymmetry in the decay $B_d^0 \to K^{\ast 0} \mu^+ \mu^-$ making allowance for the fact that the SD contributions to the amplitude of this decay are described by the effective weak-interaction Hamiltonian in its most general form. The LD effects were taken into account by including the contributions of vector mesons $\rho(770)$, $\omega(782)$, $\phi(1020)$, $J/\psi$, and $\psi(2S)$ into the $B_d^0 \to K^{\ast 0}(\to K^{\ast -}\pi^+) \mu^+ \mu^-$ decay. Information on the amplitudes of the processes $B_d^0 \to K^{\ast 0} V$, where $V = \rho(770)$, $\omega(782)$, $\phi(1020)$, $J/\psi$, and $\psi(2S)$, was taken from either available experimental data or, otherwise, theoretical predictions.

2. CP-Asymmetry and Amplitudes of Decay $B_d^0 \to K^{\ast 0} \mu^+ \mu^-$

2.1. Differential decay rate

The differential rate of the decay $B_d^0 \to K^{\ast 0} \mu^+ \mu^-$ can be written as follows:

$$\frac{d\Gamma}{dq^2} = \frac{1}{2} (3 J_{1s} - 2 J_{2s}) + \frac{1}{4} (3 J_{1c} - 2 J_{2c}),$$

(1)

where $q^2$ is the squared invariant mass of a muon pair, and the functions $J_{1s}$, $J_{1c}$, $J_{2s}$, and $J_{2c}$ are defined by the formulas

$$J_{1s} = \frac{2 + \beta_\mu^2}{4} (|A_{1L}|^2 + |A_{1\perp}|^2 + |A_{1R}|^2) +$$

$$+ (|A_{1\perp}|^2) + \beta_\mu^2 \left( |A_{1L}|^2 + |A_{1\perp}|^2 + |A_{1R}|^2 \right) +$$

$$+ |A_{1\perp}|^2 + |A_{1L}|^2 + |A_{1R}|^2 + \frac{4m_\mu^2}{q^2} \Re(A_{1L} \times$$

$$\times A_{1L}^* + A_{1\perp} A_{1\perp}^*) + \frac{8m_\mu^2}{q^2} (|A_{1L}|^2 + 2|A_{1\perp}|^2 + 2|A_{1R}|^2) +$$

$$+ (|A_{1\perp}|^2 + |A_{1L}|^2 + |A_{1R}|^2) +$$

$$\times \Im \left( (A_{1L} + |A_{1L}|^2) (A_{1L} + A_{1\perp} + A_{1R}^* - A_{1R}^2) \right) +$$

$$+ (A_{1\perp} + A_{1L} + A_{1R}) (A_{1\perp} + A_{1L}^* + A_{1R}^2 - A_{1R}^2))^*,$$

$$J_{1c} = |A_{0L}|^2 + |A_{0R}|^2 + (1 + \beta_\mu^2) (|A_{0R}|^2 +$$

$$+ |A_{0L}|^2) + 4\beta_\mu^2 (|A_{0L}|^2 - |A_{0\perp}|^2 + |A_{0R}|^2) +$$

$$+ \frac{4m_\mu^2}{q^2} (|A_{0L}|^2 + 2 \Re(A_{0L} A_{0}\perp + A_{0R} A_{0R}^*) + 4|A_{0R}|^2 +$$

$$- A_{0L}^2 + A_{0\perp}^2 + A_{0R}^2) +$$

$$+ \frac{4m_\mu^2}{q^2} (|A_{0L}|^2 + 2 \Re(A_{0L} A_{0R} + A_{0L} A_{0\perp}^*) + 4|A_{0R}|^2 +$$

$$- A_{0L}^2 + A_{0\perp}^2 + A_{0R}^2) +$$

$$+ \Im(A_{0L} A_{0\perp} + A_{0R} A_{0R}^*),$$

$$J_{2s} = \frac{\beta_\mu^2}{4} (|A_{1L}|^2 + |A_{1\perp}|^2 + |A_{1R}|^2 + |A_{1R}|^2 -$$

$$- 4(|A_{1L}|^2 + A_{0\perp}^2 + |A_{1\perp}|^2 +$$

$$+ |A_{1R}|^2 - A_{0\perp}^2 + |A_{1\perp}|^2\})^2),$$

$$J_{2c} = -\beta_\mu^2 (|A_{0L}|^2 + |A_{0R}|^2 - 4(|A_{0L}^2 - A_{0R}^2| +$$

$$+ |A_{0R} + A_{0\perp}|^2)),$$

where $\beta_\mu \equiv \sqrt{1 - 4m_\mu^2/q^2}$, and $m_\mu$ is the muon mass. The functions $J_{1s}$, $J_{1c}$, $J_{2s}$, and $J_{2c}$ depend on the amplitudes of the $B_d^0 \to K^{\ast 0} \mu^+ \mu^-$ decay, namely, $A_{0L}(R)$, $A_{0\perp}(R)$, $A_{0\perp}(\mu)$, $A_{0R}$, $A_{1L}(R)$, $A_{1\perp}(R)$, $A_{1L}(\mu)$, $A_{1\perp}(\mu)$, $A_{1R}(R)$, and $A_{1\perp}(\mu)$. The explicit expressions for those amplitudes depend on the effective weak-interaction Hamiltonian for the transition process $b \to s \mu^+ \mu^-$. In the most general case, this Hamiltonian can depend on the scalar, pseudoscalar, tensor, axial-vector, and tensor operators of this transition. If so, the non-resonant amplitudes of the $B_d^0 \to K^{\ast 0} \mu^+ \mu^-$, in the naive factorization approach, look like

$$A_{S\perp}(R) = -N((C_S - C_{\perp}) \mp (C_{T} - C_{\perp})) \sqrt{\lambda} A_0(q^2),$$

$$A_t = -2N \lambda \left( \frac{C_{\text{eff}} - C_{\text{eff}}^{10}}{2m_K \sqrt{q^2}} \right) A_0(q^2),$$

$$A_{0\text{eff}}(R) = -N \frac{C_0(q^2)}{2m_K \sqrt{q^2}} \left( C_{\text{eff}} - C_{\text{eff}}^{10} \mp \right.$$

$$\left. (C_{\text{eff}}^{10}) + 2 \frac{m_K^2 \rho(q^2)}{q^2} (C_{\text{eff}} - C_{\text{eff}}^{10}) \right),$$

$$A_{0\text{eff}}(\perp) = N \sqrt{2} C_{\perp}(q^2) \left( C_{\text{eff}} - C_{\text{eff}}^{10} \mp \right.$$

$$\left. (C_{\text{eff}}^{10}) + 2 \frac{m_K^2 \rho(q^2)}{q^2} (C_{\text{eff}} + C_{\text{eff}}^{10}) \right),$$

$$A_{1\text{eff}}(R) = -N \sqrt{2} C_{\perp}(q^2) \left( C_{\text{eff}} + C_{\text{eff}}^{10} \mp \right.$$

$$\left. (C_{\text{eff}}^{10}) + 2 \frac{m_K^2 \rho(q^2)}{q^2} (C_{\text{eff}} + C_{\text{eff}}^{10}) \right),$$

$$A_{1\text{eff}}(\perp) = N \sqrt{2} C_{\perp}(q^2) \left( C_{\text{eff}} + C_{\text{eff}}^{10} \mp \right.$$

$$\left. (C_{\text{eff}}^{10}) + 2 \frac{m_K^2 \rho(q^2)}{q^2} (C_{\text{eff}} + C_{\text{eff}}^{10}) \right),$$

$$A_{t}(R) = \frac{iN}{2m_K \sqrt{q^2}} \left( C_T - C_{\text{eff}} \mp (C_{T5} - C_{T5}^{10}) \right) \left( m_B^2 - q^2 \right) +$$

$$+ q^2 \sqrt{m_B^2 - m_K^2} T_2(q^2) - \frac{C_T - C_{\text{eff}}}{m_B^2 - m_K^2} \left( m_B^2 - q^2 \right),$$

$$A_{t}(\perp) = -iN \left( C_T - C_{\text{eff}} \mp (C_{T5} - C_{T5}^{10}) \right) \times$$

$$\times \left( m_B^2 - m_K^2 \right) \sqrt{\frac{2}{q^2}} T_2(q^2),$$

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\[ A_{L}^{(R)} = iN(C_T + C'_T \mp (C_{T5} + C'_{T5})) \sqrt{\frac{2\lambda}{q^2}} T_1(q^2), \]
\[ A_{0}^{(R)} = -iN(C_T - C'_T \mp (C_{T5} - C'_{T5})) \sqrt{\frac{2\lambda}{q^2}} T_1(q^2), \]
\[ A_{0}^{(L)} = iN(C_T + C'_T \mp (C_{T5} + C'_{T5})) \times (m_B^2 - m_{K^*}^2) \sqrt{\frac{2}{q^2}} T_2(q^2), \]
\[ A_{0}^{(L)} = \frac{iN}{2m_{K^*}} (C_T + C'_T \mp (C_{T5} + C'_{T5})) \left( (m_B^2 - q^2 + 3m_{K^*}^2)T_2(q^2) - \frac{\lambda}{m_B^2 - m_{K^*}^2} T_3(q^2) \right). \]

Here, $\overline{m}_b$ is the effective $\overline{\text{MS}}$-mass of a $b$-quark on the scale $\mu_b$; and $C_B, C'_B, C_P, C'_P, C_{T5}^{\text{eff}}, C_{T5}'^{\text{eff}}, C_{B5}^{\text{eff}}, C_{B5}'^{\text{eff}}, C_{T7}^{\text{eff}}, C_{T7}'^{\text{eff}}, C_{T}, C_T', C_{T5},$ and $C_{T5}'$ are the Wilson coefficients in the most general form of the effective weak-interaction Hamiltonian for the transition process $b \to s \mu^+\mu^-$. Note that, owing to the identity $\sigma_{\mu\gamma} \equiv -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$ ($\epsilon_{0123} = 1$), the contribution from tensor operators to the decay amplitudes actually depends on two coefficients, namely, $C_T$ and $C_{T5}'$, which are related to the coefficients $C_T, C_T', C_{T5},$ and $C_{T5}'$, by the formulas
\[ \tilde{C}_T = \frac{1}{2} \left( C_T + C'_T + C_{T5} + C'_{T5} \right), \]
\[ \tilde{C}_{T5}' = \frac{1}{2} \left( -C_T + C'_T - C_{T5} - C'_{T5} \right). \]

In the SM framework, those coefficients have definite values; namely, $C_B, C'_B, C_P, C'_P, C_{T5}, C_{T5}', C_{T5}^{\text{eff}}, C_{B5}^{\text{eff}}, C_{T7}^{\text{eff}},$ and $C_{T7}^{\text{eff}}$ equal zero; whereas $C_{T5}'^{\text{eff}}, C_{B5}'^{\text{eff}},$ and $C_{T7}'^{\text{eff}}$ in the next-to-leading-order approximation on the scale $\mu_b = 4.8$ GeV are [18]
\[ \tilde{C}_T = \frac{1}{2} \left( C_T + C'_T + C_{T5} + C'_{T5} \right), \]
\[ \tilde{C}_{T5}' = \frac{1}{2} \left( -C_T + C'_T + C_{T5} + C'_{T5} \right), \]
where $\overline{m}_s$ is the effective $\overline{\text{MS}}$-mass of an $s$-quark on the scale $\mu_b = 4.8$ GeV, and
\[ C_{T5}^{\text{eff}} = -0.2923, \quad \overline{m}_bC_{T5}^{\text{eff}} = \overline{m}_sC_{T5}^{\text{eff}}, \]
\[ C_{B5}^{\text{eff}} = 4.0749 + Y(q^2), \quad C_{T7}^{\text{eff}} = -4.3085, \]
where $Y(q^2) = h(q^2, m_e) \left( \frac{1}{3}C_1 + C_2 + 6C_3 + 60C_5 \right) - \frac{1}{2} h(q^2, m_b) \left( 7C_3 + 4C_4 + 3C_5 + \frac{64}{3}C_6 \right) - \frac{1}{2} h(q^2, 0) \left( C_3 + \frac{4}{3}C_4 + 16C_5 + \frac{64}{3}C_6 \right) - \frac{V_{ub}V_{us}^*}{V_{cd}V_{ts}^*} \left( \frac{4}{3}C_1 + C_2 \right) \left( h(q^2, 0) - h(q^2, m_e) \right) + \frac{4}{3} C_3 + \frac{64}{9} C_5 + \frac{64}{27} C_6.
\]
\[ h(q^2, m_q) = -\left( \frac{4}{9} \ln \frac{m_q^2}{\mu_b^2} - \frac{2}{3} \right) - \frac{1}{4} (2 + z) \sqrt{z - 1} \left( \text{arctan} \frac{1}{\sqrt{z - 1}}, \quad z > 1, \right) \]
\[ - \frac{1}{4} \left( 2 + z \right) \sqrt{z - 1} \left( \ln \frac{1 + \sqrt{z - 1}}{\sqrt{z - 1}} - \frac{i\pi}{2}, \quad z \leq 1, \right) \]
\[ h(q^2, 0) = -\frac{4}{9} \ln \frac{q^2}{\mu_b^2} + \frac{8}{27} + \frac{i\pi}{9}. \]

Here, $V_{ij}$ are the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix [40, 41], $z = 4m_q^2/q^2$, and $m_c$ and $m_b$ are the pole masses of $c$- and $b$-quarks, respectively. In the SM framework, in the next-to-leading-order approximation on the scale $\mu_b = 4.8$ GeV, the Wilson coefficients $C_1$ to $C_6$ are equal [18]
\[ C_1 = -0.2632, \quad C_2 = 1.0111, \quad C_3 = -0.0055, \]
\[ C_4 = -0.0806, \quad C_5 = 0.0004, \quad \text{and} \quad C_6 = 0.0009. \]

In addition, $\lambda \equiv (m_B^2 - q^2)^2 - 2(m_B^2 + q^2) m_{K^*}^2 + m_{K^*}^4$, where $m_{K^*}$ is the vector $K^{*0}$-meson mass, $m_B$ the $B^0_d$-meson mass, and
\[ N = V_{ub}V_{ts}^* \frac{G_F \alpha_{em}}{32\pi^2} \sqrt{\frac{\beta_{\mu}}{m_B^2}} \int \frac{q^2 \sqrt{\lambda}}{m_B^2} \beta_{\mu} \left( \beta^2 \right), \]
where $G_F$ is the Fermi constant, and $\alpha_{em}$ the effective intensity of the electromagnetic interaction on the scale $\mu_b$. The model parameters are listed in Table 1. The quantities $A_0(q^2), A_1(q^2), A_2(q^2), V(q^2), T_1(q^2), T_2(q^2)$, and $T_3(q^2)$ are the $B \to K^*$ transition form factors, and
\[ C_0(q^2) = (m_B^2 - q^2 - m_{K^*}^2)(m_B + m_{K^*}) A_1(q^2) - \lambda \frac{A_2(q^2)}{m_B + m_{K^*}}, \]

\[ C_\parallel(q^2) = (m_B - m_{K^*}) A_1(q^2), \]
\[ C_\perp(q^2) = \frac{V(q^2)}{m_B + m_{K^*}}, \]
\[ \kappa_0(q^2) = C_0^{-1}(q^2) - \left( \frac{m_B^2 - q^2 + 3m_{K^*}^2}{m_B^2 - m_{K^*}^2} \right) T_2(q^2) - \lambda \frac{T_3(q^2)}{m_B^2 - m_{K^*}^2}, \]
\[ \kappa_\parallel(q^2) = (m_B - m_{K^*}) \frac{T_2(q^2)}{A_1(q^2)}, \]
\[ \kappa_\perp(q^2) = (m_B + m_{K^*}) \frac{T_1(q^2)}{V(q^2)}. \]

While calculating the CP-asymmetry in the \( B^0 \rightarrow K^{*0} \rightarrow K^0 \mu^+\mu^- \) decay, the transition form factors from work [43] were used.

The differential rate of the CP-conjugate decay process \( B^0 \rightarrow K^{*0} \rightarrow K^0 \mu^+\mu^- \) looks like
\[
\frac{d\Gamma}{dq^2} = \frac{1}{2} \left( 3 J_{1s} - J_{2s} \right) + \frac{1}{4} \left( 3 J_{1c} - J_{2c} \right),
\]
where the functions \( J_i \) are obtained from \( J_i \) by changing all weak interaction phases to the corresponding opposite values. To analyze the violation of CP-invariance in the \( B^0 \rightarrow K^{*0} \mu^+\mu^- \) decay, it is expedient to measure the CP-asymmetry
\[
A_{CP} = \frac{\left( \frac{d\Gamma}{dq^2} - \frac{d\Gamma}{dq^2} \right)}{\left( \frac{d\Gamma}{dq^2} + \frac{d\Gamma}{dq^2} \right)},
\]
or the \( q^2 \)-integrated quantity
\[
\langle A_{CP} \rangle = \frac{\left( \frac{d\Gamma}{dq^2} - \frac{d\Gamma}{dq^2} \right)}{\left( \frac{d\Gamma}{dq^2} + \frac{d\Gamma}{dq^2} \right)},
\]
where
\[
\langle J \rangle \equiv \int \frac{d\Gamma}{dq^2} J(q^2).
\]

### 2.2. Transverse amplitudes with the contribution of vector resonances

Now, we add the LD contributions that arise from the \( B^0 \rightarrow K^{*0} V \) decay (here \( V = \rho^0, \omega, \phi, J/\psi(1S), \psi(2S) \)), and other mesons that decay afterward according to the scheme \( V \rightarrow \mu^+\mu^- \), to the non-resonant amplitude of the \( B^0 \rightarrow K^{*0} \mu^+\mu^- \) decay. The vector meson dominance (VMD) model is used, which is based on the Lagrangian
\[
\mathcal{L}_{V} = -\frac{e}{2} F_{\mu\nu} \sum_{V} \frac{f_{V} Q_{V}}{m_{V}} V_{\mu\nu}, \tag{2}
\]
where \( V_{\mu\nu} \equiv \partial_{\mu} V_{\nu} - \partial_{\nu} V_{\mu}, \ F_{\mu\nu} \equiv \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} \) is the electromagnetic field tensor, \( m_V \) the mass of a vector \( V \)-meson, and \( Q_{V} \) the effective electric charge of quarks in the vector meson:
\[
Q_{\rho} = \frac{1}{\sqrt{2}}, \quad Q_{\omega} = \frac{1}{3\sqrt{2}}, \quad Q_{\phi} = \frac{1}{3}.
\]

The decay constant of the neutral vector meson, \( f_V \), can be found from the \( V \rightarrow e^+e^- \) decay width, using the formula
\[
\Gamma_{V\rightarrow e^+e^-} = \frac{4\pi\alpha^2}{3m_V} f_V^2 Q_V^2.
\]

Lagrangian (2) is obviously gauge-invariant and gives rise to the \( V \gamma^* \) vertex
\[
\langle \gamma(q); \mu|V(q); \nu \rangle = -\frac{e f_{V} Q_{V}}{m_{V}} (q^2 g^{\mu\nu} - q^{\rho} q^{\nu}),
\]
where \( q \) is the 4-momentum of a virtual photon (vector meson), and \( g^{\mu\nu} \) the metric tensor. The parameters of vector resonances are quoted in Table 2.

Using the VMD concept, let us calculate the total amplitude of the decay, which consists of the non-resonant and resonant components,
\[
A_{0L(R)} = A_{0L(R)}^{NR} + \frac{4\pi^2 m_{B}^2 |N|}{m_{K^*} \sqrt{q^2}} \sum_{V} C_{V} D_{V}^{-1}(q^2) \times \left( m_{B}^2 - q^2 + m_{K^*}^2 \right) S_{V}^1 + \lambda S_{V}^2 \right), \tag{3}
\]

**Table 1. Parameters taken for the calculation of observables [42]**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{\mu\nu} )</td>
<td>((-4.04 + i0.07) \times 10^{-2})</td>
</tr>
<tr>
<td>( \alpha_{V} )</td>
<td>(0.015658) GeV</td>
</tr>
<tr>
<td>( m_{\rho} )</td>
<td>(3.17958) GeV</td>
</tr>
<tr>
<td>( m_{\phi} )</td>
<td>(4.78) GeV</td>
</tr>
<tr>
<td>( m_{K^*} )</td>
<td>(0.89594) GeV</td>
</tr>
</tbody>
</table>

\( m_{K^*} \) is the mass of the \( K^* \) meson, \( \lambda \) is a parameter related to the decay constant of the \( K^* \) meson, \( m_{B} \) is the mass of the \( B \) meson, and \( q^2 \) is the square of the four-momentum of the virtual photon. The parameters \( \alpha_{V} \) and \( \alpha_{V} \) are related to the decay constant of the \( V \) meson, \( m_{\rho} \) is the mass of the \( \rho \) meson, \( m_{\phi} \) is the mass of the \( \phi \) meson, and \( m_{K^*} \) is the mass of the \( K^* \) meson. The parameter \( \lambda \) is a parameter related to the decay constant of the \( K^* \) meson.
\[ A_{\text{LL}(R)} = A_{\text{LR}(R)}^{\text{NR}} - 8\sqrt{2}\pi^2m^4_V|\mathcal{N}| \times \]
\[ \sum_V C_V D_V(q^2)S_V, \quad (4) \]
\[ A_{\text{LL}(R)} = A_{\text{LR}(R)}^{\text{NR}} + 4\sqrt{2}\pi^2m_V|\mathcal{N}| \times \]
\[ \sum_V C_V D_V^{-1}(q^2)S_V, \quad (5) \]

where \( D_V(q^2) = q^2 - m_V^2 + i\eta \Gamma_V(q^2) \) is a standard Breit-Wigner function for the shape of a \( V \)-meson resonance with the energy-dependent width \( \Gamma_V(q^2) \).

In Eqs. (3)–(5), \( S_V^i \) (\( i = 1, 2, 3 \)) are the invariant amplitudes of the \( B_d^0 \rightarrow K^{*0}V \) decay; their values are quoted in Table 3. Those amplitudes were discussed in work [38] with the account for modern data [42] and not taking, as was done in work [39], we calculated the integrated values of vector mesons such as \( \psi(2S) \), and others, the widths are assumed to be constant.

**Table 2. Masses, total widths, widths of decays into a lepton pair, and constants \( f_V \) of vector mesons [42] (experimental uncertainties are not indicated)**

<table>
<thead>
<tr>
<th>( V )</th>
<th>( m_V, ) MeV</th>
<th>( \Gamma_V, ) MeV</th>
<th>( \Gamma_{V\rightarrow\ell^+\ell^-}, ) keV</th>
<th>( f_V, ) MeV</th>
</tr>
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<td>( \rho^0 )</td>
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<td>149.1</td>
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<td>( \omega )</td>
<td>782.65</td>
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<td>( \phi )</td>
<td>1019.455</td>
<td>4.26</td>
<td>1.27</td>
<td>228.6</td>
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<tr>
<td>( J/\psi )</td>
<td>3096.906</td>
<td>0.9029</td>
<td>5.55</td>
<td>416.4</td>
</tr>
<tr>
<td>( \psi(2S) )</td>
<td>3686.109</td>
<td>0.304</td>
<td>2.35</td>
<td>295.6</td>
</tr>
<tr>
<td>( \psi(3770) )</td>
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<td>27.3</td>
<td>0.265</td>
<td>100.4</td>
</tr>
<tr>
<td>( \psi(4040) )</td>
<td>4039</td>
<td>80</td>
<td>0.86</td>
<td>187.2</td>
</tr>
<tr>
<td>( \psi(4160) )</td>
<td>4153</td>
<td>103</td>
<td>0.83</td>
<td>186.5</td>
</tr>
<tr>
<td>( \psi(4415) )</td>
<td>4421</td>
<td>62</td>
<td>0.58</td>
<td>160.8</td>
</tr>
</tbody>
</table>

**Table 3. Branching ratios [42] and invariant amplitudes of decay processes \( B_d^0 \rightarrow K^{*0}\rho^0, \)
\( B_d^0 \rightarrow K^{*0}\omega, \)
\( B_d^0 \rightarrow K^{*0}\phi, \)
\( B_d^0 \rightarrow K^{*0}J/\psi, \)
and \( B_d^0 \rightarrow K^{*0}\psi(2S) \)**

<table>
<thead>
<tr>
<th>( \rho^0 )</th>
<th>( \omega )</th>
<th>( \phi )</th>
<th>( J/\psi )</th>
<th>( \psi(2S) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^6\text{BR}(B_d^0 \rightarrow K^{*0}\rho^0, V) )</td>
<td>3.4</td>
<td>20</td>
<td>9.8</td>
<td>1340</td>
</tr>
<tr>
<td>( 10^4</td>
<td>S_V^1</td>
<td>^2 )</td>
<td>1.40</td>
<td>0.93</td>
</tr>
<tr>
<td>( 10^4</td>
<td>S_V^2</td>
<td>^2 )</td>
<td>2.64</td>
<td>1.82</td>
</tr>
<tr>
<td>( 10^4</td>
<td>S_V^3</td>
<td>^2 )</td>
<td>2.76</td>
<td>1.77</td>
</tr>
<tr>
<td>( \delta_V^\omega - \delta_V^\phi ) (rad)</td>
<td>2.81</td>
<td>2.11</td>
<td>2.40</td>
<td>-2.86</td>
</tr>
<tr>
<td>( \delta_V^{J/\psi} - \delta_V^\phi ) (rad)</td>
<td>-0.37</td>
<td>-1.17</td>
<td>-0.84</td>
<td>0.90</td>
</tr>
<tr>
<td>( \delta_V^\psi - \delta_V^{J/\psi} ) (rad)</td>
<td>2.80</td>
<td>2.10</td>
<td>2.39</td>
<td>3.01</td>
</tr>
</tbody>
</table>

In order to calculate the contribution of a resonance to the amplitude of \( B_d^0 \rightarrow K^{*0}\mu^+\mu^- \) decay, we have to know the amplitudes of the decays \( B_d^0 \rightarrow K^{*0}\rho, B_d^0 \rightarrow K^{*0}\omega, B_d^0 \rightarrow K^{*0}\phi, B_d^0 \rightarrow K^{*0}J/\psi, B_d^0 \rightarrow K^{*0}\psi(2S) \), and so forth. In this connection, we note that the amplitude of the B-meson decay into two vector mesons is experimentally determined by the branching ratio, four polarization parameters (two polarization fractions and two relative phases), and the common phase \( \delta_V^\psi \).

Information concerning the amplitudes of the processes \( B_d^0 \rightarrow K^{*0}\rho, B_d^0 \rightarrow K^{*0}J/\psi, \) and \( B_d^0 \rightarrow K^{*0}\psi(2S) \) can be obtained from experimental data [42]. We used those amplitudes to calculate the invariant amplitudes \( S_V^i \) (see work [39]). Unfortunately, a complete angular analysis of the \( B_d^0 \rightarrow K^{*0}\rho \) and \( B_d^0 \rightarrow K^{*0}\omega \) processes has not been carried out yet. Therefore, for light vector \( \rho \) - and \( \omega \)-mesons, we can use experimental data only for the relative width and the longitudinal polarization fraction that concerns the parallel and transverse polarizations of a vector meson. They can be evaluated using the relationship of the naive factorization analysis

\[ A_0 : \Delta_- : \Delta_+ = 1 : \frac{\Lambda_{\text{QCD}}}{m_b} : \left( \frac{\Lambda_{\text{QCD}}}{m_b} \right)^2, \]

where \( \Lambda_h(h = 0, -, +) \) are the spiral amplitudes of the corresponding B-meson decays, and \( \Lambda_{\text{QCD}} \) is the QCD energy scale parameter. As relative phases, we use predictions from work [44]. The phase \( \delta_V^\psi \) was put equal to zero for all channels of the \( B_d^0 \rightarrow K^{*0}V \) decay.

At the same time, for heavier vector \( c\bar{c} \)-mesons, such as \( \psi(3770) \) and others, both theoretical predictions for the amplitudes of decays like \( B_d^0 \rightarrow K^{*0}\psi(3770) \) and experimental observations are absent. Therefore, the contributions from those processes were not included into the amplitudes.

### 3. Calculation Results for CP-Asymmetry in \( B_d^0 \rightarrow K^{*0}\mu^+\mu^- \) Decay

In order to compare the CP-asymmetry in the \( B_d^0 \rightarrow K^{*0}\mu^+\mu^- \) decay with LHCb experimental data [45], we calculated the integrated values \( (A_{\text{CP}}) \), having selected the interval limit as was done in work [45]. Calculations were made for the SM, taking \( A_{\text{SM}}^{\text{CP}} \), and not taking, \( A_{\text{SM,SM}}^{\text{NR,SM}} \), the contributions of vector mesons \( \rho^0, \omega, \phi, J/\psi(1S), \) and \( \psi(2S) \) to the CP-asymmetry of this decay.
We also calculated the CP-asymmetry in the $B^0_d \to K^{*0} \mu^+\mu^-$ decay for two NP scenarios beyond the SM. In scenario (a), besides the SM operators, we have tensor operators as well. It will be recalled that, according to the analysis made in work [29], the Wilson coefficients of the tensor operators satisfy the inequality $|C_T|^2 + |C_{T'}|^2 \leq 0.5$. For the values of Wilson coefficients $C_T = C_{T5} = C_{T'} = C'_{T5} = 0.5i$, which satisfy the inequality given above, the value of CP-asymmetry will be denoted as $A^{\text{NR,(a)}}_{\text{CP}}$ if the contribution from vector mesons is neglected, and as $A^{(a)}_{\text{CP}}$ if it is taken into account. In NP scenario (b) [30], $\delta C_T = 1.5 + 0.3i$, $\delta C_{T9} = -8.2i$, $\delta C_{T10} = 8 - 2i$, and $\delta C'_{T10} = -1.5 + 2i$, whereas the other Wilson coefficients are the same as in the SM. The corresponding results are denoted below as $A^{\text{NR,(b)}}_{\text{CP}}$, if the contribution from vector mesons is neglected, and as $A^{(b)}_{\text{CP}}$, if it is taken into account.

The results of calculations are quoted in Table 4. From this table, one can see that, first, in the framework of the SM, the CP-asymmetry is rather small and varies from 0.001 to 0.004 depending on the $q^2$-interval. Despite that the contribution of vector resonances can reach 50% (see columns 2 and 3), the asymmetry remains very small. Second, in NP scenario (a) beyond the SM, the CP-asymmetry increases a little to 0.002–0.008, and the resonant contribution can be substantial, especially in the interval of small invariant masses (columns 4 and 5). Third, NP scenario (b) is evidently a substantial modification of the SM. Therefore, it is not of surprise that the CP-asymmetry increases by more than an order of magnitude and can reach a value of 0.09 in the interval of large invariant masses of a leptonic pair (columns 6 and 7).

At last, if the theoretical results are compared with the experimental data of LHCb Collaboration (the last column in Table 4), it is necessary to mark that the experimental errors are still very large, so that the adequate theoretical model cannot be chosen. We may hope that the future LHC experiments will allow the CP-asymmetry in the $B^0_d \to K^{*0} \to K^-(\to \pi^+)\mu^+\mu^-$ process to be measured with a much higher accuracy.

### 4. Conclusions

The contributions from the $B^0_d \to K^{*0} \to K^-\pi^+)\mu^+\mu^-$ processes with vector mesons $V = \rho(770)$, $\omega(782)$, $\phi(1020)$, $J/\psi$, $\psi(2S)$, and others, which decay into a $\mu^+\mu^-$ pair, into the CP-asymmetry of the $B^0_d \to K^{*0} \to K^-\pi^+)\mu^+\mu^-$ decay induced by the flavor-changing neutral current have been analyzed in this work. In order to describe the $b \to s \mu^+\mu^-$ transition, the most general form for the effective weak-interaction Hamiltonian was used, which included scalar, pseudo-scalar, vector, axial vector, and tensor operators.

Expressions were obtained for the amplitudes of the $B^0_d \to K^{*0} \to K^-\pi^+)\mu^+\mu^-$ decay with regard for the non-resonant amplitudes depending on the Wilson coefficients in the effective Hamiltonian and the resonant amplitudes associated with intermediate vector mesons.

Predictions for the CP-asymmetry in the $B^0_d \to K^{*0} \to K^-\pi^+)\mu^+\mu^-$ decay were made in the framework of the SM and for two NP models beyond the SM. The contributions of resonances to the CP-asymmetry turned out rather substantial in some intervals of the lepton invariant mass, both in the SM and the NP models. It should also be noted that the CP-asymmetry is sensitive to the choice of a NP
model and can reach values of the order of $10^{-1}$, whereas it is much smaller in the SM, being of the order of $10^{-3}$. The results obtained were compared with the experimental data of LHCb Collaboration. The results of the researches carried out in this work can be useful for experiments on the LHC.

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