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## THE EFFECT OF HIGHER-ORDER MESONIC INTERACTIONS ON THE CHIRAL PHASE TRANSITION AND THE CRITICAL TEMPERATURE

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*In the present work, higher-order mesonic interactions are included in the linear sigma model at a finite temperature. The effective potential is minimized in the calculations of the sigma and pion effective masses. The field equations have been solved in the mean-field approximation, by using the extended iteration method at a finite temperature. The order of chiral phase transition, the effective sigma and pion masses, and the effective mesonic potential are investigated as functions of the temperature. We find that the chiral phase transition satisfies the Goldstone theorem below the critical temperature point, when the minimization condition is satisfied in the chiral limit. The value of the critical temperature is reduced as compared with that of the original model in agreement with lattice QCD results. The modified model is compared to with models in other works.*

*Keywords:* finite-temperature field theory, chiral symmetry, meson properties.

### 1. Introduction

The QCD phase transition is of great interest because of its possible occurrence in the early Universe. The QCD phase transition is also of interest due to its relevance to the heavy-ion collision experiments at RHIC and LHC. The lattice QCD calculation [1] indicates that the restoration of the chiral symmetry occurs at a temperature of the order of 150 MeV, which is the temperature to be tested in these experiments.

The appropriate framework for studying the chiral phase transitions is the finite-temperature field theory. Within this framework, the finite-temperature effective potential is an important and often used theoretical tool. The use of such techniques goes back to the 1970s when Kirzhnits and Linde [2, 3] first proposed that broken symmetries at zero temperature could be restored at finite temperatures. There are two main methods of studying the chiral phase transition, namely, lattice QCD methods and effective field theories. Lattice QCD can tell us many things about the phase transition, but it cannot be used to study the phase transition dynamics [4]. Hence, there is a need for models that have no such restriction. One of the effective models to describe baryon properties is

the linear sigma model, which was suggested by Gell-Mann and Levy [5] for nucleons interacting via the sigma ( $\sigma$ ) and pion ( $\pi$ ) exchanges. The linear sigma model clarifies how the structure of a nucleon respects the constraints imposed by chiral symmetry. The spontaneous and explicit chiral symmetry breakings require the existence of the pion mass. The model and its extensions provide a good description of the hadron properties at zero temperature, as explained in Refs. [6–10]. At a finite temperature, the model provides a good description of the phase transition, by using the Hartree approximation [11–14] within the Cornwall–Jackiw–Tomboulis (CJT) formalism [15].

In recent years, higher-order multi-quark interactions have played an important role in studying the chiral phase transition and the critical temperature in chiral quark models. The Nambu–Jona-Lasinio (NJL) model has been extended to six-quark interactions. The effect of these interactions on the phase transition has been investigated in Refs. [16, 17]. Kashiwa *et al.* [18] extended the NJL model to eight-quark interactions and studied the effect of these interactions on the critical temperature point and the phase transition. In the same way, Deb and Bhat-tacharyya [19] investigated eight-quark interactions in the Polyakov–Nambu–Jona-Lasinio model, when the chemical potential is included. Higher-order mesonic

interactions are investigated to provide a description of nucleon properties at a finite temperature in the linear sigma model [20]. In addition, Abu-Shady and Mansour [21] considered the effect of the quantization of fields at a finite temperature in the linear sigma model on the behavior of nucleon properties.

The aim of this work is to study the effect of higher-order mesonic interactions on the meson masses, the order of chiral phase transition, and the critical temperature in the linear sigma model at a finite temperature.

This paper is organized as follows: the linear sigma model with the effective mesonic potential at a finite temperature is presented in Section 2. Next, the numerical calculations and the discussion of results are given in Section 3. Finally, the summary and the conclusion are presented in Section 4.

## 2. Chiral Quark Sigma Model with the Effective Potential

### 2.1. Chiral quark sigma model with the effective normal mesonic potential

The interactions of quarks via  $\sigma$ - and  $\pi$ -meson can be described at a finite temperature as in Ref. [22]. The Lagrangian density is

$$L(r) = i\bar{\Psi}\partial_\mu\gamma^\mu\Psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\pi\cdot\partial^\mu\pi) + g\bar{\Psi}(\sigma + i\gamma_5\tau\cdot\pi)\Psi - U_1^{\text{eff}}(\sigma, \pi, T), \quad (1)$$

where

$$U_1^{\text{eff}}(\sigma, \pi, T) = U_1^{T(0)}(\sigma, \pi) + \frac{7\pi^2 T^4}{90} + \left(\frac{m_\sigma^2 - m_\pi^2}{24f_\pi^2}\right)(\sigma^2 + \pi^2)T^2 + \left(\frac{m_\sigma^2 - m_\pi^2}{24f_\pi^2}\right)T^2\left(\sigma^2 + \pi^2 - \frac{\nu^2}{2}\right), \quad (2)$$

with

$$U_1^{T(0)}(\sigma, \pi) = \frac{\lambda^2}{4}(\sigma^2 + \pi^2 - \nu^2)^2 - m_\pi^2 f_\pi \sigma. \quad (3)$$

In Eq. (2),  $U_1^{T(0)}(\sigma, \pi)$  is the usual meson-meson interaction potential at the zero-temperature, where  $\Psi$ ,  $\sigma$ , and  $\pi$  are the quark, sigma, and pion fields, respectively. The effective  $U_1^{\text{eff}}(\sigma, \pi, T)$  is derived in details in Ref. [23] and references therein at the chiral

limit. We extended it to the explicit chiral symmetry as in Ref. [22]. In the mean-field approximation, the meson fields are treated as time-independent classical fields. This means that we replace the powers and the products of meson fields by their corresponding powers and products of their expectation values. The meson-meson interactions in Eq. (3) lead to the hidden chiral  $SU(2) \times SU(2)$  symmetry with  $\sigma(r)$  taking on the vacuum expectation value

$$\langle\sigma\rangle = f_\pi, \quad (4)$$

where  $f_\pi = 93$  MeV is the pion decay constant. The final term in Eq. (3) is included to break the chiral symmetry explicitly. This leads to a partial conservation of the axial-vector isospin current (PCAC). The parameters  $\lambda^2$  and  $\nu^2$  can be expressed in terms of  $f_\pi$  and the masses of mesons, as follows:

$$\lambda^2 = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2}, \quad (5)$$

$$\nu^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda^2}. \quad (6)$$

### 2.2. The effective mesonic potential with higher-order mesonic interactions

In this subsection, we give the effective potential with higher-order mesonic interactions in the linear sigma model as in Ref. [20]. The temperature-dependent effective potential in the one-loop approximation takes the form

$$U_2^{\text{eff}}(\sigma, \pi, T) = U_2^{T(0)}(\sigma, \pi) + \frac{7\pi^2 T^4}{90} + \left(\frac{m_\sigma^2 - m_\pi^2}{24f_\pi^2}\right)(\sigma^2 + \pi^2)T^2 + \left(\frac{m_\sigma^2 - m_\pi^2}{24f_\pi^2}\right)T^2\left(\sigma^2 + \pi^2 - \frac{\nu^2}{2}\right). \quad (7)$$

Here, the meson potential  $U_2^{T(0)}$  at the zero temperature takes the same form as in [20]:

$$U_2^{T(0)}(\sigma, \pi) = \frac{\lambda^2}{4}A((\sigma^2 + \pi^2)^2 - B\nu^2)^2 - m_\pi^2 f_\pi \sigma. \quad (8)$$

Here, the potential satisfies the chiral symmetry as  $m_\pi \rightarrow 0$  and has eight-point interactions corresponding to the four parameters  $\lambda^2$ ,  $\nu^2$ ,  $A$ , and  $B$ . Applying the minimizing conditions  $\left\{\frac{\partial U_2^{T(0)}}{\partial\sigma}\right\}_{\sigma=f_\pi, \pi=0} = 0$ ,

$\left. \frac{\partial^2 U_2^{T(0)}}{\partial \sigma^2} \right|_{\sigma=f_\pi, \pi=0} = m_\sigma^2$ ,  $\left. \frac{\partial^2 U_2^{T(0)}}{\partial \pi^2} \right|_{\sigma=f_\pi, \pi=0} = m_\pi^2$

and PCAC, we get

$$A = \frac{m_\sigma^2 - 3m_\pi^2}{4f_\pi^4 (m_\sigma^2 - m_\pi^2)}, \quad (9)$$

and

$$B = f_\pi^2 \left( 1 - \frac{2m_\pi^2 (m_\sigma^2 + m_\pi^2)}{(m_\sigma^2 - 3m_\pi^2)^2} \right). \quad (10)$$

By introducing the dimensionless quantities  $x^2 = \frac{\pi^2}{f_\pi^2}$  and  $y^2 = \frac{\sigma^2}{f_\pi^2}$ , the effective potential can be rewritten as

$$\begin{aligned} U_2^{\text{eff}}(y, x, T) &= U_2^{T(0)}(y, x) + \frac{7\pi^2 T^4}{90} + \\ &+ \left( \frac{m_\sigma^2 - m_\pi^2}{24f_\pi^2} \right) T^2 ((yf_\pi)^2 + (xf_\pi)^2) + \\ &+ \left( \frac{m_\sigma^2 - m_\pi^2}{24f_\pi^2} \right) T^2 \left( (yf_\pi)^2 + (xf_\pi)^2 - \frac{\nu^2}{2} \right). \end{aligned} \quad (11)$$

The finite-temperature vacuum can be defined by minimizing the effective potential as

$$\left. \frac{\partial U_2^{\text{eff}}(\sigma, \pi, T)}{\partial \sigma} \right|_{\sigma=\sigma_0(T), \pi=0} = 0. \quad (12)$$

Equation (12) represents the condition necessary to satisfy the spontaneous breaking of chiral symmetry, thereby satisfying the Goldstone theorem, where  $\sigma_0(T) = f_\pi(1 + \delta(T))$ . We can rewrite Eq. (12) in dimensionless variables as follows:

$$\begin{aligned} &\left. \frac{\partial U_2^{\text{eff}}(\sigma, \pi, T)}{\partial \sigma} \right|_{\sigma=\sigma_0(T), \pi=0} = \\ &= \frac{1}{f_\pi} \left. \frac{\partial U_2^{\text{eff}}(y, x, T)}{\partial y} \right|_{y=y_0, x=0} = 0. \end{aligned} \quad (13)$$

Using Eq. (11) and Eq. (13), we obtain

$$2A \lambda^2 f_\pi^6 y_0^7 - 2AB \lambda^2 \nu^2 y_0^3 + \frac{1}{3} \lambda^2 T^2 y_0 + m_\pi^2 = 0. \quad (14)$$

Here,  $y_0 = 1 + \delta(T)$ . In the chiral limit ( $m_\pi = 0$ ), Eq. (14) takes the form

$$\frac{1}{4} y_0^6 - \frac{1}{4} y_0^2 + \frac{T^2}{6f_\pi^2} = 0. \quad (15)$$

The square of the sigma mass is obtained as the second derivative of the effective potential as in Refs. [23, 24]:

$$\begin{aligned} m_\sigma^2(T) &= \left. \frac{\partial^2 U_2^{\text{eff}}(\sigma, \pi)}{\partial \sigma^2} \right|_{\sigma=\sigma_0(T), \pi=0} = \\ &= \frac{1}{f_\pi^2} \left. \frac{\partial^2 U_2^{\text{eff}}(y, x)}{\partial y^2} \right|_{y=y_0, x=0}, \\ m_\sigma^2(T) &= 14A \lambda^2 f_\pi^6 y_0^6 - 6AB \lambda^2 \nu^2 f_\pi^2 y_0^2 + \frac{1}{3} \lambda^2 T^2. \end{aligned} \quad (16)$$

In the chiral limit,

$$m_\sigma^2(T) = m_\sigma^2 \left( \frac{7}{4} y_0^6 - \frac{3}{4} y_0^2 + \frac{1}{6} \frac{T^2}{f_\pi^2} \right). \quad (17)$$

Similarly,

$$\begin{aligned} m_\pi^2(T) &= \left. \frac{\partial^2 U_2^{\text{eff}}(\sigma, \pi, T)}{\partial \pi^2} \right|_{\sigma=\sigma_0(T), \pi=0} = \\ &= \frac{1}{f_\pi^2} \left. \frac{\partial^2 U_2^{\text{eff}}(y, x, T)}{\partial x^2} \right|_{y=y_0, x=0}, \end{aligned} \quad (18)$$

$$m_\pi^2(T) = 2A \lambda^2 f_\pi^6 y_0^6 - 2AB \lambda^2 \nu^2 f_\pi^2 y_0^2 + \frac{1}{3} \lambda^2 T^2. \quad (19)$$

In the chiral limit,

$$m_\pi^2(T) = m_\sigma^2 \left( \frac{1}{4} y_0^6 - \frac{1}{4} y_0^2 + \frac{T^2}{6f_\pi^2} \right), \quad (20)$$

Substituting Eq. (15) into Eqs.(17) and (20), we obtain the following sigma and pion masses in the chiral limit:

$$m_\sigma^2(T) = m_\sigma^2 \left( \frac{3}{2} (1 + \delta(T))^6 - \frac{1}{2} (1 + \delta(T))^2 \right), \quad (21)$$

and

$$m_\pi^2(T) = 0. \quad (22)$$

In view of Eqs. (21) and (22), we note that the sigma mass is finite, and the pion mass is zero. Therefore, the Goldstone theorem is satisfied at low temperatures. This method is used in Ref. [23]. By substituting  $y_0 = 1 + \delta(T)$  into Eq. (14), we obtain

$$\delta(T) = \frac{-2A \lambda^2 f_\pi^6 + 2AB \lambda^2 \nu^2 f_\pi^2 - \frac{1}{3} \lambda^2 T^2 + m_\pi^2}{14A \lambda^2 f_\pi^6 - 6AB \lambda^2 \nu^2 f_\pi^2 + \frac{1}{3} \lambda^2 T^2}. \quad (23)$$

### 3. Discussion of the Results

In this work, we concentrate only on the thermal effects and ignore quantum corrections. It is important to examine the linear sigma model in the presence of higher-order mesonic interactions. The higher-order mesonic potential was suggested in Ref. [20] as a

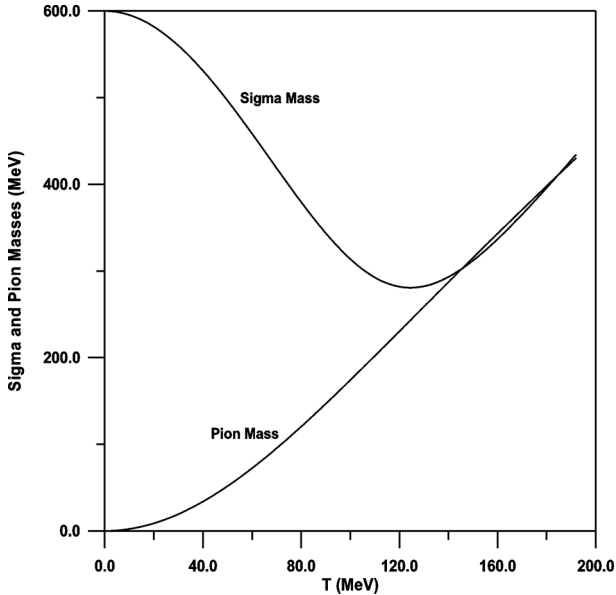


Fig. 1. Sigma  $m_\sigma(T)$  and pion  $m_\pi(T)$  masses as functions of the temperature  $T$  for two models in the chiral limit. The Goldstone theorem is not satisfied

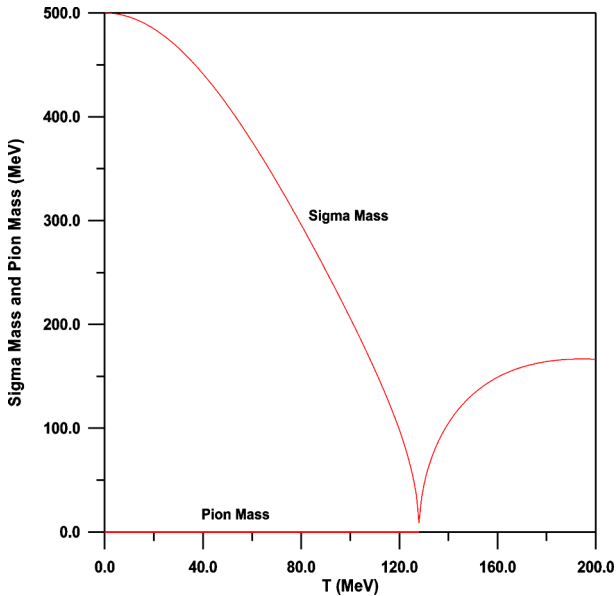


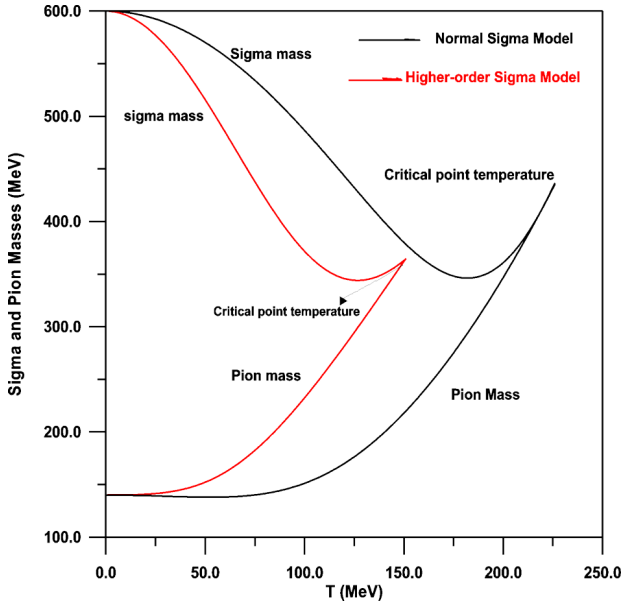
Fig. 2. Sigma  $m_\sigma(T)$  and pion  $m_\pi(T)$  masses are plotted as functions of the temperature  $T$  for the higher-order mesonic model in the chiral limit ( $m_\pi = 0$ ). The Goldstone theorem is satisfied

good means of describing the nucleon properties at non-zero temperatures. So we need to examine the effect of higher-order mesonic interactions on the me-

son properties, phase transition, and critical temperature at non-zero temperatures. We obtained the gap equations (Eqs. (16) and (19)) by minimizing the effective mesonic potential  $U_2^{\text{eff}}(\sigma, \pi, \mathbf{T})$ . This method is used in the previous works such as Refs. [23, 24]. For numerical computation, we used the model parameters at the zero temperature as the initial conditions; namely, we took  $m_\pi = 140$  MeV,  $m_\sigma = 500 \rightarrow 700$  MeV, and  $f_\pi = 93$  MeV.

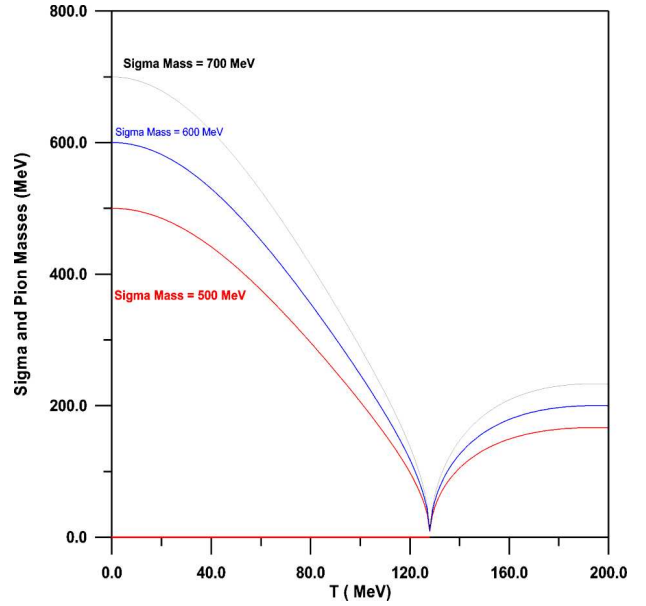
We investigate the effect of the higher-order mesonic contributions on the sigma and pion masses, the order of phase transition, and the effective potential. In addition, the effect of the sigma mass as a free parameter is investigated. In Figs. 1 and 2, we show the calculated sigma and pion masses in the chiral limit ( $m_\pi = 0$ ). In Fig. 1, the sigma and pion masses are plotted as functions of the temperature when the minimization condition in Eq. (15) has not been applied to the sigma and pion masses (Eqs. (16) and (19)). We note that the sigma mass decreases with increasing temperature, and the pion mass increases with the temperature. The two masses cross at the critical temperature, where the restoration of chiral symmetry is appeared. We note that the Goldstone theorem is not satisfied in this case. This behavior is in agreement with the findings of other authors [11, 14], who found that the pion is massive below the critical temperature, when the Hartree approximation is applied. In Fig. 2, the effective sigma and pion masses are plotted as functions of the temperature in the chiral limit. The sigma and pion masses satisfy the minimization condition in Eq. (15), which leads to a zero pion mass below the critical temperature and to a sigma mass, which decreases with increasing temperature. Therefore, the Goldstone theorem is satisfied. We conclude that the minimization condition of the spontaneous breaking symmetry is necessary to satisfy the Goldstone theorem at lower temperatures. This conclusion agrees with the findings of Nemoto *et al.* [24], who showed that the Goldstone theorem is satisfied when the condition in Eq. (15) is applied, where the pion mass is defined as the curvature of an effective potential. Hong *et al.* [23] showed also that the Goldstone theorem is satisfied when the spontaneous breaking of a symmetry of the system is conserved.

In Fig. 3, the sigma and pion masses are plotted as functions of the temperature in the presence

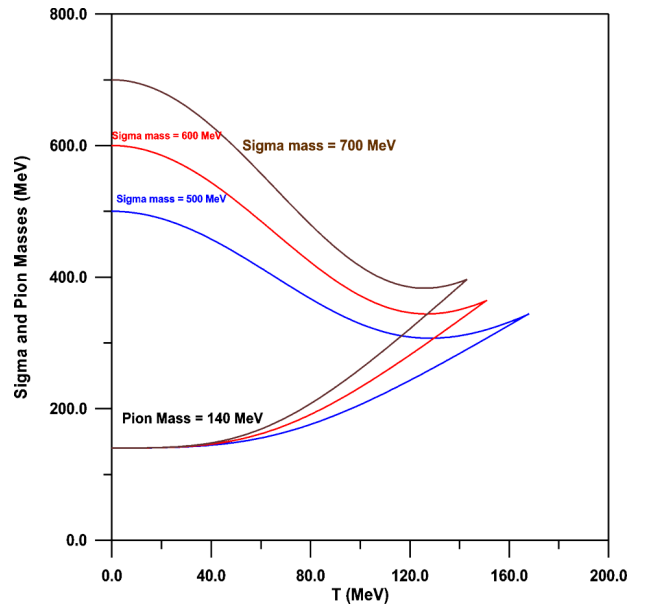


**Fig. 3.** Sigma  $m_\sigma(T)$  and pion  $m_\pi(T)$  masses are plotted as functions of the temperature  $T$  in the above models in the presence of the explicit symmetry-breaking term. At temperature  $T = 0$ , the pions appear with the observed masses ( $m_\pi = 140$  MeV)

of the explicit symmetry-breaking term  $m_\pi \neq 0$ . At the zero temperature, the sigma and pion masses appear with their experimental masses  $m_\sigma = 600$  MeV and  $m_\pi = 140$  MeV. We note that the inclusion of higher-order mesonic interactions leads to a reduction in the critical temperature from 226 MeV to 150 MeV. In Figs. 4 and 5, we investigate the effect of the sigma mass on the critical point temperature. In Fig. 4, we note that the critical point temperature is independent of the sigma mass in the chiral limit. The opposite is observed in Fig. 5, where we note that the increase in the sigma mass reduces the value of critical point temperature. Increasing the sigma mass from  $m_\sigma = 500$  to 700 MeV corresponds to decreasing the critical temperature from  $T_c = 161$  MeV to  $T_c = 127$  MeV. The present critical value  $T_c = 151$  at  $m_\sigma = 600$  MeV. In comparison, the Wuppertal–Budapest group [26] found that the transition temperature for the chiral restoration of  $u, d$  quarks equals  $T_c = 151$  MeV. Therefore, the present result of  $T_c$  is a good agreement with lattice QCD results. In addition, we found that the Nambu–Jona–Lasinio [18] model, when extended to include higher-orders of the sigma field



**Fig. 4.** Sigma  $m_\sigma(T)$  and pion  $m_\pi(T)$  masses are plotted as functions of the temperature in the chiral limit using the higher-order sigma model for different values of sigma masses. At the temperature  $T = 0$ , the pions appear massless ( $m_\pi = 0$ ). The Goldstone theorem is satisfied



**Fig. 5.** Sigma  $m_\sigma(T)$  and pion  $m_\pi(T)$  masses are plotted as functions of the temperature in the presence of the explicit symmetry-breaking term. At the temperature  $T = 0$ , the pions appear with the observed masses ( $m_\pi = 140$  MeV)

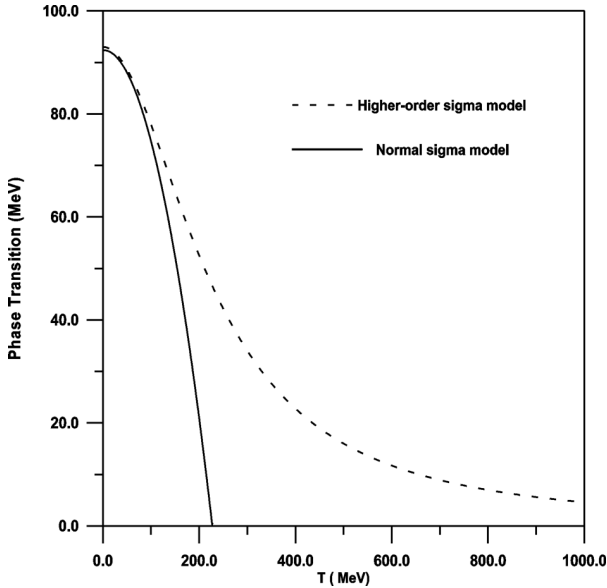


Fig. 6. The phase transition  $\Phi = (\sigma^2 + \pi^2)^{1/2}$  is plotted as a function of the temperature in the normal and higher-order sigma models in the chiral limit

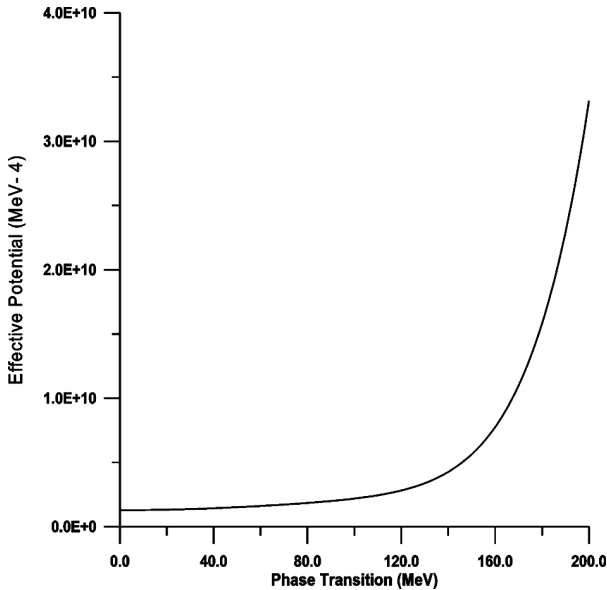


Fig. 7. The effective mesonic potential  $U_2^{\text{eff}}(\sigma, \pi, T)$  is plotted as a function of the phase transition at the fixed value of temperature  $T = 200$  MeV

( $\sigma^4$ ), leads to a reduction in the critical point temperature to  $T_c = 180$  MeV. Therefore, the present result is in agreement with the Nambu–Jona–Lasinio model.

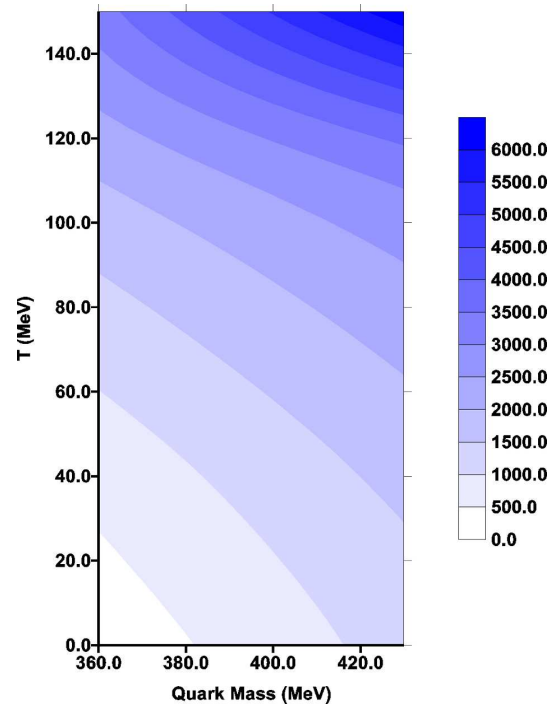
In Fig. 6, we examine the effect of a finite temperature on the chiral phase transition  $\Phi = (\sigma^2 + \pi^2)^{1/2}$ . It is clear that the quantity  $\Phi$  plays the role of the order parameter of the phase transition. In a previous work [22], we investigated the order of the phase transition and found that it is equal to 2. We note that the inclusion of the higher-order mesonic interactions in the normal sigma model converts the phase transition from a second-order phase transition to a crossover. The interpretation of the transition becoming a crossover is given in Ref. [11], which states that the phase transition vanishes at very high values of temperature. Kashiwa *et al.* [18] showed that the inclusion of a vector interaction  $w^2$  makes the phase transition smoother, but the higher-order interaction  $\sigma^4$  makes the phase transition sharper in the NJL model. In addition, they found that the  $w^2$  interaction tends to change the phase transition from a first-order one to a crossover. Moreover, the phase transition is converted from a first-order transition to a crossover by including the term  $\sigma^4 + w^2$  in the NJL model. The Wuppertal–Budapest group [26] found that the phase transition is a crossover. Therefore, the result in Fig. 6 gives similar results that the phase transition changes from a second-order phase transition to a crossover. In Fig. 7, the higher-order mesonic effective potential is plotted as a function of the phase transition. We note that the effective mesonic potential increases with the phase transition. The potential has no minima at  $\Phi \neq 0$ , which can be interpreted as a crossover [27]. Therefore, we can interpret the phase transition as a crossover in agreement with the result obtained in Fig. 6.

In Refs. [11, 14, 24], the authors ignored the effect of the fermion sector in the linear sigma model at a finite temperature. In this work, we consider this sector and its effect on the effective potential. So the effective potential is plotted in the normal potential and the higher-order potential which are defined by Eq. (2) and Eq. (7), respectively. By comparison of the two contours in Figs. 8 and 9, we note that the values of effective potential are reduced in comparison with the normal potential by about 53% at the higher values of temperature. This indicates that the higher-order mesonic contributions and the quark mass play an important role at higher values of temperature. This means that increasing the mesonic contributions in the normal sigma model

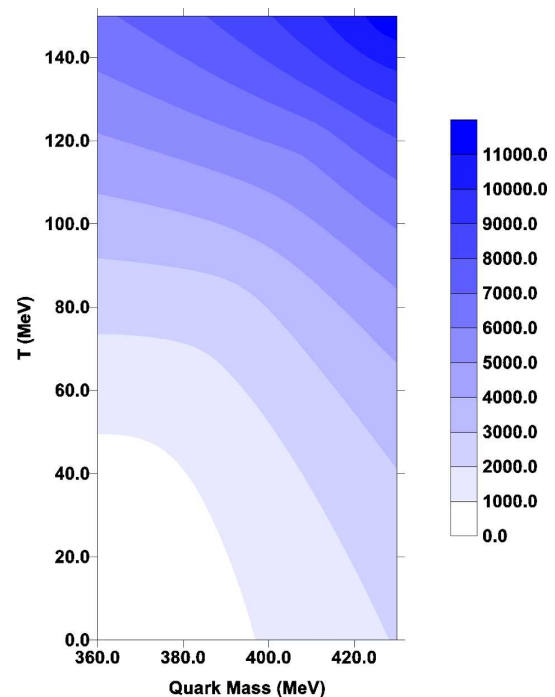
leads to the increase of the coupling between the meson fields. In addition, the increase of the quark mass leads to an increase of the coupling constant between quark and meson fields. Therefore, this leads to the strong decrease in the higher-order potential which has an effect on the behavior of the nucleon mass at finite temperatures and then the deconfinement phase transition [20].

#### 4. Comparison with Other Works

It is interesting to compare the present results with the results of other groups. Here, we compare our results with the results of the Nambu–Jona–Lasinio model with its extension and with QCD lattice calculations [26]. The chiral phase transition was studied with the use of the NJL model with a Polyakov loop in Refs. [28, 29]. Hansen *et al.* [28] studied the properties of the scalar and pseudoscalar mesons at finite temperatures and the chemical potential, by using the PNJL model, and the role of pions as Goldstone bosons. This means that the Goldstone theorem is satisfied in their calculations. In the present work, we find that the Goldstone theorem is satisfied in the chiral limit. So the present results are in good agreement with Ref. [28]. Deb *et al.* [29] investigated the phase diagram and the location of the critical end point (CEP), where the CEP is the point that separates the crossover from the first-order phase transition within the NJL and PNJL models. They found that the CEP point is shifted to a lower chemical potential and a higher temperature in the presence of eight-quark interactions. In the present work, we do not include the chemical potential, hence the CEP point is not calculated in our model. Osipov *et al.* [17] investigated eight-quark interactions in the framework of the NJL model and the Hooft Lagrangian. They found that the critical point temperature is reduced to a lower value when eight-quark interactions are included. Therefore, the present result is in agreement with their result. We find that the critical point temperature is shifted to a lower value when eight-mesonic interactions are included. Also, Hiller *et al.* [30] found that the critical temperature is shifted to a lower value when six-quark interactions are considered in the chiral phase transition. Akoi *et al.* [26] performed lattice calculations and found that the phase transition is a crossover in the absence of a chemical potential. We find that the phase transition



*Fig. 8.* Dependence of the higher-order effective potential on the quark mass and a finite temperature



*Fig. 9.* Dependence of the normal effective potential on the quark mass and the finite temperature

is a crossover when higher-order mesonic interactions are included. In Ref. [23], the authors investigated the Goldstone theorem below the critical temperature in the linear sigma model in the chiral limit. They found that the Goldstone theorem is satisfied, which is in agreement with the present results.

## 5. Summary and Conclusion

The chiral phase transition, the sigma and pion masses, and the effective mesonic potential have been examined in the framework of the extended linear sigma model, in which higher-order mesonic interactions are taken into account. We calculated the sigma and pion masses by minimizing the potential. This technique has been used in previous studies such as Ref. [22]. We find that the critical temperature is reduced to lower values of the temperature when the higher-order mesonic interactions are included in the linear sigma model, leading a good agreement with lattice QCD [26]. We find that the Goldstone theorem is satisfied in the chiral limit at low temperatures. A comparison with the other calculations and lattice QCD calculations is presented. The effective sigma and pion masses are investigated in the chiral limit and in the presence of an explicit symmetry-breaking term. Moreover, we find that the second order phase transition becomes a crossover in the chiral limit, which agrees with lattice QCD [26]. Hence, we conclude that the higher-order mesonic interactions play an important role in a change of the meson properties at finite temperatures.

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ВПЛИВ МЕЗОННИХ ВЗАЄМОДІЙ  
ВИСОКИХ ПОРЯДКІВ НА КІРАЛЬНИЙ ФАЗОВИЙ  
ПЕРЕХІД І КРИТИЧНУ ТЕМПЕРАТУРУ

Резюме

У даній роботі мезонні взаємодії високих порядків враховано в лінійній сигма-моделі при кінцевій температурі. При обчисленні мас сигма-частинки і піона ефективний потенціал мінімізується. Рівняння поля вирішені в наближенні середнього поля розширеним методом ітерацій при кінцевій температурі. Порядок кірального фазового переходу, ефективні маси сигма-частинки і піона і ефективний мезонний потенціал досліджені як функції температури. Знайдено, що кіральний фазовий перехід задовольняє теорему Голдстоуна нижче критичної точки, коли умову мінімізації виконано в кіральній границі. Величина критичної температури менше порівняно з вихідною моделлю у згоді з результатами ґраткової КХД. Проведено порівняння модифікованої моделі з моделями інших робіт.