A. LENGYEL, Z. TARICS
Institute of Electron Physics
*(21, Universytets'ka Str. Uzhgorod, 88017, Ukraine; e-mail: alexander-lengyel@rambler.ru, tarics1@rambler.ru)*

PACS 12.40.Nn, 13.85.Lg

# TRUNCATED QCD-POMERON AT LHC ENERGIES

*The eikonalized elastic proton-proton and proton-antiproton scattering amplitude $F(s,t)$ as a function of the available increasing energy is constructed with the use of the suggestion of a finite sum of ladder diagrams calculated in QCD with a certain number of s-channel gluon rungs and, correspondingly, the powers of logarithms in the total cross section. Explicit expressions for the total cross section involving three and four rungs (four and five prongs) with $\ln^3(s)$ and $\ln^4(s)$ as the highest terms, respectively) are fitted to the all available proton-proton and proton-antiproton total cross section data. Predictions for the pp total cross section at the LHC energy are given.*

*Keywords:* proton-proton scattering, proton-antiproton scattering, QCD-Pomeron, LHC energy.

## 1. Introduction

The renewed interest in high-energy soft interactions is largely triggered by the recent LHC measurements by the TOTEM [1] measurements of *pp* scattering. The accessible energies offer new possibilities to check the basic theoretical concepts, such as unitarity and the properties of (multi) Pomeron exchanges.

In the present paper, we develop further and calculate the effects coming from the Pomeron being a finite sum of reggeized gluon ladders, different from the "QCD or hard Pomeron", which is an infinite sum of such ladders [2–4], resulting in the so-called supercritical behavior of the total cross section. In that approach, the main contribution to the inelastic amplitude and to the absorptive part of the elastic amplitude in the forward direction arises from the multi-Regge kinematics in the limit $s \to \infty$ in the leading logarithmic approximation. In the next-to-leading logarithmic approximation, corrections require also the contribution from the quasi-multi-Regge kinematics [5]. Hence, the subenergies between neighboring $s$-channel gluons must be large enough to remain in the Regge domain. At finite total energies, this implies that the amplitude is represented by a finite sum of $N$ terms [6], where $N$ increases like $\ln s$, rather than by the solution of the BFKL integral equation [2–4], though the possibility of the presence of a hard Pomeron in hadron-hadron scattering should not be rejected [7]. The interest in the first few terms of the series is related to the fact that the energies reached by the present accelerators are not high enough to accommodate for a large number of $s$-channel gluons that eventually hadronize and give rise to clusters of secondary particles [8]. Consequently, one can expand the "supercritical" Pomeron $\sim s^{\alpha_P(0)}$ in powers of $\ln(s)$.

In Ref. [6], a model for the Pomeron at $t = 0$ based on the idea of a finite sum of ladder diagrams in QCD was suggested. The opening channels (in $s$) were considered as threshold effects, the relevant prongs being separated in rapidity by $\ln s_0$, where $s_0$ is a parameter related to the average subenergy in the ladder. Within the "finite gluon ladder approach" to the Pomeron (see [6] and references therein), several options are possible. In Ref. [6], a system of interconnected equations was solved with several free parameters, including the value of $s_0^i$, that determine the opening of each threshold (prong). In [9], a unitarization procedure was also included: the QCD-inspired amplitude was treated as a Born term subject to a subsequent unitarization procedure.

In the present paper, we use the above-mentioned "finite gluon ladder approach" to the unitarized QCD-inspired amplitude of [9]. Explicit expressions for the total cross section involving three and four rungs are fitted to the $\bar{p}p$ and $pp$ total cross section data from $\sqrt{s} = 5$ GeV up to highest energy data [1], [10] for several possible $s_0$. For the sake of completeness, we predict the $pp$ total cross section resulting from this model at the highest LHC energy.

## 2. Total Cross Sections from a Finite Sum of Gluon Ladders

Following [6], we write the Pomeron contribution to the total cross section in the form

$$\sigma_t^{(P)}(s) = \sum_{i=0}^{N} f_i \, \theta(\bar{s} - \bar{s}_0^i) \, \theta(\bar{s}_0^{i+1} - \bar{s}), \tag{1}$$

where

$$f_i = \sum_{j=0}^{i} a_{ij} L^j, \tag{2}$$

$\bar{s}_0$ is the prong threshold, $\theta(x)$ is the step function and $L \equiv \ln(\bar{s})$. Here, by $\bar{s}$ and $\bar{s}_0$ respectively, $s/(1\,\mathrm{GeV}^2)$ and $s_0/(1\,\mathrm{GeV}^2)$ are implied. The main assumption in Eq. (1) is that the widths of the rapidity gaps $\ln(\bar{s}_0)$ are the same along the ladder. The functions $f_i$ are polynomials in $L$ of degree $i$, corresponding to finite gluon ladder diagrams in QCD, where each power of the logarithm collects all the relevant diagrams. When $s$ increases and reaches a new threshold, a new prong opens adding a new power in $L$. In the energy region between two neighboring thresholds, the corresponding $f_i$, given in Eq. (1), is supposed to represent adequately the total cross section.

In Eq. (1), the sum over $N$ is a finite one, since $N$ is proportional to $\ln(s)$, where $s$ is the present squared c.m. energy. Hence, this model is quite different from the "canonical" approach [2], where, in the limit $s \to \infty$, the infinite sum of the leading logarithmic contributions gives rise to an integral equation for the amplitude.

By imposing the requirement of continuity (of the cross section and of its first derivative), one constrains the relevant parameters.

The same procedure can be repeated for any number of gaps.

Notice that the values of parameters depend on the energy range of the fitting procedure. For example, the values of parameters in $f_0$ fitted in "their" range, i.e. for $s \le s_0$, will get modified in $f_1$ with the higher energy data and, correspondingly, higher order diagrams included.

As the first attempt, only three rapidity gaps that correspond to two gluon rungs in the ladder were considered [6]. Fits to the $p\bar{p}$ and $pp$ data were performed up to the highest energy Tevatron data 1.8 TeV. The value of the rapidity gap turned out to be $\sqrt{s_0} \approx \approx 12$ GeV, i.e. the value, for which the energy range considered is covered with equal rapidity gaps uniformly.

## 3. Explicit Iterations of BFKL

From the theoretical point of view, the phenomenological model of Section 2 corresponds to the explicit evaluation of gluonic ladders with an increasing number of $s$-channel gluons in QCD. This correspondence is far from literal, since each term of the BFKL series takes only the dominant logarithm as $s \to \infty$ into account. In the following, we give speciific expressions for the forward high energy scattering amplitudes for hadrons in the form of an expansion in powers of large logarithms in the leading logarithmic approximation.

We start from the known results obtained in [3], where an explicit expression for the total cross section for hadron-hadron scattering has been obtained. In the high energy limit, it is convenient to introduce the Mellin transform of the amplitude

$$\mathcal{A}(\omega, t) = \int_0^\infty d\tilde{s}\tilde{s}^{-\omega-1} \frac{\mathrm{Im}_s \mathcal{A}(s,t)}{s}, \quad \tilde{s} = \frac{s}{m^2}. \tag{3}$$

To obtain the total cross section, the ansatz of Ref. [11] for the impact factor of a hadron in terms of its form factor,

$$F_0(k) = ak^2 e^{-bk^2}, \tag{4}$$

was used [9]. As a result,

$$\sigma_t(s) = \frac{\pi a^2}{2c} \left\{ 1 + 2(\ln 2)\rho + \left[\frac{\pi^2}{12} + 2(\ln 2)^2\right]\rho^2 + \right.$$
$$\left. + \frac{1}{3}\left[\frac{\pi^2}{2}(\ln 2) + 4(\ln 2)^3 - \frac{3}{4}\zeta(3)\right]\rho^3 + \ldots \right\}, \tag{5}$$

where $\rho$ is defined as

$$\rho = \frac{3\alpha_s}{\pi} \ln \tilde{s}, \tag{6}$$

and the Riemann's zeta function $\zeta(3) \approx 1.202$. This approach was used [9] to predict the total cross section at the LHC energy. The strong coupling $\alpha_s$ is assumed to be frozen at a suitable scale set, for example, by the external particles.

Now, we consider another Pomeron contribution corresponding to four rungs:

$$\sigma_t = (\pi a^2/(2b)[1+c_1\rho+c_2\rho^2+(c_3/3)\rho^3+(c_4/12)\rho^4+...], \tag{7}$$

where

$$c_1 = 2(\ln 2), \quad c_2 = \frac{\pi^2}{12} + 2(\ln 2)^2, \\ c_3 = \frac{\pi^2}{2}(\ln 2) + 4(\ln 2)^3 - \frac{3}{4}\zeta(3). \tag{8}$$

$$c_4 = \int_0^1 \frac{dz}{1-z} \int_0^1 \frac{dt}{1-t} \Bigg[ R(zt) + \\ + \frac{1}{t} R\left(\frac{z}{t}\right) + \frac{1}{z} R\left(\frac{t}{z}\right) + \frac{1}{tz} R\left(\frac{1}{tz}\right) - \\ -2R(z) - \frac{2}{z} R\left(\frac{1}{z}\right) - 2R(t) - \frac{2}{t} R\left(\frac{1}{t}\right) - 2R(1)\Bigg], \tag{9}$$

where

$$R(a) = \int_0^1 \frac{dx}{1-x} \left[ \frac{1}{1+ax} \ln \frac{(1+ax)^2}{ax} + \right. \\ \left. + \frac{1}{a+x} \ln \frac{(a+x)^2}{ax} - \frac{2}{1+a} \ln \frac{(1+a)^2}{a} \right], \tag{10}$$

$$R(1) = 2(\ln 2)^2 + \frac{\pi^2}{12}. \tag{11}$$

There, $c_4 = 10.145$.

## 4. Fit with the Unitarized QCD-inspired Pomeron

Apart the calculations of the total cross section involving fits of a variety of Pomeron models, it is interesting to estimate the value of $s_0$, a basic parameter in the finite series of QCD diagrams. Each set is "active" in its rapidity gap, i.e. the parameters $a_{ij}$ in (2) should be fitted in each energy interval separately, and the relevant solutions should be matched by imposing the continuity of the total cross section and its first derivative. Here, we present the result of a fit to the existing experimental data (including the new LHC data [1]) and predict the values of total cross section at the next LHC energy with the unitarized contribution of the explicit BFKL iteration. As will be seen below, the values of $s_0$ are quite constrained by the fits.

First of all, we must put $\sqrt{s_0} > 5$ GeV in the framework of our approach. Second, each rapidity gap must contains at least one experimental point. As a convenient minimal value of rapidity gap, we chose $\sqrt{s_0} = 6.8$ GeV, and, in estimating $\sigma_{\text{tot}}(pp)$, we are restricted by the maximal power $ln^4(s)$. From the experimental data $\sqrt{s_0} \le 24$ GeV, one expects a high-quality description of the total cross section with the Pomeron contribution of (1), $i = 0$, including relevant secondary Regge-pole terms.

To cover the whole fitted range, we choose the interval 10.9 GeV $\le \sqrt{s_0} \le$ 24 GeV, where four gaps are sufficient to estimate the total cross section at the maximal LHC energy. We have

$$f_3(s) = a_{30} + a_{31}L + a_{32}L^2 + a_{33}L^3 \tag{12}$$

for

$$\bar{s}_0^3 \le \bar{s} \le \bar{s}_0^4. \tag{13}$$

For the remaining part of the studied region 6.8 GeV $\le \sqrt{s_0} \le$ 10.9 GeV, it is necessary to take five gaps into account to estimate the total cross section at the LHC energy. In this case,

$$f_4(s) = a_{40} + a_{41}L + a_{42}L^2 + a_{43}L^3 + a_{44}L^4. \tag{14}$$

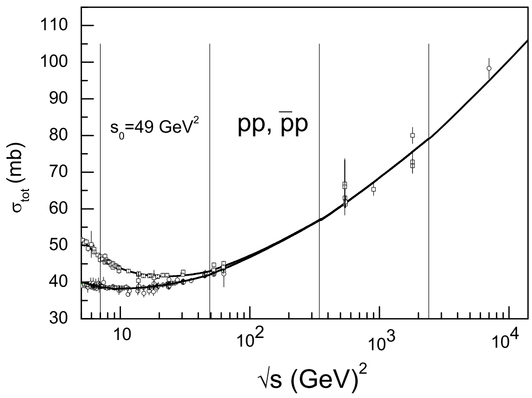
for

$$\bar{s}_0^4 \le \bar{s} \le \bar{s}_0^5. \tag{15}$$

Below, we perform such calculations in the framework of the eikonal formalism and compare the results with the experimental data.

We start from Eq. (7) for the $pp$ and $\bar{p}p$ total cross section. Supplying that expressing with an exponential $t$-dependence, we get the elastic scattering amplitude

$$F_{\text{Born}}(s,t) = A(-i\tilde{s})^{1+\alpha' t}[a_0 + a_1\gamma \ln(-i\tilde{s}) + \\ + a_2\gamma^2 \ln^2(-i\tilde{s}) + a_3\gamma^3 \ln^3(-i\tilde{s})]e^{Bt}, \tag{16}$$




Total $\bar{p}p$ (upper curve) and $pp$ cross sections from the unitarized (eikonalized) version of the model. The vertical lines corresponds to the boundaries of rapidity gaps

where $\alpha'$ and $B$ are fitting parameters,

$$\gamma = \frac{2\alpha_s}{\pi}; \tag{17}$$

and

$$a_0 = 1 + \frac{\pi^2}{4}\left(\frac{\pi^2}{12} + 2\ln^2 2\right)\gamma^2, \tag{18}$$

$$a_1 = \frac{\pi^2}{4}\left[\frac{\pi^2}{2}\ln 2 + 4\ln^3 2 - \frac{3}{4}\zeta(3)\right]\gamma^2 + 2\ln 2, \tag{19}$$

$$a_2 = \frac{\pi^2}{12} + 2\ln^2 2, \tag{20}$$

**Values of the fitted parameters and calculated total cross sections at the LHC energies. *A* in (16), (31) for different *i* rungs**

| Parameter | 3 rungs | | 4 rungs | |
|---|---|---|---|---|
| | value | error | value | error |
| $A_0$ | 3.61 | 1.11 | 4.83 | 0.25 |
| $A_1$ | 1.77 | 0.92 | 3.44 | 0.25 |
| $A_2$ | 1.06 | 0.64 | 2.63 | 0.26 |
| $A_3$ | 0.59 | 0.46 | 2.09 | 0.27 |
| $A_4$ | – | – | 1.66 | 0.27 |
| $\alpha_s$ | 0.145 | 0.049 | 0.084 | 0.007 |
| $g_f$ | –6.17 | 0.53 | –7.67 | 0.32 |
| $\alpha_f(0)$ | 0.681 | 0.028 | 0.633 | 0.011 |
| $g_\omega$ | 3.61 | 0.17 | 3.72 | 0.18 |
| $\alpha_\omega(0)$ | 0.463 | 0.015 | 0.452 | 0.015 |
| $s_0$, GeV$^2$ | 121.0 | – | 49.0 | – |
| $\sigma(pp)$, mb | | | | |
| 7 TeV | 92.2 | 1.5 | 94.8 | 1.5 |
| 14 TeV | 102.6 | 2.1 | 106.6 | 1.9 |

$$a_3 = \frac{1}{3}\left[\frac{\pi^2}{2}\ln 2 + 4\ln^3 2 - \frac{3}{4}\zeta(3)\right], \tag{21}$$

$$A = -\frac{a^2}{8c}. \tag{22}$$

In the eikonalization procedure, we follow Ref. [12], according to which the Pomeron amplitude

$$F_P(s,t) = is\int_0^\infty bdbJ_0\left(b\sqrt{-t}\right)\left(1 - e^{i\chi(b,s)}\right), \tag{23}$$

where $J_0$ is the zero-order Bessel function and the eikonal $\chi$ is

$$\chi(s,b) = \frac{1}{s}\int_0^\infty \sqrt{-t}d\sqrt{-t}I_0(b\sqrt{-t})F_{\text{Born}}(s,t). \tag{24}$$

Inserting the expression for the Pomeron into Eq. (24) and expanding the exponential in (23), we find that the eikonalized Pomeron amplitude

$$F_P = 2is\xi\sum_{k=1}^{\infty}\frac{1}{kk!}\left(-\frac{\xi}{\mu}\right)^{k-1}e^{\mu t/k}. \tag{25}$$

The corresponding forward Pomeron amplitude is

$$F_P(s,t=0) = 2is\mu[C + \ln(\xi/\mu) + E_1(\xi/\mu)], \tag{26}$$

where

$$\mu = B + \alpha'\ln(-i\tilde{s}), \tag{27}$$

$$\xi = \frac{A}{2m^2}(\xi_0 + \xi_1 + \xi_2 + \xi_3), \tag{28}$$

and

$$\xi_i = a_i\gamma^i\ln^i(-i\tilde{s}), \tag{29}$$

$C = 0.577216$ is the Euler constant, and $E_1$ is the asymptotic form of the first-order exponential integral:

$$E_1 = \frac{\exp(-\xi/\mu)}{\xi/\mu}\left[1 - \frac{1}{\xi/\mu} + \frac{2}{(\xi/\mu)^2} - \frac{6}{(\xi/\mu)^3} + ...\right]. \tag{30}$$

For the case of four rungs, the Born amplitude is

$$F_{\text{Born}}(s,t) = A(-i\tilde{s})^{1+\alpha' t}\times$$

$$\times[a_0 + a_1\gamma\ln(-i\tilde{s}) + a_2\gamma^2\ln^2(-i\tilde{s})+$$

$$+a_3\gamma^3\ln^3(-i\tilde{s}) + a_4\gamma^4\ln^4(-i\tilde{s})]e^{Bt}, \quad (31)$$

where

$$a_0 = 1 + \left(\frac{\pi\gamma}{2}\right)^2\left(\frac{\pi^2}{12} + 2\ln^2 2\right) + 5c_4\left(\frac{\pi\gamma}{2}\right)^4, \quad (32)$$

$$a_1 = \frac{\pi^2}{4}\left[\frac{\pi^2}{2}\ln 2 + 4\ln^3 2 - \frac{3}{4}\zeta(3)\right]\gamma^2 + 2\ln 2. \quad (33)$$

$$a_2 = \frac{3\pi^2}{2}c_4\gamma^2 + \frac{\pi^2}{12} + 2\ln^2 2, \quad (34)$$

$$a_3 = \frac{1}{3}\left[\frac{\pi^2}{2}\ln 2 + 4\ln^3 2 - \frac{3}{4}\zeta(3)\right], \quad (35)$$

$$a_4 = \frac{1}{12}c_4\gamma^4, \quad (36)$$

where $A$, $\mu$, $\gamma$ and $\xi_i$ are the same,

$$\xi = \frac{A}{2m^2}(\xi_0 + \xi_1 + \xi_2 + \xi_3 + \xi_4). \quad (37)$$

The obtained eikonalized Pomeron terms are appended by a contributions from secondary Reggeons, $\rho$ and $\omega$:

$$F_R^{\pm}(s, t = 0) = g_f\tilde{s}^{\alpha_f(0)} \pm ig_\omega\tilde{s}^{\alpha_\omega(0)}, \quad (38)$$

where the $+(-)$ sign corresponds to $\bar{p}p(pp)$ scattering, the resulting forward amplitude being

$$F_{pp}^{\bar{p}p}(s, t = 0) = F_P(s, t = 0) + F_R^{\pm}(s, t = 0). \quad (39)$$

For the total cross section, the norm

$$\sigma = \frac{4\pi}{s}\mathrm{Im}F_{pp}^{\bar{p}p}(s, t = 0) \quad (40)$$

was used.

Consider the fitting procedure in the approach with different numbers of rungs (power of log's) in more details. In the case of 3 rungs, for values $\sqrt{s_0}$ within the interval 10.9–24 GeV, the whole range of the data 5 GeV–7 TeV is covered. We have chosen the parameters $B = 0.116$ GeV$^{-2}$ and $\alpha' = 0.134$ GeV$^{-2}$ [9].

The parameters of secondary reggeons are the same for the whole fitted experimental data region. Notice that, in this approach for the Pomeron contribution, there is only one free parameter in every separate interval. Therefore, it is sufficient for its determination, in principle, to have one experimental point. In our case, this condition is fulfilled.

To estimate $s_0$ in the remaining part of the whole investigated region 6.8 GeV $\leq \sqrt{s_0} \leq$ 10.9 GeV, we added the new LHC [1] data. In our calculations, $s_0$ is not a free parameter. Instead, we have performed a series of fits and have chosen a value $s_0$ for the cases where $\chi^2/\mathrm{dof} \approx 1$. We find that this condition is valid in the whole interval 6.8 GeV $\leq$ $\leq \sqrt{s_0} \leq$ 10.9 GeV, where $\alpha_s$ is rather small $\approx$0.08. The prediction of the total cross section at the 7-TeV LHC energy crosses the experimental value within the error bars. In the case of three rungs, however, the predicted value is unacceptably low. In Table, two representative fits for $s_0 = 49$ GeV$^2$ and $s_0 = 121$ GeV$^2$ are quoted. Throughout this paper, we chose the fits with $(\chi^2/\mathrm{dof}) \leq 1.0$. The best fit is quoted in Figure.

## 5. Conclusions

Our main goal was an adequate picture of the Pomeron exchange at $t = 0$. In our opinion, it is neither an infinite sum of gluon ladders, as in the BFKL approach [2–4], nor its power expansion. In fact, the finite series – call it "threshold approach" considered in Secs. 2 and 4 and in the previous papers [6], realizes a nontrivial dynamical balance between the total reaction energy and the subenergies equally partitioned between the multiperipheral ladders.

The gap width $s_0$ is an important physical parameter independent of the model presented above. We have fitted it and obtained the value, which exceeds our previous estimates [6]. Another goal of the present investigation was the comparison and the prediction of the proton-proton total cross section at the LHC energies with a QCD-inspired Pomeron model. The model fits give values of $\sigma(s)$ compatible with the experiment at 7 TeV within the error bars and consequently cannot be ruled out.

*This work was done in the framework of the Hungarian Academy of Sciences Fellowship for Transfrontiery Hungarian Science. We are thank V. Bitev, L. Jenkovszky, and E. Kuraev for discussions and the valuable collaboration.*

1. TOTEM Collaboration, G. Antchev *et al.*, Europhys. Lett. **96**, 21002 (2011).

2. E.A. Kuraev, L.N. Lipatov, and V.S. Fadin, Sov. Phys. JETP **45**, 199 (1977).
3. Ya.Ya. Balitsky and L.N. Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978).
4. L.N. Lipatov, Sov. Phys. JETP **63**, 904 (1986).
5. V.S. Fadin and L.N. Lipatov, Phys. Lett. B **429**, 127 (1998) and references therein.
6. R. Fiore, L.L. Jenkovszky, A.I. Lengyel, F. Paccanoni, and A. Papa, arXiv: hep-ph/0002100 9; R. Fiore, L. Jenkovszky, E. Kuraev, A. Lengyel, F. Paccanoni, and A. Papa, Phys. Rev. D **63**, 056010 (2001).
7. J.R. Cudell, E. Martynov, O. Selyugin, and A. Lengyel, Phys. Lett. B **587**, 78 (2004).
8. A. Achilli, R. Godbole, A. Grau, and Y.N. Srivastava, *QCD Mini-Jet Contribution to the Total Cross-Section*, hep-ph/0907.0949; R.M. Godbole, A. Grau, G. Pancheri, and Y.N. Srivastava, *QCD and Total Cross-Sections: Photons and Hadrons*, hep-ph/0812.4705.
9. R. Fiore, L. Jenkovszky, E. Kuraev, A. Lengyel, and Z. Tarics, Phys. Rev. D **81**, 056001 (2010), hep-ph/090160.
10. See, http://www.ppds.ihep.su:8001/hadron.html
11. E. Levin and M. Ryskin, Sov. J. Nucl. Phys. **34**, 619 (1981); Phys. Rep. **189**, 267 (1990).
12. P. Kroll, Forts. d. Phys. **24**, 565 (1976).


Received 19.06.12

*А. Лендел, З. Торич*

ОБРІЗАНИЙ КХД-ПОМЕРОН ПРИ ЕНЕРГІЯХ ВГК


Р е з ю м е

Ейконалізована амплітуда $F(s,t)$ пружного протон-протонного та антипротон-протонного розсіяння побудована на припущенні скінчених драбинних глюонних діаграм, розрахованих в КХД з певним числом глюонних сходинок і відповідно степенів логарифма у повному перерізі залежно від доступної зростаючої енергії. Точний вираз повного перерізу, що включає три або чотири сходинки з $\ln^3(s)$ та $\ln^4(s)$, як члени вищих степенів відповідно, використано для опису усіх наявних експериментальних даних повних перерізів протон-протонного та антипротон-протонного розсіяння. Зроблено передбачення протон-протонного повного перерізу при енергіях Великого гадронного коллайдера.