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(Shahid Motahary Str., Islamic Azad Univ., Bushehr, Iran; e-mail: ehphysics75@iaubushehr.ac.ir)**RELATIVISTIC LASER-PLASMA INTERACTIONS.  
MOVING SOLITARY WAVES IN PLASMA CHANNELS  
AND THE KINETIC DISPERSION RELATION  
OF CHERENKOV RADIATION**PACS 52.38.-r, 52.38.Hb

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*The propagation of an intense laser beam in a preformed plasma channel is studied. Considering a propagating Gaussian laser pulse in a relativistic plasma channel which has a parabolic density profile, the evolution equation of the laser spot size is derived analytically and solved numerically. The governing equation includes the effects of relativistic corrections to the ponderomotive self-channeling, preformed channel focusing, and self-focusing. In order to investigate the conditions for the existence of electromagnetic solitary waves, the solutions of the evolution equation for the laser spot size are discussed in terms of a relativistic effective potential. Some solitary wave solutions are illustrated numerically. The relativistic corrections to the dispersion relation of Cherenkov emission in dusty plasma is presented briefly. In the low-velocity limit, all the expressions in the present study are reduced to their associated counterparts in the nonrelativistic regime, as should be.*

*Key words:* plasma channels, solitons, relativistic plasma, Cherenkov radiation.

**1. Introduction**

The nonlinear interaction of plasmas with high intensity lasers is of great current interest [1–7]. The possibility of reaching the extreme power levels with such setups is one of the promising aspects of laser-plasma systems [8] and holds the potential of overcoming the laser intensity limit  $h \approx 10^{25}$  W/cm<sup>2</sup> [9]. As the field strength approaches the critical Schwinger value  $E_{\text{crit}} \approx 10^{16}$  V/cm [10], there is a possibility of the photon-photon scattering, even within a plasma [11], since the ponderomotive force due to the intense laser pulse gives rise to plasma channels [12–14].

The main nonlinear effects in the propagation of intense electromagnetic pulses through a plasma arise from the relativistic variation of the electron mass (relativistic nonlinearity) and from a perturbation in

the electron density, which takes place because of the ponderomotive forces due to the radiation fields (strict nonlinearity). Both these effects change the effective dielectric constant of the plasma medium for the propagation of electromagnetic waves and lead to a coupling between the transverse electromagnetic wave and the longitudinal waves in the plasma medium [15].

The study of the formation and propagation of relativistic electromagnetic solitary waves and their effects on a plasma due to the highly nonlinear processes of strong electromagnetic wave coupling with the plasma wave is important to understand many aspects of the laser-plasma interaction such as the fast ignition scheme, laser wake field acceleration, and laser overdense penetration [16–22].

It is well known that the characteristic distance for the propagation of a directed radiation beam in vacuum is the Rayleigh range,  $Z_p$ . On the other

hand, although a laser pulse in a uniform plasma can guide itself by the effect of relativistic self-focusing and ponderomotive self-channeling, the diffraction would dominate over these effects, when the laser power is smaller than the critical power  $P_c = 17(\omega_0/\omega_p)^2(\text{GW})$ , where  $\omega_p$  is the plasma frequency, and  $\omega_0$  is the laser frequency. It has been shown that a preformed plasma channel can prevent the diffraction and allow the propagation of an intense laser pulse through many Rayleigh lengths without disruption.

In Ref. [16], Zhang *et al.* studied the necessary conditions for the existence of the electromagnetic solitary waves in plasma channels. They presented the equation of evolution of the pulse spot size and illustrated some solitary wave solutions numerically. When the velocity of a charged particle passing through a dielectric medium exceeds the phase speed of light in that medium, the molecules of the medium will be polarized. Then the molecules may emit a specific type of emission, viz., Cherenkov emission, when they turn back to their ground state. The Cherenkov wakes excited by intense laser drivers in a perpendicularly magnetized plasma are a potential source of high-power terahertz radiation. Cherenkov radiation is possible under certain conditions in the plasma and nuclear environment. Since the plasma is a diffuser, different frequencies are emitted at different angles. Cherenkov radiation is emitted mostly in the forward direction, because a significant speed is required to enable the exit of radiation out of the plasma. When the plasma is deposited with a laser pulse, the energy exits in the minimum time interval between the plasma energy transition and plasma wave life. Cherenkov wave creates some of dusty plasma waves: acoustic-dust and acoustic ion-dust ones. These shear waves support Alfvén ones and are the cause for the Mach cone observed in Saturn's dense rings.

In a recent research paper, El-Bendary *et al.* [23], by using a kinetic method based on the Vlasov equation, found a dispersion relation for a 1D Cherenkov wave in an inhomogeneous plasma. Considering some restrictive conditions, they estimated the growth rate of kinetic Cherenkov waves.

Here, we will extend the previous works in the non-relativistic limit [16, 23] and study the effect of the relativistic flow velocity of a plasma on the solitary wave solutions.

The aim of this paper is to investigate the existence of relativistic solitary waves of a Gaussian laser pulse in a preformed plasma channel with a parabolic density profile. The organization of this paper is as follows: In Section 2, considering the appropriate equations, we obtain the differential equation describing the evolution of the laser spot size. The governing equation presents the effects of ponderomotive self-channeling, preformed channel focusing, and self-focusing with relativistic corrections. Section 3 is devoted to the numerical analysis. Some solitary wave solutions are illustrated. In Section 4, the relativistic corrections to the kinetic dispersion relation for the Cherenkov radiation are discussed briefly. Section 5 summarizes the finding of this study.

## 2. Evolution Equations for the Laser Spot Size

The normalized vector potential with a slowly varying complex envelope for a circularly polarized laser pulse propagating the plasma channel with a parabolic density profile of the form  $n(r) = n_0(1 + r^2/r_{\text{ch}}^2)$  can be written as [16]

$$\mathbf{a}(r, z, t) = \frac{a(r, z, t)}{2} (\hat{\mathbf{e}}_x + i\hat{\mathbf{e}}_y) \exp[i(k_0 z - \omega_0 t)] + \text{c.c.}, \quad (1)$$

where  $\hat{\mathbf{e}}_x$  and  $\hat{\mathbf{e}}_y$  show the unit vectors along the  $x$  and  $y$  directions, respectively, in a Cartesian coordinate system,  $n_0$  is the initial axial electron density,  $r_{\text{ch}}$  is the effective channel radius,  $a(r, z, t)$  is the complex amplitude, and  $k_0$  and  $\omega_0$  are the laser center wave number and the frequency, respectively. In the relativistic regime with the use of the Coulomb gauge  $\nabla \cdot \mathbf{a} = 0$ , we present the wave equation for the laser field as

$$\left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{a} = k_p^2 \left( 1 + \gamma_\nu \frac{(1 - \beta_0 \nu)}{\gamma_0 (\beta_0 - \nu)^2} \frac{r^2}{r_{\text{ch}}^2} - \frac{|\mathbf{a}|^2}{2} + \gamma_\nu^2 \frac{(1 - \beta_0 \nu)^2}{\gamma_0^2 (\beta_0 - \nu)^4} \nabla_\perp^2 \frac{|\mathbf{a}|^2}{2} \right) \mathbf{a}, \quad (2)$$

where  $k_p = \omega_p/c$  is the plasma wave number, and  $\gamma_\nu = (1 - \nu^2)^{-1/2}$  is the relativistic factor associated with the velocity of the wave and should not be confused with the particle relativistic factor,  $\gamma = (1 - \beta^2)^{-1/2}$  related to the fluid velocity of the plasma. We have  $\beta = u/c$ ,  $\nu = V/c$ , and the subscript 0 represents the quantities at infinity. We note that, in the derivation of Eq. (2), the long pulse

limit, i.e.,  $\omega_p \tau_L \gg 1$  is used. The quantities  $\omega_p = (4\pi n_0 e^2 / m_e)^{1/2}$  and  $\tau_L$  are the plasma frequency and the laser pulse duration, respectively. It is worth to note that, in the weakly relativistic limit, Eq. (2) reduces to its counterpart in [16]. Let us substitute Eq. (1) into Eq. (2) and assume that the complex amplitude of the vector potential has a solution with the Gaussian transverse profile

$$a(r, z) = a_r(z) \exp\left[\frac{-r^2}{r_s^2(z)}\right] \exp[i(b(z)r^2 + \Phi(z))]. \quad (3)$$

Then the relativistic equation describing the evolution of the laser spot size is given as

$$\begin{aligned} \frac{\partial^2 r_s}{\partial z^2} &= \frac{\gamma_\nu(1 - \beta_0\nu)(1 - p)}{\gamma_0(\beta_0 - \nu)^2} \frac{(1 - p)}{r_s^3} - N_c r_s - \\ &- \frac{\gamma_\nu^2(1 - \beta_0\nu)^2}{\gamma_0^2(\beta_0 - \nu)^4} \frac{a_0^2}{2r_s^5}, \end{aligned} \quad (4)$$

where  $a_r(z)$ ,  $r_s(z)$ ,  $b(z)$ , and  $\phi(z)$  are the real amplitude, spot size, spatial chirp parameter, and phase shift of a laser pulse, respectively. The quantity  $p = k_p^2 a_0^2 r_0^2 / 16$  is the normalized laser power,  $N_c = \gamma_0^2 k_p^2 r_0^4 / 4r_{ch}^2$  is a parameter related to the effect of reformed channel focusing, and the dimensionless variables  $z/Z_R \rightarrow z$  and  $r_s/r_0 \rightarrow r_s$ , where  $Z_R = k_0^2 r_0^2 / 2$  is the Rayleigh length, are used. Now, considering a collimated incident laser pulse, i.e.,  $b_0 = (\partial r_s / \partial z)_{z=0} = 0$  and the initial condition  $r_s = 1$  at  $z = 0$  and integrating Eq. (4), we get

$$\frac{1}{2} \left( \frac{\partial r_s}{\partial z} \right)^2 + V(r_s) = 0, \quad (5)$$

where

$$\begin{aligned} V(r_s) &= \frac{\gamma_\nu(1 - \beta_0\nu)(1 - p)}{\gamma_0(\beta_0 - \nu)^2} \frac{(1 - p)}{2r_s^2} + \frac{N_c r_s^2}{2} - \\ &- \frac{\gamma_\nu^2(1 - \beta_0\nu)^2}{\gamma_0^2(\beta_0 - \nu)^4} \frac{a_0^2}{8r_s^4} - V_0, \end{aligned} \quad (6)$$

in which

$$V_0 = \frac{\gamma_\nu(1 - \beta_0\nu)(1 - p)}{2\gamma_0(\beta_0 - \nu)^2} + \frac{N_c}{2} - \frac{\gamma_\nu^2(1 - \beta_0\nu)^2 a_0^2}{8\gamma_0^2(\beta_0 - \nu)^4}. \quad (7)$$

We note that, for  $\beta_0 = 0$ , Eqs. (4)–(7) can be obviously reduced to the expressions for the nonrelativistic limits [16].

### 3. Solution and Results

The evolution equation, Eq. (5), can be used to study the variation of the spot size of a laser beam. Putting  $V(r_s) = 0$  and defining  $r_s^2 \equiv R_s$ , one obtains a cubic equation for the spot size as

$$\begin{aligned} N_c R_s^3 - 2V_0 R_s^2 + \frac{1}{2} (N_p + a_0^2) R_s - \\ - \frac{1}{16} \left( \frac{N_p + a_0^2}{1 + p} \right)^2 a_0^2 = 0, \end{aligned} \quad (8)$$

in which

$$N_p = \frac{4\gamma_\nu(1 - \beta_0\nu)(1 - p)}{2\gamma_0(\beta_0 - \nu)^2} - a_0^2. \quad (9)$$

Then the roots of the relativistic effective potential function can be easily found, by using the solutions of the cubic equation. Solving Eq. (8) gives three solutions as

$$r_{s1} = 1, \quad (10)$$

$$r_{s2} = \left[ \left( N_p + \sqrt{N_p^2 - 16N_c a_0^2} \right) / 8N_c \right]^{1/2}, \quad (11)$$

$$r_{s3} = \left[ \left( N_p - \sqrt{N_p^2 - 16N_c a_0^2} \right) / 8N_c \right]^{1/2}. \quad (12)$$

Three cases can be considered:

- (1) if  $p > 1 - \eta a_0 \sqrt{N_c} - \eta^2 a_0^2 / 4$ , where  $\eta = \frac{\gamma_\nu(1 - \beta_0\nu)}{\gamma_0(\beta_0 - \nu)^2}$ , the equation has three real roots:  $r_{s1} = 1$  and  $r_{s2} = r_{s3} = \sqrt{\eta a_0 / 2\sqrt{N_c}} > 1$ ;
- (2) if  $p < 1 - N_c - \eta^2 a_0^2 / 2$ , the equation  $V(r_s) = 0$  has three distinct real roots  $r_{s3} < r_{s2} < r_{s1} = 1$ ;
- (3) if  $1 - N_c - \eta^2 a_0^2 / 2 \leq p \leq 1 - \eta a_0 \sqrt{N_c} - \eta^2 a_0^2 / 4$ , three kinds of cases can be considered:

(3-1) if  $N_c = N_c^*$ , where the critical channel parameter  $N_c^* = \eta^2 a_0^2 / 4$ , the equation  $V(r_s) = 0$  has three real roots: twofold root  $r_{s1} = r_{s2} = 1$  and  $r_{s3} < 1$ ;

(3-2) if  $N_c > N_c^*$ , three types can be discussed as follows:

(3-2-1) if  $p > 1 - \eta a_0 \sqrt{N_c} - \eta^2 a_0^2 / 4$ , the equation  $V(r_s) = 0$  has only one real root, i.e.,  $r_{s1} = 1$ ;

(3-2-2) if  $1 - N_c - \eta^2 a_0^2 / 2 < p < 1 - \eta a_0 \sqrt{N_c} - \eta^2 a_0^2 / 4$ , the equation  $V(r_s) = 0$  has three real roots: twofold root  $r_{s2} < r_{s1} < r_{s3}$ ;

(3-2-3) if  $p = 1 - N_c - \eta^2 a_0^2 / 2$ , then the equation  $V(r_s) = 0$  has three unequal real roots:  $r_{s1} = 1 < r_{s3} < r_{s2}$ ;

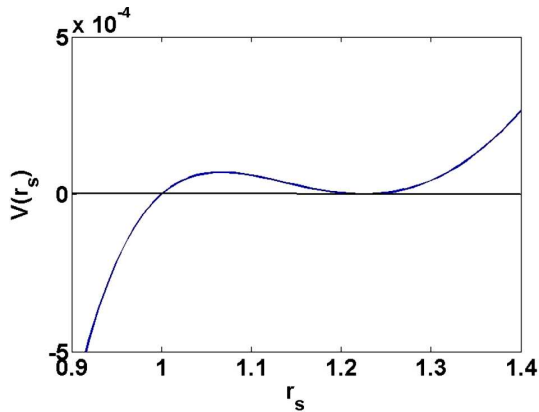


Fig. 1. Relativistic effective potential  $V(r_s)$  as a function of the spot size  $r_s$  for  $N_c = 0.3$  and  $p = 0.85$

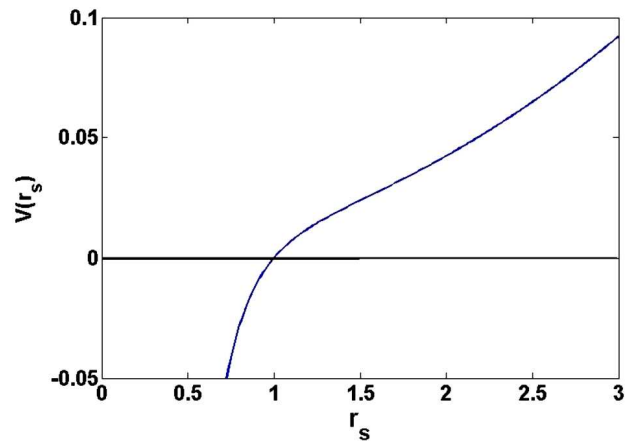


Fig. 4. Relativistic effective potential  $V(r_s)$  as a function of the spot size  $r_s$  for  $N_c = 0.4$  and  $p = 0.75$

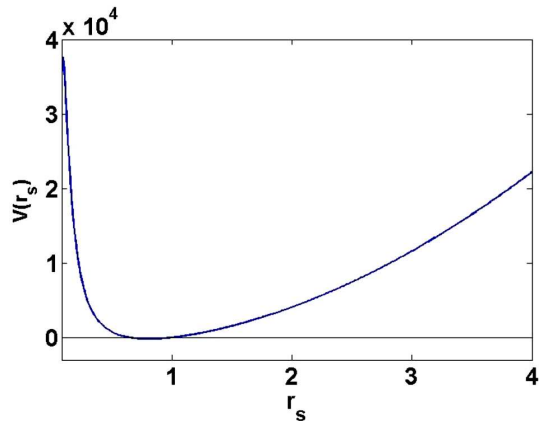


Fig. 2. Relativistic effective potential  $V(r_s)$  as a function of the spot size  $r_s$  for  $N_c = 0.3$  and  $p = 0.95$

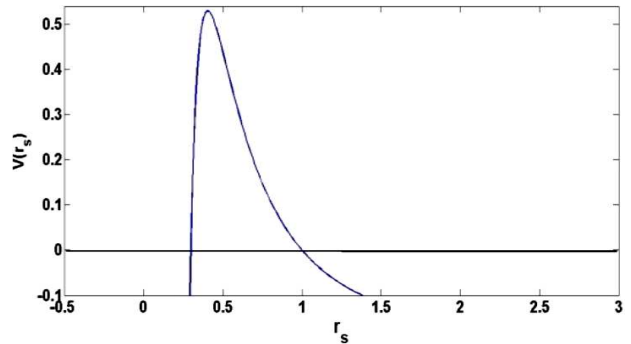


Fig. 5. Relativistic effective potential  $V(r_s)$  as a function of the spot size  $r_s$  for  $N_c = 0.45$  and  $p = 0.9$

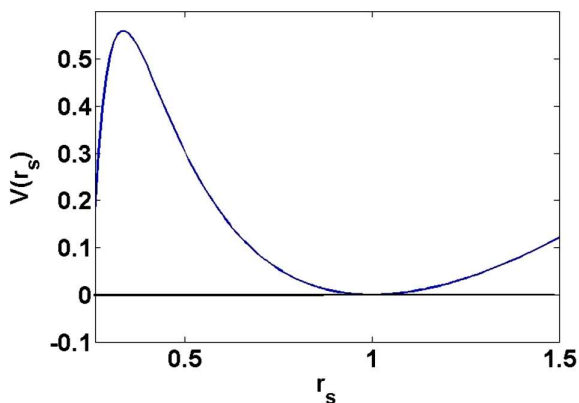
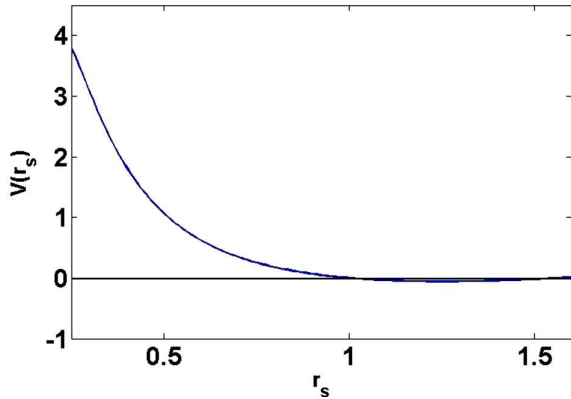


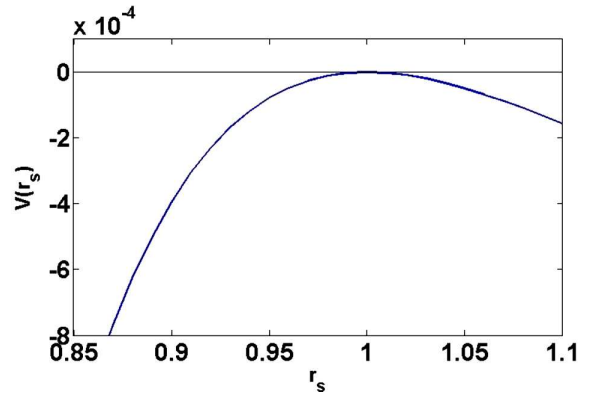
Fig. 3. Relativistic effective potential  $V(r_s)$  as a function of the spot size  $r_s$  for  $N_c = 0.04$  and  $p = 0.95$

(3-3) if  $N_c < N_c^*$ , the following results are given:  
 (3-3-1) if  $p = 1 - \eta a_0 \sqrt{N_c} - \eta^2 a_0^2 / 4$ , the equation  $V(r_s) = 0$  has triple root, i.e.,  $r_{s1} = r_{s2} = r_{s3} = 1$ ;  
 (3-3-2) if  $1 - N_c - \eta^2 a_0^2 / 2 < p < 1 - \eta a_0 \sqrt{N_c} - \eta^2 a_0^2 / 4$ , the equation  $V(r_s) = 0$  has only one real root, i.e.,  $r_{s1} = 1$ ;  
 (3-3-3) if  $p = 1 - N_c - \eta^2 a_0^2 / 2$ , the equation  $V(r_s) = 0$  has three real roots: twofold roots:  $r_{s1} = 1$  and  $r_{s2} = r_{s3} = \sqrt{\eta a_0 / 2 \sqrt{N_c}} < 1$ .

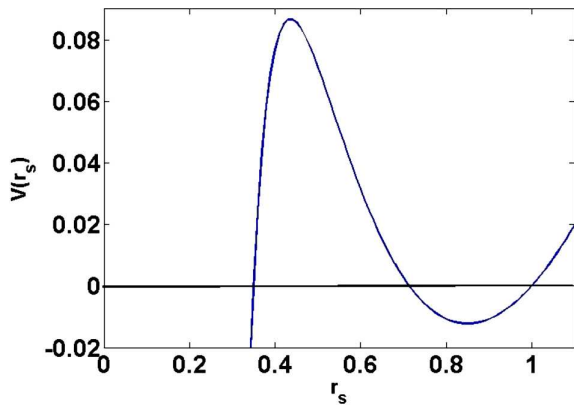
Figures 1–9 show the variations of the potential,  $V$ , for various values of  $p$  and  $N_c$  corresponding to the cases (1)-(3-3-3), respectively. In all cases, the fixed parameters  $\nu = 0.4, \beta = 0.8$ , and  $a_0 = 0.3$  are considered. In Fig. 3, the position  $r_s = 1$  is stable. In this case, (3-1), the particle will be at rest. This case could be related to a constant spot size. In Figs. 1, 4, 5, 8, and 9, the position  $r_s = 1$  is unstable. These cases correspond to the catastrophic focusing. In fact,



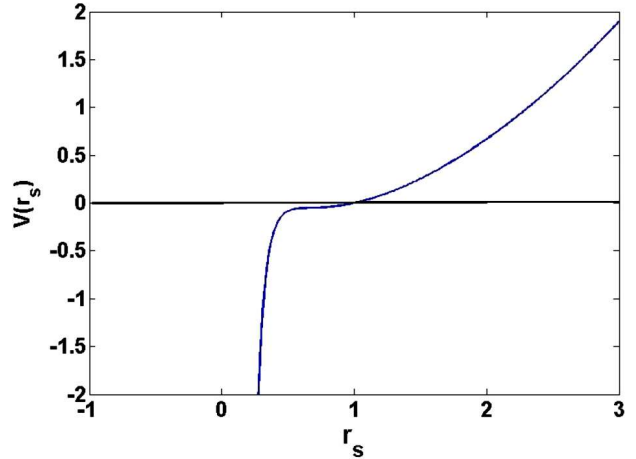
**Fig. 6.** Relativistic effective potential  $V(r_s)$  as a function of the spot size  $r_s$  for  $N_c = 0.45$  and  $p = 0.65$



**Fig. 8.** Relativistic effective potential  $V(r_s)$  as a function of the spot size  $r_s$  for  $N_c = 0.02$  and  $p = 1.02$



**Fig. 7.** Relativistic effective potential  $V(r_s)$  as a function of the spot size  $r_s$  for  $N_c = 0.02$  and  $p = 1.01$



**Fig. 9.** Relativistic effective potential  $V(r_s)$  as a function of the spot size  $r_s$  for  $N_c = 0.02$  and  $p = 1$

in this position, the particle will move to the position  $r_s \rightarrow 1$  for the certain parameters introduced in these cases. As is clear from Figs. 2 and 6, the particle will move periodically between  $r_{s1}$  and  $r_{s2}$  in cases (2) and (3-2-3), which shows the characteristic feature of periodic solutions. Finally, in the case of (3-3-1), Fig. 7, the particle is in the critical state.

#### 4. Relativistic Kinetic Dispersion Relation of Cherenkov Radiation

In this section, we present the relativistic corrections to the kinetic dispersion relation for the Cherenkov radiation waves briefly. We start with the Vlasov equation

$$\frac{\partial f_\alpha(\mathbf{x}, \mathbf{u}, t)}{\partial t} + \mathbf{V}(\mathbf{u}) \cdot \nabla f_\alpha(\mathbf{x}, \mathbf{u}, t) + \frac{q_\alpha}{m_\alpha} \times$$

$$\times \left[ \mathbf{E} + \frac{1}{c} (\mathbf{V}(\mathbf{u}) \times \mathbf{H}) \right] \frac{\partial f_\alpha(\mathbf{x}, \mathbf{u}, t)}{\partial \mathbf{V}} = 0. \quad (13)$$

Here,  $m$  is the mass,  $c$  the speed of light in vacuum,  $q$  the charge, and  $E$  and  $H$  are the electric and magnetic field vectors, respectively. The quantity  $\alpha$  represents the species of the plasma. Now, in the case where the thermal pressure is much smaller than the magnetic one, the kinetic dispersion relation of the system with regard for relativistic corrections can be written as

$$1 + \gamma_v^2 (\beta - V) \gamma_0 k_x^{-2} \sum_\alpha \frac{\omega_{p\alpha}^2}{v_{th,\alpha}^2} (1 + i s_\alpha) = 0, \quad (14)$$

where  $s_\alpha = \sqrt{\pi} L_\alpha Z_\alpha W(Z_\alpha) A(\mu_\alpha)$ ,  $\mu = k_\perp^2 r_L^2$ ,  $r_L = \nu_{th}/\omega_c$  is the Larmor radius,  $\omega_c = qH/mc$  the cy-

clotron frequency,  $\nu_{th} = (k_B T/m)^{1/2}$  the thermal velocity,  $\omega_p = (4\pi n e^2/m)^{1/2}$  the plasma frequency,  $n$  the number density,  $A = I_0 e^{-\mu}$  describes the effect of the magnetic field parameter,  $I_0$  the zero-order modified Bessel function,  $z = \frac{\omega - \mathbf{k}U}{\sqrt{2}k_s \nu_{th}}$ , and

$$L = 1 - \frac{k_x \nu_{th}^2}{\omega_c(\omega - \mathbf{k}U)} \frac{d}{dx} (\ln n + \ln T) \quad (15)$$

represents the effects of the temperature and the inhomogeneity of the plasma, and  $k_x$  and  $k_\perp$  are the components of the wave number which are oriented in parallel and perpendicularly to the magnetic field, respectively. Note that, in the limit  $\beta \rightarrow 0$ , Eq. (14) includes the nonrelativistic dispersion relation of Cherenkov radiation [23].

## 5. Conclusions

Assuming a circularly polarized Gaussian laser pulse propagating in a plasma channel with a parabolic density profile, we have obtained a relativistic effective potential and its governing equation. Then, by analyzing the differential equation of the pulse spot size, we have investigated the conditions for the existence of electromagnetic solitary waves. We have illustrated some solitary wave solutions numerically. Furthermore, the relativistic corrections to the kinetic dispersion relation for the Cherenkov radiation are presented. To this end, we started with the Vlasov equation and have investigated the dispersion relation in the case where the thermal pressure is much smaller than the magnetic one.

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РЕЛЯТИВІСТСЬКІ ЛАЗЕР-ПЛАЗМА ВЗАЄМОДІЇ.  
СОЛІТОНИ, ЩО РУХАЮТЬСЯ В КАНАЛАХ ПЛАЗМИ,  
І КІНЕТИЧНЕ ДИСПЕРСІЙНЕ СПІВВІДНОШЕННЯ  
ДЛЯ ЧЕРЕНКОВСЬКОГО ВИПРОМІНЮВАННЯ

Резюме

Вивчено поширення інтенсивного лазерного пучка в плазмовому каналі. Розглянуто поширення гаусового лазерно-

го імпульсу в релятивістському плазмовому каналі з параболічним профілем густини. Рівняння еволюції для розміру лазерної плями отримано аналітично і вирішено чисельно. Його рішення в термінах релятивістського ефективного потенціалу застосовані для знаходження умов існування електромагнітних солітонів. Визначальне рівняння описує ефекти релятивістських поправок до пондеромоторних самоканалюванню, фокусуванню і самофокусуванню попередньо створеного каналу. Дано кількісний опис деяких солітонних рішень. Знайдено релятивістські поправки до закону дисперсії черенковського випромінювання в зашпленій плазмі. У межах малих швидкостей усі результати роботи переходять до відповідних нерелятивістських виразів.