V.I. PIPA, A.I. LIPTUGA

V.E. Lashkaryov Institute of Semiconductor Physics, Nat. Acad. of Sci. of Ukraine
(41 Nauky Ave., Kyiv 03028, Ukraine)

ROTATION OF A THIN HEATED
PLATE BY ITS OWN THERMAL RADIATION

The results of the theoretical investigation of a heated solid plate rotation caused by its own thermal radiation are presented. The rotational moment of the plate with uniform temperature is due to the asymmetry of a recoil of photons of thermal radiation from different areas of its wide faces. The time dependences of the rotation angle of a rectangular plate with stable temperature and at radiative cooling are calculated. The rotation angle dependences on the plate geometric and optical parameters are studied. It is shown that the rotation angle depends nontrivially on the plate thickness because of the thickness-dependent plate emittance. The numerical calculations are performed for a silicon plate. The biggest rotation effect (rotation angle of 118.5° for 70 s) is obtained for a mirror black plate 0.1 mm thick at a preset plate temperature of 350 K and a background temperature of 300 K. The considered model can be useful to develop thermal motors, whose rotors consist of a set of heated plates with photon recoil asymmetry.

K e y w o r d s: thermal radiation, photon recoil, thin film rotation.

1. Introduction

Thermal radiation (TR) emitted by a heated body exerts a mechanical effect on it. Due to the photon recoil, a force (opposite in direction to the TR flux) acts on this body. If the intensities of radiation from various areas of body's surface differ from each other, then moments of forces may act on the body as well. A heated free body may execute a translational, as well as rotational, motion under the photon recoil depending on the TR asymmetry. This effect is inherent to a wide range of objects, from very heavy and bulky (rotation of asteroids by radiation [1], an anomaly in the acceleration of space probes "Pioneer 10 and 11" [2, 3]) to very light and small ones (optically levitating nanoparticles [4], Janus particles [5], Rytov pinwheels [6], *etc.*). The present work deals with calculating the rotation of a heated solid plate by its own TR.

2. Results and Discussion

Let us consider the rotation of a thin rectangular plate heated uniformly to the temperature T, while the environment temperature is T_1. In the xyz-coordinate system, the plate wide faces (edge lengths

l_x, l_z) are located vertically in the planes xz, $y = \pm l_y/2$. We suppose that l_x, $l_z \gg l_y$ and so consider the radiation pressure on the wide faces only. The plate cross-section, as well as the plate and environmental TR intensities, are sketched in Fig. 1.

To illustrate heat flows, the plate cross-section is presented in the horizontal xy-plane coinciding with the figure plane. The rotation z-axis is directed perpendicularly to that plane. However, it should be noted that we consider the plate rotation in the horizontal plane around the vertical z-axis.

In the presented model, the coefficients of radiation reflection from the different halves of plate wide faces are unequal. For the face a ($y = -l_y/2$) on the left ($-l_x/2 \leqslant x \leqslant 0$) and right ($0 \leqslant x \leqslant l_x/2$) halves, they are R_1 and R_2, respectively. The location of such areas on the face b ($y = l_y/2$) is opposite to that on the face a. As a result, the total intensities of radiation (TR from the plate and the reflected background radiation) from the different halves of the wide face are unequal. The pressure exerted by radiation with such an asymmetry leads to the free plate rotation around the z-axis. It should be noted that the plate rotation direction depends on the ratios between R_1 and R_2, as well as between T and T_1.
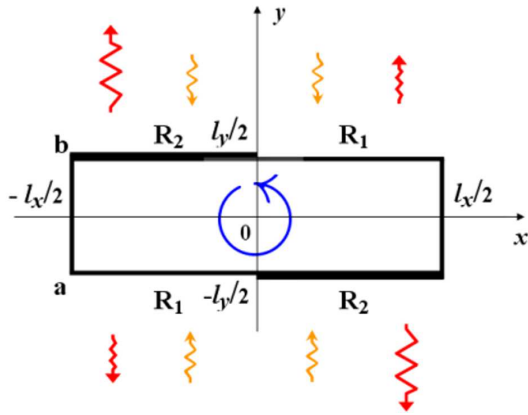
Fig. 1. Diagram (cross-section) of a heated rotating plate. The wide faces (*a*) and (*b*) are divided into two halves each that differ in the reflection coefficients (R_1 and R_2) and TR intensities (arrows of different lengths aimed away from the plate). The arrows directed toward the plate represent the background radiation. The round arrow represents the plate rotation direction around the axis passing through the center of inertia

Let $J(\lambda, \vartheta)d\lambda d\Omega$ designate the density of the normal TR energy flow in the wavelength interval $d\lambda$ into a solid angle element $d\Omega$ at an angle ϑ to the normal to the plate surface. In accordance with the Kirhhoff's law, $J(\lambda, \vartheta) = A(\lambda, \vartheta)J_0(\lambda, \vartheta)$. Here, A is the plate emittance (equal to its absorptance), $J_0 = 4\pi\hbar c^2 \lambda^{-5} N \cos\vartheta$, and N is the Planck function.

Let us calculate radiation pressure on the plate right half for $d\lambda = 1$ and $d\Omega = 1$. TR from that plate part exerts the pressure $A_1 J_0(T)/c$ on the face *b* (and $A_2 J_0(T)/c$ on the face *a*); pressure values of background radiation on these surfaces are $B_1 J_0(T_1)/c$ and $B_2 J_0(T_1)/c$, respectively. Here, $A_k = 1 - R_k - D$, $B_k = 1 + R_k + D$, D is the plate transmission coefficient, $k = 1, 2$. The resulting pressure (a difference between the values of the pressure on the right halves of faces *a* and *b*) is

$$P = c^{-1} \iint (R_1 - R_2)(J_0(T) - J_0(T_1))d\lambda d\Omega. \quad (1)$$

The opposite pressure of the same value is applied to the plate left half. If $R_1 > R_2$, then the above pressure is applied to the right half of face *a* and left half of face *b* of the heated plate, so it can rotate around the *z*-axis in the direction shown in Fig. 1.

Let the plate start to rotate at the instant $t = 0$. In the cylindrical coordinate system, the plate position

is determined by the azimuth angle $\varphi(t)$ that can be found from the equation for angular momentum of the plate

$$I_z \frac{d^2\varphi}{dt^2} = K \quad (2)$$

with initial conditions $\varphi(0) = 0$, $\dot{\varphi}(0) = 0$. Here, $I_z = ml_x^2/12$ is the plate moment of inertia in the approximation $(l_y/l_x)^2 \ll 1$, m is the plate mass, $K = Pl_x^2 l_z/4$ is the moment of pressure force couple (Eq. (1)). Action of other forces (e.g., friction) is ignored.

We consider two temperature conditions at $t > 0$: 1) the temperature $T(t = 0) = T_0$ remains constant and 2) the plate temperature decreases due to radiative cooling. The temperature of a heated unfixed plate can be maintained (e.g., by electric current).

By calculating the difference between the radiant energy input and output for the plate, we obtain that, at $t > 0$, $T(t)$ obeys the equation

$$\rho l_y C_p \frac{dT}{dt} = -\iint (A_1 + A_2)(J_0(T) - J_0(T_1))d\lambda d\Omega, \quad (3)$$

where ρ is the plate density, C_p is the thermal capacity. Other cooling mechanisms are ignored.

The maximum pressure (Eq. (1)) is achieved, if $R_1 = 1$ and $R_2 = 0$. For such a plate, the transmission coefficient $D = 0$; one (mirror) half of each plate face does not emit at all ($A_1 = 0$), while another half emits as a blackbody ($A_2 = 1$). In this case, we obtain $P = P_{bb}$ from Eq. (1), and the right-hand side of Eq. (3) is $-cP_{bb}$, where

$$P_{bb} = \sigma(T^4 - T_1^4)/c, \quad (4)$$

and σ is the Stefan–Boltzmann constant.

Besides this extreme case, let us consider also the plate (Fig. 1), for which the reflection coefficient $R_1 = 1$, and $R_2 \equiv R(\lambda, \vartheta)$ is determined by the dielectric properties of a plate material. In this case, $A_1 = 0$, $A_2 = 1 - R$, and the right-hand side of Eq. (3) is expressed in terms of the pressure (Eq. (1)) as $-cP(T, T_1)$.

Let us designate the pressure (Eq. (1)) at $t = 0$ as $P_0 = P(T_0, T_1)$. If the initial temperature T_0 is maintained constant, then Eq. (2) yields

$$\varphi = \frac{3P_0}{2\rho l_x I_y} t^2. \quad (5)$$

Let the plate be cooled at $t > 0$. For the time interval, in which T decreases but slightly $((T_0 - T)/T_0 \ll \ll 1)$, Eqs. (1)–(3) yield:

$$\varphi = \frac{3P_0}{\gamma^2 \rho l_x l_y} \left(e^{-\gamma t} + \gamma t - 1 \right), \tag{6}$$

where γ is determined as

$$\gamma = \frac{8\pi^2 c^3 \hbar^2}{\rho C_p l_y k_B T_0^2} \iint (1 - R) N(T_0) \times$$
$$\times (1 + N(T_0)) \lambda^{-6} \cos \vartheta d\lambda d\Omega. \tag{7}$$

For a mirror black plate $(R_1 = 1, R_2 = 0)$:

$$P_0 = P_{bb}(T_0, T_1), \quad \gamma = 4\sigma T_0^3 / (\rho l_y C_p). \tag{8}$$

From Eqs. (5) and (6), it follows that the rotation angle does not depend on the plate width l_z and increases (as $1/l_x$), as the plate length l_x decreases. The $\varphi(t)$ dependence on the plate thickness l_y is more complicated. Besides the explicit $\varphi(t) \infty 1/l_y$ and $\gamma \infty 1/l_y$ dependences caused by the plate mass, these quantities are also determined by the thickness dependence of the plate emittance.

The time dependences of the plate rotation angle calculated from Eqs. (5) and (6) are presented in Figs. 2 and 3, respectively. In each figure, the curves illustrate the $\varphi(t)$ dependences for silicon plates that have different reflection coefficients and whose thicknesses differ by ten times. The calculations were performed for plates with the length $l_x = 2$ cm, whose thicknesses were considerably over the wavelengths corresponding to the maxima of plate TR spectra and background radiation. This makes it possible to neglect the interference phenomena in the plate TR [7, 8]. The following parameters were used: the silicon density $\rho = 2.32$ g/cm^3, thermal capacity $C_p = 7.56 \times 10^6$ erg/(g$\cdot$K), refractive index $n = 3.4$, radiation absorption coefficient $\alpha(\lambda) \infty \lambda^2$, and $\alpha = 15$ cm^{-1} for $\lambda = 10$ μm.

Figure 2 presents the $\varphi(t)$ dependence for a uniformly accelerated rotation of the plate $(T_0(t) = = \text{const.})$. It can be seen from curves 1 and 3 for plates, whose pressure P_0 does not depend on l_y, that the rotation angle of a thin plate is over that of a thick one by ten times. For plates with $R_2 \equiv R(\lambda, \vartheta)$ (curves 2 and 4 in Fig. 2), the pressure P_0 depends on l_y; in this case, the ratio between the rotation angles of thin and thick plates is less than ten.
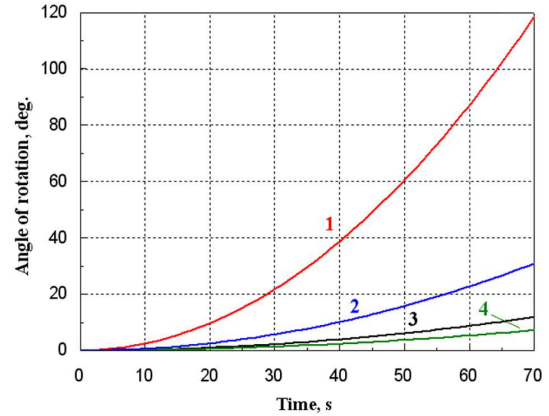

Fig. 2. Time dependence of the silicon plate rotation angle at the constant plate temperature $T = 350$ K and background temperature $T_1 = 300$ K: 1 and 3 – $l_y = 0.1$ mm and 1 mm, respectively, $R_1 = 1$, $R_2 = 0$; 2 and 4 – $l_y = 0.1$ mm and 1 mm, respectively, $R_1 = 1$, R_2 is the reflection coefficient of a polished plate
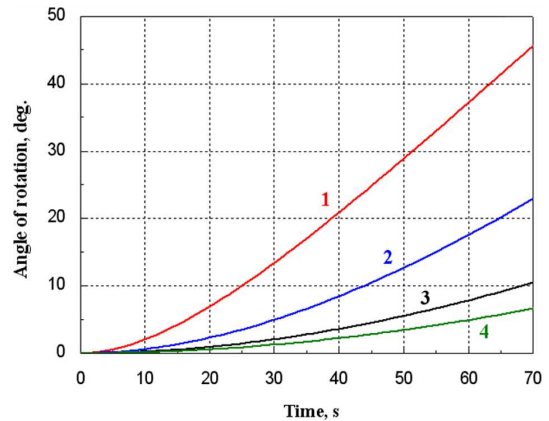

Fig. 3. Time dependences of the silicon plate rotation angle, as the plate temperature decreases. The initial plate temperature $T = 350$ K, the background temperature T_1 is constant $(T_1 = 300$ K). Curves 1–4 are calculated for the same thicknesses l_y and reflection coefficients R_1 and R_2 as for the corresponding curves in Fig. 2

Figure 3 shows the $\varphi(t)$ dependences of heated plates, whose temperature decreases because of their radiative cooling. In this case, the rotation angle dependence on the thickness l_y is also determined by the radiative cooling rate dependence on l_y.

By applying the known well-developed technologies of layer deposition onto a polished silicon plate surface, it is rather easy to provide proper conditions (the reflection coefficient close to unity or zero over

a wide spectral range). One should deposit a metal (or metamaterial) layer in the first case [9, 10] or an absorptive layer in the second case [11, 12]. Layers' thicknesses and, respectively, masses are sufficiently small as compared with those for the considered silicon plate, so they were ignored in our calculations. It should also be noted that the rotation by its own TR can be demonstrated not only by semiconductor plates but by those made of other materials as well.

3. Conclusions

We have studied theoretically a possibility to rotate a thin rectangular silicon plate by the photon recoil of its thermal radiation. The rotational moment in a plate with uniform temperature is due to a distinction between the emittances from different plate areas located symmetrically about the rotation axis. The dependence of the rotation angle on geometric, optical, and thermodynamic parameters of a plate is investigated. It is shown that the rotation angle does not depend on the plate width l_z and increases proportionally with $1/l_x$, as the plate length l_x decreases. The rotation angle depends nontrivially on the plate thickness because of the thickness dependence of the plate emittance. At a preset temperature regime (constant plate temperature of 350 K and background temperature of 300 K), the biggest effect of rotation (118.5° for a time of 70 s) is obtained for a mirror black plate 0.1 mm thick. The model considered can be useful for the development of a thermal motor, whose rotor consists of a set of heated plates with a photon recoil asymmetry.

1. M. Kaasalainen, J. Durech, B.D. Warner, Y.N. Krugly, N.M. Gaftonyuk. Acceleration of the rotation of asteroid 1862 Apollo by radiation torques. *Nature* **446**, 420 (2007).
2. V. Toth, S.G. Turyshev. Thermal recoil force, telemetry, and the Pioneer anomaly. *Phys. Rev. D* **79**, 043011 (2009).
3. S.G. Turyshev, V.T. Toth, G. Kinsella, S.C. Lee, S.M. Lok, J. Ellis. Support for the thermal origin of the Pioneer anomaly. *Phys. Rev. Lett.* **108**, 241101 (2012).
4. V. Jain, J. Gieseler, C. Moritz, C. Dellago, R. Quidant, L. Novotny. Direct measurement of photon recoil from a levitated nanoparticle. *Phys. Rev. Lett.* **116**, 243601 (2016).
5. M. Liu, T. Zentgraf, Y. Liu, G. Bartal, X. Zhang. Light-driven nanoscale plasmonic motors. *Nature Nanotechnology* **5**, 570 (2010).
6. M.T.H. Reid, O.D. Miller, A.G. Polimeridis, A.W. Rodriguez, E.M. Tomlinson, S.G. Johnson. Photon torpedoes and Rytov pinwheels: Integral-equation modeling of non-equilibrium fluctuation-induced forces and torques on nanoparticles. arXiv preprint arXiv:1708.01985 (2017).
7. K.Y. Guga, A.G. Kollyukh, A.I. Liptuga, V.A. Morozhenko, V.I. Pipa. Features of thermal radiation of plane-parallel semiconductor wafers. *Semiconductors* **38**, 507 (2004).
8. O.G. Kollyukh, A.I. Liptuga, V. Morozhenko, V.I. Pipa. Effect of the optical characteristics of semiconductor resonator structures on the amplitudes of their thermal radiation lines. *Optics Communications* **260**, 607 (2006).
9. O.S. Heavens. *Optical Properties of Thin Solid Films* (Dover, 1991).
10. J.-B. Brückner, J. Le Rouzo, L. Escoubas, G. Berginc, O. Calvo-Perez, N. Vukadinovic, F. Flory. Metamaterial filters at optical-infrared frequencies. *Optics Express* **21**, 16992 (2013).
11. J. Lehman, E. Theocharous, G.E.C. Pannell. Gold-black coatings for freestanding pyroelectric detectors. *Measurement Science and Technology* **14**, 916 (2003).
12. I. Mellouki, N. Bennaji, N. Yacoubi. IR characterization of graphite black-coating for cryogenic detectors. *Infrared Physics and Technology* **50**, 58 (2007).

В.І. Піпа, А.І. Ліптуга

ОБЕРТАННЯ ТОНКОЇ НАГРІТОЇ
ПЛАСТИНИ ЗА РАХУНОК ВЛАСНОГО
ТЕПЛОВОГО ВИПРОМІНЮВАННЯ

Р е з ю м е

Представлено результати теоретичного дослідження обертання твердотільної пластини за рахунок власного теплового випромінювання. Обертальний момент пластини з однорідною температурою зумовлено асиметрією фотонної віддачі теплового випромінювання від різних ділянок її широких граней. Розраховано часові залежності кута повороту прямокутної пластини в умовах, коли її температура не змінюється, а також в режимі її радіаційного охолодження. Досліджується залежність кута повороту від геометричних та оптичних параметрів пластини. Показано, що кут повороту нетривіальним чином залежить від товщини пластини через товщинну залежність її випромінювальної здатності. Чисельні розрахунки виконано для кремнієвої пластини. Найбільший ефект обертання (кут повороту 118,5° за час 70 с) отримано для дзеркально-чорної пластини товщиною 0,1 мм при заданій сталій температурі пластини 350 К і температурі фону 300 К. Розглянута модель може бути корисною для розроблення теплових двигунів, ротори яких складаються з набору нагрітих пластин з асиметрією фотонної віддачі.