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THE PROPERTIES OF HEAVY QUARKONIUM STUDIED USING THE NIKIFOROV–UVAROV METHOD FOR STRONGLY COUPLED QUARK-GLUON PLASMA AT FINITE TEMPERATURE

The N-dimensional radial Schrödinger equation with generalized two-body potential is solved, and the energy eigen values are calculated using the Nikiforov–Uvarov (NU) method in the N-dimensional space. The results are applied for the mass spectra of charmonium and bottomonium at finite temperature. The effect of dimensionality number on quarkonium mass is investigated. The dissociation temperature of different states of charmonium and bottomonium are calculated.

Keywords: charmonium, bottomonium, strongly coupled quark-gluon plasma, quarkonium.

1. Introduction

At the critical temperature (T_c), there occurs a phase transition from the confined hadronic phase to a deconfined partonic phase. This phase of deconfined quarks and gluons is called quark-gluon plasma (QGP) [1]. At a temperature near T_c, QGP can be considered as strongly coupled and termed as strongly coupled quark-gluon plasma (SCQGP) [2]. The bound state properties in quark-gluon plasma can be studied using two different approaches. The first one is a direct calculation using QCD [3] and the another one is the use of a potential model [4, 5, 6]. The nonrelativistic interaction potential model can reproduce the experimentally observed mass spectrum of bound states [7, 8]. This approach uses the potential as a function of the relative separation between the particles, and the binding energy can be calculated mathematically by solving the Schrödinger equation. The radial Schrödinger equation and its solution play an important role in many fields of physics such as the high-energy physics, nuclear physics, particle physics, *etc.* The N-dimensional nonrelativistic radial Schrödinger equation (SE) has been solved for the interaction potential for different bound states in SCQGP. For solving the SE, the interaction potential should be known. A confining potential is a mathematical representation of a force that governs the dynamics of the particles in a particular system. Depending on the nature of interaction among the particles, a large number of confining potential has been used. Some of them are Harmonic [9], Cornell [10], Coulomb [11], Coulomb perturbed [12], ring-shaped [13], double-ring shaped [14], Gaussian, modified Gaussian [15], Energy-dependent harmonic [16], *etc.* Heavy quarkonia are considered as hard probes of QGP [17]. The dissociation of heavy quarkonia at finite temperatures could explain the observed quarkonia suppression in heavy ion collisions [18]. There are

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many literature works for solving the Schrödinger equation at finite temperature [19, 20]. In [21], the SE was solved using the modified internal potential to study QGP. In [19], the SE was solved using Funke–Hecke theorem. In [18], the authors solved the SE and studied heavy quarkonium properties using an analytical exact iteration method (AEIM). The aim of the present work is to solve the N-dimensional radial SE with generalized two-body potential at a finite temperature to obtain the energy eigen values using the Nikiforov–Uvarov (NU) method [22]. We try to find out the mass spectra and dissociation temperatures of heavy quarkonium states. In addition, the influence of the dimensionality number has been investigated for the binding energy and mass spectra of heavy quarkonium at finite temperatures.

1.1. A brief description of NU method

Consider a second-order differential equation of the form

$$\Psi''(s)+\frac{\tilde{\tau}(s)}{\sigma(s)}\Psi'(s)+\frac{\tilde{\sigma}(s)}{\sigma^2(s)}\Psi(s)=0, \quad (1)$$

where $\sigma(s)$ and $\tilde{\sigma}(s)$ are polynomials of maximum second degree and $\tilde{\tau}(s)$ is a polynomial of maximum first degree with an appropriate $s=s(r)$ coordinate transformation.

If

$$\Psi(s)=\Phi(s)\chi(s) \quad (2)$$

substituting Eq. (2) in Eq. (1)

$$2\Phi'(s)\chi'(s)+\Phi(s)\chi''(s)+\Phi''(s)\chi(s)+ \\ +\frac{\tilde{\tau}(s)}{\sigma(s)}\Big(\Phi(s)\chi'(s)+\Phi'(s)\chi(s)\Big)+\frac{\tilde{\sigma}(s)}{\sigma^2(s)}\Phi(s)\chi(s)=0.$$

Rearranging the terms, we get

$$\phi(s)\chi''(s)+\Big(2\Phi'(s)+\frac{\tilde{\tau}(s)}{\sigma(s)}\Phi'(s)\Big)\chi'(s)+ \\ +\Big(\Phi''(s)+\frac{\tilde{\tau}(s)}{\sigma(s)}\Phi'(s)+\frac{\tilde{\sigma}(s)}{\sigma^2(s)}\Phi(s)\Big)\chi(s)=0.$$

Dividing throgh out by $\Phi(s)$

$$\chi''(s)+\Big(\frac{2\Phi'(s)}{\Phi(s)}+\frac{\tilde{\tau}(s)}{\sigma(s)}\Big)\chi'(s)+ \\ +\left(\frac{\Phi''(s)}{\Phi(s)}+\frac{\tilde{\tau}(s)}{\sigma(s)}\frac{\Phi'(s)}{\Phi(s)}+\frac{\tilde{\sigma}(s)}{\sigma^2(s)}\right)\chi(s)=0.$$

The coefficient of $\chi'(s)$ is taken in the form $\tau(s)/\sigma(s)$, where $\tau(s)$ is a polynomial of degree atmost one. Then

$$\frac{2\Phi'(s)}{\Phi(s)}+\frac{\tilde{\tau}(s)}{\sigma(s)}=\frac{\tau(s)}{\sigma(s)}. \quad (3)$$

This happens only if

$$\frac{\Phi'(s)}{\Phi(s)}=\frac{\pi(s)}{\sigma(s)}. \quad (4)$$

Therefore,

$$\frac{2\pi(s)}{\sigma(s)}+\frac{\tilde{\tau}(s)}{\sigma(s)}=\frac{\tau(s)}{\sigma(s)}.$$

That is,

$$2\pi(s)+\tilde{\tau}(s)=\tau(s).$$

Thus,

$$\pi(s)=\frac{1}{2}\left(\tau(s)-\tilde{\tau}(s)\right)$$

and

$$\tau(s)=\tilde{\tau}(s)+2\pi(s);\quad \tau'(s)<0. \quad (5)$$

The term $\Phi''(s)/\Phi(s)$ in the coefficient of $\chi(s)$ can be written as

$$\frac{\Phi''(s)}{\Phi(s)}=\left(\frac{\Phi'(s)}{\Phi(s)}\right)'+\left(\frac{\Phi'(s)}{\Phi(s)}\right)^2.$$

Using Eq. (4),

$$\frac{\Phi''(s)}{\Phi(s)}=\left(\frac{\pi(s)}{\sigma(s)}\right)'+\left(\frac{\pi(s)}{\sigma(s)}\right)^2. \quad (6)$$

Now, the coefficient of $\chi(s)$ is transformed into a more suitable form,

$$\frac{\Phi''(s)}{\Phi(s)}+\frac{\tilde{\tau}(s)}{\sigma(S)}\frac{\Phi'(s)}{\Phi(s)}+\frac{\tilde{\sigma}(s)}{\sigma^2(s)}=\frac{\overline{\sigma}(s)}{\sigma^2(s)}. \quad (7)$$

Substituting Eq. (6) in Eq. (7),

$$-\frac{\pi(s)\sigma'(s)}{\sigma^2(s)}+\frac{\pi'(s)}{\sigma(s)}+\frac{\pi^2(s)}{\sigma^2(s)}+\frac{\pi(s)\tilde{\tau}(s)}{\sigma^2(s)}+\frac{\tilde{\sigma}(s)}{\sigma^2(s)}=\frac{\overline{\sigma}(s)}{\sigma^2(s)}.$$

Therefore,

$$\overline{\sigma}(s)=\tilde{\sigma}(s)+\pi^2(s)+\pi(s)\Big(\tilde{\tau}(s)-\sigma(s)\Big)+\pi'(s)\sigma(s).$$

Substituting the right-hand sides of Eq. (3), Eq. (6), and Eq. (7), Eq. (1) reduces to an equation of the hypergeometric type as

$$\chi''(s) + \frac{\tau(s)}{\sigma(s)}\chi'(s) + \frac{\overline{\sigma}(s)}{\sigma^2(s)}\chi(s) = 0, \tag{8}$$

with

$$\sigma(s) = \pi(s)\frac{\Phi(s)}{\Phi'(s)}. \tag{9}$$

As a consequence of the algebraic transformations above, the functional form of Eq. (1) is protected in a systematic way. If the polynomial $\overline{\sigma}(s)$ in Eq. (8) is divisible by $\sigma(s)$, we can write

$$\overline{\sigma}(s) = \lambda\sigma(s),$$

where λ is a constant. Then Eq. (8) is reduced to

$$\sigma(s)\chi''(s) + \tau(s)\chi'(s) + \lambda\chi(s) = 0. \tag{10}$$

To determine the polynomial $\pi(s)$,

$$\overline{\sigma}(s) = \tilde{\sigma}(s) + \pi^2(s) +$$
$$+ \pi(s)(\tilde{\tau}(s) - \sigma(s)) + \pi'(s)\sigma(s) = \lambda\sigma(s).$$

That is,

$$\tilde{\sigma}(s) + \pi^2(s) + \pi(s)(\tilde{\tau}(s) - \sigma(s)) - K\sigma(s) = 0,$$

with

$$K = \lambda - \pi'(s). \tag{11}$$

The solution of this quadratic equation for $\Pi(s)$ gives

$$\pi(s) = \frac{\sigma'(s) - \overline{\tau}(s)}{2} \pm$$
$$\pm\sqrt{\left(\frac{\sigma'(s) - \overline{\tau}(s)}{2}\right)^2 - \overline{\sigma}(s) + K\sigma(s)}, \tag{12}$$

where $\pi(s)$ is a polynomial of the first degree. The values of K in the square root of Eq. (12) can be calculated if the expressions under the square root are square of expressions. This is possible if its discriminant is zero. $\chi(s) = \chi_n(s)$ is a polynomial of the n^{th} degree which satisfies the hypergeometric equation, in order to obtain an eigen value solution through the NU method, a relation between λ and λ_n must be set up which is given by

$$\lambda = \lambda_n = -n\tau'(s) - \frac{n(n-1)}{2}\sigma''(s), \quad n = 0, 1, 2.... \tag{13}$$

$\chi_n(s)$ is a hypergeometric-type function whose polynomial solutions are given by the Rodrigue's relation

$$\chi_n(s) = \frac{B_n}{\rho_n}\frac{d^n}{ds^n}(\sigma''(s)\rho(s)), \tag{14}$$

where B_n is a normalization constant and $\rho(s)$ is the weight function that satisfies the equation

$$\frac{d}{ds}\omega(s) = \frac{\tau(s)}{\sigma(s)}\omega(s); \quad \omega(s) = \sigma(s)\rho(s). \tag{15}$$

1.2. Solution of the N-dimensional Schrödinger equation at finite temperature

The SE for two-particles interacting via a spherically symmetric potential $\phi(r)$ in the N- dimensional space is given by

$$\left[\frac{d^2}{dr^2} + \frac{N-1}{r}\frac{d}{dr} - \frac{l(l+N-2)}{r^2} + \right.$$
$$\left. + 2\mu(E_{nl} - \Phi(r))\right]\Psi(r) = 0, \tag{16}$$

where $\phi(r)$ is given by [23]

$$\phi(r) = \frac{Q}{4\pi r}\left(\cos(k_i r) + \frac{1}{\sqrt{3}}\sin(k_i r)\right)e^{-k_r r}. \tag{17}$$

Now, the N-dimensional SE can be written as

$$\left[\frac{d^2}{dr^2} + 2\mu\left[E - \frac{Q}{4\pi r}\left[\left(\cos(k_i r) + \frac{1}{\sqrt{3}}\sin(k_i r)\right)e^{-k_r r}\right] - \right.\right.$$
$$\left.\left. - \frac{l(l+N-2) + \frac{N^2-4N+3}{4}}{2\mu r^2}\right]\right]R(r) = 0 \tag{18}$$

or

$$\left[\frac{d^2}{dr^2} + 2\mu\left(E - \frac{Q}{4\pi}\left[\frac{1}{r} + \left(-k_r + \frac{k_i}{\sqrt{3}}\right) + \right.\right.\right.$$
$$+ \left(\frac{k_r^2}{2} - \frac{k_i k_r}{\sqrt{3}} - \frac{k_i^2}{2}\right)r +$$
$$\left. + \left(-\frac{k_r^3}{6} + \frac{k_i k_r^2}{2\sqrt{3}} - \frac{k_i^2 k_r}{2} - \frac{k_i^3}{6\sqrt{3}}\right)r^2\right] -$$
$$\left.\left. - \frac{l(l+N-2) + \frac{N^2-4N+3}{4}}{2\mu r^2}\right)\right]R(r) = 0. \tag{19}$$

Putting $r = \frac{1}{x}$, the equation becomes

$$\left[\frac{d^2}{dx^2} + \frac{2x}{x^2}\frac{d}{dx} + \frac{2\mu}{x^4}\left(E - \frac{Q}{4\pi}\left[x + \left(-k_r + \frac{k_i}{\sqrt{3}}\right) + \right.\right.\right.$$

$$+\left(\frac{k_r^2}{2}-\frac{k_ik_r}{\sqrt{3}}-\frac{k_i^2}{2}\right)\frac{1}{x}+$$
$$+\left(-\frac{k_r^3}{6}+\frac{k_ik_r^2}{2\sqrt{3}}-\frac{k_i^2k_r}{2}-\frac{k_i^3}{6\sqrt{3}}\right)\frac{1}{x^2}\Bigg]-$$
$$-\frac{l(l+N-2)+\frac{N^2-4N+3}{4}}{2\mu}x^2\Bigg)\Bigg]R(x)=0 \tag{20}$$

or

$$\left[\frac{d^2}{dx^2}+\frac{2x}{x^2}\frac{d}{dx}+\frac{2\mu}{x^4}\left[E-Ax-B-\frac{C}{x}-\frac{D}{x^2}-\right.\right.$$
$$\left.\left.-\frac{l(l+N-2)+\frac{N^2-4N+3}{4}}{2\mu}x^2\right]\right]R(x)=0, \tag{21}$$

where

$$A=\frac{Q}{4\pi},$$
$$B=\frac{Q}{4\pi}\left(-k_r+\frac{k_i}{\sqrt{3}}\right),$$
$$C=\frac{Q}{4\pi}\left(\frac{k_r^2}{2}-\frac{k_ik_r}{\sqrt{3}}-\frac{k_i^2}{2}\right),$$
$$D=\frac{Q}{4\pi}\left(-\frac{k_r^3}{6}+\frac{k_ik_r^2}{2\sqrt{3}}-\frac{k_i^2k_r}{2}-\frac{k_i^3}{6\sqrt{3}}\right).$$

Now, expand $\frac{C}{x}$ and $\frac{D}{x^2}$ in a power series around the characteristic radius r_0 of the meson up to the second-order, Setting $y=x-\delta$, where $\delta=\frac{1}{r_0}$. Now, we expand $\frac{C}{x}$ and $\frac{D}{x^2}$ in a power series around $y=0$ [19] as

$$\frac{C}{x}=C\left(\frac{3}{\delta}-\frac{3x}{\delta^2}+\frac{x^2}{\delta^3}\right) \tag{22}$$

and

$$\frac{D}{x^2}=D\left(\frac{6}{\delta^2}-\frac{8x}{\delta^3}+\frac{3x^2}{\delta^4}\right). \tag{23}$$

Substituting Eq. (22) and (23) in Eq. (21), we get

$$\left[\frac{d^2}{dx^2}+\frac{2x}{x^2}\frac{d}{dx}+\frac{2\mu}{x^4}\left[E-Ax-B-\right.\right.$$
$$-C\left(\frac{3}{\delta}-\frac{3x}{\delta^2}+\frac{x^2}{\delta^3}\right)-D\left(\frac{6}{\delta^2}-\frac{8x}{\delta^3}+\frac{3x^2}{\delta^4}\right)-$$
$$\left.\left.-\frac{l(l+N-2)+\frac{N^2-4N+3}{4}}{2\mu}x^2\right]\right]R(x)=0 \tag{24}$$

or we can write

$$\left[\frac{d^2}{dx^2}+\frac{2x}{x^2}\frac{d}{dx}+\right.$$
$$\left.+\frac{2\mu}{x^4}\left[-D_1+D_2x-D_3x^2\right]\right]R(x)=0, \tag{25}$$

where

$$D_1=-\left[E-B-\frac{3C}{\delta}-\frac{6D}{\delta^2}\right],$$
$$D_2=-A+\frac{3C}{\delta^2}+\frac{8D}{\delta^3},$$
$$D_3=\frac{C}{\delta^3}+\frac{3D}{\delta^4}+\frac{l(l+N-2)+\frac{N^2-4N+3}{4}}{2\mu}.$$

Comparing Eq. (1) and Eq. (25), we get

$$\overline{\tau}(s)=2x,\quad \sigma(s)=x^2,$$
$$\overline{\sigma}(s)=2\mu\left(-D_1+D_2x-D_3x^2\right).$$

By following NU method,

$$\pi=\sqrt{2a-2bx+(K+2c_1)x^2}, \tag{26}$$

where

$$a=2\mu D_1,\quad b=2\mu D_2,\quad c_1=2\mu D_3.$$

The constant K is chosen such as the discriminant of the function under the square root is zero i.e.,

$$\begin{aligned}&\Delta=4b^2-8a(K+2c_1)=0,\\&b^2=2a(K+2c_1),\\&K=\frac{b^2}{2a}-2c_1,\\&\pi=\frac{2a-bx}{\sqrt{2a}}.\end{aligned} \tag{27}$$

Thus,

$$\tau=2x\pm\frac{2(2a-bx)}{\sqrt{2a}}. \tag{28}$$

We choose the positive sign in the above equation for bound state solutions.

$$\tau'=2-\frac{2b}{\sqrt{2a}}. \tag{29}$$

By using Eq. (11), we obtain

$$\lambda=\frac{b^2}{a}-2c_1-\frac{b}{\sqrt{2a}}, \tag{30}$$

$$\lambda_n = -n\left(2 - \frac{2b}{\sqrt{2a}}\right) - n(n-1). \quad (31)$$

From Eq. (13),

$$\lambda = \lambda_n,$$

$$\frac{b^2}{a} - 2c_1 - \frac{b}{\sqrt{2a}} = -n\left(2 - \frac{2b}{\sqrt{2a}}\right) - n(n-1),$$

$$\frac{b^2}{2a} - \frac{b}{\sqrt{2a}}(1+2n) = -n^2 - n + 2c_1,$$

$$\left(\frac{b}{\sqrt{2a}} - \frac{(2n+1)}{2}\right)^2 = \frac{1}{4} + 2c_1,$$

$$\frac{b}{\sqrt{2a}} - \frac{(2n+1)}{2} = \sqrt{\frac{1}{4} + 2c_1},$$

$$\frac{b}{\sqrt{2a}} = \frac{(2n+1)}{2} \pm \sqrt{\frac{1}{4} + 2c_1},$$

$$\frac{b^2}{2a} = \left(\frac{(2n+1)}{2} \pm \sqrt{\frac{1}{4} + 2c_1}\right)^2,$$

$$a = \frac{b^2}{2\left(\frac{(2n+1)}{2} \pm \sqrt{\frac{1}{4} + 2c_1}\right)^2},$$

$$D_1 = \frac{2\mu D_2^2}{\left((2n+1) \pm \sqrt{1 + 8\mu D_3}\right)^2},$$

$$-E + B + \frac{3C}{\delta} + \frac{6D}{\delta^2} = \left(2\mu\left(-A + \frac{3C}{\delta^2} + \frac{8D}{\delta^3}\right)^2\right) \Big/ \left(\left((2n+1) \pm \sqrt{1 + 8\mu\left(\frac{C}{\delta^3} + \frac{3D}{\delta^4} + \frac{l(l+N-2) + \frac{N^2-4N+3}{4}}{2\mu}\right)}\right)^2\right).$$

The energy eigen values at finite temperature in the N-dimensional space can be written as

$$E_{nl}^N = B + \frac{3C}{\delta} + \frac{6D}{\delta^2} - \left(2\mu\left(-A + \frac{3C}{\delta^2} + \frac{8D}{\delta^3}\right)^2\right) \Big/ \left(\left((2n+1) \pm \sqrt{1 + \frac{8\mu C}{\delta^3} + \frac{24\mu D}{\delta^4} + 4\left(l(l+N-2) + \frac{N^2-4N+3}{4}\right)}\right)^2\right). \quad (32)$$

Equation (32) gives the expression for the binding energy of the bound state of quarks.

2. Binding Energy of Quarkonium

Using Eq. (32),

$$E_{n,l}^N = B + \frac{3C}{\delta} + \frac{6D}{\delta^2} - \left\{m\left(-A + \frac{3C}{\delta^2} + \frac{8D}{\delta^3}\right)^2\right\} \Big/ \left((2n+1) \pm \sqrt{1 + \frac{4mC}{\delta^3} + \frac{12mD}{\delta^4} + 4\left(l(l+N-2) + \frac{N^2-4N+3}{4}\right)}\right)^2. \quad (33)$$

Equation (33) gives the expressions for binding the energy of quarkonium.

3. Variation of Binding Energy of Quarkonium with Temperature

By considering the temperature dependence of α_s from the literature [23], we have

$$\alpha_s(T) = \frac{6\pi}{(33-2n_f)\ln(T/\Lambda_T)} \times \left(1 - \frac{3(153-19n_f)}{(33-2n_f)^2}\frac{\ln(2\ln(T/\Lambda_T))}{\ln(T/\Lambda_T)}\right).$$

The binding energy of quarkonia such as charmonium and bottomonium, is calculated in the N-dimensional space for any state at finite temperature using Eq. (33). Here, $m = M_c$ for charmonium and $m = M_b$ for bottomonium mesons. The variation of binding energy with temperature for different states of quarkonia is shown in Fig. 3.

4. Mass Spectra of Quarkonia

For calculating the mass of heavy quarkonium, the following relation is used [22].

$$M = 2M_q + E_{nl}^N. \quad (34)$$

Substituting Eq. (33) into Eq. (34), the mass spectra for different states of heavy quarkonia as a function of the temperature is given by

$$M = 2m + B + \frac{3C}{\delta} + \frac{6D}{\delta^2} - \left\{m\left(-A + \frac{3C}{\delta^2} + \frac{8D}{\delta^3}\right)^2\right\} \Big/ \left((2n+1) \pm \sqrt{1 + \frac{4mC}{\delta^3} + \frac{12mD}{\delta^4} + 4\left(l(l+N-2) + \frac{N^2-4N+3}{4}\right)}\right)^2. \quad (35)$$

The mass spectra of different states of heavy quarkonia as functions of the temperature are plotted in Fig. 6.

5. Dissociation Temperature of Heavy Quarkonium in the N-Dimensional Space

When the binding energy of a quarkonium falls below a certain temperature, that state is weakly bound,

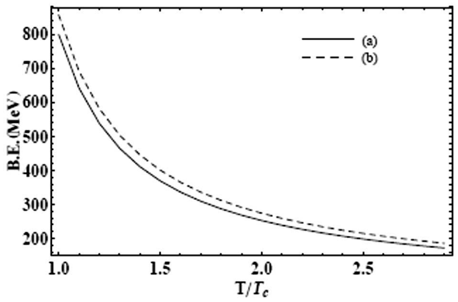


Fig. 1. Variation of J/ψ binding energy on temperature $T(a)$ and variation of χ_c binding energy on temperature T (NU Method) (b)

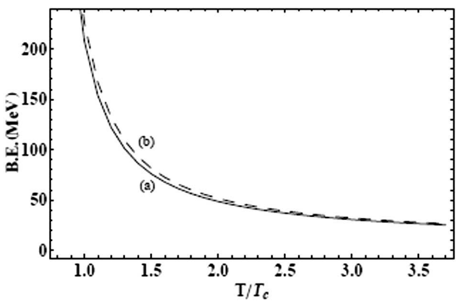
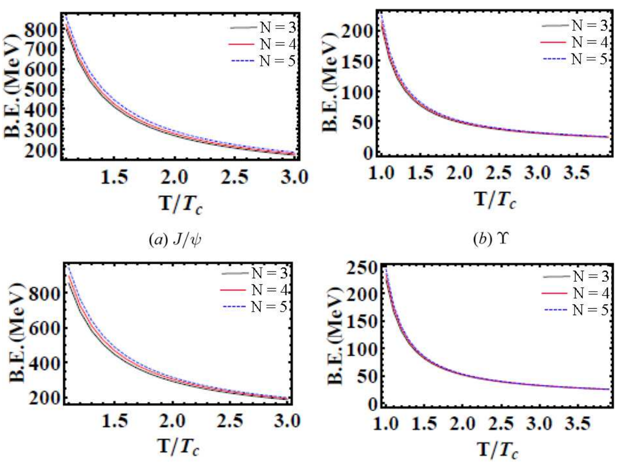
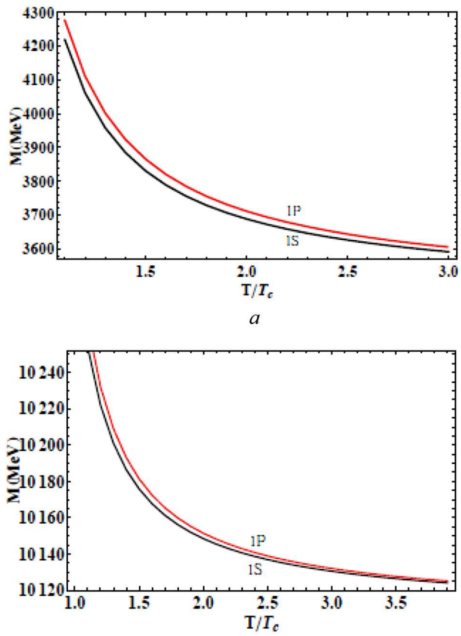


Fig. 2. Variation of Υ binding energy on temperature T (a) and Variation of χ_b binding energy on temperature T (NU method) (b)



(a) J/ψ (b) Υ (c) χ_c (d) χ_b

Fig. 3. Variation of quarkonium binding energy on temperature T for different values of N (NU method)



a

b

Fig. 4. The mass spectra of charmonium and bottomonium as a function of temperature T for 1S and 1P states (NU method)

and it can be destroyed by thermal fluctuations by transferring energy. The dissociation temperature is defined as the smallest temperature where no resonance structure can be observed in reality. The value of binding energy needs not to be reached zero for the dissociation of quarkonia. There are a lot of previous works which have determined the dissociation temperatures for different states of quarkonia. In [24], the dissociation temperatures were calculated from the thermal width. In [4], the upper bound for the dissociation temperature of quarkonium states was found by using the condition $\Gamma(T) \geq 2E_{\text{bin}}(T)$. In [12], the upper bound and lower bound for the dissociation temperatures have been calculated by the conditions $E_{\text{bin}} = T$ and $E_{\text{bin}} = 3T$, respectively. In [10, 25], the condition of vanishing binding energy have used for calculating the dissociation temperature for different states of heavy quarkonium. In this paper, we use the condition $E_{\text{bin}} = T$ to find out the dissociation temperatures for the different states of quarkonium. We have obtained the dissociation temperatures for different states of quarkonium using the NU method and are summarized in Table 1, Table 2, and Table 3.

6. Results and Discussions

Figures 1 and 2 show the variation of the binding energy of heavy quarkonia with the temperature for 1S and 1P states, respectively, where binding energy is calculated using the Nikiforov–Uvarov method. It has been observed that the binding energy becomes weaker with increasing the temperature. This result agrees with the literature [4, 10]. Figure 3 shows the dependence of the binding energy on the dimensionality number (N). It is clear from the figure that the binding energy increases with the dimensionality number. This result also agrees with [10].

Figure 4 shows the mass spectra of heavy quarkonia with the temperature for 1S and 1P states. It is clear from the figure that the mass spectra decrease with increasing temperature for 1S and 1P states. The values of the 1P state are larger than the values of the 1S state. This also agrees with [10].

Figure 5 shows the behavior of the mass spectra of J/ψ state and Υ state with the temperature for two different values of constituent quark mass. The increase of constituent quark mass leads to increasing mass spectra of charmonium for 1S state. Thus, the mass spectra of heavy quarkonia increase with

Table 1. **The dissociation temperature (T_D) with $T_c = 175$ MeV for the quarkonium states using $M_c = 1710$ MeV and $M_b = 5050$ MeV at N = 3**

State	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'
T_D	$1.72T_c$	$1.81T_c$	$1.84T_c$	$1.04T_c$	$1.06T_c$	$1.07T_c$

Table 2. **The dissociation temperature (T_D) with $T_c = 175$ MeV for the quarkonium states using $M_c = 1710$ MeV and $M_b = 5050$ MeV at N = 4**

State	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'
T_D	$1.77T_c$	$1.82T_c$	$1.86T_c$	$1.07T_c$	$1.08T_c$	$1.09T_c$

Table 3. **The dissociation temperature (T_D) with $T_c = 175$ MeV for the quarkonium states using $M_c = 1710$ MeV and $M_b = 5050$ MeV at N = 5**

State	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'
T_D	$1.8T_c$	$1.85T_c$	$1.87T_c$	$1.08T_c$	$1.09T_c$	$1.1T_c$

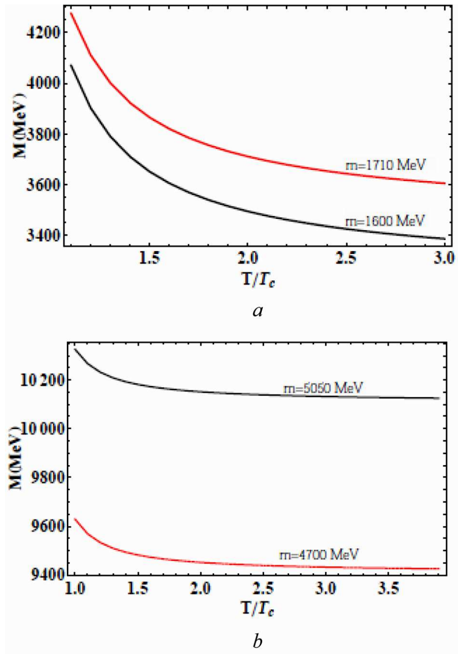


Fig. 5. The mass spectra of charmonium as a function of the temperature T for 1P state for different charm quark mass (*a*) and the mass spectra of bottomonium as a function of the temperature T for 1P state for different bottom quark mass (NU method) (*b*)

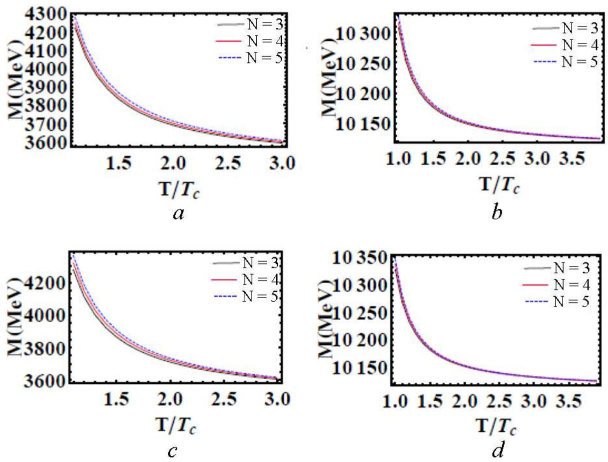


Fig. 6. The mass spectra of charmonium as a function of the temperature T for 1S state for different values of N (*a*), the mass spectra of bottomonium as a function of the temperature T for 1S state for different values of N (*b*), the mass spectra of charmonium as a function of the temperature T for 1P state for different values of N (*c*) and the mass spectra of bottomonium as a function of the temperature T for 1P state for different values of N (NU method) (*d*)

increasing quark mass. This also agrees with the previous findings [11]. Figure 6 shows the variation of mass spectra with dimensionality number. The increase in dimensionality number increases the mass spectra as in [10]. The values of the binding energy and mass spectra of different quarkonium states agree with the previous reports for the lower dimension. As the dimensionality number increases, these values also increase.

In Table 1, we have summarized the dissociation temperatures for different states of $c\bar{c}$ and $b\bar{b}$ at N = 3. It is observed that the dissociation temperature becomes slightly higher when calculated through NU method. The dissociation temperature of J/ψ state agrees with [26]. The dissociation temperature of χ_b agrees with [3]. In Table 2 and Table 3, we have summarized the dissociation temperatures for different states of $c\bar{c}$ and $b\bar{b}$ at N = 4 and N = 5. The above three tables display the effect of dimensionality number on the dissociation temperature of different quarkonium states. It is clear from the tables that the increasing dimensionality number leads to increasing dissociation temperature for different quarkonium states.

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ДОСЛІДЖЕННЯ ВЛАСТИВОСТЕЙ ВАЖКОГО КВАРКОНІЮ МЕТОДОМ НІКІФОРОВА–УВАРОВА ДЛЯ СИЛЬНОВЗАЄМОДІЮЧОЇ КВАРК-ГЛЮОННОЇ ПЛАЗМИ ПРИ СКІНЧЕННІЙ ТЕМПЕРАТУРІ

Отримано розв'язки N-вимірного радіального рівняння Шрьодінгера з узагальненим двочастинковим потенціалом та розраховано власні значення енергії методом Нікіфорова–Уварова в N-вимірному просторі. Результати застосовано для оцінок спектрів мас чармонію та боттомонію при скінченних температурах. Досліджено вплив вимірності на масу кварконію. Розраховано температури дисоціації для різних станів чармонію та боттомонію.

Ключові слова: чармоній, боттомоній, сильновзаємодіюча кварк-глюонна плазма, кварконій.