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POTENTIAL OF THE MODIFIED THOMAS–FERMI METHOD AND ITS ANALYTICAL REPRESENTATION BY THE EXAMPLE OF ^{16}O NUCLEUS INTERACTION WITH ^{56,58,60,62,64}Ni ISOTOPES

Nucleon density distributions and nucleus-nucleus interaction potentials of the ^{16}O nucleus with the ^{56,58,60,62,64}Ni isotopes have been calculated in the framework of the modified Thomas–Fermi method, i.e., considering all terms up to the second order in $\hbar$ in the quasi-classical series expansion of the kinetic energy. Skyrme forces dependent on the nucleon density are used as the nucleus-nucleus interaction. A successful parametrization of the obtained potential has been found, which allowed the latter to be presented in an analytical form.

K e y w o r d s: nucleus-nucleus interaction potential, modified Thomas–Fermi method, nucleon density distribution, repulsive core, analytical representation.

1. Introduction

One of the main tasks of theoretical nuclear physics since its appearance is the study of peculiarities of the interaction between atomic nuclei. In order to calculate such fundamental characteristics of nuclear reactions as the cross-sections of various processes, it is necessary to know the potential energy of nuclear interaction [1–4]. From this point of view, of particular interest is the information on the magnitude and radial dependence of this quantity at small distances between the nuclei. Unfortunately, the potential of nucleus-nucleus interaction, especially its nuclear component, still remains not well determined nowadays.

Generally speaking, the potential of nucleus-nucleus interaction can be qualitatively divided into the nuclear, Coulomb, and centrifugal parts. The properties of the two latter have been studied quite well. But the situation with the nuclear part is much more complicated. A large number of various models are currently used for its approximation [1–25], and the corresponding heights of the barrier in the potential of nucleus-nucleus interaction, which affect the mechanism of nuclear reactions, can differ substantially among those models. For this reason, the information about the nucleus-nucleus interaction potential and the barrier heights is principally important for the proper description of the reaction process.

In this work, among all the methods used to construct the nucleus-nucleus interaction potential, we chose the quasi-classical approach. In this approach, the nucleon- and energy-density distributions are calculated with the use of the modified Thomas–Fermi method with density-dependent Skyrme forces [4, 7, 8, 10, 11, 13–25]. At present, there are a large number of successful Skyrme interaction parametrizations. In this work, we use the SkP parametrization

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[26] and account for all possible terms up to $\hbar^2$ in the quasi-classical series expansion of the kinetic energy. Previous calculations carried out for specific nuclear problems both by us and other authors showed that this is a rather accurate approximation, so we will use it in our further works. Under such conditions, the modified Thomas–Fermi approach with Skyrme forces well describes the distribution of nucleon density, the binding energies, the mean square radii, and many other characteristics of the ground and excited states of atomic nuclei [26–32].

In the modified Thomas–Fermi approximation with Skyrme forces, the nucleus-nucleus potential at small distances between the surfaces of colliding nuclei reveals a potential barrier arising as a result of the Coulomb repulsion and nuclear attraction between the nuclei. At shorter distances between the nuclei, the potential energy gradually increases. However, in the modified Thomas–Fermi approximation with Skyrme forces, the nucleus-nucleus potential has a repulsive core at rather small distances between the nuclei, namely, when the densities of the nuclei overlap significantly [7,10,13,14,17–22]. This repulsive core is associated with a substantial incompressibility of the nuclear matter [13, 14, 19, 22]. Note that the repulsive potential at short distances between the nuclei also exists in the proximity potential [5] and in the microscopic approach [11, 16, 21]. Elastic scattering taking the potential core into account was considered in works [13, 14, 19, 22–25, 35, 36]. The repulsive term in the potential also made it possible to describe the deep subbarrier inhibition of the fusion of heavy nuclei [37–39].

The structure of the paper is as follows. The mathematical apparatus required to implement the selected approach is presented in Sections 2 and 3. Section 4 contains a discussion of the obtained results and the corresponding conclusions.

2. Calculation of the Potential in the Framework of the Modified Thomas–Fermi Method

As was already mentioned, the nucleus-nucleus interaction potential $V(R)$ consists of the nuclear, $V_N(R)$, Coulomb, $V_{\rm COUL}(R)$, and centrifugal, $V_l(R)$, components, which depend on the distance R between the centers of mass of the nuclei:

$$V(R) = V_N(R) + V_{\rm COUL}(R) + V_l(R). \quad (1)$$

For the Coulomb and centrifugal parts, we used commonly known expressions that can be found, e.g., in works [20, 23, 24].

Let us calculate the nuclear part $V_N(R)$ of the interaction potential in the framework of the extended Thomas–Fermi method, namely, considering all terms up to the second order in $\hbar$ in the semiclassical series expansion of the kinetic energy [4, 7, 8, 10, 11, 13–25]. As the nucleus-nucleus interaction, we use the density-dependent Skyrme forces, namely, the SkP parametrization [26]. In our calculations, we deal with the approximation of "frozen" densities, which is quite applicable for energies near the barrier values.

The nucleus-nucleus interaction potential is determined as the difference between the energies of a system of two nuclei, when they are located at a finite, $E_{12}(R)$, and the infinite, $E_{1(2)}$, distance from each other [8, 10],

$$V(R) = E_{12}(R) - (E_1 + E_2). \quad (2)$$

Note that if the distance between the nuclei is infinite, the energy of the system is the sum of binding energies for separate nuclei,

$$E_{12} = \int \epsilon[\rho_{1p}(\mathbf{r}) + \rho_{2p}(\mathbf{r}, R), \rho_{1n}(\mathbf{r}) + \\ + \rho_{2n}(\mathbf{r}, R)]d\mathbf{r}, \quad (3)$$

$$E_{1(2)} = \int \epsilon\left[\rho_{1(2)p}(\mathbf{r}), \rho_{1(2)n}(\mathbf{r})\right] d\mathbf{r}. \quad (4)$$

Here, $\rho_{1(2)n}$ and $\rho_{1(2)p}$ denote the neutron and proton, respectively, densities in nucleus 1(or 2); $\epsilon[\rho_{1(2)p}(\mathbf{r}), \rho_{1(2)n}(\mathbf{r})]$ is the energy density, and R denotes the distance between the centers of mass of the nuclei.

The energy density can be considered as the sum of the kinetic, potential, and Coulomb parts. If Skyrme forces [24–29, 31] are used, its form is well known:

$$\epsilon = \epsilon_{\rm kin} + \epsilon_{\rm pot} + \epsilon_{\rm coul} = \\ = \frac{\hbar^2}{2m}\tau + \frac{1}{2}t_0\left[\left(1+\frac{1}{2}x_0\right)\rho^2 - \left(x_0+\frac{1}{2}\right)(\rho_n^2+\rho_p^2)\right] + \\ + \frac{1}{12}t_3\rho^\alpha\left[\left(1+\frac{1}{2}x_3\right)\rho^2 - \left(x_3+\frac{1}{2}\right)(\rho_n^2+\rho_p^2)\right] + \\ + \frac{1}{4}\left[t_1\left(1+\frac{1}{2}x_1\right) + t_2\left(1+\frac{1}{2}x_2\right)\right]\tau\rho +$$

$$+\frac{1}{4}\left[t_2\left(x_2+\frac{1}{2}\right)-t_1\left(x_1+\frac{1}{2}\right)\right](\tau_n\rho_n+\tau_p\rho_p)+$$
$$+\frac{1}{16}\left[3t_1\left(1+\frac{1}{2}x_1\right)-t_2\left(1+\frac{1}{2}x_2\right)\right](\nabla\rho)^2-$$
$$-\frac{1}{16}\left[3t_1\left(x_1+\frac{1}{2}\right)+t_2\left(x_2+\frac{1}{2}\right)\right]((\nabla\rho_p)^2+(\nabla\rho_n)^2)+$$
$$+\frac{1}{2}W_0[J\nabla\rho+J_n\nabla\rho_n+J_p\nabla\rho_p]+\epsilon_{\text{coul}}. \quad (5)$$

Here, $\epsilon_{\text{kin}}=\frac{\hbar^2}{2m}\tau$ denotes the kinetic energy density (this is the first term in the expression), ϵ_{pot} is the potential energy density, ϵ_{coul} is the Coulomb energy density, $\rho=\rho_n+\rho_p$ is the total nucleon density, and the quantities t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, α, and W_0 are the Skyrme interaction parameters. The terms proportional to t_0 and t_3 correspond to zero-radius forces: the term proportional to t_0 is associated with attraction, whereas the term with t_3 describes the repulsion and increases, as the density of the nuclear matter grows, which prevents the collapse of nuclear systems. The terms proportional to t_1 and t_2 make corrections to the finite radius of the nuclear force action; as the nucleon density grows, the contribution of these members to the total energy increases. The constants x_0, x_1, x_2, and x_3 describe exchange effects and are associated with the spin and isospin asymmetries; and W_0 is the spin-orbit interaction constant.

The kinetic energy density with an accuracy up to the second order in $\hbar$ has the form $\tau=\tau_{\text{TF}}+\tau_2$ [7, 8, 10, 11, 24, 28, 29], where, in turn, $\tau=\tau_n+\tau_p$ is the sum of the densities of the kinetic energies of protons and neutrons. Here [28, 29],

$$\tau_{\text{TF},n(p)}=k\rho_{n(p)}^{5/3} \quad (6)$$

is the kinetic energy density of neutrons (protons) in the Thomas–Fermi approximation, $k=\frac{3}{5}(3\pi^2)^{2/3}$, and τ_2 is the complete expression for the second-order (in $\hbar$) gradient correction,

$$\tau_{2n(p)}=b_1\frac{(\nabla\rho_{n(p)})^2}{\rho_{n(p)}}+b_2\nabla^2\rho_{n(p)}+$$
$$+b_3\frac{\nabla f_{n(p)}\nabla\rho_{n(p)}}{f_{n(p)}}+b_4\rho_{n(p)}\frac{\nabla^2 f_{n(p)}}{f_{n(p)}}+$$
$$+b_5\rho_{n(p)}\frac{(\nabla f_{n(p)})^2}{f_{n(p)}^2}+\frac{b_6}{h_m^2}\rho_{n(p)}\left(\frac{\mathbf{W}_{n(p)}}{f_{n(p)}}\right)^2, \quad (7)$$

where $b_1=1/36$, $b_2=1/3$, $b_3=1/6$, $b_4=1/6$, $b_5=-1/12$, and $b_6=1/2$ are numerical coefficients; $h_m=\hbar^2/2m$; and the last term in formula (7) takes the spin-orbit interaction into account. The following notations were also introduced:

$$\mathbf{W}_{n(p)}=\frac{\delta\varepsilon(r)}{\delta\mathbf{J}_{n(p)(r)}}=\frac{W_0}{2}\nabla(\rho+\rho_{n(p)}), \quad (8)$$

and the quantity $f_{n(p)}$ is expressed via the parameters of Skyrme forces as follows:

$$f_{n(p)}=1+\frac{2m}{\hbar^2}\left[\frac{1}{4}\left[t_1\left(1+\frac{x_1}{2}\right)+t_2\left(1+\frac{x_2}{2}\right)\right]\rho+\right.$$
$$\left.+\frac{1}{4}\left[t_2\left(x_2+\frac{1}{2}\right)-t_1\left(x_1+\frac{1}{2}\right)\right]\rho_{n(p)}\right], \quad (9)$$

where x_1, x_2, t_1, t_2, and W_0 are the Skyrme force constants depending on the parametrization choice. The contribution made by the Thomas–Fermi term dominates, especially in the nucleus bulk, but the gradient corrections begin to play a substantial role at the nucleus surface.

The Coulomb interaction energy E_{coul} can be written in the approximation describing the interaction of two uniformly charged spheres, which was considered in detail in work [40]. Let Q_1 and Q_2 be the charges of the spheres ($Q_{1(2)}=eZ_{1(2)}$), R_1 and R_2 their radii, and R the distance between their centers. Then we consider three cases for the Coulomb term E_{coul}:

1) at $0\le R\le R_1-R_2$,

$$E_{\text{coul}}(R)=\frac{3}{2}\frac{Q_1Q_2}{R_1^3}\left(R_1^2-\frac{1}{3}R^2-\frac{1}{5}R_2^2\right); \quad (10a)$$

2) at $R_1-R_2\le R\le R_1+R_2$,

$$E_{\text{coul}}(R)=\frac{Q_1Q_2}{R}\left\{1-\frac{y^4}{160R_1^3R_2^3}[y^2-6y(R_1+R_2)+\right.$$
$$\left.+30R_1R_2]\right\}, \quad (10b)$$

where $y=R_1+R_2-R$; and

3) at $R_1+R_2\le R$,

$$E_{\text{coul}}(R)=\frac{Q_1Q_2}{R}. \quad (10c)$$

In this paper, we analyzed the interaction of the ^{16}O nucleus with the ^{56,58,60,62,64}Ni isotopes. The

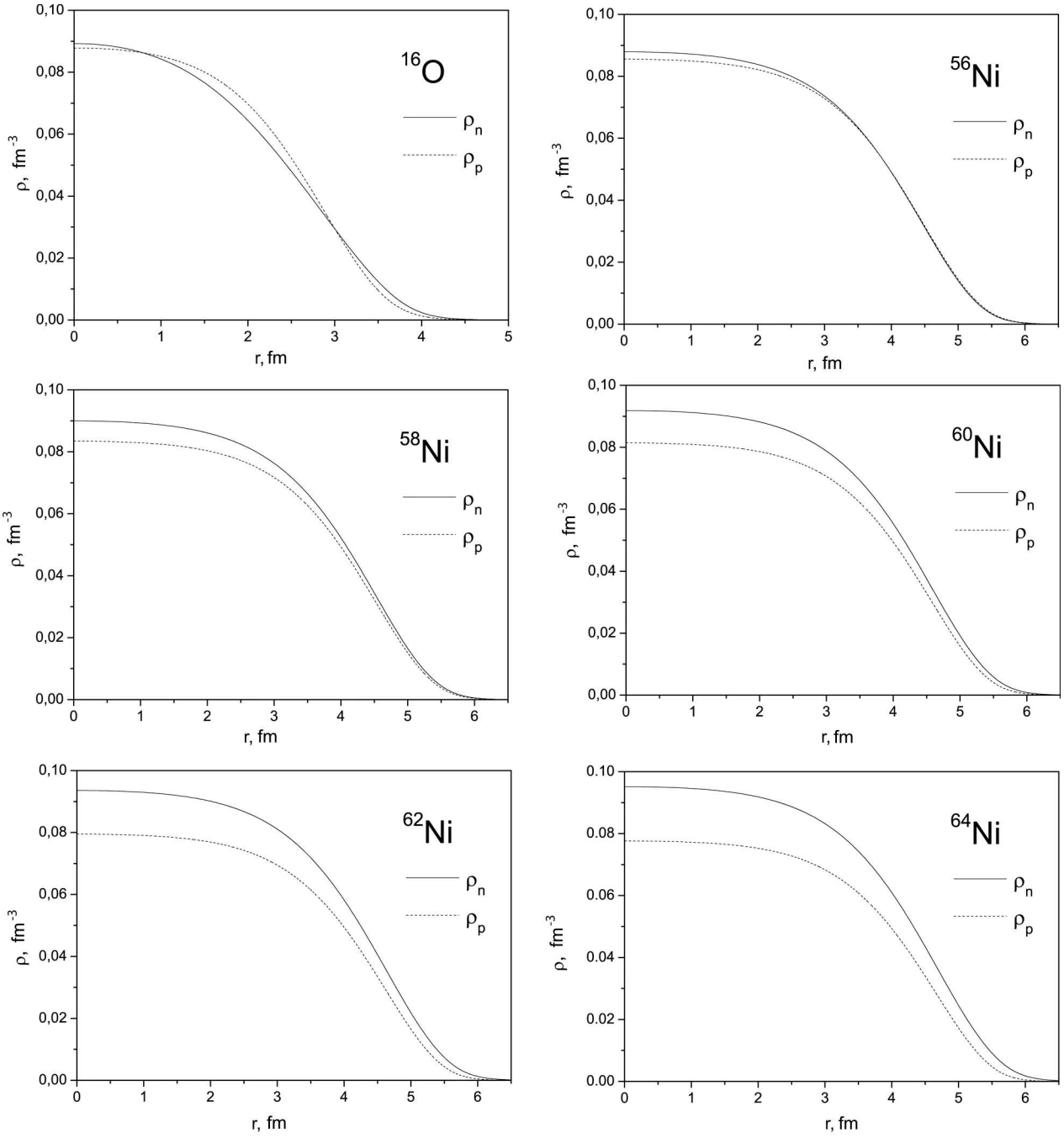

Fig. 1. Nucleon density distributions for the ^{16}O nucleus and ^{56,58,60,62,64}Ni isotopes calculated in the framework of the modified Thomas–Fermi method

nucleus-nucleus interaction potential for those systems was calculated in the framework of the modified Thomas–Fermi approach. For this purpose, it is necessary, first of all, to know the distributions of nucleon densities in the interacting nuclei. We used the nucleon densities obtained in the framework of the

same modified Thomas–Fermi approach with Skyrme forces, for which we applied the SkP parameterization [26]. The distributions of nucleon densities in the ^{16}O nucleus and the ^{56,58,60,62,64}Ni isotopes obtained in this way are shown in Fig. 1.

Knowing the nucleon densities, we obtained an expression for the energy density and calculated the nucleus-nucleus interaction potential in the framework of the modified Thomas–Fermi approach with Skyrme forces, formulas (1)–(9). Figure 2 demonstrates the total interaction potentials obtained for the ^{16}O nucleus and the ^{56,58,60,62,64}Ni isotopes, and Figure 3 exhibits their nuclear parts. The resulting potentials have a quite realistic form with a substantial repulsive core at small distances. A regular isotopic behavior of the presented dependences can also be observed.

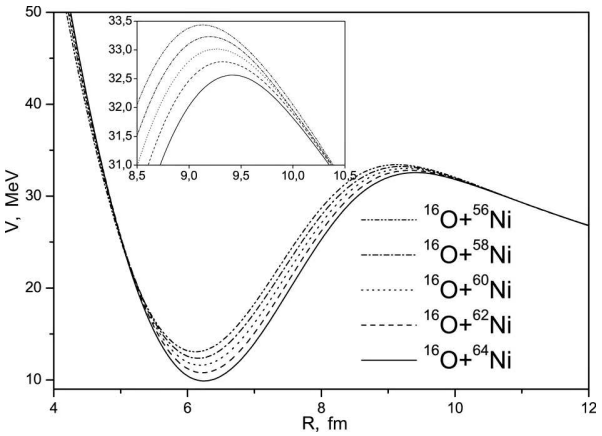

Fig. 2. Total nucleus-nucleus interaction potentials between the ^{16}O nucleus and the ^{56,58,60,62,64}Ni isotopes calculated in the framework of the modified Thomas–Fermi method and considering the Coulomb energy in form (10)

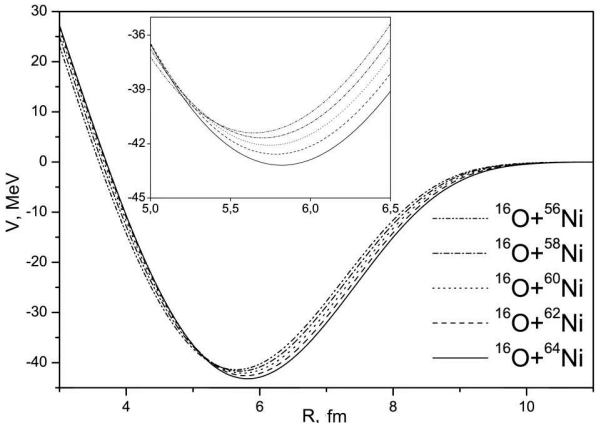

Fig. 3. The nuclear part of the nucleus-nucleus interaction potentials between the ^{16}O nucleus and the ^{56,58,60,62,64}Ni isotopes calculated in the framework of the modified Thomas–Fermi method

3. Analytical Presentation for the Interaction Potential

For the convenience of calculations, it would be very useful to present the obtained potential in the analytical form. At the same time, for an adequate description of the parameters of nuclear reactions, it is very important to account for the presence of the repulsive core, which imposes certain requirements on the form of potential parametrization. Bearing all that in mind, the application of the conventional Woods–Saxon parametrization is not suitable. In order to give a more realistic form to our analytical potential, let us consider an additional term in it, which is similar in its form to the expression for the kinetic energy in the Thomas–Fermi method and should provide the necessary repulsion at short distances. We do this in a certain analogy with what was done in work [20], where we dealt with double-convolution potentials and considerably improved the results obtained in this way. In other words, the general expression for the potential now reads

$$V_{\rm FIT}(R) = V_{\rm WS}(R) + V_{\rm kin}(R). \tag{11}$$

Here, $V_{\rm WS}(R)$ is the well-known Woods–Saxon potential,

$$V_{\rm WS}(R) = \frac{-V_0}{1+\exp\dfrac{(R-R_0)}{d_0}}, \tag{12}$$

and $V_{\rm kin}(R)$ is the kinetic term in the form given by the Thomas–Fermi method. In the latter, the kinetic energy is proportional to $\rho^{5/3}$ [Eq. (6)], so the kinetic term in the potential is approximated using the well-

Parameters of the analytical approximation of the potential for the considered reactions

Reaction	V_0, MeV	r_0, fm	d_0, fm	V_c, MeV	C fm	a fm
^{16}O + ^{56}Ni	50.8174	7.1761	0.6786	19.4348	3.7577	1.0130
^{16}O + ^{58}Ni	51.8171	7.2093	0.6872	19.7405	3.7967	1.0347
^{16}O + ^{60}Ni	52.2549	7.2590	0.6895	19.7649	3.8381	1.0387
^{16}O + ^{62}Ni	52.7468	7.3199	0.6912	19.6698	3.8702	1.0339
^{16}O + ^{64}Ni	53.1439	7.3585	0.6943	19.6309	3.9116	1.0394

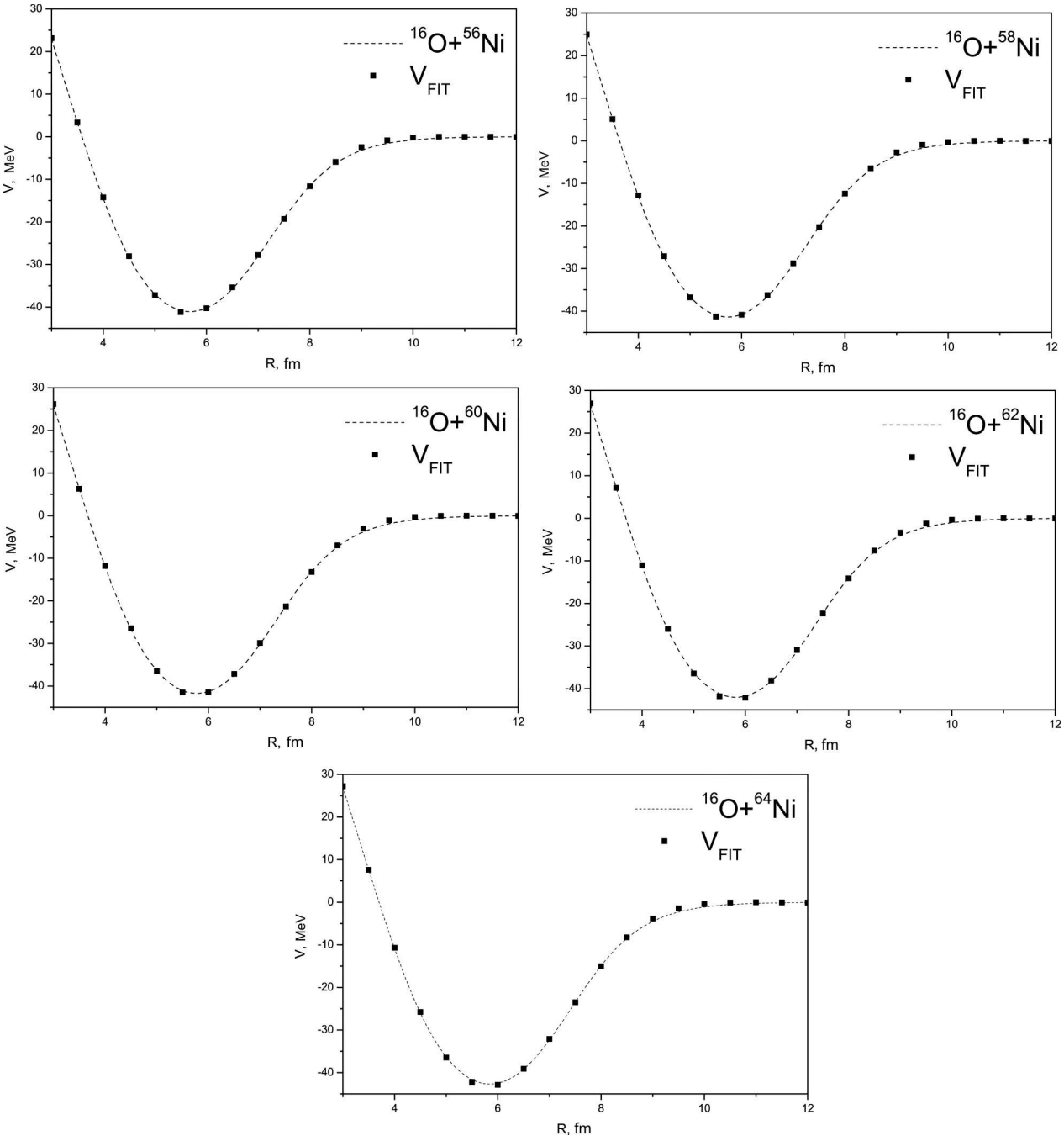

Fig. 4. Nucleus-nucleus interaction potentials for the ^{16}O nucleus and the ^{56,58,60,62,64}Ni isotopes calculated in the framework of the modified Thomas–Fermi method and their analytical approximations $V_{\rm FIT}$ [Eq. (14)]

known Fermi distribution for the density,

$$V_{\rm kin}(R) = \left(\frac{-V_c}{1+e^{\frac{(R-C)}{a}}}\right)^{5/3}. \qquad (13)$$

As a result, our analytical potential acquires the final form

$$V_{\rm FIT}(R) = \frac{-V_0}{1+e^{\frac{(R-R_0)}{d_0}}} + \left(\frac{-V_c}{1+e^{\frac{(R-C)}{a}}}\right)^{5/3}. \qquad (14)$$

Formula (14) contains six fitting parameters: V_0, R_0, d_0, V_c, C, and a. Their values can be determined by minimizing the most accurate description of the realistic potential found in the framework of the modified Thomas–Fermi approach with Skyrme forces. The resulting parameters of the potential for the reactions considered in this work are quoted in Table.

In Fig. 4, we see the approximations of the nuclear parts of interaction potentials between the ^{16}O nucleus and the ^{56,58,60,62,64}Ni isotopes, which were calculated in the framework of the modified Thomas–Fermi approach with Skyrme forces making use of expression (14). The approximation turned out so accurate that the deviations are practically unobservable on the plot scale. Thus, the proposed form of fitting potential very well describes the realistic nucleus-nucleus interaction potential obtained by numerical calculations.

4. Conclusions

The nucleus-nucleus interaction potentials between the ^{16}O nucleus and the ^{56,58,60,62,64}Ni isotopes have been calculated in the framework of the modified Thomas–Fermi approach with density-dependent Skyrme forces. The corresponding nucleon densities are obtained in the framework of the same approach. The SkP parametrization [26] is used for Skyrme forces. The obtained potentials contain a repulsive core, which is important, in particular, when calculating the elastic scattering cross-sections. A successful parametrization of the nucleus-nucleus interaction potential is found, which well describes the magnitude and form of the potential in the modified Thomas–Fermi approach with density-dependent Skyrme forces. This parametrization can be used to calculate specific parameters of nuclear reactions [36].

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ПОТЕНЦІАЛ МОДИФІКОВАНОГО МЕТОДУ ТОМАСА–ФЕРМІ ТА ЙОГО АНАЛІТИЧНЕ ПРЕДСТАВЛЕННЯ НА ПРИКЛАДІ ВЗАЄМОДІЇ ^{16}O ТА ІЗОТОПІВ ^{56,58,60,62,64}Ni

Густини розподілу нуклонів і потенціали ядро-ядерної взаємодії для ядра ^{16}O та ізотопів ^{56,58,60,62,64}Ni було розраховано в рамках модифікованого методу Томаса–Фермі, з урахуванням усіх доданків до членів другого порядку по $\hbar$ у квазикласичному розкладі кінетичної енергії. В ролі нуклон-нуклонної взаємодії використовувалися сили Скірма, залежні від густини нуклонів. Для одержаного потенціалу знайдено вдалу параметризацію, що дозволяє представити його в аналітичній формі.

Ключові слова: потенціал ядро-ядерної взаємодії, модифікований метод Томаса–Фермі, розподіл густини нуклонів, кор відштовхування, аналітичне представлення.