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ON GOLDSTONE FIELDS WITH SPIN HIGHER THAN 1/2 [1]

We review the properties of 3d non-linear models of vector and vector-spinor Goldstone fields associated with the spontaneous breaking of certain higher-spin counterparts of supersymmetry (so-called Hietarinta algebras), whose Lagrangians are of the Volkov–Akulov type. At the quadratic order, these Lagrangians contain, respectively, the Chern–Simons and Rarita–Schwinger terms. The vector Goldstone model has a propagating degree of freedom which, in a decoupling limit, is a quartic Galileon scalar field (similar to those appearing in models of modified gravity). On the other hand, the vector-spinor goldstino retains the gauge symmetry of the Rarita–Schwinger action and eventually reduces to the latter by a non-linear field redefinition. We thus find that, in three space-time dimensions, the free Rarita–Schwinger action is invariant under a hidden rigid symmetry generated by fermionic vector-spinor operators and acting non-linearly on the Rarita–Schwinger goldstino.

Keywords: higher-spin symmetries, spontaneous symmetry breaking, non-linear realizations.

1. Introduction

The spontaneous breaking of symmetries plays an important role in various fields of physics. The Goldstone theorem [1] states that if a rigid (or global) symmetry is broken spontaneously in a certain field theory, there always appear massless fields, called Nambu–Goldstone fields [1, 2] whose number is associated with the number of broken symmetries. The most known examples are the spontaneous breaking of internal symmetries such as $U(1)$, whose associated Goldstone fields are space-time scalars [1] and the spontaneous breaking of the global supersymmetry generating a massless spin-1/2 field [3, 4] called Volkov–Akulov goldstino. When a local (gauge) symmetry is broken, the Nambu–Goldstone fields are effectively absorbed by corresponding gauge fields which become massive due to the Brout–Englert–Higgs mechanism [5, 6]. The first description of this effect in supergravity was given in [7, 8].

The Volkov–Akulov model is the first example of the appearance of Nambu–Goldstone fields carrying a non-zero spin due to the spontaneous breaking of supersymmetry, which is a non-trivial generalization of the space-time (Poincaré) symmetry. One can then ask a natural question whether Nambu–Goldstone fields with spins s higher than $1/2$ can also exist? Such fields should be associated with the breaking of symmetries whose generators transform as rank-s tensors under the Lorentz group [2].

One possible example might be the spontaneous breaking of Poincaré translations which would give rise to spin-1 goldstones. In this context, one may wonder, if models of massive gravity (see, e.g., [9–12] for a review and references) could be regarded as effective theories arising due to a mechanism similar to the Brout–Englert–Higgs one associated with the spontaneous breaking of the diffeomorphism invariance. Nambu–Goldstone fields of else higher spins might arise as a result of the spontaneous breaking of higher-spin symmetries which underly the gauge theories of higher-spin fields. Higher-spin field theories

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[2] Strictly speaking, as the single characteristic of the angular momentum of a field, the notion of spin is an attribute of four-dimensional theories. However, as is often adopted in the higher-spin literature for any space-time dimension, we loosely call symmetric tensor fields $A_{a_1\ldots a_s}$ of rank s as integer spin-s fields and the symmetric-tensor spinor fields $\Psi^{\alpha}_{a_1\ldots a_s}$ as half-integer spin $s+\frac{1}{2}$ fields.

(see, e.g., [13–37] for a review and references) are regarded as possible candidates for a consistent ultraviolet completion of quantum gravity. On the other hand, the string theory, which is the main candidate on the role of quantum gravity, has an infinite tower of higher-spin fields of increasing mass proportional to the square root of the string tension. These higher-spin fields are indispensable for the renormalizability (or even finiteness) of the string theory. For quite awhile, there has been conjectured [38–41] that the string theory might be a spontaneously broken phase of the underlying higher-spin gauge theory, with a string tension generated via the Brout–Englert–Higgs effect. However, so far a specific realization of this assumption has not been carried out because of a very high complexity of the problem related, in particular, to the fact that the consistent higher-spin gauge theories are based on infinite-dimensional higher-spin algebras. So, the questions such as how to break higher-spin (HS) symmetries, are they broken spontaneously, and what is the mechanism remain to be answered.

In such a situation, one can try to ask a bit simpler question, namely, can one construct consistent non-linear models of higher-spin Nambu–Goldstone fields associated with the spontaneous breaking of HS symmetries? Such models would describe, in a universal way, the result of the spontaneous HS symmetry breaking without the knowledge of a physical mechanism that leads to this breaking. To address this question, we chose, in [42], a simplified set-up based on simpler but yet non-trivial finite HS algebras (similar to supersymmetry) constructed in [43]. The D-dimensional Hietarinta algebras have the following generic structure:

$$\begin{aligned}
&\{Q_\alpha^{a_1\ldots a_n}, Q_\beta^{b_1\ldots b_m}\} = f_{\alpha\beta}^{a_1\ldots a_n, b_1\ldots b_m, c} P_c,\\
&[S^{a_1\ldots a_p}, S^{b_1\ldots b_q}] = f^{a_1\ldots a_n, b_1\ldots b_m, c} P_c,\\
&[Q_\alpha^{a_1\ldots a_n}, P_b] = 0, \quad [S^{b_1\ldots b_q}, P_a] = 0, \quad [Q, S] = 0,
\end{aligned} \tag{1}$$

where $a, b, c = 0, 1 \ldots, D-1$ are vector indices, α, β are spinor indices, $Q_\alpha^{a_1\ldots a_n}$ are fermionic tensor-spinor generators, $S^{a_1\ldots a_p}$ are bosonic tensor generators, and P_c is the translation generator. The generators are transformed under certain representations of the Lorentz group $S = SO(1, D-1)$. The structure constants $f_{\alpha\beta}^{a_1\ldots a_n, b_1\ldots b_m, c}$ and $f^{a_1\ldots a_n, b_1\ldots b_m, c}$ are $SO(1, D-1)$-invariant and constructed with the use of the Minkowski metric, Levi-Civita tensor, and gamma matrices. The list of the Hietarinta algebras can be further extended by algebras with commuting tensor-spinor generators forming a "bosonic supersymmetry" (see, e.g., [44, 45]).

Algebras (1) are finite-dimensional higher-spin algebras. This distinguishes them from the more familiar infinite-dimensional higher-spin algebras in which the (anti)-commutators of higher-spin generators involve the generators carrying yet higher spins

$$[T_{s_1}, T_{s_2}\} = T_{s_1+s_2-2} + T_{s_1+s_2-4} + \ldots + T_{|s_1-s_2|+2}.$$

At least in some cases, the Hietarinta algebras may be obtained by Inönü–Wigner contractions of infinite HS algebras.

In this contribution, I will review the results of [42]. We will use the Hietarinta algebras to construct (á la Volkov–Akulov) non-linear models for associated Nambu–Goldstone fields. Though these Lagrangians appeared already in [43], we do not aware of any study of these models, as far as their structure and physical consistency are concerned. We will see that even in the simplest $3d$ cases of spin-1 and spin-3/2 goldstones (to which we restrict our consideration), the models have interesting and peculiar features.

2. Volkov–Alulov Goldstino Model

To set-up the stage, let us review the Volkov–Akulov construction of a non-linear Lagrangian for a spin-1/2 goldstino [3, 4] describing a result of the spontaneous breaking of the $N = 1$, $D = 3$ supersymmetry. The latter has the form

$$\begin{aligned}
&\{Q_\alpha, Q_\beta\} = 2\,(\Gamma^a C^{-1})_{\alpha\beta}\, P_a,\\
&[Q_\alpha, P_a] = 0,
\end{aligned} \tag{2}$$

where now $a = 0, 1, 2$, $\alpha = 1, 2$, Q_α are Grassmann-odd Majorana spinor generators, $\Gamma_\alpha^{a\,\beta}$ are the Dirac matrices in the Majorana representation, $C^{\alpha\beta} = -C^{\beta\alpha}$ is the charge conjugation matrix, and $C^{-1}_{\alpha\beta}$ is its inverse.

The Volkov–Akulov construction works as follows. First, one identifies the Cartan one-form which is invariant under transformations generated by (2)

$$\begin{aligned}
&\Omega = -ig^{-1}dg = E^a P_a + E^\alpha Q_\alpha,\\
&g = e^{ix^a P_a} e^{i\theta^\alpha Q_\alpha},
\end{aligned} \tag{3}$$

where

$$E^a = dx^a + i\bar\theta\Gamma^a d\theta, \quad E^\alpha = d\theta, \tag{4}$$

x^a is a coordinate of the $3D$ Minkowski space, and θ^α is its Grassmann-odd counterpart.

Components (4) of the one-form are invariant under the following rigid supersymmetry transformations with parameter ϵ^α of the coordinates:

$$\theta'^\alpha = \theta^\alpha + \epsilon^\alpha, \quad x'^a = x^a - i\bar\epsilon\Gamma^a\theta. \tag{5}$$

The second step is to promote the Grassmann-odd coordinate θ^α to a spinorial field $\psi^\alpha(x)$ such that the Cartan-form components (4) take the form

$$E^a = dx^a + if^{-2}\bar\psi\Gamma^a d\psi(x), \quad E^\alpha = f^{-1}d\psi^\alpha(x), \tag{6}$$

where we have introduced the parameter f of mass-dimension $m^{\frac{3}{2}}$ characterizing the supersymmetry breaking scale. Now, due to (5), the infinitesimal supersymmetry variation of $\psi^\alpha(x)$ has the form

$$\delta\psi^\alpha = \epsilon^\alpha + if^{-2}(\bar\epsilon\Gamma^b\psi)\partial_b\psi^\alpha. \tag{7}$$

The above expression tells us that ψ^α is transformed as a typical Nambu–Goldstone field, whose variation is characterized by the constant shift ϵ^α and the nonlinear second term. The Volkov–Akulov action for ψ^α which is invariant under (7) has the form

$$S = \frac{f^2}{6}\int E^a\wedge E^b\wedge E^c\,\varepsilon_{abc} = -f^2\int d^3x\,\det E_b^a, \tag{8}$$

where

$$E_b^a = \delta_b^a + if^{-2}\bar\psi\Gamma^a\partial_b\psi.$$

Substituting this expression into the action, we get its explicit form

$$S_{1/2} = \int d^3x\Bigg(-f^2 - i\,\psi\,\Gamma^a\,\partial_a\psi + \frac{f^{-2}}{2}\,\varepsilon^{abc}\,(\psi\psi)\,\partial_a\psi\,\Gamma_b\,\partial_c\psi\Bigg), \tag{9}$$

where $\psi\,\psi \equiv \psi^\alpha\,C_{\alpha\beta}\,\psi^\beta \equiv \psi^\alpha\,\psi_\alpha$.

The second term in this action is the standard massless Majorana field Lagrangian, and the third term describes the quartic self-interaction of ψ^α required by the supersymmetry of the action. In what follows, we will skip the first constant term in the action, which, however, becomes important, when the goldstino couples to gravity, since it gives a positive contribution to the cosmological constant (see [46,47] for a review and references).

3. Spin-1 Goldstone and Galileon

Let us now use the Volkov–Akulov prescription to construct a $3d$ spin-1 Goldstone model which describes the spontaneous breaking of the simplest Hietarinta symmetry based on the algebra of the following form:

$$[M^{ab}\,, S^c] = i(\eta^{bc}\,S^a - \eta^{ac}\,S^b), \tag{10}$$

$$[S^a, S^b] = 2i\,\varepsilon^{abc}P_c, \quad [S^a, P_b] = 0, \tag{11}$$

where M^{ab} are generators of the Lorentz group $SO(1,2)$, and P_a are Poincaré translations, while S^a is another vector generator whose commutator closes on the translations, resembling a bosonic counterpart of the supersymmetry algebra (2). This similarity allows us to apply the Volkov–Akulov prescription. We associate a vector field $A_a(x)$ to the generators S^a and construct the Cartan one-form

$$E^a = dx^a + f^{-2}\,\varepsilon^{abc}A_b(x)\,dA_c(x) = $$
$$= dx^m(\delta_m^a + f^{-2}\,\varepsilon^{abc}A_b(x)\,\partial_m A_c(x)) \equiv dx^m E_m^a \tag{12}$$

which is invariant under the following transformations:

$$x'^a = x^a - f^{-2}\,\varepsilon^{abc}\,s_b\,A_c(x), \quad A'_a(x') = A_a(x) + s_a, \tag{13}$$

where s_a is a constant vector parameter. The infinitesimal transformation of the form of the Goldstone field $A_a(x)$,

$$\delta A_a(x) = s_a + f^{-2}\,\varepsilon^{dbc}s_b\,A_c(x)\,\partial_d A_a(x), \tag{14}$$

shows that it is transformed non-linearly under the symmetry. Note that the commutator of two variations closes on the translation of A_a in accordance with the structure of algebra (11),

$$[\delta_2\,, \delta_1]\,A_a(x) = 2\,f^{-2}\,\varepsilon^{dbc}\,(s_b^1\,s_c^2)\,\partial_d A_a(x). \tag{15}$$

The action which is invariant under (14) has the following form:

$$S_1 = -f^2\int d^3x\,(\det E_d^a - 1) = \int d^3x\Big(\varepsilon^{abc}A_a\,\partial_b A_c - \frac{f^{-2}}{2}\,\varepsilon^{abc}\varepsilon^{def}\,A_a\,A_d\,\partial_e A_b\,\partial_f A_c\Big). \tag{16}$$

Note that the sixth-order term in A_a (and its derivatives) in (16) vanishes, while the quadratic term is the Abelian Chern–Simons action. The latter is known to be invariant under the $U(1)$ variations $\delta A_a = \partial_a \lambda(x)$ of the vector field. This gauge symmetry makes the Chern–Simons field non-dynamical. As such, there arises a natural question whether the gauge invariance of the Chern–Simons action extends to the quartic term of (16). The answer to this question is negative, i.e. the non-linear action is not gauge-invariant, and, hence, the field $A_a(x)$ propagates a degree of freedom. To identify this degree of freedom, we apply the Stückelberg trick to make the action (16) formally gauge-invariant, by replacing $A_a(x)$ with

$$\hat{A}_a(x) = A_a(x) - f^{\frac{1}{2}} \partial_a \varphi(x), \tag{17}$$

where $\varphi(x)$ is the Stückelberg field ensuring the gauge invariance of $\hat{A}_a(x)$ under the variations $\delta A_a = \partial_a \lambda$ and $\delta\varphi = f^{-\frac{1}{2}}\lambda$. The factor $f^{\frac{1}{2}}$ in (17) is chosen to perform an appropriate limit $f \to \infty$ in the action (16) for $\hat{A}_a$ such that the terms with $A_a(x)$ and $\varphi(x)$ decouple from each other. In this decoupling limit, the action reduces to

$$S(\hat{A}_a)|_{f\to\infty} = \int d^3x \Bigg(\varepsilon^{abc} A_a\, \partial_b A_c - - \frac{1}{2}\, \varepsilon^{abc}\varepsilon^{\mathrm{def}}\, \partial_a\varphi\, \partial_d\varphi\, \partial_e\partial_b\varphi\, \partial_f\partial_c\varphi\Bigg). \tag{18}$$

Somewhat surprisingly, one recognizes, in the quartic φ-dependent term of this action, the quartic Lagrangian for a so-called Galileon scalar field [48] which appears in modified theories of gravity. The Galileon Lagrangian modulo total derivatives can be written in different forms

$$\mathcal{L}(\varphi) = \frac{1}{2}\, \varphi\, \varepsilon^{abc}\varepsilon^{def}\, \partial_a\partial_d\varphi\, \partial_e\partial_b\varphi\, \partial_f\partial_c\varphi = = -3\,\varphi\, \det(\partial_a\partial^b\varphi) = = -\frac{1}{2}\varphi\, ((\Box\varphi)^3 - 3\,\Box\varphi\, \partial_a\partial^b\varphi\, \partial_b\partial^a\varphi + + 2\, \partial_a\partial^b\varphi\, \partial_b\partial^c\varphi\, \partial_c\partial^a\varphi)). \tag{19}$$

The Lagrangian is invariant under the Galilean symmetry with $\phi \to \phi + c + c_a x^a$, where c and c_a are constant parameters.

Note that, in the model under consideration, the propagating (Galileon) scalar mode of the field $A_a(x)$ appears only at the non-linear level. It does not have the conventional quadratic kinetic term, since the quadratic part of action (16) is gauge-invariant. Note also that, in spite of the presence of higher derivatives, the Galileon Lagrangians contain only second-order time derivatives of $\varphi(x)$ and, hence, do not have Ostrogradskii ghosts, though they may still be haunted by instabilities. In this respect, it is instructive to look at the Hamiltonian which can be derived from action (16) (see [42] for more details on the Hamiltonian analysis)

$$\mathcal{H} = 6\, f^{-2}\, (A_0)^2 \det \partial_i A_j, \tag{20}$$

where A_0 is the time component of the Goldstone field, and $i, j = 1, 2$ are spatial indices. The corresponding Galileon Hamiltonian is

$$\mathcal{H}^{\text{Galileon}} = 6\, (\partial_0\varphi)^2 \det \partial_i\partial_j\varphi. \tag{21}$$

Because of the presence of the determinants, the Hamiltonians are not positive definite and not bounded from below. Hence, fluctuations around certain zero-energy configurations of the fields may have negative energies leading to instabilities. This is known for more general Galileon Lagrangians (see, e.g., [49, 50]). Some of these instabilities can be removed by imposing the appropriate initial and boundary conditions on the fields.

4. Spin-3/2 Goldstino

We now pass to the consideration of a $3d$ model describing a fermionic goldstino field of spin-3/2. It is based on the following Hietarinta algebra:

$$\{Q^a_\alpha, Q^b_\beta\} = 2\, C_{\alpha\beta}\, \varepsilon^{abc} P_c, \quad [Q^a_\alpha, P_b] = 0, \tag{22}$$

where Q^a_α are Grassmann-odd tensor-spinor generators to which we associate the spin-3/2 goldstino field $\chi^\alpha_a(x)$.

The Cartan one-form which we will use for the construction of the action is

$$E^a = dx^d E^a_d = dx^d\, (\delta^a_d + i f^{-2}\, \varepsilon^{abc}\, \chi_b\, \partial_d \chi_c). \tag{23}$$

It is invariant under the variations

$$\delta\chi^\alpha_a(x) = \zeta^\alpha_a + i\, f^{-2} \varepsilon^{dbc}\, \zeta^\beta_b\, \chi_{c\beta}(x)\, \partial_d \chi^\alpha_a(x) \tag{24}$$

generated by (22), where ζ^α_a is a constant parameter. Note that, as for the spin-1/2 and the spin-1 case, the commutator of two variations (24) closes on the translations off the mass shell, i.e. without the use of the equations of motion:

$$[\delta_2, \delta_1]\, \chi^\alpha_a = \xi^d\, \partial_d \chi^\alpha_a, \quad \xi^d = 2\, i\, f^{-2}\, \varepsilon^{dbc}\, \zeta^1_b\, \zeta^2_c. \tag{25}$$

The spin-3/2 goldstino action has the form

$$\begin{aligned} S_{3/2} &= -f^2 \int d^3x\, (\det E^a_d - 1) = \\ &= \int d^3x\, \big(i\, \varepsilon^{abc}\, \chi_a\, \partial_b \chi_c + \\ &+ \frac{f^{-2}}{4}\, \varepsilon^{abc} \varepsilon^{dfg}\, (\chi_a\, \partial_{[b}\chi_{|c)})\, (\chi_{d|}\, \partial_{f]}\chi_g) + \\ &+ \frac{i\, f^{-4}}{6}\, \varepsilon^{a'b'c'}\, (\varepsilon^{abc}\varepsilon^{def} - \varepsilon^{abf}\varepsilon^{dec}) \times \\ &\times (\chi_c\, \partial_{a'}\chi_f)\, (\chi_a\, \partial_{b'}\chi_b)\, (\chi_d\, \partial_{c'}\chi_e)\big)\,. \end{aligned} \tag{26}$$

We see that the quadratic term in (26) is the action for a $D = 3$ Rarita–Schwinger spin-$\frac{3}{2}$ free massless field which is invariant under conventional (linearized) local supersymmetry variations $\delta\chi^\alpha_a = \partial_a \epsilon^\alpha(x)$. Because of this local symmetry, the Rarita–Schwinger field is non-dynamical in the three-dimensional space-time, as well as the Chern–Simons vector field. Now, the question is whether the non-linear action (26) retains this local symmetry, probably modified by non-linear terms? In contrast to the spin-1 case, it turns out that now the answer is positive. Implicitly, the gauge transformation of the field χ^α_a under which action (26) is invariant looks as follows:

$$\begin{aligned} &\delta\chi^\alpha_a + \frac{if^{-2}}{3}\, \varepsilon^{dfg}\, \partial_a \Big(\chi^\alpha_d\, (\chi_f\, \delta\chi_g)\Big) + \\ &+ i\, f^{-2}\, \varepsilon^{dfg}\, (\delta\chi_d\, \partial_a \chi_f)\, \chi^\alpha_g = \partial_a \epsilon^\alpha. \end{aligned} \tag{27}$$

From this relation, by an iteration procedure which exploits the Grassmann-odd nature of χ^α_a, one can get the gauge variation of χ_a to all orders in f^{-2}:

$$\begin{aligned} \delta\chi^\alpha_a &= \partial_a \Big(\epsilon^\alpha - \frac{if^{-2}}{3}\, \varepsilon^{dfg}\, \chi^\alpha_d\, (\chi_f\, \partial_g \epsilon)\Big) - \\ &- if^{-2}\, \varepsilon^{dfg}\, (\partial_d \epsilon\, \partial_a \chi_f)\, \chi^\alpha_g + \mathcal{O}(f^{-4}). \end{aligned} \tag{28}$$

Moreover, it turns that the non-linear field redefinition

$$\hat{\chi}^\alpha_a = \chi^\alpha_a + \frac{if^{-2}}{3}\, \varepsilon^{dfg}\, \chi^\alpha_d\, (\chi_f\, \partial_a \chi_g) \tag{29}$$

reduces action (26) to the manifestly gauge-invariant quadratic action for the Rarita–Schwinger field $\hat{\chi}^\alpha_a$

$$S_{RS} = i \int d^3x\, \varepsilon^{abc}\, \hat{\chi}_a\, \partial_b \hat{\chi}_c. \tag{30}$$

Note that relation (29) is invertible. Using the iteration procedure, one can express χ_a in terms of $\hat{\chi}_a$.

We have thus learned that, due to the fact that action (26) is invariant under the rigid spin-3/2 supersymmetry variations of the goldstino χ_a (24), relation (30) is also non-manifestly invariant under this symmetry with the corresponding variation of $\hat{\chi}_a$ derived from (29) having the following form:

$$\begin{aligned} &\delta\hat{\chi}^\alpha_a = \zeta^\alpha_a + i\, f^{-2}\, \varepsilon^{dbc}\, (\zeta_b\, \hat{\chi}_c)\, \partial_d \hat{\chi}^\alpha_a + \frac{if^{-2}}{3} \times \\ &\times \varepsilon^{dbc} \Big((\hat{\chi}_b\, \partial_a \hat{\chi}_c)\, \zeta^\alpha_d + (\zeta_b\, \partial_a \hat{\chi}_c) \hat{\chi}^\alpha_d\Big) + \mathcal{O}(f^{-4}), \end{aligned} \tag{31}$$

$$[\delta_2, \delta_1]\, \hat{\chi}^\alpha_a \equiv \xi^d\, \partial_d \hat{\chi}^\alpha_a, \quad \xi^d = 2\, i\, f^{-2}\, \varepsilon^{dbc}\, \zeta^1_b\, \zeta^2_c.$$

Therefore, the 3*d* Rarita–Schwinger field is the non-dynamical goldstino associated with the spontaneous breaking of the Hietarinta symmetry (22).

5. Conclusion

The simplest examples of the spontaneous breaking of Hietarinta symmetries and the corresponding Goldstone models turn out to be peculiar non-linear generalizations of the Chern–Simons and Rarita–Schwinger Lagrangians.

The Chern–Simons goldstone propagates a scalar mode which is a Galileon field that appears in modified theories of gravity. It would be of interest to consider couplings of this spin-1 Goldstone to a 3d bi-gravity theory which is invariant under the local symmetry associated with algebra (10) and (11) and to study the generation of the graviton field mass due to this coupling. In [51], it was noticed that algebra (11) is dual to the so-called Maxwell algebra [52, 53] in three space-time dimensions. The latter has the commutation relations

$$[P_a, P_b] = 2i\epsilon_{abc} S^c, \quad [S_a, P_b] = 0. \tag{32}$$

We see that, in (32), the roles of P_a and S_a get interchanged with respect to (11), but the algebras are formally identical. What is changed is the physical meaning of fields associated with these generators. The Chern–Simons Maxwell gravity model based on (32) was constructed and studied in [54–56], while its Hietarinta counterpart and its higher-spin generalizations have been considered in [42,51]. These actions are a natural basis for studying the effects of the local symmetry breaking in these models due to the presence of spin-1 Nambu–Goldstone fields.

In the spin-3/2 case, the spontaneous breaking of the rigid spin-3/2 Hietaring symmetry retains a local symmetry of the Rarita–Schwinger Lagrangian. The non-linear spin-3/2 goldstino Lagrangian reduces to the quadratic Rarita–Schwinger Lagrangian upon a non-linear field redefinition. This causes the 3d spin-3/2 goldstino be a non-propagating field. It would be of interest to couple the spin-3/2 goldstino to conventional 3d (super)gravity and hypergravity dealing with spin-2 and spin-5/2 gauge fields [57–63], and to study properties of these models.

It would be also of interest to generalize the construction reviewed in this contribution to higher-dimensional models of higher-spin Nambu–Goldstone fields. In this respect, the recent results on the study of algebraic structures involving Hietarinta-like algebras (see, e.g., [64–66]) might be useful.

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ПОЛЯ ГОЛДСТОУНА ЗІ СПІНОМ, ВИЩИМ ЗА 1/2

Р е з ю м е

Ми розглядаємо властивості 3d-нелінійних моделей векторних та векторно-спінорних полів Голдстоуна, пов'язаних зі спонтанним порушенням деяких вищих спінових аналогів суперсиметрії (так звані алгебри Гітарінти), лагранжіани яких мають тип Волкова–Акулова. У квадратичному порядку ці лагранжіани містять, відповідно, члени Черна–Саймонса і Раріти–Швінгера. Векторна модель Голдстоуна має еволюціонуючу ступінь вільності, яка на межі роз'єднання являє собою скалярне поле галілеона (подібне до тих, що з'являються в моделях модифікованої гравітації). З іншого боку, вектор-спінорне голдстіно зберігає калібрувальну симетрію дії Раріти–Швінгера і, зрештою, зводиться до останнього шляхом нелінійного переозначення поля. Таким чином, ми знаходимо, що в трьох просторово-часових вимірах вільна дія Раріти–Швінгера є інваріантною при перетвореннях прихованої симетрії, що породжується ферміонними векторно-спінорними операторами і діє нелінійно на голдстіно Раріти–Швінгера.