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DIVERGENCES IN THE EFFECTIVE LOOP INTERACTION OF THE CHERN–SIMONS BOSONS WITH LEPTONS. THE UNITARY GAUGE CASE

We will consider the extension of the Standard Model (SM) with Chern–Simons type interaction. This extension has a new vector massive boson (Chern–Simons bosons). There is no direct interaction between the Chern–Simons bosons and fermions of the SM. Using only three-particle dimension-4 interaction of the Chern–Simons bosons with vector bosons of the SM, we consider effective loop interaction of a new vector boson with leptons. We consider the renormalizability of this loop interaction and conclude that, for the computation of loop diagrams in the unitary gauge, we can not eliminate the divergences in the effective interaction of the Chern–Simons bosons with leptons.

K e y w o r d s: beyond the Standard Model, extensions of gauge sector, Chern–Simons theories.

1. Introduction

Despite the success of the Standard Model (SM) [1] in describing numerous collider experiments, SM is not a complete theory, because it cannot explain phenomena such as active neutrino oscillation (see, e.g., [2–4]), baryon asymmetry of the Universe (see, e.g., [5–7]), dark matter (see, e.g., [8–10]). In all likelihood, the SM needs to be extended to include new particles and new interactions.

It may turn out that SM has a hidden sector with many new particles and some of these particles are not relevant to solving problems of the SM. We do not detect new particles, because they are quite heavy or light but very feebly interact with particles of the SM. If the new particles are quite heavy and can not be produced at current accelerators, we hope for that they will be detected at more powerful future accelerators such as FCC [11, 12]. But if the new particles are light, they can be found at existing accelerators nowadays, see, e.g., [13–15], in the intensity frontier experiments such as MATHUSLA [16], FACET [17], FASER [18, 19], SHiP [20, 21], NA62 [22–24], DUNE [25, 26], LHCb [27], *etc.*

We do not know what type of particles of the new physics they will be. They can be scalar [28–30], pseudoscalar (axionlike) [31–34], fermion [35–38], or vector (dark photons) [39–41] particles, see reviews [16, 21]. In this paper, we consider the extension of the Standard Model with Chern–Simons type interaction with a new massive vector bosons (Chern–Simons bosons or CS bosons in the follow-

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ing). The Chern–Simons interactions appear in various theoretical models, including extra dimensions and the string theory, see, e.g., [42–47]. The minimal gauge-invariant Lagrangian of the interaction of the CS bosons with SM particles has a form of 6-dimension operators [21, 48]:

$$\mathcal{L}_1 = \frac{C_Y}{\Lambda_Y^2} X_\mu (\mathfrak{D}_\nu H)^\dagger H B_{\lambda\rho}\, \epsilon^{\mu\nu\lambda\rho} + \text{h.c.}, \quad (1)$$

$$\mathcal{L}_2 = \frac{C_{SU(2)}}{\Lambda_{SU(2)}^2} X_\mu (\mathfrak{D}_\nu H)^\dagger F_{\lambda\rho} H\, \epsilon^{\mu\nu\lambda\rho} + \text{h.c.}, \quad (2)$$

where Λ_Y, $\Lambda_{SU(2)}$ are new scales of the theory; C_Y, $C_{SU(2)}$ are new dimensionless coupling constants; $\epsilon^{\mu\nu\lambda\rho}$ is the Levi-Civita symbol ($\epsilon^{0123} = +1$); X_μ – CS vector bosons; H – scalar field of the Higgs doublet; $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, $F_{\mu\nu} = -ig \sum_{i=1}^{3} \frac{\tau^i}{2} V_{\mu\nu}^i$ – field strength tensors of the $U_Y(1)$ and $SU_W(2)$ gauge fields of the SM. The gauge-invariance of Lagrangians (1), (2) is achieved, because X_μ is the Stueckelberg field [49, 50].

After the electroweak symmetry breaking, Lagrangians (1), (2) generate (among other terms of higher dimensions) Lagrangian of three fields interactions in the form of 4-dimension operators:

$$\mathcal{L}_{\text{CS}} = c_z \epsilon^{\mu\nu\lambda\rho} X_\mu Z_\nu \partial_\lambda Z_\rho + c_\gamma \epsilon^{\mu\nu\lambda\rho} X_\mu Z_\nu \partial_\lambda A_\rho + \\ + \left\{ c_w \epsilon^{\mu\nu\lambda\rho} X_\mu W_\nu^- \partial_\lambda W_\rho^+ + \text{h.c.} \right\}, \quad (3)$$

where A_μ is the electromagnetic field; $W_\mu^\pm$ and Z_μ are fields of the weak interaction; and c_z, c_γ, c_w are some dimensionless independent coefficients. Coefficients c_z and c_γ are real, but c_w can be complex. As one can see, there is no direct interaction of the CS vector boson X_μ with fields of the matter.

If one rewrites the coefficients before operators in (1), (2) as $C_Y/\Lambda_Y^2 = C_1/v^2$ and $C_{SU(2)}/\Lambda_{SU(2)}^2 = C_2/v^2$, where $C_1 = c_1 + \text{i}c_{1\text{i}}$, and $C_2 = c_2 + \text{i}c_{2\text{i}}$ are dimensionless coefficients, then, in a unitary gauge, one can obtain

$$c_z = -c_{1\text{i}} g' + \frac{c_2}{2} g^2, \quad c_\gamma = c_{1\text{i}} g + \frac{c_2}{2} g g', \quad (4)$$

$$c_w = \frac{c_2 + \text{i}c_{2\text{i}}}{2} g^2 \equiv \Theta_{W1} + \text{i}\Theta_{W2}. \quad (5)$$

Effective interaction of the CS bosons with quarks of different flavors was considered in [51–53]. It was shown that, in this case, the divergent part of the loop diagrams (containing only $W^\pm$ bosons) is proportional to a non-diagonal element of the unity matrix V^+V (V is the Cabibbo–Kobayashi–Maskawa matrix) and is removed. Note that the interaction of the CS boson with down-type quarks of different flavors turned out to be dominant, whereas the interaction with up-type quarks of different flavors can be neglected, because it is proportional to $(V^+V)_{u_n,u_m} \times \sum f(m_{d_i}, m_W) \ll 1$, where the summation is performed over the mass of down-type quarks, and $f(m_{d_i}, m_W)$ is almost constant because of $m_{d_i} \ll m_W$.

It is interesting to note that if we take into account that active neutrinos are a superposition of massive neutrino states that are defined by Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [3] then the interaction of the CS bosons will neutrinos will be similar to the case of interaction with quarks. In this case, decays of the CS bosons in neutrinos with different lepton flavors are possible. These decays will be strongly suppressed for the same reason as the interaction of the CS boson with up-type quarks but will not contain divergences.

Consideration of the effective interaction of CS bosons with quarks makes it possible to construct an effective Lagrangian of the interaction of the CS bosons with quarks of different flavors and compute the GeV-scale CS bosons' production in meson decays. At the same time, it was shown that effective interaction of the CS bosons with quarks of the same flavors or with leptons contains divergence. This divergence can not be removed via counterterms of the CS boson interaction with fermions, because the initial Lagrangian does not contain these terms.

The question of the effective interaction of the CS bosons with fermions of the same flavor is very important. Solving this problem will make it possible to find all decay channels of the CS boson and compute the CS boson's lifetime, which, in turn, will allow us to compute the sensitivity region of the intensity frontier experiments for searching the CS boson.

In order to avoid unnecessary complications taking into account the CKM matrix, in this paper, we will consider the effective interaction of the CS bosons only with leptons [1] and compute all corre-

[1] We will consider neutrinos as massless particles of SM, otherwise, we would have to deal with elements of PMNS matrix, which would clutter up the calculations.

sponding loop diagrams (containing $W^{\pm}$, Z bosons, and photons) in the unitary gauge. We will be interested in the processes of decay of the CS boson into a lepton pair $\ell^{+}\ell^{-}$ ($\ell = e, \mu, \tau$) and neutrinos. We want to check whether there will be a cancellation of divergences when accountrng for all corresponding diagrams.

2. Lepton Pair Production in the CS Boson Decays. Direct Computations

In the unitary gauge, to compute the amplitude of the CS boson's decay into leptons, one has to consider the following diagrams presented in Fig. 1, where for the vertex of the CS boson interaction with vector fields (3), we have the following rules, see Fig. 2

$$XWW \qquad -(c_W p - c_W^{*} k)_{\lambda}\epsilon^{\mu\nu\lambda\rho}, \tag{6}$$

$$XZZ \qquad -c_Z p_{\lambda}\epsilon^{\mu\nu\lambda\rho}, \tag{7}$$

$$XZA \qquad -c_{\gamma} p_{\lambda}\epsilon^{\mu\nu\lambda\rho}. \tag{8}$$

It is also necessary to take into account that the diagram with vertex XZZ in Fig. 1, b and Fig. 1, f actually corresponds to two diagrams (line with a derivative from Z can be from the left side or the right side of the diagram).

2.1. Production via XWW interaction

Lepton pair production via XWW interaction is described by the diagrams a and e in Fig. 1. The amplitude of the lepton pair production via diagram e in Fig. 1 is identically equal to zero due to the presence of the Levi-Civita symbol in the vertex of XWW interaction. The amplitude of the lepton pair production in the CS boson decays via diagram a in Fig. 1 is given by

$$M_{fi}^{WW} = \frac{g^2}{2}\,\bar{\ell}\,(p')\,\hat{P}_R I_W^{\bar{\mu}} \hat{P}_L \ell(-p)\,\epsilon_{\bar{\mu}}^{\lambda x}, \tag{9}$$

where g is $SU(2)_L$ coupling, $P_{L(R)} = (1-(+)\gamma^5)/2$,

$$I_W^{\bar{\mu}} = \int \frac{d^4k}{(2\pi)^4}\,\gamma^{\mu} G_{\nu}(-p-k) D^{W}_{\mu\bar{\rho}}(k+q)\times$$
$$\times\left[c_{\omega}(k+q)_{\bar{\lambda}} + c_{\omega}^{*} k_{\bar{\lambda}}\right] D^{W}_{\bar{\nu}\nu}(k)\,\gamma^{\nu}\epsilon^{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}} \tag{10}$$

and

$$G_f(p) = \frac{m_f + \not{p}}{m_f^2 - p^2}, \quad D^{V}_{\mu\nu}(k) = g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{M_V^2} \tag{11}$$

are propagators for vector field V in unitary gauge and for fermion f.

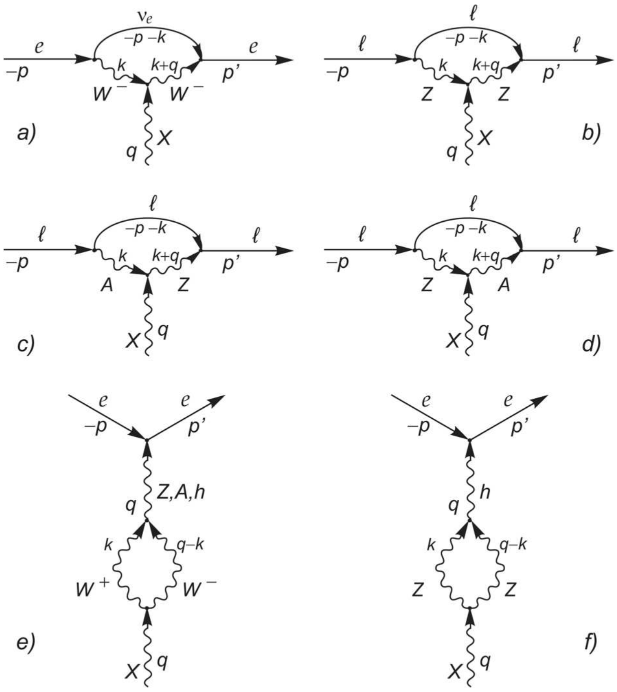

Fig. 1. Diagrams of the CS boson's decay into leptons in the unitary gauge

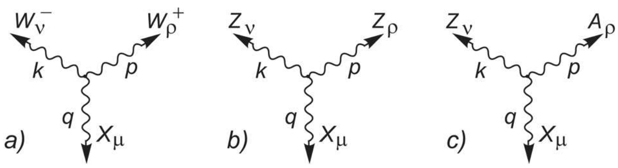

Fig. 2. Vertex diagrams of the interaction of the CS boson with vector fields of the SM in accordance with (3)

After computation using the technique of α (Schwinger) representation, see, e.g., [54], one can get similarly to [53]:

$$I_W^{\bar{\mu}} = \hat{\Lambda}_0^{\nu\ell W}\Big\{\gamma_{\bar{\rho}}(\mathcal{P} - \not{p})\gamma_{\bar{\nu}}\left\{\Theta_W^1(q-2\mathcal{P})_{\bar{\lambda}} + i\Theta_W^2 q_{\bar{\lambda}}\right\} +$$
$$+\frac{q_{\bar{\lambda}}}{M_W^2}\mathcal{P}_{\bar{\nu}}\gamma_{\bar{\rho}}\left[c_w^{*}\left\{2(p\mathcal{P}) + \not{q}\,\mathcal{P} - \not{q}\,\not{p}\right\} - 2\Theta_W^1\mathcal{P}^2 +\right.$$
$$\left. + 2i\Theta_W^2\,\not{p}\,\mathcal{P}\right]\Big\}\epsilon^{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}} + \hat{\Lambda}_1^{\nu\ell W}\Big\{(-i\Theta_W^1)\gamma_{\bar{\rho}}\gamma_{\bar{\lambda}}\gamma_{\bar{\nu}} +$$
$$+\frac{q_{\bar{\lambda}}}{M_W^2}\left[i\frac{c_w^{*}}{2}\gamma_{\bar{\rho}}\not{q}\,\gamma_{\bar{\nu}} - 6i\Theta_W^1\gamma_{\bar{\rho}}\mathcal{P}_{\bar{\nu}} -\right.$$
$$\left. -\Theta_W^2\gamma_{\bar{\rho}}\,\not{p}\gamma_{\bar{\nu}}\right]\Big\}\epsilon^{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}}, \tag{12}$$

where

$$\mathcal{P} = xp + yq \tag{13}$$

and $\hat{\Lambda}_0^{fgV}$, $\hat{\Lambda}_1^{fgV}$ are integral operators acting on some function:

$$\hat{\Lambda}_0^{fgV}u(x,y) = \frac{i\pi^2}{(2\pi)^4}\int\limits_0^1 dx \int\limits_0^{1-x} dy \frac{u(x,y)}{D^{fgV}(x,y)}, \tag{14}$$

$$\hat{\Lambda}_1^{fgV}u(x,y) =$$

$$= \frac{-\pi^2}{(2\pi)^4}\int\limits_0^1 dx \int\limits_0^{1-x} dy\, u(x,y) \ln\frac{\Lambda^2 x}{D^{fgV}(x,y)}, \tag{15}$$

$$D^{fgV}(x,y) = xm_f^2 - x(1-x)m_g^2 +$$

$$+ (1-x)M_V^2 - y(1-x-y)M_X^2, \tag{16}$$

Λ is some constant with dimension of mass (it should be put to infinity in the final result, $\Lambda \to \infty$). So, the divergent part of the loop diagram is hidden in the operator $\hat{\Lambda}_1^{fgV}$.

2.2. Production via XZZ interaction

Lepton pair production via XZZ interaction is described by the diagrams b and f in Fig. 1. The amplitude of the lepton pair production via diagram f in Fig. 1 is identically equal to zero due to the presence of the Levi-Civita symbol in the vertex of XZZ interaction. The amplitude of the lepton pair production via diagram b in Fig. 1 is given by

$$M_{fi}^{ZZ} = \frac{g^2}{4\cos^2\theta_W}\,\bar{\ell}(p')\,\hat{\bar{P}}_Z I_Z^{\bar{\mu}} \hat{P}_Z\,\ell(-p)\epsilon_{\bar{\mu}}^{\lambda x}, \tag{17}$$

where

$$\hat{P}_Z = t_3^\ell(1-\gamma^5) - 2q_\ell \sin^2\theta_W, \tag{18}$$

$$\hat{\bar{P}}_Z = t_3^\ell(1+\gamma^5) - 2q_\ell \sin^2\theta_W \tag{19}$$

and

$$I_Z^{\bar{\mu}} = \int \frac{d^4k}{(2\pi)^4}\gamma^\mu G_\ell(-p-k)D^Z_{\mu\bar{\rho}}(k+q)c_Z(2k+q)_{\bar{\lambda}} \times$$

$$\times D^Z_{\bar{\nu}\nu}(k)\gamma^\nu \epsilon^{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}}, \tag{20}$$

where $G_f(p)$ and $D^V_{\mu\nu}$ were defined in (11), θ_W is Weinberg angle, q_ℓ is electric charge of lepton ℓ in the units of proton charge and t_3^ℓ is the third component of the weak isospin ($+1/2$ for neutrinos and $-1/2$ for electrically charged leptons).

One can get

$$I_Z^{\bar{\mu}} = m_\ell c_Z \hat{\Lambda}_0^{\ell\ell Z}\left\{\gamma_{\bar{\rho}}\gamma_{\bar{\nu}}(q-2\mathcal{P})_{\bar{\lambda}} + \frac{q_{\bar{\lambda}}}{M_Z^2}\mathcal{P}_{\bar{\nu}}\, \not{q}\,\gamma_{\bar{\rho}}\right\} \times$$

$$\times \epsilon^{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}} + c_Z\hat{\Lambda}_0^{\ell\ell Z}\Big\{\gamma_{\bar{\rho}}(\not{\mathcal{P}} - \not{p})\gamma_{\bar{\nu}}(q-2\mathcal{P})_{\bar{\lambda}} +$$

$$+ \frac{q_{\bar{\lambda}}}{M_Z^2}\mathcal{P}_{\bar{\nu}}\gamma_{\bar{\rho}}\left(-2\mathcal{P}^2 + 2(p\mathcal{P}) + \not{q}\,\not{\mathcal{P}} - \not{q}\,\not{p}\right)\Big\}\epsilon^{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}} +$$

$$+ ic_Z\hat{\Lambda}_1^{\ell\ell Z}\left\{-\gamma_{\bar{\rho}}\gamma_{\bar{\lambda}}\gamma_{\bar{\nu}} + \frac{q_{\bar{\lambda}}}{M_Z^2}\left[\frac{\gamma_{\bar{\rho}}\not{q}\gamma_{\bar{\nu}}}{2} - 6\gamma_{\bar{\rho}}\mathcal{P}_{\bar{\nu}}\right]\right\}\epsilon^{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}}, \tag{21}$$

where $\hat{\Lambda}_0^{\ell\ell Z}$, $\hat{\Lambda}_1^{\ell\ell Z}$ are defined in (14) and (15).

2.3. Production via XZA interaction

The amplitude of the lepton pair production via XZA interaction is described by the diagrams c,d in Fig. 1

$$M_{fi}^{AZ} = -\frac{gq_f e}{2\cos\theta_W}\,\bar{\ell}(p')I^{\bar{\mu}}_{ZA}\ell(-p)\epsilon_{\bar{\mu}}^{\lambda x}, \tag{22}$$

where

$$I^{\bar{\mu}}_{ZA} = c_\gamma\int\frac{d^4k}{(2\pi)^4}\Big[\hat{\bar{P}}_Z\gamma^\mu G_\ell(-p-k)D^Z_{\mu\bar{\rho}}(k+q)\times$$

$$\times k_{\bar{\lambda}}D^\gamma_{\bar{\nu}\nu}(k)\gamma^\nu + \gamma^\mu G_\ell(-p-k)D^\gamma_{\mu\bar{\rho}}(k+q)(k+q)_{\bar{\lambda}}\times$$

$$\times D^Z_{\bar{\nu}\nu}(k)\gamma^\nu\hat{P}_Z\Big]\epsilon^{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}} = c_\gamma\hat{\bar{P}}_Z I^{\bar{\mu}}_{ZA1} + c_\gamma I^{\bar{\mu}}_{ZA2}\hat{P}_Z, \tag{23}$$

$\hat{P}_Z$, $\hat{\bar{P}}_Z$ are defined by (19), $G_f(p)$, and $D^V_{\mu\nu}$ were defined in (11).

One can get

$$I^{\bar{\mu}}_{ZA1} = -m_f\left[\hat{\Lambda}_0^{1,\ell Z\gamma}\left\{\gamma_{\bar{\rho}} + \frac{q_{\bar{\rho}}}{M_Z^2}(\not{\mathcal{P}} - \not{q})\right\}\gamma_{\bar{\nu}}\mathcal{P}_{\bar{\lambda}} +\right.$$

$$\left.+\ \hat{\Lambda}_1^{1,\ell Z\gamma}\frac{i}{2}\frac{q_{\bar{\rho}}}{M_Z^2}\gamma_{\bar{\lambda}}\gamma_{\bar{\nu}}\right]\epsilon^{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}} + \hat{\Lambda}_0^{1,\ell Z\gamma}\Big\{\gamma_{\bar{\rho}}(\not{p} - \not{\mathcal{P}})\gamma_{\bar{\nu}} +$$

$$+\frac{q_{\bar{\rho}}}{M_Z^2}\left[(\not{\mathcal{P}}\,\not{p} - \mathcal{P}^2)\gamma_{\bar{\nu}} - \not{q}(\not{p} - \not{\mathcal{P}})\gamma_{\bar{\nu}}\right]\Big\}\mathcal{P}_{\bar{\lambda}}\epsilon^{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}} +$$

$$+\hat{\Lambda}_1^{1,\ell Z\gamma}\frac{i}{2}\Big\{-\gamma_{\bar{\rho}}\gamma_{\bar{\lambda}} + \frac{q_{\bar{\rho}}}{M_Z^2}(\gamma_{\bar{\lambda}}\,\not{p} +$$

$$+\ \not{q}\gamma_{\bar{\lambda}} - 6\mathcal{P}_{\bar{\lambda}})\Big\}\gamma_{\bar{\nu}}\epsilon^{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}} \tag{24}$$

and

$$I^{\bar{\mu}}_{ZA2} = -m_f\left[\hat{\Lambda}_0^{2,\ell Z\gamma}\gamma_{\bar{\rho}}\left\{\gamma_{\bar{\nu}}(\mathcal{P}_{\bar{\lambda}} - q_{\bar{\lambda}}) + \frac{q_{\bar{\lambda}}}{M_Z^2}\,\not{\mathcal{P}}\mathcal{P}_{\bar{\nu}}\right\} +\right.$$

$$\left.+\ \hat{\Lambda}_1^{2,\ell Z\gamma}\frac{i}{2}\gamma_{\bar{\rho}}\frac{q_{\bar{\lambda}}}{M_Z^2}\gamma_{\bar{\nu}}\right]\epsilon^{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}} + \hat{\Lambda}_0^{2,\ell Z\gamma}\gamma_{\bar{\rho}}\Big\{(\not{p} - \not{\mathcal{P}})(\mathcal{P}_{\bar{\lambda}} -$$

$$-\ q_{\bar{\lambda}})\gamma_{\bar{\nu}} + \frac{q_{\bar{\lambda}}}{M_Z^2}(\not{p}\,\not{\mathcal{P}} - \mathcal{P}^2)\mathcal{P}_{\bar{\nu}}\Big\}\epsilon^{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}} +$$

$$+\hat{\Lambda}_1^{2,\ell Z\gamma}\frac{i}{2}\gamma_{\bar{\rho}}\left\{-\gamma_{\bar{\lambda}}\gamma_{\bar{\nu}} + \frac{q_{\bar{\lambda}}}{M_Z^2}(\not{p}\gamma_{\bar{\nu}} - 6\mathcal{P}_{\bar{\nu}})\right\}\epsilon^{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}}, \tag{25}$$

where $\hat{\Lambda}_0^{i,\ell Z\gamma}$, $\hat{\Lambda}_1^{i,\ell Z\gamma}$ $(i = 1,2)$ are integral operators acting on some function

$$\hat{\Lambda}_0^{i,\ell Z\gamma}u(x,y) = i\frac{\pi^2}{(2\pi)^4}\int\limits_0^1 dx \int\limits_0^{1-x} dy \frac{u(x,y)}{D^{i,\ell Z\gamma}(x,y)}, \quad (26)$$

$$\hat{\Lambda}_1^{i,\ell Z\gamma}u(x,y) = $$

$$= -\frac{\pi^2}{(2\pi)^4}\int\limits_0^1 dx \int\limits_0^{1-x} dy\, u(x,y) \ln\frac{\Lambda^2 x}{D^{i,\ell Z\gamma}(x,y)}, \quad (27)$$

and

$$D^{1,\ell Z\gamma}(x,y) = x^2 m_\ell^2 + yM_Z^2 - y(1-x-y)M_X^2, \quad (28)$$

$$D^{2,\ell Z\gamma}(x,y) = x^2 m_\ell^2 + $$

$$+ (1-x-y)M_Z^2 - y(1-x-y)M_X^2. \quad (29)$$

3. Combining Divergent Parts of Diagrams in the Direct Approach

Let us look only at divergent parts of the diagrams (9), (17), (22) to find the possible conditions for cancellation of divergences. It is the parts containing corresponding operators $\hat{\Lambda}_1^j$, see (15), (27). We can see that different $\hat{\Lambda}_1^j$ operators contain equal divergent parts

$$\hat{\Lambda}_1^j = \frac{-\pi^2}{(2\pi)^4}\int\limits_0^1 dx \int\limits_0^{1-x} dy \left[\ln\frac{\Lambda^2}{M_W^2} + \ln\frac{M_W^2 x}{D^j(x,y)}\right] =$$

$$= \frac{-\pi^2}{2(2\pi)^4}\ln\frac{\Lambda^2}{M_W^2} + \hat{\Lambda}_1^{j,\text{finite}} = L + \hat{\Lambda}_1^{j,\text{finite}}. \quad (30)$$

We can write the following useful relation for the operator:

$$\hat{\Lambda}_1^j x = L/3 + \hat{\Lambda}_1^{j,\text{finite}} x, \quad (31)$$

where

$$L = \frac{-\pi^2}{2(2\pi)^4}\ln\frac{\Lambda^2}{M_W^2} \to \infty. \quad (32)$$

The part of $M_{fi} = M_{fi}^{WW} + M_{fi}^{ZZ} + M_{fi}^{AZ}$ proportional to divergent quantity L is given by

$$M_{fi}^{\text{div}} = L\,\bar{\ell}(p')I^\mu \ell(-p)\epsilon_\mu^{\lambda x}, \quad (33)$$

where

$$I^\mu = A\,\gamma_{\bar\rho}\gamma_{\bar\lambda}\gamma_{\bar\nu}\epsilon^{\bar\mu\bar\nu\bar\lambda\bar\rho} + Bq_{\bar\lambda}\gamma_{\bar\rho}\,\not{q}\gamma_{\bar\nu}\epsilon^{\bar\mu\bar\nu\bar\lambda\bar\rho} +$$

$$+ C\,q_{\bar\lambda}\gamma_{\bar\rho}\,\not{p}\gamma_{\bar\nu}\epsilon^{\bar\mu\bar\nu\bar\lambda\bar\rho} + D\,q_{\bar\rho}\gamma_{\bar\lambda}\gamma_{\bar\nu}\epsilon^{\bar\mu\bar\nu\bar\lambda\bar\rho} + E\,q_{\bar\lambda}\gamma_{\bar\rho}p_{\bar\nu}\epsilon^{\bar\mu\bar\nu\bar\lambda\bar\rho}, \quad (34)$$

$$A = \gamma^5\frac{g}{2}\left[\frac{\Theta_W^1 g}{2} + \frac{c_z g}{\cos^2\theta_W}\,t_3^f\left[t_3^f - 2q_f\sin^2\theta_W\right] + \right.$$

$$+ \left.\frac{c_\gamma q_f e}{\cos\theta_W}t_3^f\right] - \frac{g}{2}\left[\frac{\Theta_W^1 g}{2} + \right.$$

$$+ \frac{c_z g}{\cos^2\theta_W}\left(t_3^f\left[t_3^f - 2q_f\sin^2\theta_W\right] + 2q_f^2\sin^4\theta_W\right) +$$

$$+ \left.\frac{c_\gamma q_f e}{\cos\theta_W}\left(t_3^f - 2q_f\sin^2\theta_W\right)\right], \quad (35)$$

$$B = -\gamma^5\frac{g}{4M_W^2}\left[\frac{c_w^* g}{2} + c_z g t_3^f\left[t_3^f - 2q_f\sin^2\theta_W\right] + \right.$$

$$+ \left. c_\gamma q_f e\cos\theta_W t_3^f\right] + \frac{g}{4M_W^2}\left[\frac{c_w^* g}{2} + \right.$$

$$+ c_z g\left(t_3^f\left[t_3^f - 2q_f\sin^2\theta_W\right] + 2q_f^2\sin^4\theta_W\right) +$$

$$+ \left. c_\gamma q_f e\cos\theta_W\left(t_3^f - 2q_f\sin^2\theta_W\right)\right], \quad (36)$$

$$C = i\Theta_W^2\frac{g^2}{4M_W^2}(1-\gamma^5), \quad D = \frac{gq_f t_3^f e}{2\cos\theta_W}\frac{m_f}{M_Z^2}c_\gamma\,\gamma^5, \quad (37)$$

$$E = \gamma^5\frac{g}{3M_W^2}\left[\frac{\Theta_W^1 g}{2} + c_z g t_3^f\left[t_3^f - 2q_f\sin^2\theta_W\right] + \right.$$

$$+ \left. c_\gamma q_f e\cos\theta_W t_3^f\right] - \frac{g}{3M_W^2}\left[\frac{\Theta_W^1 g}{2} + \right.$$

$$+ c_z g\left(t_3^f\left[t_3^f - 2q_f\sin^2\theta_W\right] + 2q_f^2\sin^4\theta_W\right) +$$

$$+ \left. c_\gamma q_f e\cos\theta_W\left(t_3^f - 2q_f\sin^2\theta_W\right)\right]. \quad (38)$$

It would be good to have some relationships between coefficients Θ_W^1, Θ_W^2, c_γ, c_z of the interaction Lagrangian (3) like (4)–(5), under which the divergent terms of the loop diagrams are eliminated, i.e., the condition $A = B = C = D = E = 0$ would be satisfied. But that is not true. Putting $C = D = 0$, one gets $\Theta_W^2 = 0$ and $c_\gamma = 0$. In this case, the system of the other equations $A = B = E = 0$ has only a trivial solution $\Theta_W^1 = c_z = 0$.

Expressions (10), (21), and (23) contain linear divergent integrals. They will change, when the integration variable is shifted by a constant, as when considering the chiral anomaly, see [55]. However, such a change of variables will cause the value of the integrals to change only by a finite value. So, the expressions for A – E will not change.

4. Conclusions

In this paper, we considered the extension of the Standard Model with the Chern–Simons type interaction with a new massive vector particle – Chern–Simons (CS) boson. The Lagrangians of such interaction (1), (2) contain operators of dimension 6, but, after the spontaneous symmetry breaking of the Higgs field, these Lagrangians generate operators of dimension 4 among operators of higher dimensions. Limiting ourselves to considering only the three-particle dimension 4 (possibly, renormalizable) interaction of the Chern–Simons bosons with vector bosons of the SM (3), we considered the effective loop interaction of a new vector boson with leptons.

As was shown in [51–53], the effective loop interaction (with only W-bosons in the loop) of the CS bosons with fermions of different flavors (quarks) does not contain divergences, but interactions with fermions of the same flavors suffer from divergences. But the initial interaction Lagrangian (3) has no direct interaction of the CS boson with fermions; so, we cannot use counterterms to eliminate the divergences. Therefore, the interaction of the CS bosons with vector bosons of the SM (3) is self-consistent, only if divergences in the effective CS boson's interaction with fermions of the same flavors will be eliminated accounting for all appropriate loop diagrams.

Taking interaction of the CS bosons with vector fields of the SM in the form (3), we considered loop diagrams of the CS boson interaction with leptons that include all possible three-particle vertices, see Fig. 1. We concluded that, for the computation of loop diagrams in the unitary gauge, we can not eliminate the divergences in the effective interaction of the Chern–Simons bosons with fermions of the same flavors (leptons), except for the trivial case of the absence of interaction of the Chern–Simons boson with SM particles ($c_w = c_\gamma = c_z = 0$), see (3).

Note that we made these disappointing conclusions after considering the loop interaction of the CS boson with leptons in the unitary gauge. Despite theoretical expectations that calculations in the unitary and R_ξ gauges should lead to the same results, they can give different results for the SM processes in reality [2]. Examples of such situations are directly calculating Feynman diagrams of Higgs boson's loop decays $h \to \gamma\gamma$ and $h \to Z\gamma$ [57]. In particular, it has been shown that the answer depends on whether the process is calculated in the unitary gauge or the process is calculated in the R_ξ gauge, and then we put $\xi \to \infty$. On the other side, direct calculations in the R_ξ gauge give the same results as calculations in the unitary gauge using the dispersion-relation approach with a subtraction determined by the Goldstone boson equivalence theorem, see, e.g., [58]. So, the question of agreement of results of calculations in the unitary and R_ξ gauges is interesting and not trivial.

Thus, for definitive conclusions, the problem of divergences in the CS boson's interaction with fermions of the same flavors requires an additional consideration within the framework of a non-unitary gauge. Suppose, it turns out that using a non-unitary gauge does not help solve the divergence problem. In that case, this will mean that the interaction of the Chern–Simons bosons with fermions of the same flavors must either be considered within the framework of an effective field theory or it will be necessary to supplement the Lagrangian (3) with additional terms. In both cases, additional couplings appear besides c_w, c_γ and c_Z couplings, which will complicate finding manifestations of the CS bosons in experiments.

[2] It seems that, for Abelian theories, such a problem does not exist [56].

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РОЗБІЖНОСТІ У ЕФЕКТИВНІЙ ПЕТЛЬОВІЙ ВЗАЄМОДІЇ БОЗОНА ЧЕРНА–САЙМОНСА З ЛЕПТОНАМИ. ВИПАДОК УНІТАРНОГО КАЛІБРУВАННЯ

Ми розглядаємо розширення Стандартної моделі (СМ) зі взаємодією типу Черна–Саймонса. В цьому розширенні існує новий векторний масивний бозон (бозон Черна–Саймонса), який напряму́ не взаємодіє з ферміонами СМ. Використовуючи лише тричастинкову взаємодію бозонів Черна–Саймонса з векторними бозонами СМ у вигляді операторів розмірності 4, ми розглядаємо ефективну петльову взаємодію нового векторного бозона з лептонами. Ми розглядаємо перенормовність цієї петльової взаємодії та приходимо до висновку, що у випадку розрахунків петльових діаграм в унітарній калібровці не можна позбутись розбіжностей в ефективній взаємодії бозонів Черна–Саймонса з лептонами.

Ключові слова: фізика за межами Стандартної Моделі, розширення калібрувального сектора, теорії Черна–Саймонса.