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V.M. GORKAVENKO,[1] T.V. GORKAVENKO,[1] YU.A. SITENKO,[2,3]
M.S. TSARENKOVA [1]

[1] Taras Shevchenko National University of Kyiv, Ukraine
(64, Volodymyrs'ka Str., Kyiv 01601, Ukraine)
[2] Bogolyubov Institute for Theoretical Physics, Nat. Acad. of Sci. of Ukraine
(14-b, Metrologichna Str., Kyiv 03143, Ukraine)
[3] Donostia International Physics Center
(4, Paseo Manuel de Lardizabal, 20018 Donostia-San Sebastán, Gipuzkoa, Spain)

INDUCED VACUUM ENERGY DENSITY OF QUANTUM CHARGED SCALAR MATTER IN THE BACKGROUND OF AN IMPENETRABLE MAGNETIC TUBE WITH THE NEUMANN BOUNDARY CONDITION

We consider the vacuum polarization of a charged scalar matter field outside the tube with magnetic flux inside. The tube is impenetrable for quantum matter, and the perfectly rigid (Neumann) boundary condition is imposed at its surface. We write expressions for the induced vacuum energy density for the case of a space with arbitrary dimension and for an arbitrary value of the magnetic flux. We do the numerical computation for the case of a half-integer flux value in the London flux units and the (2 + 1)-dimensional space-time. We show that the induced vacuum energy of the charged scalar matter field is induced, if the Compton wavelength of the matter field exceeds the transverse size of the tube considerably. We show that the vacuum energy is periodic in the value of the magnetic flux of the tube, providing a quantum-field-theoretical manifestation of the Aharonov–Bohm effect. The dependencies of the induced vacuum energy upon the distance from the center of the tube for different values of its thickness are obtained. The results are compared to those obtained earlier in the case of the perfectly reflecting (Dirichlet) boundary condition. It is shown that the value of the induced vacuum energy density in the case of the Neumann boundary condition is greater than in the case of the Dirichlet boundary condition.

K e y w o r d s: vacuum polarization, Aharonov–Bohm effect, Casimir effect.

1. Introduction

More than 70 years ago, it was shown by Casimir [1] that the presence of external boundaries leads to changes in the vacuum energy density. First, two perfectly conducting plates at a very tiny distance apart were considered. It was shown that the difference between the vacuum expectation values leads to the emergence of a force of interaction between the plates. Since then, many setups with different shapes of boundaries' and materials have been considered. The boundary manifolds are usually chosen as a disconnected noncompact object (as the infinite plates) or, in other cases, a closed compact object (as a box or a sphere), see, e.g., [2–4]. However, there is another case that is interesting of its own accord: a connected noncompact object (e.g., an infinite tube).

As shown by Aharonov and Bohm in the framework of the first-quantized theory, see [5], the magnetic flux inside a cylindrical tube impenetrable for the matter field can interact with quantum matter outside the tube. The consequences arising from it in the framework of the second-quantized theory are the polarization of the vacuum and the induction of the vacuum current and magnetic flux outside the tube. The effect of the boundary condition at the surface of the impenetrable tube and the magnetic flux inside the tube on the vacuum of the matter field outside the tube has the name of the Casimir–Bohm–Aharonov effect [6]. The boundary condition in this setup affects the matter field outside the tube essentially.

It should be noted the problem of vacuum polarization outside the impenetrable magnetic tube has numerous physical applications. In astrophysics, it can be considered as a model of the cosmic strings, that may have appeared in the early Universe as a result of phase transitions with spontaneous gauge symmetry breaking [7–10]. In condensed matter physics, it can be considered as a model of Abrikosov–Nielsen–Olesen vortex in superconductors of the second group, see, e.g., [11, 12] or as disclinations in nanoconical structures, see, e.g., [13–16].

It should be noted that, initially, the Bohm–Aharonov effect was considered under the assumption that the transverse size of the tube is zero, which corresponds to the singular magnetic vortex, see, e.g., [6, 17–25].

In this paper, we will consider the case of charged scalar matter. In the case of finite transverse size, impenetrable magnetic tube boundary conditions can be generically parametrized with the use of a family of boundary conditions of the Robin type

$$(\cos\theta\,\psi + \sin\theta\, r\partial_r\psi)|_{r_0} = 0. \tag{1}$$

Here, the cases $\theta = 0$ and $\theta = \pi/2$ correspond to the Dirichlet and Neumann boundary conditions, respectively. For the induced vacuum energy, the case of the Dirichlet boundary condition was considered in [26–28]. For the induced vacuum current and magnetic flux, the case of the Dirichlet boundary condition was considered in [29], the case of the Neumann boundary condition was considered in [30], and the general case for the arbitrary value of the parameter θ was considered in [31].

In this paper, we will focus on the vacuum polarization of the charged scalar matter outside the impenetrable finite-thickness magnetic tube with the Neumann boundary condition at its surface.

The paper is organized as follows. In the second section, we provide a general definition of the induced renormalized vacuum energy density for the quantized charged scalar field in the case of d-dimensional space. In the third section, using numerical methods, we compute the value of the induced vacuum energy density in the simplest case of $(2 + 1)$-dimensional space-time, namely outside the impenetrable tube (it is a ring in the 2-dimensional space) of radius r_0 and a magnetic flux inside it. In the fourth section, we summarize and discuss the results.

2. Energy Density

The Lagrangian for a complex scalar field ψ in the $(d + 1)$-dimensional space-time has form

$$\mathcal{L} = (\nabla_\mu\psi)^* (\nabla^\mu\psi) - m^2\psi^*\psi, \tag{2}$$

where ∇_μ is the covariant derivative, and m is the mass of the scalar field. The operator of the quantized charged scalar field is represented in the form

$$\Psi(x^0, \mathbf{x}) = \sum\!\!\!\!\!\!\int_\lambda \frac{1}{\sqrt{2E_\lambda}} \times$$
$$\times \left[e^{-iE_\lambda x^0}\psi_\lambda(\mathbf{x})\, a_\lambda + e^{iE_\lambda x^0}\psi^*_\lambda(\mathbf{x})\, b^\dagger_\lambda\right]. \tag{3}$$

Here, $a^\dagger_\lambda$ and a_λ ($b^\dagger_\lambda$ and b_λ) are the scalar particle (antiparticle) creation and annihilation operators satisfying the commutation relation; λ is the set of parameters (quantum numbers) specifying the state; $E_\lambda = E_{-\lambda} > 0$ is the energy of the state; symbol $\sum\!\!\!\!\!\!\int_\lambda$ denotes the summation over discrete and the integration (with a certain measure) over continuous values of λ; wave functions $\psi_\lambda(\mathbf{x})$ are the solutions to the stationary equation of motion,

$$\left\{-\boldsymbol{\nabla}^2 + m^2\right\}\psi_\lambda(\mathbf{x}) = E^2_\lambda\psi(\mathbf{x}), \tag{4}$$

$\boldsymbol{\nabla}$ is the covariant differential operator in an external (background) field.

We are considering a static background in the form of a cylindrically symmetric gauge flux tube with finite transverse size. The coordinate system is chosen in such a way that the tube is along the z axis. The tube in the 3-dimensional space is obviously generalized to the $(d - 2)$-tube in the d-dimensional space by adding extra $d - 3$ dimensions as longitudinal ones. The covariant derivative is $\nabla_0 = \partial_0$, $\boldsymbol{\nabla} = \boldsymbol{\partial} - \mathrm{i}\tilde{e}\,\mathbf{V}$ with $\tilde{e}$ being the coupling constant of dimension $m^{(3-d)/2}$, and the vector potential possessing only one nonvanishing component is given by

$$V_\varphi = \Phi/2\pi, \tag{5}$$

outside the tube; here, Φ is the value of the gauge flux inside the $(d - 2)$-tube, and φ is the angle in polar (r, φ) coordinates on a plane that is transverse to the tube. The Neumann boundary condition at the surface of the tube ($r = r_0$) is imposed on the scalar field:

$$\partial_r \psi_\lambda|_{r=r_0} = 0, \tag{6}$$

i.e., the surface of the flux tube is a perfectly rigid boundary for the matter field.

The solution of (4) satisfying the boundary condition (6) outside the impenetrable tube of radius r_0 takes the form

$$\psi_{kn\mathbf{p}}(\mathbf{x}) = (2\pi)^{(1-d)/2} e^{i\mathbf{p}\mathbf{x}_{d-2}} e^{in\varphi} \Omega_{|n-\tilde{e}\Phi/2\pi|}(kr, kr_0), \tag{7}$$

where

$$\Omega_\rho(u,v) = \frac{Y'_\rho(v) J_\rho(u) - J'_\rho(v) Y_\rho(u)}{\left[J'^2_\rho(v) + Y'^2_\rho(v)\right]^{1/2}}, \tag{8}$$

and $0 < k < \infty$, $-\infty < p^j < \infty$ ($j = \overline{1, d-2}$), $n \in \mathbb{Z}$ ($\mathbb{Z}$ is the set of integer numbers), $J_\rho(u)$ and $Y_\rho(u)$ are the Bessel functions of order ρ of the first and second kinds, the prime near the function means a derivative with respect to the function argument. Solutions (7) obey the orthonormalization condition

$$\int\limits_{r>r_0} d^d\mathbf{x}\, \psi^*_{kn\mathbf{p}}(\mathbf{x}) \psi_{k'n'\mathbf{p}'}(\mathbf{x}) = \frac{\delta(k-k')}{k}\, \delta_{n,n'}\, \delta^{d-2}(\mathbf{p}-\mathbf{p}'). \tag{9}$$

The standard definition for vacuum energy density is the vacuum expectation value of the time-time component of the energy-momentum tensor

$$\varepsilon = \langle \text{vac}| \left(\partial_0 \Psi^+ \partial_0 \Psi + \partial_0 \Psi \partial_0 \Psi^+\right) |\text{vac}\rangle = \sum\!\!\!\!\!\!\int_\lambda E_\lambda \psi^*_\lambda(\mathbf{x})\, \psi_\lambda(\mathbf{x}). \tag{10}$$

This relation suffers from ultraviolet divergencies. The well-defined quantity is obtained with the help of the regularization and then renormalization procedures, see, e.g., [3].

For the regularization, one can use the zeta-function method, see, e.g., [2, 32, 33], i.e., by inserting the inverse energy in a sufficiently high power

$$\varepsilon_{reg}(s) = \sum\!\!\!\!\!\!\int_\lambda E_\lambda^{-2s} \psi^*_\lambda(\mathbf{x})\, \psi_\lambda(\mathbf{x}). \tag{11}$$

The sums (integrals) are convergent in the case of $\mathrm{Re}\, s > d/2$. Thus, the summation (integration) is performed in this case, and then the result will be analytically continued to the case of $s = -1/2$.

In our case, the magnetic field configuration in the excluded region, irrespective of the number of spatial dimensions, the renormalization procedure is reduced to making one subtraction, namely to subtract the contribution corresponding to the absence of the magnetic flux, see [24].

Now, to compute the vacuum expectation value of the energy density, we have to substitute (7) into (11) and then obtain

$$\varepsilon_{\text{ren}}(s) = (2\pi)^{1-d} \lim_{s\to-1/2} \int d^{d-2}p \int\limits_0^\infty dk\, k \times \left(\mathbf{p}^2 + k^2 + m^2\right)^{-s} \left[S(kr, kr_0, \Phi) - S(kr, kr_0, 0)\right], \tag{12}$$

where

$$S(kr, kr_0, \Phi) = \sum_{n\in\mathbb{Z}} \Omega^2_{|n-\tilde{e}\Phi/2\pi|}(kr, kr_0). \tag{13}$$

Because of the infinite range of summation, the S-function will depend only on the fractional part of the flux

$$F = \frac{\tilde{e}\Phi}{2\pi} - \left[\!\!\left[\frac{\tilde{e}\Phi}{2\pi}\right]\!\!\right], \quad (0 \le F < 1), \tag{14}$$

where $[[u]]$ is the integer part of the quantity u (i.e., the integer which is less than or equal to u). So, we get

$$S(kr, kr_0, F) = \sum_{n=0}^\infty [\Omega^2_{n+F}(kr, kr_0) + \Omega^2_{n+1-F}(kr, kr_0)] \tag{15}$$

and conclude that the induced vacuum energy density (12) depends on F, i.e., it is periodic in the flux Φ with a period equal to $2\pi\tilde{e}^{-1}$. Moreover, the value of the induced vacuum energy density is symmetric under the substitution $F \to 1 - F$.

In the absence of a magnetic flux in the tube, the S-function takes the form

$$S(kr, kr_0, 0) = \Omega^2_0(kr, kr_0) + 2\sum_{n=1}^\infty \Omega^2_n(kr, kr_0). \tag{16}$$

Unfortunately, the computation of the vacuum energy density in the case of the finite-thickness magnetic tube can not be done analytically because of the complicated form of the ψ-function (7) and requires numerical methods.

3. Numerical Evaluation of Energy Density

In this paper, we will take the simplest situation of the $(2+1)$-dimensional space-time and consider the induced vacuum energy density outside the impenetrable tube (it is a ring in the 2-dimensional space) of radius r_0 with the Neumann boundary condition at its edge and with half-integer values of the magnetic flux $F = 1/2$ inside the tube. At this flux value, we expect the maximal effect of vacuum polarization by analogy with the case of singular magnetic vortex, see, e.g., [6, 25]. Based on the results of [26–28] on the computation of the induced vacuum energy density outside the magnetic impenetrable tube with the Dirichlet boundary condition at its edge, we can conclude that we can immediately take $s = -1/2$ in (12).

Let us now briefly discuss the main ideas of numerical calculations. Expression (12) is finite and can be evaluated numerically because of the possibility to restrict the upper limit of integration and summation in it. To make numerical computations in this case, it is better to use, instead of (12), the following relation for the dimensionless quantity

$$r^3\varepsilon_{\rm ren} = \frac{1}{2\pi}\int\limits_0^{z_{\max}} dzz\,\sqrt{z^2+\Big(\frac{mr_0}{\lambda}\Big)^2}\times$$
$$\times\sum_{n=0}^{n_{\max}(z)}[2\Omega^2_{n+1/2}(z,\lambda z)-\Omega^2_n(z,\lambda z)-\Omega^2_{n+1}(z,\lambda z)], \quad (17)$$

where we introduced a dimensionless variables

$$kr = z, \quad \lambda = r_0/r, \quad \lambda\in[0,1]. \quad (18)$$

The case of $\lambda = 1$ corresponds to $r = r_0$, i.e., the point on the boundary of the tube, the case of $\lambda = 0$ corresponds to the point on the infinity $r \to \infty$ or the case of a singular tube ($r_0 = 0$).

The necessary number of terms for the summation ($n_{\max}(z)$) is defined at the fixed value of the parameter z from the condition that the summation result with a high precision did not change with an increase in the number of terms.

For small values of z, we make a direct integration of the function in (17). For large values of z, we use another approach. The integrand function in this case is a quasiperiodic [1] oscillating function (with the change of sign) with the slowly decreasing amplitude with increasing its argument. So, it is convenient to integrate over these periods separately with the right value of the parameter $n_{\max}(z)$. In such a way we get a falling series, each element of which is the value of the integral over the single period of the function. Getting a sufficiently large number of the elements of the series, we stop integrating and interpolate the series forward. The final result of the integration is the sum of the integral for small values of z, the sum of the explicitly counted elements of the series, and the sum of interpolated series. If the contribution to the overall integration result from the sum of the interpolated series is a few percent or less, then the computation result can be considered reliable. In the next step, we compute $r^3\varepsilon_{\rm ren}$ (17) for different values of the parameter λ and interpolate the obtained results.

It should be noted that, in the case of a singular magnetic vortex, the analytic expressions for the induced vacuum energy density can be obtained [23, 25]. For the case of a $(2+1)$-dimensional space-time and a half-integer magnetic flux value $F = 1/2$, it is expressed in terms of the Macdonald function $K_\rho(u)$ and modified Struve function $L_\rho(u)$

$$r^3\varepsilon^{\rm sing}_{\rm ren} = \frac{x^3}{3\pi^2}\Bigg\{\frac{\pi}{2}+\frac{K_0(2x)}{2x}-\Big(1-\frac{1}{2x^2}\Big)K_1(2x)-$$
$$-\pi x\,[K_0(2x)L_{-1}(2x)+K_1(2x)L_0(2x)]\Bigg\}, \quad (19)$$

where $x = mr$.

The result of our computation for the induced vacuum energy density outside the impenetrable magnetic tube with the Neumann boundary condition at its edge is presented in Fig. 1 as a function of the dimensionless distance from the center of the tube (mr) for the different values of the dimensionless tube radius (mr_0). For the comparison, we demonstrate also the induced vacuum energy density for the case of a singular magnetic vortex.

It is of interest also to compare the obtained induced vacuum energy density with the case of vacuum polarization outside an impenetrable magnetic tube with perfectly reflecting (Dirichlet) boundary condition at its edge. The results of the comparison are presented in Fig. 2.

4. Summary

We obtained a general relation for the computation of the vacuum polarization of the quantized charged

[1] Value of the period slowly decreases, as the function argument increases.

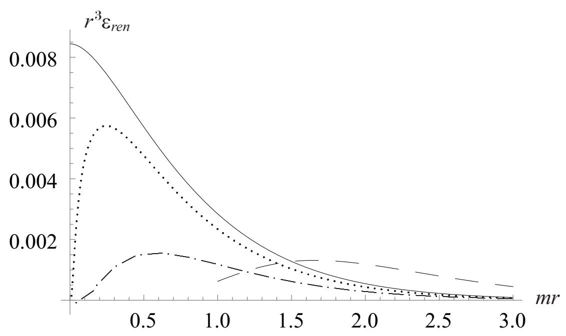

Fig. 1. Induced vacuum energy density of the charged scalar matter outside the impenetrable magnetic tube with the Neumann boundary condition at its edge for the (2+1)-dimensional space time and a half-integer magnetic flux value $F = 1/2$. Dotted line corresponds to the case of the dimensionless tube radius $mr_0 = 0.01$, dash-dotted line to the case of $mr_0 = 0.1$ and the dashed line corresponds to the induced vacuum energy density multiplied by 200, and $mr_0 = 1$. Solid line corresponds to the case of a singular magnetic vortex

scalar field in the background of a $(d-1)$-tube (it is an infinitely long tube for $d = 3$ and a ring for $d = 2$) with a static magnetic field inside in the flat $(d+1)$-dimensional space-time in the case where the tube is impenetrable for the scalar field and obeys perfectly rigid (Neumann) boundary conditions at its surface. We showed that the induced vacuum energy, in this case, (15) depends periodically on the magnetic flux inside the tube with a period equal to $2\pi\tilde{e}^{-1}$. The effect of the vacuum polarization disappears for an integer value of the magnetic flux $\Phi = 2\pi n\tilde{e}^{-1}$, $n \in \mathbb{Z}$. Thus, the induced vacuum energy depends only on the fractional part of the magnetic flux. We can see the manifestation of the Casimir–Bohm–Aharonov effect [6] in this case.

Our results confirm the statement that the Casimir–Bohm–Aharonov effect is due to the condition of the impenetrability of the tube for the matter field. Otherwise, namely in the case where a quantized matter penetrates into the region with a magnetic field, the dependence of the induced vacuum polarization effect on the magnetic flux is not periodic: the effect is determined by the value of the total magnetic flux in the tube, see, e.g., [34–40].

In the simplest case of the $(2+1)$-dimensional space-time, with the help of numerical methods, we compute the value of the vacuum energy density of the quantized charged scalar field outside the impenetrable tube of radius r_0 with the Neumann boundary conditions at its edge. We chose a half-integer value of the magnetic flux $F = 1/2$ inside the tube. At this flux value, we expect the maximal effect of the vacuum polarization by analogy with the case of a singular magnetic vortex, see, e.g., [6, 25]. Without the regularization procedure, we made computations of the vacuum energy, but due to its renormalization by subtracting the contribution corresponding to the absence of the magnetic flux, see (17).

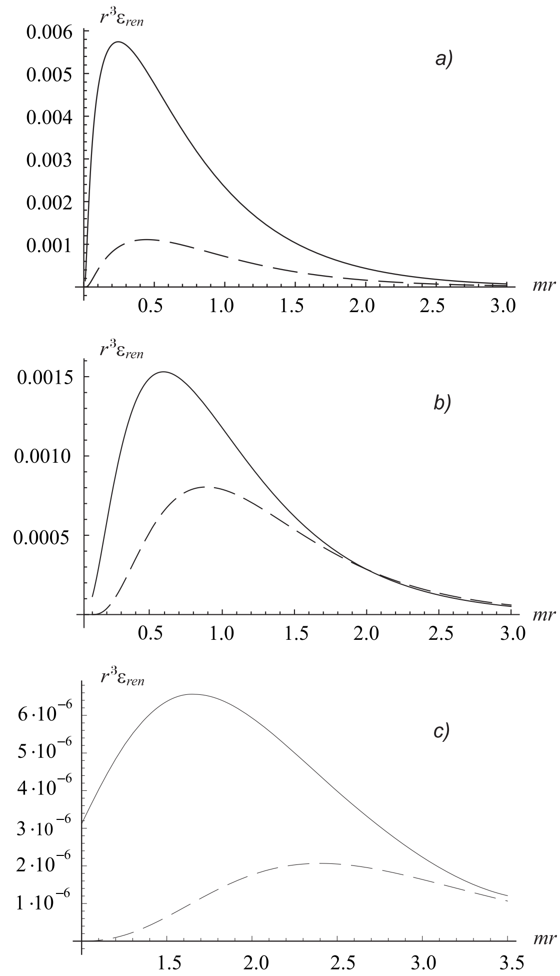

Fig. 2. Comparison of the induced vacuum energy density of the charged scalar matter outside the impenetrable magnetic tube with the Neumann (solid line) and Dirichlet (dashed line) boundary conditions at its edge for the case of a $(2+1)$-dimensional space-time and a half-integer magnetic flux value $F = 1/2$: $mr_0 = 0.01$ (a), $mr_0 = 0.1$ and the dashed line corresponds to the induced vacuum energy density multiplied by 10 (b), $mr_0 = 1$ and dashed line corresponds to the induced vacuum energy density multiplied by 1000 (c)

The results of our computations are presented in Fig. 1. One can see that, at the same dimensionless distance from the center of the tube (mr), the vacuum polarization effect is the largest in the case of the singular magnetic vortex and the exponentially quickly decrease with the growth of the tube radius. It should be noted that the effect of the vacuum polarization becomes negligible, when the radius of the tube is of order or more than the Compton wavelength of the matter field $(mr_0 \gtrsim 1)$.

The comparison of the vacuum polarization in the case of the perfectly rigid (Neumann) and perfectly reflecting (Dirichlet) boundary conditions at the tube edge is presented in Fig. 2. One can see that, for the tubes of the same thickness, the vacuum polarization effect is always the largest in the case of the Neumann boundary condition. This result is in agreement with the result in [30] in the case of induced magnetic flux.

We need to pay attention to that the convergence of the integral under the computation of the induced vacuum energy density (17) in the case of the Neumann boundary condition at the tube edge is weaker than that in the case of the Dirichlet boundary condition. The complexity of the computations strongly increases with decreasing the tube thickness. We conclude that relation (17) based on the direct usage of the field solutions (7) is not suitable for the computations of the vacuum polarization outside the impenetrable thin tube $(mr_0 \ll 1)$. In this case, more appropriate, in our opinion, should be the technique of computation with the help of a transformation in the complex plane, when the Bessel functions $J_\nu(y)$ and $Y_\nu(y)$ transform to the modified Bessel $I_\nu(y)$ and Macdonald $K_\nu(y)$ functions, see, e.g., [31].

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В.М. Горкавенко, Т.В. Горкавенко,
Ю.А. Ситенко, М.С. Царенкова

ІНДУКОВАНА ГУСТИНА ЕНЕРГІЇ ВАКУУМУ КВАНТОВАНОЇ ЗАРЯДЖЕНОЇ СКАЛЯРНОЇ МАТЕРІЇ В ПРИСУТНОСТІ НЕПРОНИКНОЇ МАГНІТНОЇ ТРУБКИ З ГРАНИЧНОЮ УМОВОЮ ТИПУ НЕЙМАНА

В роботі досліджується поляризація вакууму зарядженого скалярного поля матерії зовні трубки, яка містить магнітний потік та є непроникливою для квантованої матерії. На поверхні трубки накладено граничну умову типу Неймана. Записано вирази для індукованої густини енергії вакууму у випадку простору довільної вимірності та при довільному значенні магнітного потоку. Проведено чисельні розрахунки для випадку напівцілого значення магнітного потоку в одиницях Лондона у $(2+1)$-вимірному просторі-часі. Показано, що індукування енергії вакууму зарядженої скалярної матерії відбувається за умови, якщо комптонівська довжина хвилі поля матерії набагато перевищує поперечний розмір трубки. Показано, що енергія вакууму періодична по відношенню до значення магнітного потоку в трубці, що є квантовотеоретичним проявом ефекту Ааронова–Бома. Отримано залежності індукованої енергії вакууму від відстані до центру трубки при різних значеннях товщини трубки. Отримані результати було порівняно з результатами, отриманими раніше для випадку граничної умови типу Діріхле. Показано, що значення індукованої густини енергії вакууму у випадку граничної умови типу Неймана більші, ніж у випадку граничної умови типу Діріхле.

Ключові слова: поляризація вакууму, ефект Ааронова–Бома, ефект Казимира.