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THE GENERALIZED DRUDE–LORENTZ MODEL AND ITS APPLICATIONS IN METAL PLASMONICS

The Drude–Lorentz model has been generalized to the case of plasmons under nonmagnetic conductors located in the static magnetic, $\mathbf{H}_0$*, and electric,* $\mathbf{E}_0$*, fields by taking the spatial dispersion effects into account. It is shown that the magnetostatic field* $\mathbf{H}_0$ *and the spatial dispersion induce the appearance of two additional types of low-frequency bulk plasmons, and the dispersion of bulk plasmons of all types substantially depends on the relative orientation of the direction* $\mathbf{e}_k$ *of their propagation and the magnetostatic field vector* $\mathbf{H}_0$*. In the case of surface plasmons, the spatial dispersion leads to a two-component structure (in the metal) of their electric field* $\mathbf{E}$*, and the external electrostatic field* $\mathbf{E}_0$ *induces the spatial dispersion depending on the Hall constant* R_p*. At the same time, the orientation of the magnetostatic field* $\mathbf{H}_0$ *has a significant effect on the total dispersion of surface plasmons.*

K e y w o r d s: bulk plasmons, surface plasmons, spatial dispersion, magnetostatic field.

1. Introduction

The development of nano and stealth technologies, as well as the design of metamaterials, confirms the challenging character of researches dealing with the physical properties of plasmons in metals and semiconductors (see works [1–12] and references therein). Accounting for the nature of plasmons as macroscopic concentration fluctuations of free electric charges, their theoretical analysis is usually made with the help of the methods of electrodynamics of continuous media, the latter being described using the Drude–Lorentz material parameters [1, 2, 14–18].

The application of the Drude–Lorentz model in the plasmon theory has a long story [1, 2, 14–18]. However, despite its simplicity, this model turned out quite effective, being still used in plasmonics of continuous media [1–12]. In this work, we will show that the capabilities of the Drude–Lorentz model are limited, if external influences on the physical properties of plasmons and the effects of spatial dispersion (the wave properties of plasmons) have to be taken into consideration, which is important for practical applications of plasmonic phenomena. It is reasonable to apply the quasistatic magnetic, $\mathbf{H}_0$, and electric, $\mathbf{E}_0$, fields to control the plasmon parameters.

The aim of this work was to generalize the Drude-Lorentz model to the case of plasmons in nonmagnetic crystals located in the static magnetic, $\mathbf{H}_0$, and electric, $\mathbf{E}_0$, fields, making allowance for the spatial dispersion of the media and, on this basis, to study the orientational effects associated with the variation of the plasmon propagation direction with respect to the magnetostatic field direction.

Note that the behavior of plasmons in magnetostatic fields was considered – in particular, in works [4–10] – for special fixed geometries. (As for the systematic study of orientational effects in the plasmon dynamics that are governed by the magnetostatic field and the spatial dispersion, such issues have not been discussed in the scientific literature.) It is shown that the magnetostatic field $\mathbf{H}_0$ and the spatial dispersion give rise to the appearance of two additional types of low-frequency bulk plasmons. The cyclic frequencies of bulk plasmons of all three types depend on the direction of their propagation with respect to the direction of the magnetostatic field $\mathbf{H}_0$.

Concerning surface plasmons, the spatial dispersion leads to a two-component structure (in metal) of their electric field $\mathbf{E}$, and the external electrostatic field $\mathbf{E}_0$ to the induced spatial dispersion depending on the Hall constant R_p. At the same time, the orientation of the magnetic field $\mathbf{H}_0$ substantially affects the total dispersion of surface plasmons.

In this work, illustrative calculations were carried out for indium antimonide as an example (InSb is an n-type semiconductor with a narrow bandgap of about 0.18 eV). Owing to its unique physical properties [13], this compound is widely applied in electronics and instrument engineering. The aim of performed calculations was to form a holistic picture of the orientational dynamics of plasmons, which is described in the Fourier space by means of the dispersion equation for plasmons.

In our calculations, we used the following parameters of indium antimonide: the electron concentration $n_e \simeq 2\times10^{16}$ cm^{-3}, the effective electron mass $m^* \simeq \simeq 0.014\ m_e$, where m_e is the free-electron mass, and the electron mobility $u_e \simeq 7.8\times10^5$ cm^2/(V s), which was determined at temperatures of about $T = 300$ K [13]. At the same time, the effective mass of holes in indium antimonide is two orders of magnitude larger than the effective mass of electrons [13]. Therefore, they have little effect on the general kinetic properties of electric charge carriers [15, 16], which allowed indium antimonide to be used in this work as a model object.

Provided the indicated values of the parameters, the plasma cyclic frequency [1, 2, 13] in indium antimonide equals

$$\omega_p = \sqrt{\frac{4\pi n_e e^2}{m^*}} \simeq 6.74\times10^{13}\ \text{s}^{-1},$$

which is three orders of magnitude lower than the corresponding values in most metals. By order of magnitude, the plasmon damping parameter in indium antimonide equals $\gamma = \frac{v_\text{F}}{2l_e}$ [16, 17], where $v_\text{F} = \sqrt{\frac{2E_\text{F}}{m^*}} \simeq \simeq 9.73\times10^6$ cm/s, E_F is the Fermi energy, and l_e is the free path length of electrons. Because of the extremely high value of the electron mobility u_e [13], the parameter l_e exceeds the lattice constant in InSb by 2 to 3 orders of magnitude, which, in turn, leads to the inequality

$$\gamma \simeq 4.8\times(10^9{\div}10^{10})\ \text{s}^{-1} \ll \omega_p.$$

According to literature data [13], the Debye constant r_D in indium antimonide is of an order of $(10^{-4}{\div}10^{-5})$ cm.

While making calculations, the cyclotron frequency ω_H was chosen to be an order of magnitude lower than the plasma cyclic frequency ω_p in indium antimonide, which corresponds to the terahertz frequency interval $\left[9.42\times(10^{11}{\div}10^{12})\ \text{s}^{-1}\right]$ of electromagnetic waves in real magnetic fields ($H_0 \simeq 10^3{\div}10^5$ Oe).

2. Generalization of the Drude–Lorentz Model to the Case of a Metal in a Uniform Magnetostatic Field and Taking Spatial Dispersion Effects Into Account

The electric charge conservation law in the differential form looks like

$$\nabla\mathbf{j} + \frac{\partial\delta\rho}{\partial t} = 0, \quad \mathbf{j} = en_0\frac{\partial\mathbf{q}}{\partial t}, \quad \rho = e(n_0 + \delta n_e), \quad (1)$$

where e is the electron charge, n_0 is the average electron concentration in the metal specimen, $\mathbf{q} = \mathbf{q}(\mathbf{r}, t)$ is the local electron displacement from the equilibrium state, and $\delta n_e = \delta n_e(\mathbf{r}, t)$ is the variation of the electron concentration under the influence of various disturbances. From Eq. (1), we obtain that

$$\delta n_e = -n_0(\nabla\,\mathbf{q}). \quad (2)$$

From the course of general physics, we know the relationships between the pressure of ideal gas and the concentration of material points (the latter, in our case, are electrons):

$$\begin{aligned}\delta p_e &= \frac{2}{3}\Big\langle\frac{m^*\mathbf{v}_e{}^2}{2}\Big\rangle\delta n_e,\\ \delta\mathbf{f}_e &= -\nabla\,\delta p_e = \frac{2}{3}n_0\Big\langle\frac{m^*\mathbf{v}_e{}^2}{2}\Big\rangle\nabla^2\mathbf{q},\end{aligned} \quad (3)$$

where $\delta\mathbf{f}_e$ is the internal density of the force induced by electron density waves.

Formulas (3) are valid in the case of non-degenerate electron gas. However, since the electron gas is degenerate in real metals, then, according to works [15, 16], the following reduction of the pressure variation δp and the quantities dependent on it is required:

$$\frac{2}{3}n_0\left\langle\frac{m^*\mathbf{v}_e{}^2}{2}\right\rangle \to m^*u_p{}^2, \quad \delta\mathbf{f}_e = m^*u_p{}^2\nabla^2\mathbf{q},$$
$$u_p = \omega_p r_{\rm D}, \quad \omega_p{}^2 = \frac{4\pi e^2 n_0}{m^*}, \tag{4}$$

where ω_p is the cyclic plasma frequency, and $r_{\rm D}$ the Debye radius of electron screening in the metal.

Given the expression for $\delta\mathbf{f}_e$ (see Eqs. (4)), the dynamic equation for the specific metal polarization $\mathbf{P} = en_0\mathbf{q}$ in a magnetostatic field $\mathbf{H}_0$ can be written in the following form:

$$\frac{\partial^2\mathbf{P}}{\partial t^2} + 2\gamma\frac{\partial\mathbf{P}}{\partial t} + (\omega_0{}^2 - u_p{}^2\nabla^2)\mathbf{P} -$$
$$-\left(\frac{\partial\mathbf{P}}{\partial t}\times\boldsymbol{\omega}_H\right) = \frac{\omega_p{}^2}{4\pi}\mathbf{E}, \quad \boldsymbol{\omega}_H = \frac{e\mathbf{H}_0}{m^*c}, \tag{5}$$

where the electric field of plasmons $\mathbf{E}$ is described by the Maxwell equations (see below), γ is the plasmon damping parameter, $\boldsymbol{\omega}_H$ is the cyclotron frequency vector, and the expression $(\omega_0^2 - u_p^2\nabla^2)$ describes the wave character of plasmon propagation (the spatial dispersion of plasmons). In most real metals, $\omega_0^2 \to 0$.

The wave equation (5) has to be supplemented with a boundary condition. As a natural relationship, this is the zero value of the normal component of the plasmon electric current density vector $\mathbf{j} = \frac{\partial\tilde{\mathbf{P}}}{\partial\mathbf{t}}$ at the metal surface S,

$$(\mathbf{n}\,\mathbf{j})\Big|_{\mathbf{r}\in S} = 0, \quad \mathbf{j} = \frac{\partial\mathbf{P}}{\partial t}, \tag{6}$$

The electric field of plasmons $\mathbf{E}$ and the metal polarization $\mathbf{P}$ are approximated by plane monochromatic waves,

$$\mathbf{E} = \mathbf{E}_0\,\exp\,(i\mathbf{kr} - i\omega t) + \text{c.c},$$
$$\mathbf{P} = \mathbf{P}_0\,\exp\,(i\mathbf{kr} - i\omega t) + \text{c.c.} \tag{7}$$

After substituting expressions (7) into the dynamic equation (5), we obtain an algebraic equation that relates the amplitudes of electric field and metal polarization waves to each other,

$$-(\omega^2 - \omega_k^2 + 2i\gamma\omega)\mathbf{P}_0 + i\omega(\mathbf{P}_0\times\boldsymbol{\omega}_H) = \frac{\omega_p^2}{4\pi}\mathbf{E}_0, \tag{8}$$

where

$$\omega_k^2 = \omega_0^2 + u_p^2\mathbf{k}^2.$$

In the matrix form, Eq. (8) reads

$$D_{\alpha\beta}\;P_0^\beta = \frac{\omega_p^2}{4\pi}E_{0\alpha}, \tag{9}$$

where the matrix $D_{\alpha\beta}$ has the following structure:

$$D_{\alpha\beta} = D_{\alpha\beta}^{(s)} + iD_{\alpha\beta}^{(a)}, \tag{10}$$

where

$$D_{\alpha\beta}^{(s)} = -D_0\delta_{\alpha\beta}, \quad D_{\alpha\beta}^{(a)} = \omega e_{\alpha\beta\gamma}\omega_H{}^\gamma,$$

and

$$D_0 = \omega^2 - \omega_k{}^2 + 2i\gamma\omega.$$

Hereafter, $\delta_{\alpha\beta}$ and $e_{\alpha\beta\gamma}$ are the Kronecker delta and the Levi-Civita symbol, respectively, in the flat three-dimensional space.

The general solution of the matrix equation (9) looks like

$$\mathbf{P}_0 = \frac{\omega_p^2}{4\pi}\hat{D}^{-1}\mathbf{E}_0, \tag{11}$$

where

$$\hat{D}^{-1}{}_{\alpha\beta} = -\frac{D_0}{D_0{}^2 - \omega^2\boldsymbol{\omega}_H{}^2}\delta_{\alpha\beta} +$$
$$+\frac{\omega^2\omega_{H\alpha}\omega_{H\beta}}{D_0(D_0{}^2 - \omega^2\boldsymbol{\omega}_H{}^2)} - i\frac{\omega e_{\alpha\beta\gamma}\omega_H{}^\gamma}{D_0{}^2 - \omega^2\boldsymbol{\omega}_H{}^2}. \tag{12}$$

Then, expression (11) for the amplitude of the specific polarization vector of the metal has the form

$$\mathbf{P}_0 = -\frac{\omega_p{}^2}{4\pi}\left(\frac{D_0\mathbf{E}_0}{D_0{}^2 - \omega^2\boldsymbol{\omega}_H{}^2} -\right.$$
$$\left.-\frac{\omega^2\boldsymbol{\omega}_H(\boldsymbol{\omega}_H\,\mathbf{E}_0)}{D_0(D_0{}^2 - \omega^2\boldsymbol{\omega}_H{}^2)} - i\frac{\omega(\boldsymbol{\omega}_H\times\mathbf{E}_0)}{D_0{}^2 - \omega^2\boldsymbol{\omega}_H{}^2}\right). \tag{13}$$

From this expression, one can see that the magnetic field $\mathbf{H}_0$ induces an additional polarization of the metal, with the third term in Eq. (13) pointing to the gyroscopic effect induced by the magnetic field $\mathbf{H}_0$ [18].

The tensor of the dielectric permittivity of the metal, $\epsilon_{\alpha\beta}$, is determined in the usual way via the matrix D^{-1},

$$\epsilon_{\alpha\beta} = \delta_{\alpha\beta} + \chi_{\alpha\beta}, \quad \chi_{\alpha\beta} = \omega_p{}^2D^{-1}{}_{\alpha\beta}(\omega,\mathbf{k}). \tag{14}$$

The dependence of the tensor $\epsilon_{\alpha\beta}$ on the wave vector $\mathbf{k}$ means the account for the plasmon spatial dispersion effects.

If the magnetic field is absent – i.e., if $\boldsymbol{\omega}_H \to 0$ – and the spatial dispersion is negligibly weak, expression (12) turns into that, for the dielectric permittivity of the metal in the standard Drude–Lorentz model [1, 2],

$$\omega_H \to 0, \quad \epsilon_{\alpha\beta} \to \left(1 - \frac{{\omega_p}^2}{\omega(\omega + 2i\gamma)}\right)\delta_{\alpha\beta}. \qquad (15)$$

The tensor relationship (13) between the metal polarization and the plasmon electric field introduces qualitative features into their magnetodynamics (see below). In particular, in optics of metals characterized by the tensor of dielectric permittivity (14), one should expect such phenomena as birefringence and optical activity [18].

3. Boundary Conditions at the Metal-Insulator Interface in an External Static Electromagnetic Field

In what follows, we consider surface plasmons at the insulator-metal interface. From this point of view, a practically important application task is the study of the mechanisms of influence of external factors – in particular, these are the static electric, $\mathbf{E}_0$, and magnetic, $\mathbf{H}_0$, fields – on the value of the surface plasmon frequency.

Under the action of the electrostatic field $\mathbf{E}_0$, there arise surface electric charges at the metal surface,

$$\rho_s = \sigma\delta(\mathbf{n}\,\mathbf{r})\Big|_{\mathbf{r}\in S}, \quad \sigma = \frac{1}{4\pi}(\mathbf{E}_0\,\mathbf{n})\Big|_{\mathbf{r}\in S}, \qquad (16)$$

where $\mathbf{n}$ is the external normal vector to the metal surface S. In this paper, we consider the case where the electrostatic field $\mathbf{E}_0$ is directed perpendicularly to the metal surface. Otherwise, there arises an electric current in the metal, which, in turn, in the presence of the magnetostatic field $\mathbf{H}_0$, leads to the appearance of electric charges associated with the Hall effect on the metal surface. Their role in the theory of surface plasmons will be considered in the next paper.

The appearance of surface charges does not appreciably affect the dynamics of bulk plasmons. At the same time, they give rise to a reduction of boundary conditions for the electric field induction vector $\mathbf{D}$ and the magnetic field strength $\mathbf{H}$ at the metal surface [17], which is a substantial factor for the theory of surface plasmon-polaritons.

The reduction of boundary conditions associated with induced surface charges (16) can be described by means of the following reduction of the electric field induction vector $\mathbf{D}$ of plasmon-polaritons:

$$\mathbf{D} \to \mathbf{D} + 4\pi\mathbf{P}_s\delta(\mathbf{n}\,\mathbf{r})\Big|_{\mathbf{r}\in S}, \qquad \mathbf{P}_s = \frac{1}{4\pi}(\mathbf{E}_0\,\mathbf{n})\mathbf{q}(\mathbf{r},t)\Big|_{\mathbf{r}\in S}, \qquad (17)$$

where $\mathbf{P}_s$ is the vector of local (surface) polarization, which is determined by the surface charge density (16). The corresponding dynamic equation for $\mathbf{P}_s$ at the metal surface can be obtained by reducing the equation for bulk metal polarization to the following form;

$$\frac{\partial^2\mathbf{P}_s}{\partial t^2} + 2\gamma_s\frac{\partial\mathbf{P}_s}{\partial t} + ({\omega_s}^2 - {u_s}^2{\nabla_s}^2)\mathbf{P}_s - \left(\frac{\partial\mathbf{P}_s}{\partial t} \times \boldsymbol{\omega}_H\right) = \frac{{\omega_p}^2}{4\pi}(\sigma R_p)\mathbf{E}, \qquad (18)$$

where $R_p = \frac{1}{en_0}$ is the Hall constant. The subscript s in Eq. (18) indicates that all quantities in this equation are renormalized to their values in the near-surface metal region. At the same time, the operator $\nabla_s^2 = (\nabla \cdot \hat{n} \cdot \nabla)$ acts only in the plane tangent to the metal surface. Hereafter, $n_{\alpha\beta} = \mathbf{n}^2\delta_{\alpha\beta} - n_\alpha n_\beta$ is the tensor of vector projection onto the metal surface.

The solutions of Eq. (18) can be obtained by renaming the variables in formulas (13) and (14). Following this algorithm, we can find expressions for the local polarization $\mathbf{P}_s$ and the electric current density $\mathbf{j}_s$ at the metal surface in the form

$$\mathbf{P}_s = \frac{1}{4\pi}\hat{\chi}^{(s)}\mathbf{E}, \quad \mathbf{j}_s = \frac{\omega}{4\pi i}\mathbf{P}_s, \qquad (19)$$

where

$$\hat{\chi}^{(s)} = \frac{R_p}{4\pi}(\mathbf{n}\,\mathbf{E}_0)\hat{\chi}(\omega, \mathbf{k}_s)$$

is the surface polarizability of the metal (with $\gamma \to \gamma_s$ in the corresponding expression), and the wave vector $\mathbf{k}_s$ is tangent to the metal surface.

The surface polarization $\mathbf{P}_s$ and the surface electric current $\mathbf{j}_s$ induced by the external electric field change the boundary conditions for the normal components of the electric induction $\mathbf{D}$ and the tangential components of the magnetic field strength $\mathbf{H}$ created by plasmons [17]. To obtain the required boundary conditions, we have to solve the following Maxwell equations in the standard way [17]:

$$\begin{cases} (\nabla\,(\mathbf{D} + 4\pi\mathbf{P}_s\delta(\mathbf{n}\,\mathbf{r}))) = 0 \to \\ \to (\nabla\,\mathbf{D}) + 4\pi(\nabla\,\hat{n}\,\mathbf{P}_s)\delta(\mathbf{n}\,\mathbf{r}) = 0; \\ (\nabla\times\mathbf{H}) = \dfrac{4\pi}{c}\dfrac{\partial\mathbf{P}_s}{\partial t}\delta(\mathbf{n}\,\mathbf{r}). \end{cases} \quad (20)$$

As a result, we obtain the following boundary conditions for the formulated problem:

$$\begin{cases} ((\mathbf{D}^{(1)} - \mathbf{D}^{(2)})\,\mathbf{n}) = -4\pi(\nabla\,\hat{n}\,\mathbf{P}_s); \\ ((\mathbf{H}^{(1)} - \mathbf{H}^{(2)})\times\mathbf{n}) = \dfrac{4\pi}{c}\Big(\hat{n}\,\dfrac{\partial\mathbf{P}_s}{\partial t}\Big). \end{cases} \quad (21)$$

The superscripts (1) and (2) in Eq. (21) denote the insulator and the metal, respectively. The dependence of the boundary conditions (21) on the surface charges (16) induced by the external electric field $\mathbf{E}_0$ essentially affects the dispersion of surface plasmons and plasmon-polaritons (see below).

In the case of insulator, the influence of the electrostatic field $\mathbf{E}_0$ on its dielectric permittivity tensor is usually described by the following expression [17]:

$$\varepsilon_{\alpha\beta} \simeq \varepsilon^{(1)}_{\alpha\beta} + \chi^{(2)}_{\alpha\beta\nu}\,E^{\nu}_0 + \chi^{(2)}_{\alpha\beta\nu\mu}\,E^{\nu}_0\,E^{\mu}_0 + \dots. \quad (22)$$

Here, the first term on the right-hand side is the ordinary dielectric permittivity tensor of the insulator, whereas the second and the third ones describe the Pockels and Kerr effects, respectively. In so doing, it should be borne in mind that the Pockels and Kerr effects lead, as a rule, to the insulator anisotropy.

Hence, the magnetostatic field, the appearance of induced charges at the metal surface, and the Pockels and Kerr effects, all those factors taken in that or another combination make it possible to control the frequency of surface plasmons.

4. Bulk Plasmons

If the anisotropy and spatial dispersion effects are taken unto consideration, the electromagnetic field created by plasmon-polaritons and approximated by plane monochromatic waves

$$\begin{aligned} &\mathbf{E} = \mathbf{E}_0\exp(i\mathbf{kr} - i\omega t) + \text{c.c.}, \\ &\mathbf{H} = \mathbf{H}_0\exp(i\mathbf{kr} - i\omega t) + \text{c.c.}, \end{aligned} \quad (23)$$

satisfies the Maxwell equations

$$\begin{cases} (\mathbf{k}\times\mathbf{E}) = \dfrac{\omega}{c}\mathbf{B}, & (\mathbf{k}\,\mathbf{B}) = 0; \\ (\mathbf{k}\times\mathbf{H}) = -\dfrac{\omega}{c}\mathbf{D}, & (\mathbf{k}\,\mathbf{D}) = 0, \end{cases} \quad (24)$$

the constitutive relations

$$\begin{cases} \mathbf{B} = \hat{\mu}(\omega,\mathbf{k})\mathbf{H}; \\ \mathbf{D} = \hat{\varepsilon}(\omega,\mathbf{k})\mathbf{E} \end{cases} \quad (25)$$

and the corresponding dispersion equation

$$\begin{aligned} &(\mathbf{k}\,\hat{\eta}\,\mathbf{k})\mathbf{k}^2 - {q_0}^2(\mathrm{Tr}(\hat{\eta})(\mathbf{k}\,\hat{\eta}\,\mathbf{k}) - \\ &- (\mathbf{k}\,\hat{\eta}\,\hat{\eta}\,\mathbf{k})) + {q_0}^4\det(\hat{\eta}) = 0, \end{aligned} \quad (26)$$

where $\hat{\eta} = \hat{\mu}\,\hat{\varepsilon}$ and $q_0 = \frac{\omega}{c}$. In non-magnetic media, which are considered below, $\hat{\mu}(\omega,\mathbf{k}) = 1$.

The electric field of bulk plasmons satisfies the simplified Maxwell equations

$$\begin{cases} (\mathbf{k}\times\mathbf{E}) = 0, & (\mathbf{k}\,\mathbf{B}) = 0; \\ (\mathbf{k}\times\mathbf{H}) = 0, & (\mathbf{k}\,\mathbf{D}) = 0, \end{cases} \quad (27)$$

which can be obtained from Eqs. (24) by neglecting the retardation effects ($c \to \infty$). Equations (27) have the following solution:

$$\begin{aligned} &\mathbf{E} = \mathbf{k}A_0\,\exp(i\mathbf{kr} - i\omega t) + \text{c.c.}, \\ &(\mathbf{k}\,\hat{\varepsilon}(\omega,\mathbf{k})\,\mathbf{k}) = 0, \end{aligned} \quad (28)$$

where A_0 is an integration constant.

As concerning plasmon-polaritons, they will be considered in the next paper.

In the general case, taking the structure of dielectric permittivity into account (see Eq. (12)), the dispersion equation for bulk plasmons in Eq. (28) at $\gamma = 0$ can be rewritten in the form of the following cubic equation in the quantity ω^2,

$$\begin{aligned} &(\omega^2 - {\omega_k}^2)^3 - {\omega_p}^2(\omega^2 - {\omega_k}^2)^2 - \\ &- \omega^2({\boldsymbol{\omega}_H}^2(\omega^2 - {\omega_k}^2) - {\omega_p}^2(\boldsymbol{\omega}_H\,\mathbf{e}_k)^2) = 0, \end{aligned} \quad (29)$$

which has three real roots. Hereafter, $\mathbf{e}_k = \mathbf{k}/\,|\mathbf{k}|$.

In terms of the dimensionless variables

$$w = \frac{\omega}{\omega_p}, \quad w_k = \frac{\omega_k}{\omega_p} = r_D|\mathbf{k}|, \quad \mathbf{w}_H = \frac{\boldsymbol{\omega}_H}{\omega_p} \quad (30)$$

Eq. (29) can be rewritten in a form that is more convenient for the analysis,

$$\begin{aligned} &(w^2 - {w_k}^2)^3 - (w^2 - {w_k}^2)^2 - \\ &- w^2({\mathbf{w}_H}^2(w^2 - {w_k}^2) - (\mathbf{w}_H\,\mathbf{e}_k)^2) = 0. \end{aligned} \quad (31)$$

4.1. The magnetic field is absent ($\mathbf{w}_H = 0$) *and the spatial dispersion is substantial* ($w_k \neq 0$)

In this case, Eq. (31) takes the form

$$(w^2 - {w_k}^2 - 1)(w^2 - {w_k}^2)^2 = 0. \tag{32}$$

As a result, we obtain two solutions of Eq. (32), which correspond to high-frequency (the optical frequency interval, ω_1) and low-frequency (the ultra-high frequency (UHF) interval, ω_2) bulk plasmons (see Figs. 1 and 2, respectively),

$$\begin{cases} {w_1}^2 = 1 + {w_k}^2; \\ {w_2}^2 = {w_k}^2, \end{cases} \to \begin{cases} {\omega_1}^2 = {\omega_p}^2 + {\omega_k}^2; \\ {\omega_2}^2 = {\omega_k}^2. \end{cases} \tag{33}$$

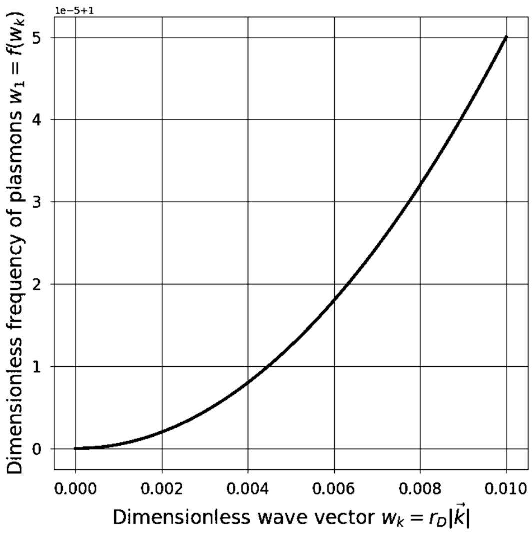

Fig. 1. Spatial dispersion of high-frequency bulk plasmons

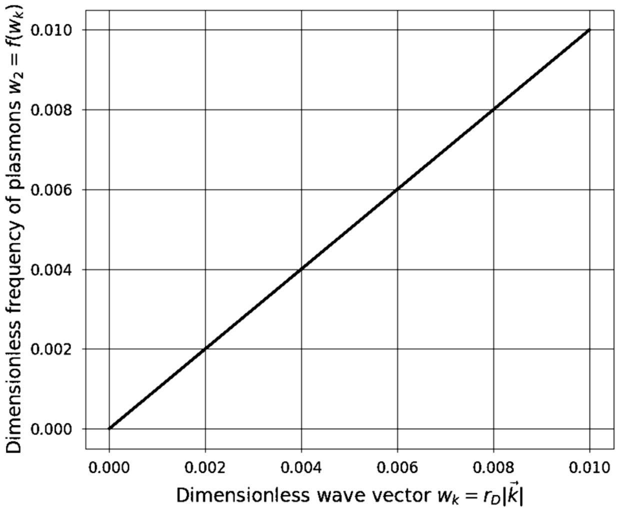

Fig. 2. Spatial dispersion of low-frequency bulk plasmons

Note that the description of plasmons with the frequency ω_1 at $\mathbf{k} =$ is reduced to the plasmons considered in the framework of the standard Drude–Lorentz model [1, 2], whereas the existence of low-frequency plasmons with the frequency ω_2 is possible exclusively due to the spatial dispersion, which, in turn, is determined by the Debye radius r_D of electron screening in the metal.

4.2. The spatial dispersion is negligibly weak ($w_k \ll 1$) *and the magnetic field is substantial* ($\mathbf{w}_H \neq 0$)

In this case, Eq. (31) takes the form

$$w^4 - (1 + {\mathbf{w}_H}^2)w^2 + (\mathbf{w}_H\, \mathbf{e}_k)^2 = 0. \tag{34}$$

Its solutions are

$$\begin{cases} {w_1}^2 = \frac{1}{2}(1 + {\mathbf{w}_H}^2) + \\ + \frac{1}{2}\sqrt{(1 + {\mathbf{w}_H}^2)^2 - 4(\mathbf{w}_H\, \mathbf{e}_k)^2}; \\ {w_2}^2 = \frac{1}{2}(1 + {\mathbf{w}_H}^2) - \\ - \frac{1}{2}\sqrt{(1 + {\mathbf{w}_H}^2)^2 - 4(\mathbf{w}_H\, \mathbf{e}_k)^2}, \end{cases} \tag{35}$$

or, in terms of physical variables,

$$\begin{cases} {\omega_1}^2 = \frac{1}{2}({\omega_p}^2 + {\boldsymbol{\omega}_H}^2) + \\ + \frac{1}{2}\sqrt{({\omega_p}^2 + {\boldsymbol{\omega}_H}^2)^2 - 4{\omega_p}^2(\boldsymbol{\omega}_H\, \mathbf{e}_k)^2}; \\ {\omega_2}^2 = \frac{1}{2}({\omega_p}^2 + {\boldsymbol{\omega}_H}^2) - \\ - \frac{1}{2}\sqrt{({\omega_p}^2 + {\boldsymbol{\omega}_H}^2)^2 - 4{\omega_p}^2(\boldsymbol{\omega}_H\, \mathbf{e}_k)^2}. \end{cases} \tag{36}$$

From Eqs. (36), one can see that the frequencies of the high-frequency, ω_1, and low-frequency, ω_2, bulk plasmons considerably depend on the relative orientation of their propagation direction $\mathbf{e}_k$ and the direction of the magnetic field $\mathbf{H}_0$ (see Figs. 3 and 4). In two limiting cases, we have

$$\begin{cases} {\omega_1}^2 = {\omega_p}^2; \\ {\omega_2}^2 = {\boldsymbol{\omega}_H}^2, \quad \text{if } (\boldsymbol{\omega}_H\, \mathbf{e}_k) = 0, \end{cases} \tag{37}$$

and

$$\begin{cases} {\omega_1}^2 = {\omega_p}^2 + {\boldsymbol{\omega}_H}^2; \\ {\omega_2}^2 = 0, \quad \text{if } (\boldsymbol{\omega}_H\, \mathbf{e}_k) = \pm|\boldsymbol{\omega}_H|. \end{cases} \tag{38}$$

Thus, we established that the frequency variation for the plasmons of both types by changing the relative orientation of their propagation direction $\mathbf{e}_k$ and the direction of the magnetic field $\mathbf{H}_0$ can be performed within an interval of $(0, |\boldsymbol{\omega}_H|)$.

Note also that the description of plasmons with the frequency ω_1 at $\mathbf{H}_0 = 0$ is reduced to the plasmons considered in the framework of the standard Drude–Lorentz model [1, 2] (the optical frequency interval), whereas the existence of low-frequency plasmons (the UHF interval) with the frequency ω_2 is possible exclusively due to the specific feature of electron interaction with the magnetic field $\mathbf{H}_0$. In this case, the relation between the induction $\mathbf{D}$ and the electric field $\mathbf{E}$ of plasmons has an essentially tensor character. Therefore, the appearance of low-frequency plasmons allows us to draw an analogy with birefringence in the optics of anisotropic crystals.

4.3. The General Case of Bulk Plasmons

In the general case, the dispersion equation for bulk plasmons (31) can be reduced to a 3rd-order polynomial, and thus it has three solutions (see Figs. 5 to 7). It is quite clear that these solutions correspond to the plasmons known from the standard Drude-Lorentz model (high-frequency plasmons) [1, 2] and to additional plasmons induced by the external magnetostatic field $\mathbf{H}_0$ (low-frequency plasmons) and the spatial dispersion (acoustic plasmons).

From Figs. 5 and 6, one can see that the spatial dispersion weakly affects the dynamics of high- and low-frequency plasmons. At the same time, the influence of the magnetostatic field $\mathbf{H}_0$ on the dynamics of acoustic plasmons is considerable if the vectors $\mathbf{k}$ and $\mathbf{H}_0$ are oriented orthogonally to each other (see Fig. 7), i.e., when the action of the Lorentz force on plasmons is maximum.

It is obvious that low-frequency plasmons lead to additional frequency intervals of metal transparency for electromagnetic waves, and this fact can be of practical importance for plasmonics.

5. Surface Plasmons at the Interface between Anisotropic Metal and Anisotropic Insulator

Let the metal-insulator interface be determined by the condition $\mathbf{r} \in (r_x, r_y, 0)$ in the Cartesian coordinate system $\{X, Y, Z\}$, and ε and ϵ be the dielectric permittivities of the insulator and the non-magnetic metal ($\hat{\mu} = 1$) that occupy the half-spaces $z > 0$ and $z < 0$, respectively. In such a geometry, the approximate (neglecting the retardation effects, $c \to \infty$) electric fields of surface plasmons are collinear to their wave vectors: $\mathbf{p}_0 = (\mathbf{k}, i\alpha)$ in the insulator and

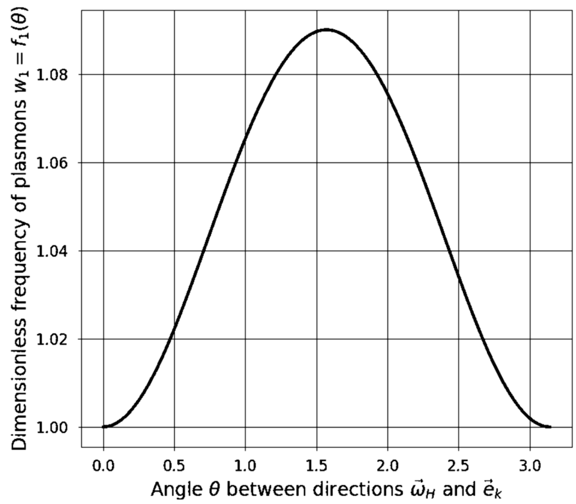

Fig. 3. Influence of the propagation direction $\mathbf{e}_k$ of high-frequency bulk plasmons with respect to the direction of the magnetostatic field vector $\mathbf{H}_0$ $(0 \leq \theta \leq \pi)$ on their cyclic frequency ω

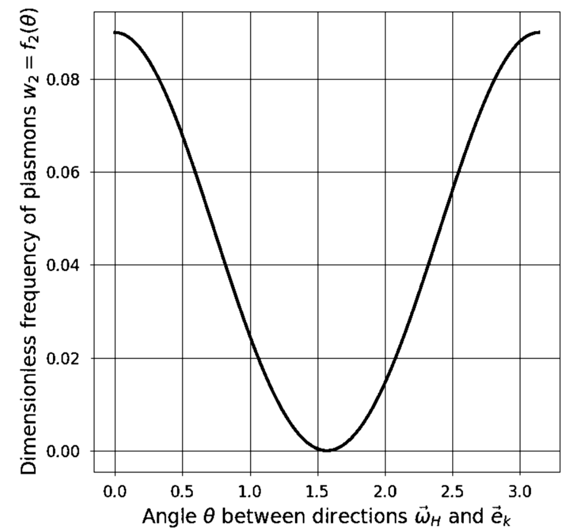

Fig. 4. Influence of the propagation direction $\mathbf{e}_k$ of low-frequency bulk plasmons with respect to the direction of the magnetostatic field vector $\mathbf{H}_0$ $(0 \leq \theta \leq \pi)$ on their cyclic frequency ω

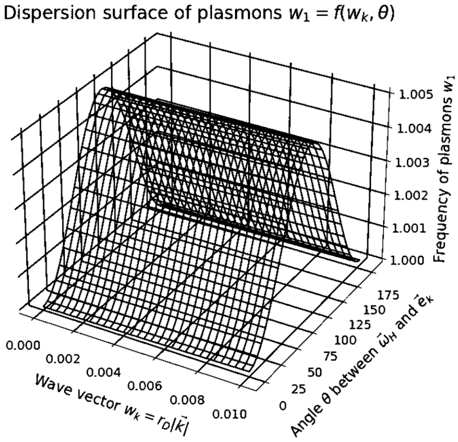

Fig. 5. Influence of the magnetostatic field $\mathbf{H}_0$ on the dispersion of high-frequency bulk plasmons

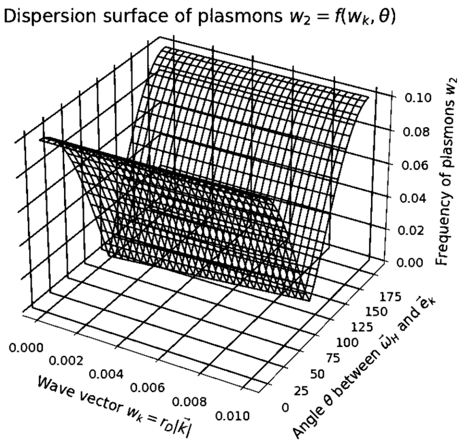

Fig. 6. Influence of the magnetostatic field $\mathbf{H}_0$ on the dispersion of low-frequency bulk plasmons

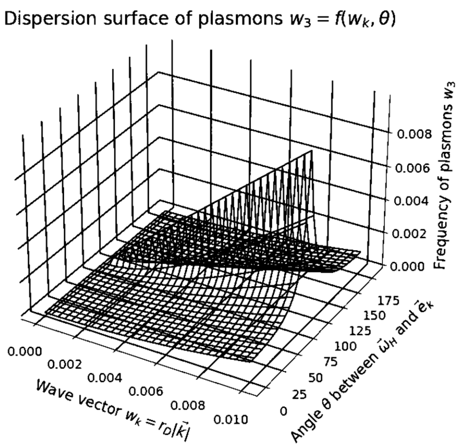

Fig. 7. Influence of the magnetostatic field $\mathbf{H}_0$ on the dispersion of acoustic bulk plasmons

$\mathbf{p}_1 = (\mathbf{k}, -i\beta)$ in the metal, i.e.,

$$\mathbf{E} = \begin{cases} \mathbf{P}_0 A \, \exp\left(i(\mathbf{P}_0\mathbf{r} - \omega t)\right) & \text{if } z > 0; \\ \mathbf{P}_1 B \, \exp\left(i(\mathbf{P}_1\mathbf{r} - \omega t)\right) & \text{if } z < 0, \end{cases} \tag{39}$$

where $\mathbf{k} = (k_x, k_y)$.

The quantities α and β describe the decay rate of the electric field when moving away from the interface between two media, and they are determined by the dispersion equations

$$(\mathbf{P}_0 \, \hat{\varepsilon}(\omega, \mathbf{P}_0) \, \mathbf{P}_0) = 0, \quad (\mathbf{P}_1 \, \hat{\epsilon}(\omega, \mathbf{P}_1) \, \mathbf{P}_1) = 0 \tag{40}$$

analogous to Eqs. (28).

Let the insulator contacting with the metal be an isotropic linear crystal with the dielectric permittivity $\varepsilon_{\alpha\beta} = \varepsilon\delta_{\alpha\beta}$. Let us consider some special cases.

5.1. Spatial dispersion in the theory of surface plasmons if the static electromagnetic field is absent

Let us analyze the role of spatial dispersion in the theory of surface plasmons provided that the static electromagnetic field is absent ($\mathbf{H}_0 = 0$, $\mathbf{E}_0 = 0$). Under those conditions, Eqs. (40) have the following solutions:

$$\alpha = |\mathbf{k}|, \\ \beta_1 = |\mathbf{k}|, \quad \beta_2 = |\mathbf{k}|\sqrt{1 - \frac{\omega^2 - \omega_p^2}{\omega_k^2}}. \tag{41}$$

The availability of two possible values of the parameter β complicates the structure of the electric field created by surface plasmons because their electric field in the metal becomes two-component,

$$\mathbf{E} = \begin{cases} \mathbf{P}_0 A \, \exp\left(i\left(\mathbf{P}_0\mathbf{r} - \omega t\right)\right) \text{ if } z > 0; \\ \mathbf{P}_1 B_1 \, \exp\left(i\left(\mathbf{P}_1\mathbf{r} - \omega t\right)\right) + \\ + \mathbf{P}_2 B_2 \, \exp\left(i\left(\mathbf{P}_2\mathbf{r} - \omega t\right)\right) \text{ if } z < 0. \end{cases} \tag{42}$$

In this case, the electrostatic boundary conditions are not enough. As an additional boundary condition, let us use the boundary condition (5) for the electric current density vector $\mathbf{j} = \frac{\partial \mathbf{P}}{\partial t}$. Then, taking into account that $\epsilon(\omega, \mathbf{p}_1) = \epsilon_1(\omega)$ if $\mathbf{p}_1{}^2 = 0$, and $\epsilon(\omega, \mathbf{p}_2) = 0$, we obtain a closed system of boundary conditions in the form

$$\begin{cases} A - B_1 - B_2 = 0; \\ \alpha\varepsilon A + \beta_1\epsilon_1 B_1 = 0; \\ \beta_1(\epsilon_1 - 1)B_1 - \beta_2 B_2 = 0. \end{cases} \tag{43}$$

From the non-triviality condition for the solution of the system of linear equations (43), we find the dispersion equation for surface plasmons in which the effects of spatial dispersion are taken into account,

$$(\alpha\varepsilon + \beta_1\epsilon_1)\,\beta_2 + \alpha\varepsilon(\epsilon_1 - 1)\,\beta_1 = 0. \qquad (44)$$

In terms of dimensionless variables (31), it can be rewritten as follows:

$$((\varepsilon+1)w^2 - 1)\sqrt{1 + w_k^2 - w^2} - \varepsilon w_k = 0. \qquad (45)$$

This equation has three roots, two of which have a physical meaning,

$$\begin{cases} {w_1}^2 = \dfrac{1}{\varepsilon+1} + \dfrac{{w_k}^2}{2} + w_k\sqrt{\left(\dfrac{w_k}{2}\right)^2 + \dfrac{\varepsilon}{\varepsilon+1}}; \\ {w_3}^2 = 1. \end{cases} \qquad (46)$$

In terms of physical variables, they can be rewritten as follows:

$$\begin{cases} {\omega_1}^2 = \dfrac{{\omega_p}^2}{\varepsilon+1} + \dfrac{{\omega_k}^2}{2} + \omega_k\sqrt{\left(\dfrac{\omega_k}{2}\right)^2 + \dfrac{\varepsilon{\omega_p}^2}{\varepsilon+1}}; \\ {\omega_3}^2 = {\omega_p}^2. \end{cases} \qquad (47)$$

The frequency $\omega_1 = \omega_1(\mathbf{k})$ corresponds to ordinary surface plasmons but taking the spatial dispersion into account (see Fig. 8). On the other hand, the frequency $\omega_3 = \omega_p$ corresponds to exotic surface plasmons (with the frequency of bulk plasmons, ω_p), which are a result of the two-component structure of their electric field in the metal.

5.2. Dispersion equation for surface plasmons at negligibly weak spatial dispersion and substantial static electromagnetic fields

Let the spatial dispersion in contacting media be negligibly weak. In this case, the dispersion equations (40) have the following solutions:

$$\alpha = |\mathbf{k}|, \quad \beta = -i|\mathbf{k}|\frac{(\mathbf{e}_k\,\hat{\eta}\,\mathbf{n})}{(\mathbf{n}\,\hat{\eta}\,\mathbf{n})} + {} + |\mathbf{k}|\sqrt{\frac{(\mathbf{e}_k\,\hat{\eta}\,\mathbf{e}_k)}{(\mathbf{n}\,\hat{\eta}\,\mathbf{n})} - \left(\frac{(\mathbf{e}_k\,\hat{\eta}\,\mathbf{n})}{(\mathbf{n}\,\hat{\eta}\,\mathbf{n})}\right)^2}, \qquad (48)$$

where $\eta_{\alpha\beta} = \frac{1}{2}(\epsilon_{\alpha\beta}+\epsilon_{\beta\alpha})$ is the symmetric part of the metal dielectric permittivity tensor $\hat{\epsilon}$, $\mathbf{n} = (0,0,1)$ is the vector directed normally to the interface between

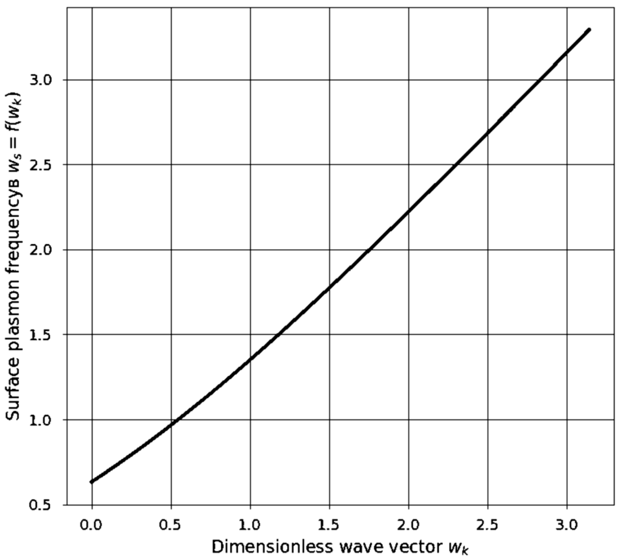

Fig. 8. Spatial dispersion of surface plasmons with the two-component (in the metal) electric field

the two media $\mathbf{r} \in (r_x, r_y, 0)$, and the unit vector $\mathbf{e}_k = \mathbf{k}/|\mathbf{k}| = (e_x, e_y, 0)$ determines the direction of motion of surface plasmons. It is quite clear that the expression given for the parameter β has a physical meaning only if the radicand in the left-hand side of Eq. (48) is positive. As for the magnetic field of surface plasmons, $\mathbf{H}$, it is negligibly low in this case ($\mathbf{H} \to 0$).

According to Eq. (21), the electric field of surface plasmons (39) must satisfy the boundary conditions

$$\begin{cases} E_{x,y}(x,y,z=+0) = E_{x,y}(x,y,z=-0); \\ D_z(x,y,z=+0) - D_z(x,y,z=-0) = \\ = -4\pi i(\mathbf{k}\,\hat{\chi}^{(s)}\,\mathbf{E}(x,y,z=-0)), \end{cases} \qquad (49)$$

where $\hat{\chi}_s$ is the surface polarizability of the metal (see Eq. (20)) under the action of the external electrostatic field $\mathbf{E}_0$. From the first equation in (49), we obtain that $A = B$ in Eq. (40), and from the second one the dispersion equation for surface plasmons,

$$\varepsilon + i(\mathbf{n}\,\hat{\xi}\,\mathbf{e}_k) + \mathrm{sign}(\mathbf{n}\,\hat{\eta}\,\mathbf{n}) \times {} \times \sqrt{(\mathbf{e}_k\,\hat{\eta}\,\mathbf{e}_k)(\mathbf{n}\,\hat{\eta}\,\mathbf{n}) - (\mathbf{e}_k\,\hat{\eta}\,\mathbf{n})(\mathbf{n}\,\hat{\eta}\,\mathbf{e}_k)} = {} = -4\pi((\mathbf{e}_k\,\hat{\chi}^{(s)}\,\mathbf{e}_k)|\mathbf{k}| - i\beta(\mathbf{e}_k\,\hat{\chi}^{(s)}\,\mathbf{n})), \qquad (50)$$

where

$$(\mathbf{n}\,\hat{\eta}\,\mathbf{n}) = 1 - \frac{{\omega_p}^2}{\omega^2}\left(\frac{\omega^2 - (\mathbf{n}\,\boldsymbol{\omega}_H)^2}{\omega^2 - {\boldsymbol{\omega}_H}^2}\right),$$

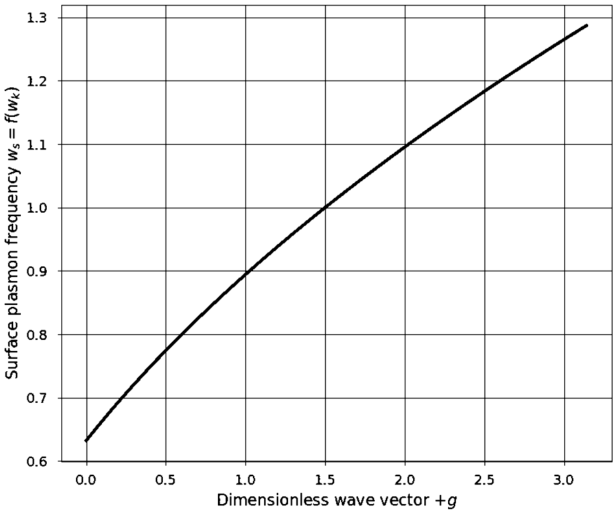

Fig. 9. Spatial dispersion of surface plasmons induced by the electrostatic field $\mathbf{E}_0$ at $g > 0$

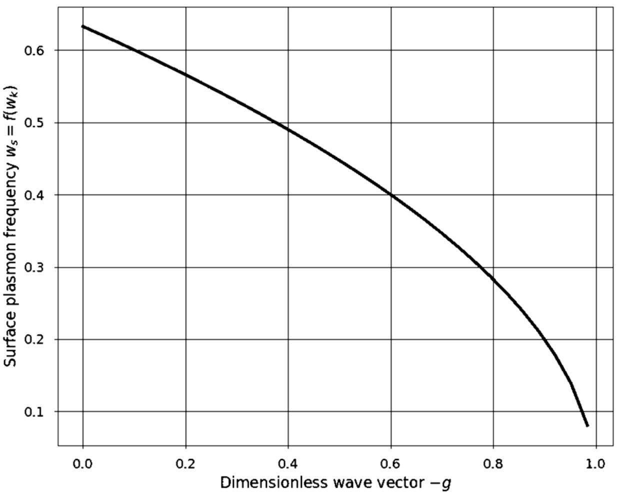

Fig. 10. Spatial dispersion of surface plasmons induced by the electrostatic field $\mathbf{E}_0$ at $g < 0$

$$(\mathbf{e}_k\,\hat{\eta}\,\mathbf{e}_k) = 1 - \frac{{\omega_p}^2}{\omega^2}\Big(\frac{\omega^2 - (\mathbf{e}_k\,\boldsymbol{\omega}_H)^2}{\omega^2 - {\boldsymbol{\omega}_H}^2}\Big),$$

$$(\mathbf{n}\,\hat{\eta}\,\mathbf{e}_k) = (\mathbf{e}_k\,\hat{\eta}\,\mathbf{n}) = \frac{{\omega_p}^2}{\omega^2}\Big(\frac{(\mathbf{n}\,\boldsymbol{\omega}_H)(\mathbf{e}_k\,\boldsymbol{\omega}_H)}{\omega^2 - {\boldsymbol{\omega}_H}^2}\Big),$$

$$(\mathbf{n}\,\hat{\xi}\,\mathbf{e}_k) = -i\frac{{\omega_p}^2}{\omega^2}\Big(\frac{\omega(\mathbf{n}\,\mathbf{e}_k\,\boldsymbol{\omega}_H)}{\omega^2 - {\boldsymbol{\omega}_H}^2}\Big),$$

and $\xi_{\alpha\beta} = \frac{1}{2}(\epsilon_{\alpha\beta} - \epsilon_{\beta\alpha})$ is the antisymmetric part of the metal dielectric permittivity tensor $\hat{\epsilon}$.

It is evident that the dispersion equation in (50) can have real solutions for the cyclic frequency ω if one of the contacting media has negative components in its dielectric permittivity tensor. In the case of surface plasmons, the condition $\mathrm{Re}(\epsilon_{\alpha\beta}) < 0$ must be satisfied.

Essential for practical applications of surface plasmons is the fact that their cyclic frequency ω can be affected by varying the intensities of the magnetostatic, $\mathbf{H}_0$ (see Eq. (14)), and electrostatic, $\mathbf{E}_0$ (see Eqs. (16) and (22)), fields. In this case, owing to the presence of surface electric charges induced by the electrostatic field $\mathbf{E}_0$, the frequency of surface plasmons becomes dependent on the magnitude of their wave vector $\mathbf{k}$ (see Eq. (50)).

In the general case, the right-hand side of the dispersion equation in (50) is complex-valued, which leads to the damping of surface plasmons. This phenomenon can be explained as occurring when the curvature radius of the electron trajectory exceeds the characteristic localization depth of surface plasmons in the metal, $\Delta Z_1 = \mathrm{Re}\big(\frac{1}{\beta}\big)$, which takes place owing to the specificity of surface plasmon dynamics in the static electromagnetic fields $\mathbf{E}_0$ and $\mathbf{H}_0$. If one takes the construction of the tensor $\hat{\chi}^{(s)}$ into account, it becomes clear that such damping of surface plasmons can be suppressed provided that only one of the static electromagnetic fields, $\mathbf{E}_0$ or $\mathbf{H}_0$, can affect the dynamics of surface plasmons (see below).

Now, let us consider some particular cases.

5.3. The magnetostatic field is absent ($\mathbf{H_0 = 0}$) and the electrostatic field $\mathbf{E}_0$ is perpendicular to the metal surface

In this case, the dispersion equation (50) for surface plasmons, if being rewritten in terms of the dimensionless variables

$$w = \frac{\omega}{\omega_p}, \quad g = \pm R_0|\mathbf{k}|, \quad R_0 = |R_p\mathbf{E}_0| \tag{51}$$

acquires the simple form

$$\varepsilon + \epsilon = \frac{g}{w^2} \tag{52}$$

and has the following solution

$$w_s = \sqrt{\frac{1+g}{\varepsilon+1}}, \quad \omega_s = \omega_p w_s = \omega_p\sqrt{\frac{1 \pm R_0|\mathbf{k}|}{\varepsilon+1}}, \tag{53}$$

where ω_s is the cyclic frequency depending on the magnitude of the surface plasmon wave vector $\mathbf{k}$ (see

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Figs. 9 and 10), R_p is the Hall constant, and the sign of the quantity g in Eqs. (51)–(53) is determined by the sign of the quantity $R_p(\mathbf{n}\cdot\mathbf{E}_0)$.

The dependence of the cyclic frequency of surface plasmons on the wave vector magnitude testifies to the transition of their dynamics from oscillatory dynamics to wave one.

5.4. The electrostatic field is absent ($\mathbf{E_0} = \mathbf{0}$) *and the magnetostatic field* $\mathbf{H_0}$ *is arbitrarily oriented with respect to the metal surface*

In this case, the right-hand side of the dispersion equation in (50) equals zero.

In three specific geometries of the problem – namely, if the vector $\boldsymbol{\omega}_H$ is collinear to one of the vectors $\mathbf{n}$, $\mathbf{e}_k$, and $(\mathbf{n}\times\mathbf{e}_k)$ – the dispersion equation becomes strongly simplified and looks like

$$\begin{gathered}\varepsilon+\operatorname{sign}\Big(1-\frac{{\omega_p}^2}{\omega^2}\Big)\times\\ \times\sqrt{\Big(1-\frac{{\omega_p}^2}{\omega^2}\Big)\Big(1-\frac{{\omega_p}^2}{\omega^2-{\boldsymbol{\omega}_H}^2}\Big)}=0,\ \text{if}\ \boldsymbol{\omega}_H||\mathbf{n},\\ \varepsilon+\operatorname{sign}\Big(1-\frac{{\omega_p}^2}{\omega^2-{\boldsymbol{\omega}_H}^2}\Big)\times\\ \times\sqrt{\Big(1-\frac{{\omega_p}^2}{\omega^2-{\boldsymbol{\omega}_H}^2}\Big)\Big(1-\frac{{\omega_p}^2}{\omega^2}\Big)}=0,\ \text{if}\ \boldsymbol{\omega}_H||\mathbf{e}_k,\\ \varepsilon+\frac{{\omega_p}^2}{\omega^2}\Big(\frac{\omega((\mathbf{n}\times\mathbf{e}_k)\,\boldsymbol{\omega}_H)}{\omega^2-{\boldsymbol{\omega}_H}^2}\Big)+\\ +\Big(1-\frac{{\omega_p}^2}{\omega^2-{\boldsymbol{\omega}_H}^2}\Big)=0,\ \text{if}\ \boldsymbol{\omega}_H||(\mathbf{n}\times\mathbf{e}_k).\end{gathered} \tag{54}$$

The physically meaningful solutions of these equations can be written as follows:

$$\begin{gathered}{\omega_{1,2}}^2=\frac{(\varepsilon^2-1){\boldsymbol{\omega}_H}^2-2{\omega_p}^2+\sqrt{(\varepsilon^2-1)^2{\boldsymbol{\omega}_H}^4+4\varepsilon^2{\omega_p}^4}}{2(\varepsilon^2-1)},\\ \text{if}\ \boldsymbol{\omega}_H||\mathbf{n},\ \boldsymbol{\omega}_H||\mathbf{e}_k,\\ \omega_{3,4}=\pm\frac{|\boldsymbol{\omega}_H|}{2}+\sqrt{\Big(\frac{|\boldsymbol{\omega}_H|}{2}\Big)^2+\frac{{\omega_p}^2}{\varepsilon+1}},\\ \text{if}\ ((\mathbf{n}\times\mathbf{e}_k)\,\boldsymbol{\omega}_H)=\mp|\boldsymbol{\omega}_H|.\end{gathered} \tag{55}$$

At $\boldsymbol{\omega}_H\to 0$, solutions (55) of dispersion equations (54) are reduced to the expression obtained in the framework of the standard Drude–Lorentz

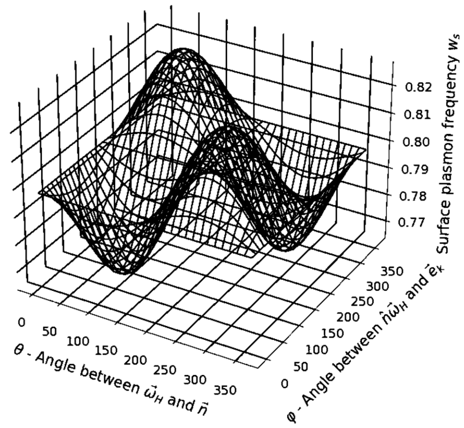

Fig. 11. Influence of the orientation of the magnetostatic field $\mathbf{H}_0$ on the surface plasmon dispersion

model [1,2]. At an arbitrary orientation of the magnetostatic field vector $\mathbf{H}_0$, the cyclic frequency ω_s of surface plasmons is determined by the projection of the position of the vector $\boldsymbol{\omega}_H$ on the S^2 sphere onto the dispersion "surface" of surface plasmons (see Fig. 11).

6. Conclusions

To summarize, in this work, it was shown that the generalization of the Drude–Lorentz model makes it possible to describe the complex influence of static electromagnetic fields and spatial dispersion on the physical characteristics of both bulk and surface plasmons. It was found that the magnetostatic field $\mathbf{H}_0$ and the spatial dispersion lead to the appearance of additional types of bulk plasmons with the dispersion depending on the relative orientation of their propagation direction $\mathbf{e}_k$ and the direction of the magnetostatic field $\mathbf{H}_0$. In the case of surface plasmons, the spatial dispersion leads to the two-component character of the electric field $\mathbf{E}$ generated by surface plasmons in the metal, and the electrostatic field $\mathbf{E}_0$ induces the spatial dispersion. At the same time, the orientation of the magnetic field $\mathbf{H}_0$ substantially affects the total dispersion of surface plasmons.

It is essential that no analogs of low-frequency bulk plasmons induced by the magnetostatic field $\mathbf{H}_0$ and the spatial dispersion arise in the case of surface plasmons. This circumstance can be explained by the fact that the curvature of the electron trajectory in the magnetostatic field $\mathbf{H}_0$ exceeds the characteristic localization size of surface plasmons in the metal, $\Delta Z_1 = \operatorname{Re}\,\beta^{-1}$.

The dependence of the physical properties of plasmons on static electromagnetic fields can be used to implement control methods in application problems of metal plasmonics.

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УЗАГАЛЬНЕНА МОДЕЛЬ ДРУДЕ–ЛОРЕНЦА
ТА ЇЇ ЗАСТОСУВАННЯ У МЕТАЛОПЛАЗМОНІЦІ

Узагальнено модель Друде–Лоренца на випадок плазмонів у немагнітних провідниках, що знаходяться у статичних магнітних $\mathbf{H}_0$ та електричних $\mathbf{E}_0$ полях із врахуванням ефектів просторової дисперсії. Показано, що магнітостатичне поле $\mathbf{H}_0$ та просторова дисперсія формують два додаткові типи низькочастотних об'ємних плазмонів, а дисперсія всіх типів об'ємних плазмонів суттєво залежить від взаємної орієнтації напряму їх розповсюдження $\mathbf{e}_k$ та вектора магнітостатичного поля $\mathbf{H}_0$. Що стосується поверхневих плазмонів, то тут просторова дисперсія приводить до двокомпонентної структури (в металі) їх електричного поля $\mathbf{E}$, а зовнішнє електростатичне поле $\mathbf{E}_0$ – до наведеної просторової дисперсії, залежної від величини постійної Холла R_p. У той самий час, орієнтація магнітостатичного поля $\mathbf{H}_0$ суттєво впливає на загальну дисперсію поверхневих плазмонів.

Ключові слова: об'ємні плазмони, поверхневі плазмони, просторова дисперсія, магнітостатичне поле.