https://doi.org/10.15407/ujpe65.3.187

M. DIACHENKO,[1] O. NOVAK,[1] R. KHOLODOV,[1] A. FOMINA[2]
[1] Institute of Applied Physics, Nat. Acad. of Sci. of Ukraine
(58, Petropavlivs'ka Str., Sumy 40030, Ukraine; e-mail: mykhailo.m.diachenko@gmail.com)
[2] Bogolyubov Institute for Theoretical Physics, Nat. Acad. of Sci. of Ukraine
(14b, Metrolohichna Str., Kyiv 03143, Ukraine)

ELECTRON-POSITRON PAIR PHOTOPRODUCTION IN A STRONG MAGNETIC FIELD THROUGH THE POLARIZATION CASCADE

The process of the e^-e^+ pair photoproduction in a strong magnetic field through the polarization cascade (the creation of an e^-e^+ pair from a single photon and its subsequent annihilation to a single photon) has been considered. The kinematics of the process is analyzed, and the expression for the general amplitude is obtained. A radiation correction to the process of pair creation at the lowest Landau levels by a single photon is found in the case where the energy of this photon is close to the threshold value. A comparison with the process of e^-e^+ pair production by one photon is made.

Keywords: e^-e^+ pair photoproduction, vacuum polarization, quantum electrodynamics, strong magnetic field.

1. Introduction

The challenging character of theoretical studies concerning the quantum electrodynamics (QED) processes – e.g., photoproduction of electron-positron pairs – occurring in the presence of a strong external magnetic field is associated with the existence of such physical objects as neutron stars, where the magnetic field reaches the critical value $H_c = m^2c^3/(e\hbar) \approx \approx 4.41 \times 10^{13}$ Gs (pulsars) or even exceeds it (magnetars). In most papers, when studying such objects, the main attention is focused on the first-order processes. In particular, the photoproduction process is considered to be the dominating mechanism of the e^-e^+ plasma generation in the magnetosphere of pulsars, thus playing a key role in the mechanisms of radiation generation by them [1–6]. In so doing, the cited authors did not take into account higher-order processes. However, under certain conditions, the latter can have a resonant character arising when an intermediate particle reaches the mass shell. In particular, the account for the physical vacuum polarization induced by the generation of a virtual e^-e^+ pair and its subsequent annihilation to a photon (polarization loop) is a poorly studied aspect of the electron-positron pair photoproduction.

The effect of vacuum polarization in the presence of an external electromagnetic field has been predicted rather long ago in works [7, 8]. In particular, in the cited works, the Heisenberg–Euler Lagrangian was derived and the birefringence effect in strong electromagnetic fields was predicted. Later, this effect was theoretically studied with the help of a polarization tensor in the framework of the Schwinger proper-time method [9–17]. In works [18, 19], the resonant case where the intermediate particles reach the mass shell was analyzed. In work [20], the polarization tensor was studied using Green's function in a magnetic field (the tensor is determined by the summation over the

Landau levels in the basis of exact solutions of the Dirac equation). The general form of the polarization operator in the lowest-Landau-level approximation was also obtained in work [21] and applied in work [22] to study the magnetic catalysis problem. In work [23], a possibility of observing the effect of vacuum birefringence in alternative configurations of an external field – namely, in the field of a coil generator with a laser emf and the field of a radiofrequency waveguide – was considered theoretically.

It should be noted that much interest has been drawn recently to the experimental verification of nonlinear QED effects, in particular, the vacuum birefringence. This effect consists in a change of the electromagnetic wave polarization in strong external electromagnetic fields, which is a result of fluctuations of the electron-positron field in vacuum. In such fields, nonlinear effects become significantly stronger, and the physical vacuum transforms into an anisotropic medium whose properties are different along and across the field.

Despite a large number of theoretical studies, there is no direct experimental confirmation of the birefringence effect for today. Attempts are being made to detect this effect under laboratory conditions; in particular, on a PVLAS (Polarization of the vacuum with laser) installation [24–26]. In this experiment, the linearly polarized laser radiation is passed through a section with a magnetic field, and the ellipticity and the rotation angle of the polarization plane that arise due to the polarization of physical vacuum are attempted to be measured. But at present, the sensitivity of devices is not sufficiently high to confirm the existence of the birefringence effect.

Lately, experiments with the application of high-power lasers are also planned to be performed. They will be carried out at HIBEF (Helmholtz International Beamline for Extreme Fields) on the XFEL (European X-ray Free Electron Laser, Germany) [27] and ELI (Extreme Light Infrastructure, Czech Republic, Romania, Hungary) [28] installations. The appearance of first results concerning the study of the polarization of the optical radiation emitted by neutron stars, where the magnetic field is close to the critical value, should also be mentioned. In particular, the polarization degree of optical photons, which can be a result of the vacuum polarization, was determined for the first time for the isolated neutron star RX J1856.5-3754 [29].

As for theoretical researches of the process of electron-positron pair photoproduction in a magnetic field, it was considered for the first time in work [30] in the approximation of ultra-relativistic particle motion. In this approximation, the charged particles are in highly excited states, and the motion of those particles is quasiclassical. Using the operator method, this problem was considered by Baier and Katkov also in the quasiclassical approximation [31,32]. In work [33], this method was applied to study the photocreation of an electron-positron pair located at low Landau levels. It is worth to note that, in work [34], the process of photogeneration of polarized particles at arbitrary Landau levels and in arbitrary magnetic fields was considered.

In works [35,36], simple analytical expressions were found for the probability of the electron-positron pair photoproduction in a magnetic field in the general quantum-relativistic case without imposing additional restrictions on the problem parameters (pulses, energies, field strengths). The results demonstrated a clear dependence on the photon Stokes parameters. In works [12, 18, 37], the optical theorem was used to derive general expressions for the probabilities of the electron-positron pair production by a photon. Note also that, in the works [38,39], the resonant case of the process of electron-positron pair generation in a magnetic field by two photons was considered. In work [40], the 1γ and resonant 2γ pair creation processes were compared at the parameter values that are typical of the neutron star magnetosphere, and the limiting concentration of cyclotron photons at which the second-order process dominates over the first-order one was determined.

We should also mention the works in which third-order processes were considered, because just such a process is dealt with in the present paper. In particular, the process of photon splitting (three external photon lines) in a magnetic field was studied in work [11]. Analogous diagrams are also used to explain the difference between the energy losses by protons and antiprotons [43], to describe the Barkas effect, and in chromodynamics (the effective $\gamma\gamma G$ vertex) [44].

In this work, the third-order process of e^-e^+ pair photoproduction in a strong magnetic field through the polarization loop will be considered. In Section 2, an expression for the total amplitude of the process of e^-e^+ pair photoproduction is derived with regard for the creation of a virtual e^-e^+ pair from a single photon and its subsequent annihilation to a single photon. In Section 3, the kinematics of the analyzed process is described, its threshold conditions are found, and the momenta of final particles are determined. In Section 4, the amplitude of the process concerned is found in the case where the pair is created at the lowest Landau levels, and the probability of the process is determined provided that the energy of the initial photon is close to the threshold value.

2. Total Amplitude of the e^-e^+ Pair Photoproduction Taking the Polarization Loop Into Account

In this paper, the calculations are carried out in the relativistic system of units ($\hbar = c = 1$). The Landau gauge with the external field 4-potential $A_{\text{ext}} = (0, 0, xH, 0)$ is also used.

According to the rules of quantum field theory, the amplitude of the process of electron-positron pair creation by a single photon making allowance for the polarization loop is determined as follows:

$$S_{fi} = e^3 \int d^4x d^4x' d^4x'' \bar{\Psi}''_e \gamma^\mu \Psi''_p D(x'' - x') \times \\ \times \operatorname{Sp}\left[\gamma_\mu G(x', x) \gamma_\nu G(x, x')\right] A^\nu. \quad (1)$$

The Feynman diagram of this process is depicted in Fig. 1, where the external and internal wavy lines correspond to the photon wave function and propagator, whereas the external and internal double solid lines correspond to the wave functions and propagators of an electron and a positron in the external magnetic field.

It should be noted that there are also other Feynman diagrams with three vertices describing the process of pair photoproduction, e.g., a triangle including two double solid and one wavy internal double solid lines (Fig. 2) and a diagram with the mass operator (Fig. 3). However, in this work, we analyze a process running near the threshold of the pair creation at ground Landau levels, which corresponds to the reaching of the mass shell by intermediate particles, i.e. to resonant conditions. In this case, the processes shown in Figs. 2 and 3 can be represented as cascades of first-order processes (pair photoproduction, synchrotron electron or positron emission, and photon absorption). In this paper, only zero Landau levels are considered. Therefore, the process of synchrotron emission from a zero level is impossible, so

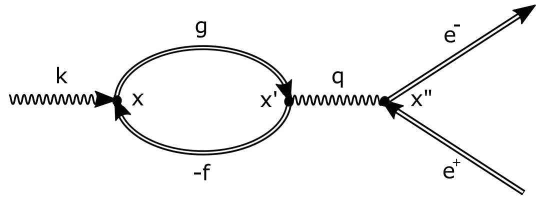

Fig. 1. Feynman diagram for the process of e^-e^+ pair production by a single photon making allowance for the polarization loop

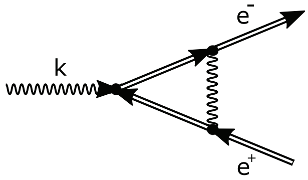

Fig. 2. Feynman diagram for the process of e^-e^+ pair photoproduction through the triangle

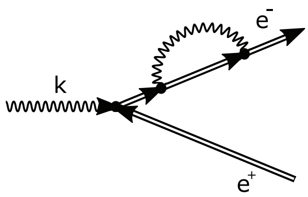

Fig. 3. Feynman diagram for the process of e^-e^+ pair photoproduction through the mass operator

that the contribution of those diagrams can be neglected. But attention should be drawn that if the process of the electron-positron pair photoproduction at arbitrary Landau levels is analyzed, the processes with the mass operator and the internal triangle have also to be taken into account.

In expression (1) for the process amplitude,

$$A^\nu = \sqrt{\frac{2\pi}{\omega V}}\, e^\nu e^{-ikx} \quad (2)$$

is the wave function of the initial photon [45]. Here, V is the normalization volume, $e^\nu = (0, \mathbf{e})$ is the 4-vector of photon polarization,

$$\mathbf{e} = \begin{pmatrix} \cos\phi\cos\theta\cos\alpha - e^{i\beta}\sin\phi\sin\alpha \\ \sin\phi\cos\theta\cos\alpha + e^{i\beta}\cos\phi\sin\alpha \\ -\sin\theta\cos\alpha \end{pmatrix}, \quad (3)$$

ϕ and θ are the azimuthal and polar angles, respectively, and α and β are polarization parameters. Expression (1) also includes the photon Green's function [45]

$$D(x''-x') = \frac{1}{(2\pi)^4}\int d^4q\, e^{-iq(x''-x')}\frac{4\pi}{q^2+i\Gamma} \qquad (4)$$

and the wave functions of an electron and a positron

$$\begin{aligned}\Psi_e &= \frac{1}{\sqrt{S}}e^{-i(E_et-p_{ey}y-p_{ez}z)}\psi_e(\zeta_e),\\ \Psi_p &= \frac{1}{\sqrt{S}}e^{i(E_pt-p_{py}y-p_{pz}z)}\psi_p(\zeta_p),\end{aligned} \qquad (5)$$

where

$$\zeta_{e,p} = m\sqrt{h}\left(x \pm \frac{p_{(e,p)y}}{m^2h}\right),$$

$$\psi_e(\zeta_e) = C_e[i\sqrt{m_e-\mu_e m}\,U_{l_e}(\zeta_e) +$$

$$+\,\mu_e\sqrt{m_e+\mu_e m}\,U_{l_e-1}(\zeta_e)\gamma^1]u_e, \qquad (6)$$

$$\psi_p(\zeta_p) = C_p[i\sqrt{m_p+\mu_p m}\,U_{l_p}(\zeta_p) -$$

$$-\,\mu_p\sqrt{m_p-\mu_p m}\,U_{l_p-1}(\zeta_p)\gamma^1]u_p, \qquad (7)$$

S is the normalization area, $\mu_{e,p}$ are the double projections of the electron and positron spins,

$$C_{e,p} = \frac{1}{2}\sqrt{\frac{\sqrt{eH}}{E_{e,p}m_{e,p}}} \qquad (8)$$

are the normalization constants,

$$m_{e,p} = m\sqrt{1+2l_{e,p}h}, \qquad (9)$$

$$u_{e,p} = \frac{1}{R_{e,p}}\left(0, \mp R^2_{e,p}, 0, p_{(e,p)z}\right) \qquad (10)$$

are constant bispinors, and

$$R_{e,p} = \sqrt{E_{e,p}-\mu_{e,p}m_{e,p}}. \qquad (11)$$

The function $G(x,x')$ in expression (1) for the amplitude is Green's function of an electron in the magnetic field. It was obtained for the first time in work [46] and, using another method, in work [47]. In addition, it was found in work [48] proceeding from the exact solutions of the Dirac equations. This function looks like

$$G(x',x) = -\frac{m\sqrt{h}}{(2\pi)^3}\int d^3g\, e^{-i\Phi}\, G_H(x',x), \qquad (12)$$

where $d^3g = dg_0dg_ydg_z$, $\Phi = g_0(t'-t) - g_y(y'-y) - g_z(z'-z)$ is the phase, $h = H/H_c$, $H_c = m^2/e = 4.41\times10^{13}$ Gs is the Schwinger critical magnetic field,

$$G_H(x',x) = \sum_n \frac{1}{g_0^2-E_g^2}((\gamma P+m)[\tau U_nU'_n +$$

$$+\,\tau^*U_{n-1}U'_{n-1}] + im\sqrt{2nh}\gamma^1[\tau U_{n-1}U'_n -$$

$$-\,\tau^*U_nU'_{n-1}]), \qquad (13)$$

$$E_g = \sqrt{m^2(1+2nh)+g_z^2}, \qquad (14)$$

$$\tau = \frac{1}{2}(1+i\gamma^2\gamma^1), \qquad (15)$$

$$P = (g_0, 0, 0, g_z), \qquad (16)$$

γ are the Dirac matrices, U_n is the Hermite function, $\rho(x) = m\sqrt{h}x + g_y/(m\sqrt{h})$ is the argument of U_n, and the primed functions in Eq. (13) depend on x'. Note that this propagator is widely used, when calculating the amplitudes of the second-order processes in which the electron plays the role of an intermediate particle (see, e.g., works [38–40, 49]).

After the integration over d^3x, d^3x', d^3x'', d^3f, and d^3q, we obtain

$$S_{fi} = C\delta^3(p_e+p_p-k)\int d^3g\,dq_x\,dx\,dx'\,dx'' \times$$

$$\times \frac{e^{ik_xx-iq_xx'+iq_xx''}}{\omega^2-k_y^2-k_z^2-q_x^2+i\Gamma}\sum_{n,n'}\frac{B}{(g_0^2-E_g^2)(f_0^2-E_f^2)}, \qquad (17)$$

where

$$C = \frac{2e^3}{S}\sqrt{\frac{2\pi}{V\omega}},$$

$$B = \bar{\psi}_e(\zeta_e)\gamma^\mu\psi_p(\zeta_p)\mathrm{Sp}[\gamma_\mu G_H(g;\rho',\rho)\gamma_\nu \times$$

$$\times\, G_H(-f;\eta,\eta')]e^\nu. \qquad (18)$$

Formula (17) is a general expression for the amplitude of a third-order process in the fine structure constant. It has a rather complicated form, because it contains infinite sums over the Landau levels of intermediate particles. Furthermore, the quantity B [see Eq. (18)] consists of 64 terms. Therefore, in what follows, we will consider a case where the magnetic field is rather strong, so that the lowest-Landau-level approximation can be applied, which significantly simplifies the calculations.

3. Kinematics of the Process

Before calculating the amplitude and probability of the process, let us consider its kinematics. The kinematics of the researched process is absolutely identical to that of the pair photoproduction, which was studied in detail in many papers, including our works [35, 36].

The conservation laws for the process concerned in a magnetic field look like

$$\begin{cases}\omega = E_e + E_p, \\ k_z = p_{ez} + p_{pz}.\end{cases} \tag{19}$$

As one can see from Eqs. (19), the conservation laws for the energy and the longitudinal component of the electron and positron momenta with respect to the magnetic field direction are obeyed. To find the threshold conditions, the following function is introduced:

$$f(p) = \omega - E_e - E_p, \tag{20}$$

where p is the z-component of the electron momentum. The energy conservation law is fulfilled, if $f = 0$. The function f can also be written in the form

$$\tilde{f} = \tilde{\omega} - \sqrt{1 + 2l_e h + \tilde{p}^2} - \sqrt{1 + 2l_p h + (\tilde{\omega} u - \tilde{p})^2}, \tag{21}$$

where the tilde (˜) over the symbol means the division by the electron mass, $u = \cos\theta$, and θ is the angle between the photon propagation and magnetic field directions. The dependences of the function $\tilde{f}$ on the electron momentum and the photon frequency in the cases of the longitudinal and transverse photon propagations with respect to the magnetic field are shown in Figs. 4 and 5, respectively, for the Landau levels $l_e = 3$ and $l_p = 1$. As one can see from Fig. 4, the function $\tilde{f}$ never equals zero. Therefore, the process of pair production is impossible, if $\theta = 0$.

The process threshold is determined from the condition that the function f equals zero at its maximum point. Probing this function with respect to its maximum, we obtain

$$p_m = \frac{m_e}{m_e + m_p}\omega u. \tag{22}$$

Expression (22) and Fig. 5 testify that if the photon propagates perpendicularly to the magnetic field,

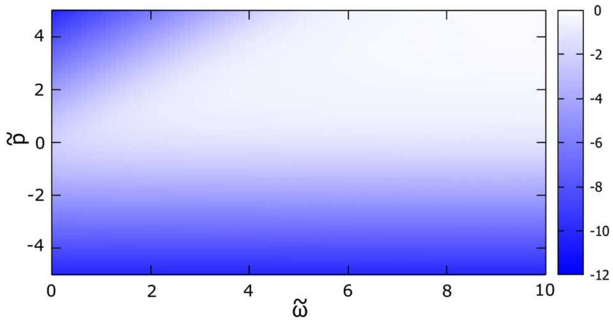

Fig. 4. Dependence of the function $\tilde{f}$ on the electron momentum $\tilde{p}$ and the photon frequency $\tilde{\omega}$ for the longitudinal photon propagation with respect to the magnetic field direction ($\theta = 0$) and for the Landau levels $l_e = 3$ and $l_p = 1$. All quantities are reckoned in the electron mass units

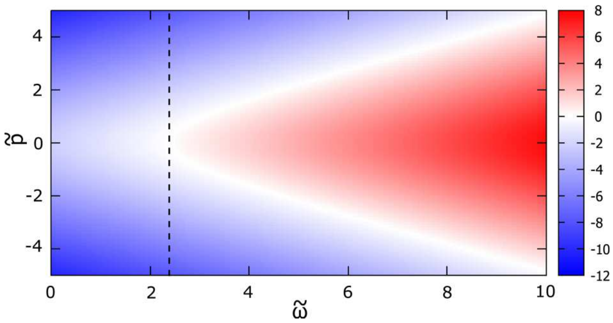

Fig. 5. The same as in Fig. 4, but for the perpendicular photon propagation with respect to the magnetic field direction ($\theta = \pi/2$)

the electron momentum always equals zero at the maximum point. Therefore, if the threshold conditions are satisfied, the electron-positron pair is created at the Landau levels, with the z-components of the electron and positron momenta, being absent in this case. Then, proceeding from Eqs. (21) and (22), the threshold photon energy can be found in the form

$$\omega_{tr} = \frac{m_e + m_p}{\sqrt{1 - u^2}}. \tag{23}$$

Hence, for a photon propagating perpendicularly to the magnetic field ($k_z = 0$), we have a simple condition for the process threshold:

$$\omega_{tr} = m_e + m_p. \tag{24}$$

The conservation laws can also be used to determine the electron momentum. In the general case, if the photon energy exceeds the threshold value, we obtain

$$p_{1,2} = \frac{m_e^2 - m_p^2 + \omega^2 v^2}{2\omega v^2} u \pm$$
$$\pm \sqrt{\frac{\omega^2}{4} - \frac{m_e^2 + m_p^2}{2v^2} + \frac{(m_e^2 - m_p^2)^2}{4\omega^2 v^4}}, \quad (25)$$

where $v = \sin\theta$. In the case $\theta = \pi/2$, the electron momentum looks like

$$p_{1,2} = \pm m \sqrt{\frac{\tilde{\omega}^2}{4} - 1 - h(l_e + l_p) + \frac{h^2}{\tilde{\omega}^2}(l_e - l_p)^2}. \quad (26)$$

For the lowest Landau levels ($l_{e,p} = 0$), we obtain

$$p_{1,2} = \pm m \sqrt{\frac{\tilde{\omega}^2}{4} - 1}. \quad (27)$$

4. Amplitude and Probability of the Process in the Case of the Lowest Landau Levels

Below, we will consider the case where a photon propagates perpendicularly to the magnetic field ($\theta = \pi/2$), and its energy is close to the threshold value, i.e. [hereafter, $\tilde{\omega} = \omega/(2\pi)$]

$$\tilde{\omega}^2 = \tilde{\omega}_{\text{th}}^2 + \delta, \quad \delta \ll 1. \quad (28)$$

In addition, we will assume that an electron-positron pair is created at the lowest Landau levels, and the intermediate particles are at the ground Landau levels,

$$n_{g,f} = 0, \quad l_{e,p} = 0. \quad (29)$$

It should be noted that the corresponding conditions are satisfied in strong magnetic fields that are close to the critical value or exceed it.

In the case concerned [Eqs. (29)], the threshold condition (23) acquires a simple form

$$\tilde{\omega}_{\text{th}} = 1, \quad \tilde{\omega}_{\text{th}} = \frac{\omega_{\text{th}}}{2m}. \quad (30)$$

Then, the z-component of the electron momentum equals

$$p = \pm m\sqrt{\delta}. \quad (31)$$

Now, integrating over x, x', x'', g_y, and q_x in expression (17), we obtain the process amplitude in the form

$$S_{fi} = -C'\Phi' \sqrt{1-\mu_e}\sqrt{1+\mu_p}\,\delta^3(p_e + p_p - k) \times$$
$$\times \int dg_0 dg_z A', \quad (32)$$

where

$$C' = \sqrt{\frac{2\pi}{V\omega E_e E_p}} \frac{\pi e^3}{Sk^2} m^2 h e^{-\frac{3\eta}{2}},$$

$$A' = \frac{\bar{u}_e \gamma^\mu u_p Sp[\gamma_\mu(\gamma g + m)\alpha\gamma_\nu(-\gamma f + m)\alpha]e^\nu}{(g_0^2 - E_g^2)(f_0^2 - E_f^2)},$$

$$\Phi' = \exp\left\{i \frac{k_x k_y - 2p_{ey}k_x}{2m^2 h}\right\},$$

$$\eta = \frac{\omega^2}{2m^2 h} \approx \frac{2}{h}.$$

In order to integrate over g_0 and g_z, let us rewrite the denominator in expression (32) with the help of the α-representation, which is analogous to the Schwinger proper-time method:

$$\frac{1}{g_0^2 - E_g^2 + i\epsilon} = -i \int_0^\infty d\alpha e^{i\alpha\left(g_0^2 - E_g^2 + i\epsilon\right)}. \quad (33)$$

The presence of a polarization loop is known to result in a divergence. Therefore, the regularization procedure has to be carried out. In this work, we use the following method [50]:

$$\frac{1}{g_0^2 - E_g^2 + i\epsilon} = -i \int_0^\infty d\alpha e^{i\alpha\left(g_0^2 - g_z^2 + i\epsilon\right)}[e^{-i\alpha m^2} - e^{-i\alpha M^2}]. \quad (34)$$

When integrating, the following change of variables is performed:

$$\alpha = \lambda\frac{1-\xi}{2}, \quad \beta = \lambda\frac{1+\xi}{2}. \quad (35)$$

Then, taking Eqs. (33)–(35) into account, the integral in expression (32) can be rewritten in the form

$$\int dg_0 dg_z A' = -e_z u_0 \pi m^2 \int_{-1}^{1} d\xi \int_0^\infty d\lambda E(\lambda, \xi) \times$$
$$\times (1 - \tilde{\omega}^2 + \tilde{\omega}^2 \xi^2), \quad (36)$$

where

$$u_0 = R_{ep}R_{pm} - R_{pp}R_{em},$$

$$
\begin{aligned}
&E(\lambda,\xi) = e^{-i\epsilon\lambda}[e^{-i\lambda A} + e^{-i\lambda B} - e^{-i\lambda C} - e^{-i\lambda D}],\\
&A = m^2(1+F),\\
&B = m^2\left(F + \frac{M^2}{m^2}\right),\\
&C = m^2\left\{F + \frac{1}{2}(1+\xi) + \frac{M^2}{2m^2}(1-\xi)\right\},\\
&D = m^2\left\{F + \frac{1}{2}(1-\xi) + \frac{M^2}{2m^2}(1+\xi)\right\},\\
&F = \tilde{\omega}^2(\xi^2-1).
\end{aligned}
$$

When the regularization procedure has been carried out, formula (36) reads

$$\int dg_0 dg_z A' = -2i\pi e_z u_0 (I-2), \tag{37}$$

where

$$
\begin{aligned}
I &= \int_{-1}^{1} \frac{d\xi}{1+F-i\tilde{\epsilon}} = \\
&= \frac{1}{\tilde{\omega}\sqrt{\tilde{\omega}^2-1}}\left[\ln\left|\frac{\tilde{\omega}-\sqrt{\tilde{\omega}^2-1}}{\tilde{\omega}+\sqrt{\tilde{\omega}^2-1}}\right| + i\pi\right], \quad \tilde{\omega} > 1.
\end{aligned} \tag{38}
$$

In case (29),

$$I - 2 \approx \frac{i\pi}{\sqrt{\delta}}. \tag{39}$$

Taking expressions (37) and (Int2) into account, we get amplitude (32) in the following form:

$$
\begin{aligned}
S_{fi} &= e_z u_0 \frac{2\pi^3 e^3}{S\Gamma\sqrt{\delta}}\sqrt{\frac{2\pi}{V\omega E_e E_p}} m^2 h \times \\
&\times \sqrt{1-\mu_e}\sqrt{1+\mu_p}\,\Phi'\delta^3(p_e + p_p - k).
\end{aligned} \tag{40}
$$

From whence, one can see that the amplitude of the electron-positron pair photoproduction in a strong magnetic field with regard for the vacuum polarization differs from zero in the case where the projections of the electron and positron spins equal $\mu_e = -1$ and $\mu_p = 1$, respectively. This conclusion is in agreement with the results obtained in works [35, 36] for the pair photoproduction in a magnetic field.

Finally, let us determine the probability of the examined process near the threshold with the help of formula (40) for the amplitude. For this purpose, let us take advantage of the well-known rule from quantum field theory [45]:

$$
\begin{aligned}
&dW = |S_{fi}|^2 dN_e dN_p,\\
&dN_{e,p} = \frac{d^2 p_{e,p} S}{(2\pi)^2}.
\end{aligned} \tag{41}
$$

Then, making use of formulas (40) and (41), we obtain the following expression for the probability of the process per unit time:

$$w_{\gamma\to\gamma\to e^-e^+} = \alpha^3 \frac{m^3 h^3}{4\Gamma^2\delta\sqrt{\delta}}(1+\xi_3), \tag{42}$$

where α is the fine structure constant.

5. Conclusions

In this work, the process of electron-positron pair photoproduction in a strong magnetic field has been studied taking the polarization cascade into consideration. A formula is obtained for the total amplitude of this process, and the corresponding probability is determined in the case where the e^-e^+ pair is created at the lowest Landau levels by a single photon. As one can see from expression (42), the probability of the process depends on the initial photon polarization, similar to the process of pair photoproduction; namely, it depends only on ξ_3 and vanishes for the normal photon polarization ($\xi_3 = -1$).

The expressions obtained in this work made it possible to estimate the influence of the physical vacuum polarization on the photoproduction process in a strong magnetic field. In the case where the photon energy is close to the threshold energy of pair photoproduction, the probability of the electron-positron pair production by a single photon looks like

$$w_{\gamma\to e^-e^+} = \alpha\frac{mh}{4\sqrt{\delta}}(1+\xi_3). \tag{43}$$

By comparing expressions (42) and (43), we obtain

$$w_{\gamma\to\gamma\to e^-e^+} = \alpha^2\frac{h^2}{\delta\tilde{\Gamma}^2} w_{\gamma\to e^-e^+}. \tag{44}$$

One can see from this expression that, in the selected case, the ratio between the probabilities of the processes of the first and third orders in the fine structure constant depends on the width of the intermediate photon state and the detuning from the process threshold value. To evaluate the proportionality factor in Eq. (44), let us put the magnetic field equal to $h = 10$ (such fields are observed near magnetars), the detuning to $\delta = 0.1$, and the intermediate state width to $\tilde{\Gamma} \sim \alpha h^2$ [40]. Then the sought coefficient equals 0.1.

This work was executed in the framework of the scientific programs Nos. 0116U005848 and 0118U003535 of the National Academy of Sciences of Ukraine. The authors are grateful to Professor V. V. Skalozub for the idea of a research, valuable comments, and discussion of the results obtained.

1. J.K. Daugherty, A.K. Harding. Electromagnetic cascades in pulsars. *Astrophys. J.* **252**, 337 (1982).
2. P.A. Sturrock, A.K. Harding. Cascade model of gamma-ray bursts. *Astrophys. J.* **346**, 950 (1989).
3. J.K. Daugherty, A.K. Harding. Gamma-ray pulsars: Emission from extended polar cap cascades. *Astrophys. J.* **458**, 278 (1996).
4. P.A. Sturrock, A.K. Harding. A model of pulsars. *Astrophys. J.* **164**, 529 (1971).
5. J.K. Daugherty. A.K. Harding. Pair production in superstrong magnetic fields. *Astrophys. J.* **273**, 761 (1983).
6. A.K. Harding. Physics in strong magnetic fields near neutron stars. *Science* **251**, 1033 (1991).
7. H. Euler, B. Kockel, The scattering of light by light in the Dirac theory. *Naturwissenschaften* **23**, 246 (1935).
8. W. Heisenberg, H. Euler. Folgerungen aus der Diracschen Theorie des Positrons. *Z. Phys.* **98**, 714 (1936).
9. J. Schwinger. On gauge invariance and vacuum polarization. *Phys. Rev.* **82**, 664 (1951).
10. I.A. Batalin, A.E. Shabad. Green's function of a photon in a constant homogeneous electromagnetic field of general form. *JETP* **33**, 483 (1971).
11. S. Adler. Photon splitting and photon dispersion in a strong magnetic field. *Ann. Phys.* **67**, 599 (1971).
12. W. Tsai. Vacuum polarization in homogeneous magnetic fields. *Phys. Rev. D* **10**, 2699 (1974).
13. V.M. Katkov. Polarization operator of a photon in a magnetic field. *Zh. Èksp. Teor. Fiz.* **150**, 229 (2016) (in Russian).
14. W. Tsai, T. Erber. Propagation of photons in homogeneous magnetic fields: Index of refraction. *Phys. Rev. D* **15**, 1132 (1975).
15. K. Kohri, S. Yamada. Polarization tensors in strong magnetic fields. *Phys. Rev. D* **65**, 043006 (2002).
16. K. Hattori, K. Itakura. Vacuum birefringence in strong magnetic fields: (I) Photon polarization tensor with all the Landau levels. *Ann. Phys.* **330**, 23 (2013).
17. K. Hattori, K. Itakura. Vacuum birefringence in strong magnetic fields: (II) Complex refractive index from the lowest Landau level. *Ann. Phys.* **334**, 58 (2013).
18. A. Shabad. Photon dispersion in a strong magnetic field. *Ann. Phys.* **90**, 166 (1975).
19. M. Diachenko, O. Novak, R. Kholodov. A cascade of e^-e^+ pair production by a photon with subsequent annihilation to a single photon in a strong magnetic field. *Laser Phys.* **26**, 066001 (2016).
20. M. Diachenko, O. Novak, R. Kholodov. Vacuum birefringence in supercritical magnetic fields. *Ukr. J. Phys.* **64**, 179 (2019).
21. G. Calucci, R. Ragazzon. Nonlogarithmic terms in the strong field dependence of the photon propagator. *J. Phys. A* **27**, 2161 (1994).
22. V.P. Gusynin, V.A. Miransky, I.A. Shovkovy. Dimensional reduction and catalysis of dynamical symmetry breaking by a magnetic field. *Nucl. Phys. B* **462**, 249 (1996).
23. M. Diachenko, O. Novak, E. Padusenko, R. Kholodov. Vacuum birefringence in the fields of a current coil and a guided electromagnetic wave. *Ukr. J. Phys.* **63**, 979 (2018).
24. E. Zavattini, G. Zavattini, G. Ruoso *et al.* New PVLAS results and limits on magnetically induced optical rotation and ellipticity in vacuum. *Phys. Rev. D* **77**, 032006 (2008).
25. F. Della Valle, E. Milotti, A. Ejlli *et al.* First results from the new PVLAS apparatus: A new limit on vacuum magnetic birefringence. *Phys. Rev. D* **90**, 092003 (2014).
26. F. Della Valle, A. Ejlli, U. Gastaldi *et al.* The PVLAS experiment: measuring vacuum magnetic birefringence and dichroism with a birefringent Fabry–Perot cavity. *Eur. Phys. J. C* **76**, 24 (2016).
27. H.P. Schlenvoigt, T. Heinzl, U. Schramm *et al.* Prospects for studying vacuum polarisation using dipole and synchrotron radiation. *Phys. Scr.* **91**, 023010 (2016).
28. O. Tesileanu, D. Ursescu, R. Dabu, N. V. Zamfir. Extreme light infrastructure. *J. Phys.: Conf. Ser.* **420**, 012157 (2013).
29. R.P. Mignani, V. Testa, D. Gonzalez Caniulefet *et al.* Evidence for vacuum birefringence from the first optical-polarimetry measurement of the isolated neutron star RX J1856.5–3754. *Mon. Not. Roy. Astron. Soc.* **465**, 492 (2017).
30. N.P. Klepikov. Radiation of photons and electron-positron pairs in a magnetic field. *Zh. Èksp. Teor. Fiz.* **26**, 19 (1954) (in Russian).
31. V.N. Baier, V.M. Katkov. Processes involved in the motion of high energy particles in a magnetic field. *Sov. Phys. – JETP* **26**, 854 (1968).
32. V.N. Baier, V.M. Katkov. Quasiclassical theory of bremsstrahlung by relativistic particles. *Sov. Phys. – JETP* **28**, 807 (1969).
33. V.N. Baier, V.M. Katkov. Pair creation by a photon in a strong magnetic field. *Phys. Rev. D* **75**, 07300 (2007).
34. L. Semionova, D. Leahy. Remarks concerning pair creation in strong magnetic fields. *Astron. Astrophys.* **373**, 272 (2001).
35. O.P. Novak, R.I. Kholodov. Polarization effects in the photon-induced process of electron-positron pair cre-

ation in a magnetic field, studied in the ultra-quantum-mechanical approximation. *Ukr. J. Phys.* **53**, 185 (2008).
36. O.P. Novak, R.I. Kholodov. Spin-polarization effects in the processes of synchrotron radiation and electron-positron pair production by a photon in a magnetic field. *Phys. Rev. D* **80**, 025025 (2009).
37. V.V. Skobelev. On photon propagation in a magnetic field. *Zh. Èksp. Teor. Fiz.* **73**, 1301 (1977) (in Russian).
38. M. Dyachenko, O. Novak, R. Kholodov. Resonant threshold two-photon e^-e^+ pair production onto the lowest Landau levels in a strong magnetic field. *Ukr. J. Phys.* **59**, 849 (2014).
39. M.M. Diachenko, O.P. Novak, R.I. Kholodov. Resonant generation of an electron–positron pair by two photons to excited Landau levels. *JETP* **121**, 813 (2015).
40. M.M. Diachenko, O.P. Novak, R.I. Kholodov. Pair production in a magnetic and radiation field in a pulsar magnetosphere. *Mod. Phys. Lett. A* **30**, 1550111 (2015).
41. O.P. Novak, R.I. Kholodov, P.I. Fomin. Electron-positron pair production by an electron in a magnetic field near the process threshold. *JETP* **110**, 978 (2010).
42. O.P. Novak, R.I. Kholodov. Electron-positron pair production by an electron in a magnetic field in the resonant case. *Phys. Rev. D* **86**, 105013 (2012).
43. M.M. Diachenko, R.I. Kholodov. Energy losses of positive and negative charged particles in electron gas. *Mod. Phys. Lett. A* **32**, 1750031 (2017).
44. V. Skalozub. Induced color charges, effective $\gamma\gamma G$ vertex in QGP. Applications to heavy-ion collisions. *Ukr. J. Phys.* **64**, 754 (2019).
45. V.B. Berestetskii, E.M. Lifshitz, L.P. Pitaevskii, *Relativistic Quantum Theory* (Pergamon Press, 1982).
46. A. Chodos, K. Everding, D.A. Owen. QED with a chemical potential: The case of a constant magnetic field. *Phys. Rev. D* **42**, 2881 (1990).
47. V.P. Gusynin, V.A. Miransky, I.A. Shovkovy. Dynamical chiral symmetry breaking by a magnetic field in QED. *Phys. Rev. D* **52**, 4747 (1995).
48. D.B. Melrose, A.J. Parle. Quantum electrodynamics in strong magnetic fields. I. Electron states. *Aust. J. Phys.* **36**, 755 (1983).
49. P.I. Fomin, R.I. Kholodov. To the theory of resonance quantum-electrodynamic processes in an external magnetic field. *Ukr. Fiz. Zh.* **44**, 1526 (1999) (in Ukrainian).
50. N.N. Bogoliubov, D.V. Shirkov. *Introduction to the Theory of Quantized Field.* (Interscience, 1959).

Received 11.11.19.
Translated from Ukrainian by O.I. Voitenko

М. Дяченко, О. Нова, Р. Холодов, А. Фоміна

ФОТОНАРОДЖЕННЯ ЕЛЕКТРОН-ПОЗИТРОННОЇ ПАРИ ЧЕРЕЗ ПОЛЯРИЗАЦІЙНИЙ КАСКАД У СИЛЬНОМУ МАГНІТНОМУ ПОЛІ

Р е з ю м е

У роботі досліджено процес фотонародження e^-e^+ пари з врахуванням поляризаційного каскаду (народження та послідовної анігіляції пари в один фотон) в сильному магнітному полі. Проаналізовано кінематику та отримано вираз для загальної амплітуди процесу. Знайдено радіаційну поправку при народженні пари фотоном на найнижчі рівні Ландау для випадку, коли енергія початкового фотона має значення, близьке до порогового. Проведено порівняння з процесом народження e^-e^+ пари одним фотоном.