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INFLUENCE OF MAGNETOSTATIC FIELD ON THE EXCITATION MECHANISM OF HYBRID PLASMON-POLARITONS IN SEMICONDUCTORS

The influence of a magnetostatic field on the dynamics of quasiparticles (plasmons and plasmon-polaritons) in semiconductors with a direct current has been analyzed. The counterflow of the electron and hole continua gives rise to the appearance of unstable hybrid quasiparticles that are intrinsically coupled to both electrons and holes. It has been demonstrated that the dispersion relationship, as well as the increment of growth and the decrement of damping of the amplitudes of the dynamical variables describing the hybrid quasiparticles, strongly depend on the stationary drift velocity of charge carriers associated with the direct current and on the relative orientation of the quasiparticle wave vector and the magnetostatic field vector. In particular, the direct current is the physical origin of the appearance of unstable quasiparticles in the terahertz frequency interval in semiconductors, whereas the magnetostatic field induces additional frequency bands of hybrid quasiparticles, with the number of the bands being governed by the relative orientation of the quasiparticle wave vector and the magnetostatic field vector. The combined effect of these two factors on the dynamics of unstable hybrid quasiparticles in semiconductors provides flexible control over their behavior, which can be used to solve problems in applied terahertz radiophysics.

Keywords: plasma, electrons, holes, electric field, magnetostatic field, electric charge density, electric current density, plasmon frequency, effective mass, polarization, plasmons, polaritons, cyclic frequency, wave vector, dielectric permittivity, dispersion equation, frequency interval, spatial dispersion, instability, increment of growth, decrement of damping.

1. Introduction

This work is a logical continuation of the cycle of Refs. [1–3] aimed at studying plasmons and plasmon-polaritons, both bulk and surface, in metals and semiconductors. In those works, the challenging character of research dealing with plasmons and plasmon-polaritons has been demonstrated, and a detailed review of relevant scientific publications has been presented.

We note in particular that considerable attention is currently paid to external influences, especially those of a magnetostatic field [4, 8], on the state of electron-hole plasma (*eh*-plasma) in semiconductors, and those effects have versatile manifestations in interesting physical phenomena. In particular, the photogenerated *eh*-plasma in a semiconductor located in a quantized external magnetostatic field (in a quantum well) was studied in experimental-theoretical work [4], and "giant" bursts of coherent radiation were detected therein.

In Ref. [5], a two-dimensional (2D) quantum *eh*-plasma (in InGaAs quantum wells) in a strong magnetic field, at which an inversion of level population was realized, was considered, and a series of bursts were observed. The authors of the cited work emphasized that the magnetostatic field and a high population degree affect the spectral and temporal characteristics of collective recombination.

In paper [6], the distribution of surface electromagnetic waves along the interface between the semicon-

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ductor quantum eh-plasma and the vacuum in the presence of an external magnetostatic field was considered theoretically, taking into account quantum effects (tunneling) and the electron and hole degeneration. A new type of waves, which is absent in the classical approximation, was discovered. It was also shown that the magnetostatic field and quantum effects can substantially modify the dispersion of surface waves.

In Ref. [7], large electron-hole droplets in germanium (Ge) were experimentally studied at low temperatures and optical generation of eh-plasma. Microwave resonances in magnetostatic fields were found in the droplets about 1 mm in size. The results of the work testify to the fact that the magnetostatic field affects the dynamics and dimensions of macroscopic eh-structures.

In research [8], it was shown that the quantum-degenerate non-equilibrium (at the Fermi limit) eh-plasma in semiconductor quantum wells can generate collective spontaneous radiation emission, especially under the action of a magnetostatic field.

The aim of this work is to continue the study of the excitation mechanism of hybrid plasmons and plasmon-polaritons (hybrid quasiparticles) in semiconductors [3], which is associated with the combined effect of the counter-motion of electrons and holes. This motion arises when a direct electric current flows through a semiconductor subjected to an external magnetostatic field.

In what follows, we will apply the terminology introduced in Ref. [3]. In particular, the term "hybrid quasiparticles" has a direct physical meaning here. As shown in Ref. [3], unstable states of eh-plasma do not exist in semiconductors with a single type of electric charge carriers. There are also no such states in the absence of the counter-motion of electrons and holes. Therefore, the use of the term "hybrid quasiparticles" to describe the unstable state of eh-plasma is justified.

An important fact from the application point of view is that the frequency bands of hybrid quasiparticles in semiconductors correspond [3] to the terahertz frequency interval of electromagnetic waves, $\omega = (10^{11} \div 1.5 \times 10^{13})$ Hz.

The main attention in this work is devoted to the influence of a magnetostatic field $\mathbf{H_0}$ on the excitation mechanism of hybrid quasiparticles in semiconductors. We show that the field $\mathbf{H_0}$ considerably affects the qualitative and quantitative parameters of hybrid quasiparticles in semiconductors; namely, the number of their frequency bands increases several times.

The basic mathematical model used to describe the dynamics of hybrid quasiparticles in semiconductors was formulated in Ref. [3]. Here, it was extended by taking into account the influence of the Lorentz force acting on moving electrons and holes in the semiconductor due to the presence of the magnetostatic field $\mathbf{H_0}$.

2. Dielectric Permittivity of a Semiconductor with a Direct Electric Current and Located in a Magnetostatic Field

As is known [9, 10], in solid-state plasmonics, an adequate model of the dielectric permittivity of the medium is required for the analysis of the physical properties of quasiparticles $\widehat{\varepsilon} = \widehat{\varepsilon}(\omega, \mathbf{k})$, where ω is the cyclic frequency, and $\mathbf{k}$ is the wave vector of quasiparticles. When constructing the dielectric permittivity of a semiconductor with a direct electric current flow in a magnetostatic field $\mathbf{H_0}$, we will use the algorithm verified in Ref. [3]. Therefore, similarly to what was done in that work, we choose such macroscopic indicators as the electron and hole concentrations, $n_{e,h} = n_{e,h}(t, \mathbf{r})$, and the velocities of motion $\mathbf{v_{e,h}} = \mathbf{v_{e,h}}(\mathbf{t}, \mathbf{r})$ of infinitesimal macroscopic volumes filled with electrons (e) and holes (h), as dynamic quantities. The dynamic quantities $n_{e,h}$ and $\mathbf{v_{e,h}}$, in turn, determine the densities of negative and positive electric charges

$$\rho_{fe} = -qn_{fe}, \quad \rho_{fh} = qn_{fh}, \tag{1}$$

where $q = -e$ and e is the electron charge, as well as the electric current densities

$$\mathbf{j}_{fe} = -qn_{fe}\mathbf{v}_{fe}, \quad \mathbf{j}_{fh} = qn_{fh}\mathbf{v}_{fh}, \tag{2}$$

which satisfy the law of electric charge conservation (the law of continuity) [11, 12].

Below, as in Ref. [3], we assume that the concentrations $n_{f(e,h)}$ of electric charges and the velocities $\mathbf{v}_{f(e,h)}$ of electrons and holes have two components; namely, the stationary complnents $n_{0(e,h)}$ and $\mathbf{v}_{0(e,h)}$, and the fluctuation components $n_{e,h}$ and $\mathbf{v}_{e,h}$, associated with fluctuations in the electric charge density. Accordingly, the dynamic variables of the problem take the following representation:

$$\begin{aligned} &n_{fe} = n_{0e} + n_e(t,\mathbf{r}), \quad \mathbf{v}_{fe} = \mathbf{v}_{0e} + \mathbf{v}_e(t,\mathbf{r}),\\ &n_{fh} = n_{0h} + n_h(t,\mathbf{r}), \quad \mathbf{v}_{fh} = \mathbf{v}_{0h} + \mathbf{v}_h(t,\mathbf{r}),\\ &n_e \ll n_{0e} = \text{const}, \quad n_h \ll n_{0h} = \text{const},\\ &\mathbf{v}_e \ll \mathbf{v}_{0e} = \text{const}, \quad \mathbf{v}_h \ll \mathbf{v}_{0h} = \text{const}. \end{aligned} \tag{3}$$

From formulas (2) and (3), we obtain the following expressions for the electric current densities:

$$\begin{aligned} &\mathbf{j}_{fe} \simeq -q(n_{0e}\mathbf{v}_{0e} + n_{0e}\mathbf{v}_e + n_e\mathbf{v}_{0e}),\\ &\mathbf{j}_{fh} \simeq q(n_{0h}\mathbf{v}_{0h} + n_{0h}\mathbf{v}_h + n_h\mathbf{v}_{0h}). \end{aligned} \tag{4}$$

A closed system of dynamic equations for the posed problem in the linear approximation, similarly to Ref. [3], has to include the laws of electric charge conservation, the equations of motion for infinitely small macroscopic volumes filled with electrons and holes, in which Lorentz forces arising due to the presence of the external magnetostatic field $\mathbf{H}_0$ are made allowance for, and the quasi-electrostatic Maxwell equation [11, 12]. Hence, the final system of dynamic equations will differ from the similar system used in work [3] only by the presence of Lorentz forces and the forces produced by the constant electric Hall field [11, 12],

$$\mathbf{E}_{(e,h)} = \frac{1}{c}\left[\mathbf{v}_{0(e,h)} \times \mathbf{H}_0\right] = \text{const}, \tag{5}$$

in the dynamic equations for the vectors $\mathbf{v}_{0(e,h)}$. In the linear approximation, those corrections have no significant effect.

Analogously to Ref. [3], solutions to the system of dynamic equations for the posed problem will be sought in the form of plane monochromatic waves,

$$n_e, n_h, \mathbf{v}_e, \mathbf{v}_h, \mathbf{E} \sim e^{i(\omega t - (\mathbf{k}\cdot\mathbf{r}))}. \tag{6}$$

In this case, the system of differential equations, similarly to Ref. [3], can be reduced to a system of linear algebraic equations

$$\begin{cases} [\omega - (\mathbf{k}\cdot\mathbf{v}_{0e})]\; n_e - (\mathbf{k}\cdot\mathbf{v}_e)\, n_{0e} = 0,\\ [\omega - (\mathbf{k}\cdot\mathbf{v}_{0h})]\; n_h - (\mathbf{k}\cdot\mathbf{v}_h)\, n_{0h} = 0,\\ \left(\left[\omega - (\mathbf{k}\cdot\mathbf{v}_{0e}) + i\omega_{0e}^{\times}\right]\cdot\mathbf{v}_e\right) = +\dfrac{iq}{m_e^*}\mathbf{E},\\ \left(\left[\omega - (\mathbf{k}\cdot\mathbf{v}_{0h}) + i\omega_{0h}^{\times}\right]\cdot\mathbf{v}_h\right) = -\dfrac{iq}{m_h^*}\mathbf{E},\\ i\varepsilon_\circ\,(\mathbf{k}\cdot\mathbf{E}) = 4\pi q\,(n_e - n_h), \end{cases} \tag{7}$$

where

$$\omega_{0(e,h)}^{\times} = \begin{bmatrix} 0 & -\omega_{0(e,h)_z} & \omega_{0(e,h)_y}\\ \omega_{0(e,h)_z} & 0 & -\omega_{0(e,h)_x}\\ -\omega_{0(e,h)_y} & \omega_{0(e,h)_x} & 0 \end{bmatrix},$$

$$\omega_{0(e,h)}^{\times}\mathbf{v}_{(e,h)} = \left[\boldsymbol{\omega}_{0(e,h)} \times \mathbf{v}_{(e,h)}\right],$$

$$\boldsymbol{\omega}_{0(e,h)} = \pm\frac{q\mathbf{H}_0}{m_{(e,h)}^* c}$$

are the Larmor frequencies of electrons (e) and holes (h), and $m_{(e,h)}^*$ are their effective masses.

From the first two equations in system (7), we can express the electron and hole concentrations $n_{(e,h)}$ in terms of the electron and hole velocities $\mathbf{v}_{(e,h)}$ as follows:

$$\begin{aligned} &n_{(e,h)} = \frac{n_{0(e,h)}}{\omega_{(e,h)}}(\mathbf{k}\cdot\mathbf{v}_{(e,h)}),\\ &\omega_{(e,h)} = \omega - (\mathbf{k}\cdot\mathbf{v}_{0(e,h)}). \end{aligned} \tag{8}$$

The solutions $\mathbf{v}_{(e,h)}$ of the third and fourth equations in system (7) can be written in the form

$$\mathbf{v}_{(e,h)} = \pm\frac{iq}{m_{(e,h)}^*}\left(\widehat{\Omega}_{(e,h)}^{-1}\mathbf{E}\right). \tag{9}$$

Here the tensor $\widehat{\Omega}_{(e,h)}^{-1}$ is inverse to the tensor

$$\widehat{\Omega}_{(e,h)} = \omega_{(e,h)} + i\omega_{0(e,h)}^{\times}$$

and looks like

$$\widehat{\Omega}_{(e,h)}^{-1} = \frac{\omega_{(e,h)} - i\omega_{0(e,h)}^{\times}}{\omega_{(e,h)}{}^2 - \boldsymbol{\omega}_{0(e,h)}{}^2} - {} - \frac{\boldsymbol{\omega}_{0(e,h)} \otimes \boldsymbol{\omega}_{0(e,h)}}{\omega_{(e,h)}\left(\omega_{(e,h)}{}^2 - \boldsymbol{\omega}_{0(e,h)}{}^2\right)}$$

or

$$\widehat{\Omega}_{(e,h)}^{-1} = \frac{\omega_{(e,h)}}{\Delta_{(e,h)}^2} - \frac{\boldsymbol{\omega}_{0(e,h)} \otimes \boldsymbol{\omega}_{0(e,h)}}{\omega_{(e,h)}\Delta_{(e,h)}^2} - \frac{i\omega_{0(e,h)}^{\times}}{\Delta_{(e,h)}^2},$$

where

$$\Delta_{(e,h)}^2 = \omega_{(e,h)}^2 - \left[\boldsymbol{\omega}_{0(e,h)}\right]^2.$$

After substituting expression (9) for the velocities $\mathbf{v}_{(e,h)}$ of electron and hole motion into Eq. (8), we obtain the dependences of the electron and hole concentrations on the electric field $\mathbf{E}$,

$$n_{(e,h)} = \pm\frac{iqn_{0(e,h)}}{\varepsilon_\circ m_{(e,h)}^*\omega_{(e,h)}}\left(\mathbf{k}\cdot\widehat{\Omega}_{(e,h)}^{-1}\mathbf{E}\right). \tag{10}$$

If we substitute this expression into the last equation in the system of linear algebraic equations (7), we obtain the key formula

$$\mathbf{k}\cdot\left(\varepsilon_\circ - \frac{{\omega_{pe}}^2}{\omega_e}\widehat{\Omega}_e^{-1} - \frac{{\omega_{ph}}^2}{\omega_h}\widehat{\Omega}_h^{-1}\right)\mathbf{E} = 0 \tag{11}$$

to find the semiconductor permittivity; here

$$\omega_{p(e,h)}^2 = 4\pi\frac{n_{0(e,h)}q^2}{\varepsilon_\circ m^*_{(e,h)}}$$

are the plasmon cyclic frequencies of electrons and holes.

Now we write down Maxwell's equation for the potential electric field [11, 12],

$$(\nabla\mathbf{D}) = 0, \quad \mathbf{D} = \widehat{\varepsilon}\,\mathbf{E},$$

and specify it for the case of plane monochromatic waves. As a result, we obtain the following expression:

$$-i(\mathbf{k}\cdot\widehat{\varepsilon}\,\mathbf{E}) = 0. \tag{12}$$

By comparing this expression with formula (11), we arrive at the required formula for the dielectric constant of a semiconductor located in a magnetostatic field and through which a direct electric current flows:

$$\widehat{\varepsilon} = \varepsilon_\circ - \frac{\omega_{pe}^2}{\omega_e}\widehat{\Omega}_e^{-1} - \frac{\omega_{ph}^2}{\omega_h}\widehat{\Omega}_h^{-1}. \tag{13}$$

For further application of the semiconductor dielectric permittivity $\widehat{\varepsilon}$, in order to analyze the physical properties of hybrid quasiparticles and their dependence on the magnetostatic field $\mathbf{H}_0$, it is expedient to express formula (13) in the form

$$\widehat{\varepsilon} = \widehat{\varepsilon}^{(\mathbf{v}_0)} + \widehat{\chi}^{(\boldsymbol{\omega}_0)}, \tag{14}$$

where

$$\widehat{\varepsilon}^{(\mathbf{v}_0)} = \varepsilon_\circ - \varkappa_e - \varkappa_h, \quad \widehat{\chi}^{(\boldsymbol{\omega}_0)} = \widehat{\chi}^{(s)} + i\widehat{\chi}^{(a)},$$

$$\widehat{\chi}^{(s)} = \frac{\varkappa_e}{{\omega_e}^2}\boldsymbol{\omega}_{0e}\otimes\boldsymbol{\omega}_{0e} + \frac{\varkappa_h}{{\omega_h}^2}\boldsymbol{\omega}_{0h}\otimes\boldsymbol{\omega}_{0h},$$

$$\widehat{\chi}^{(a)} = \frac{\varkappa_e}{\omega_e}\omega_{0e}^{\times} + \frac{\varkappa_h}{\omega_h}\omega_{0h}^{\times},$$

$$\varkappa_{(e,h)} = \frac{{\omega_{p(e,h)}}^2}{{\Delta_{(e,h)}}^2}.$$

Whence, one can see that the dielectric permittivity $\widehat{\varepsilon}$ has a tensor character. The tensor $\widehat{\varepsilon}$ has a scalar part $\widehat{\varepsilon}^{(\mathbf{v}_0)}$ and a tensor part $\widehat{\chi}^{(\boldsymbol{\omega}_0)}$; the latter in turn consists of a symmetric, $\widehat{\chi}^{(s)}$, and an antisymmetric, $i\widehat{\chi}^{(s)}$, part.

3. Dispersion Law for Quasiparticles in a Semiconductor with a Direct Electric Current and Located in a Magnetostatic Field

To determine the dispersion equation for quasi-particles in semiconductors located in a magnetostatic field and through which a constant electric current flows, it is necessary to use Maxwell's wave equation [11, 12]

$$\widehat{W}\mathbf{E} = 0, \quad \widehat{W} = \mathbf{k}^2 - \mathbf{k}\otimes\mathbf{k} - \frac{\omega^2}{c^2}\widehat{\varepsilon} \tag{15}$$

for a plane monochromatic electromagnetic field in the Fourier space $(\omega, \mathbf{k})$.

In the case of isotropic medium, using the scalar and vector multiplications of wave equation (15) by the wave vector $\mathbf{k}$, this equation can be split (see Ref. [3]) into two independent equations that describe plasmons (the potential electric field) and plasmon-polaritons (the vortex electric field). But in the case of anisotropic medium, such an approach cannot be fully implemented. In particular, if we multiply wave equation (15) by the wave vector $\mathbf{k}$, we obtain the equation

$$-\frac{\omega^2}{c^2}\left(\mathbf{k}\cdot\widehat{\varepsilon}\,\mathbf{E}\right) = 0 \tag{16}$$

describing plasmons. At the same time, it is not possible to obtain an equation that would describe plasmon-polaritons from Eq. (15). Due to the tensor character of the dielectric permittivity $\widehat{\varepsilon}$, the potential and vortex electric fields are mixed. Therefore, wave equation (15) describes a combination of quasiparticles consisting of plasmons and plasmon-polaritons.

Let us derive a dispersion equation for plasmons in which the electric field is described by the formulas $\mathbf{E} = -\boldsymbol{\nabla}\varphi \to \mathbf{E} = \mathbf{k}\varphi_0$ and $\varphi_0 = \mathrm{const}$. By substituting these formulas into Eq. (16), we obtain the dispersion equation for plasmons,

$$(\mathbf{k}\,\widehat{\varepsilon}\,\mathbf{k}) = 0. \tag{17}$$

The dispersion equation of the combination of quasiparticles follows from the condition that the system of linear algebraic equations (15) has nontrivial solutions, i.e., from vanishing its main determinant,

$$\det\left(\widehat{W}\right) = 0. \tag{18}$$

The specification of condition (18) looks like

$$\mathbf{k}^2\big(\mathbf{k}\,\widehat{\varepsilon}\,\mathbf{k}\big) - \frac{\omega^2}{c^2}\Big(\big(\mathbf{k}\,\widehat{\varepsilon}\,\mathbf{k}\big)\,\mathrm{tr}\,\big(\widehat{\varepsilon}\big) - \big(\mathbf{k}\,\widehat{\varepsilon}\widehat{\varepsilon}\,\mathbf{k}\big)\Big) + \\ + \frac{\omega^4}{c^4}\det\big(\widehat{\varepsilon}\big) = 0. \tag{19}$$

Now, applying representation (14) for the tensor $\widehat{\varepsilon}$, we obtain the following final formula for the dispersion law for a quasiparticle combination of plasmons and plasmon-polaritons:

$$\varepsilon^{(\mathbf{v}_0)}\Big(\mathbf{k}^2 - \frac{\omega^2}{c^2}\varepsilon^{(\mathbf{v}_0)}\Big)^2 + \\ + \Big(\big(\mathbf{k}\,\widehat{\chi}^{(s)}\mathbf{k}\big) - \frac{\omega^2}{c^2}\varepsilon^{(\mathbf{v}_0)}\mathrm{tr}\big(\widehat{\chi}^{(s)}\big)\Big)\times \\ \times\Big(\mathbf{k}^2 - \frac{\omega^2}{c^2}\varepsilon^{(\mathbf{v}_0)}\Big) + \frac{\omega^2}{c^2}\big[\mathbf{k}\times\boldsymbol{\chi}^{(a)}\big]^2 - \\ - \frac{\omega^4}{c^4}\boldsymbol{\chi}^{(a)2}\Big(\varepsilon^{(\mathbf{v}_0)} + \mathrm{Tr}\big(\widehat{\chi}^{(s)}\big)\Big) = 0, \tag{20}$$

where

$$\boldsymbol{\chi}^{(a)} = \varkappa_e\frac{\boldsymbol{\omega}_{0e}}{\omega_e} + \varkappa_h\frac{\boldsymbol{\omega}_{0h}}{\omega_h}.$$

Dispersion equations (17) and (20) can be simplified in the case of special relative orientations of the vectors $\mathbf{k}$ and $\boldsymbol{\omega}_0 \sim \mathbf{H}_0$. In particular, the simplified version of dispersion equation (17) for plasmons takes the following form:

$$\mathbf{k}^2\varepsilon^{(\mathbf{v}_0)} + (\mathbf{k}\,\widehat{\chi}^{(s)}\mathbf{k}) = 0. \tag{21}$$

Dispersion equation (20) for the quasiparticle combination can be simplified if the vectors $\mathbf{k}$ and $\boldsymbol{\omega}_0$ are collinear ($\mathbf{k} \parallel \boldsymbol{\omega}_0$) or perpendicular ($\mathbf{k}\perp\boldsymbol{\omega}_0$) to each other. For instance, if $\mathbf{k} \parallel \boldsymbol{\omega}_0$, Eq. (20) takes the form

$$\Big(\varepsilon^{(\mathbf{v}_0)} + \mathrm{Tr}\big(\widehat{\chi}^{(s)}\big)\Big)\times \\ \times\Big(\Big(\mathbf{k}^2 - \frac{\omega^2}{c^2}\varepsilon^{(\mathbf{v}_0)}\Big)^2 - \frac{\omega^4}{c^4}\boldsymbol{\chi}^{(a)2}\Big) = 0. \tag{22}$$

The first factor in Eq. (22) coincides with dispersion equation (21) for plasmons, provided that $\mathbf{k} \parallel \boldsymbol{\omega}_0$, i.e., its equality to zero determines the dispersion equation for plasmons. Obviously, the equality to zero of the second factor in Eq. (22) determines two dispersion branches for plasmon-polaritons.

If the condition $\mathbf{k}\perp\boldsymbol{\omega}_0$, is satisfied, then dispersion equation (20) takes the following form:

$$\varepsilon^{(\mathbf{v}_0)}\Big(\mathbf{k}^2 - \frac{\omega^2}{c^2}\varepsilon^{(\mathbf{v}_0)}\Big)^2 - \\ - \frac{\omega^2}{c^2}\Big(\varepsilon^{(\mathbf{v}_0)}\mathrm{tr}\big(\widehat{\chi}^{(s)}\big) - \boldsymbol{\chi}^{(a)2}\Big)\Big(\mathbf{k}^2 - \frac{\omega^2}{c^2}\varepsilon^{(\mathbf{v}_0)}\Big) - \\ - \frac{\omega^4}{c^4}\boldsymbol{\chi}^{(a)2}\mathrm{tr}\big(\widehat{\chi}^{(s)}\big) = 0. \tag{23}$$

In this case, one cannot consider plasmons and plasmon-polaritons separately.

In the absence of magnetic field, $\boldsymbol{\omega}_0 \sim \mathbf{H}_0 = 0$, and dispersion equation (20) takes the expected form of the dispersion equation for electromagnetic waves in an isotropic semiconductor,

$$\varepsilon^{(\mathbf{v}_0)}\Big(\mathbf{k}^{\,2} - \frac{\omega^2}{c^2}\varepsilon^{(\mathbf{v}_0)}\Big)^2 = 0.$$

4. Dispersion Law for Plasmons in a Semiconductor with a Direct Electric Current and Located in a Magnetostatic Field

To simplify the analysis of the dispersion equation for plasmons in the given problem formulation, we specify Eq. (21) in the form

$$\mathbf{k}^2\big(\varepsilon_\circ - \varkappa_e - \varkappa_h\big) + \\ + \Big(\frac{(\mathbf{k}\cdot\boldsymbol{\omega}_{0e})^2}{{\omega_e}^2}\varkappa_e + \frac{(\mathbf{k}\cdot\boldsymbol{\omega}_{0h})^2}{{\omega_h}^2}\varkappa_h\Big) = 0, \tag{24}$$

or

$$\big(\varepsilon_\circ - \varkappa_e - \varkappa_h\big) + \\ + \Big(\frac{{\boldsymbol{\omega}_{0e}}^2}{{\omega_e}^2}\varkappa_e + \frac{{\omega_{0h}}^2}{{\omega_h}^2}\varkappa_h\Big)\cos^2(\theta) = 0, \tag{25}$$

where θ is the angle between the directions of the vectors $\mathbf{k}$ and $\boldsymbol{\omega}_0 \sim \mathbf{H}_0$.

The corresponding analysis showed that dispersion equation (25) has 12 solutions in the general case. Some of them are real-valued, and the others are complex conjugate. Therefore, a detailed analysis of the physical properties of plasmons can be done here only using the methods of computational mathematics.

To specify the problem parameters for model calculations, we chose germanium (Ge) as a semiconductor, in which the effective masses of electrons

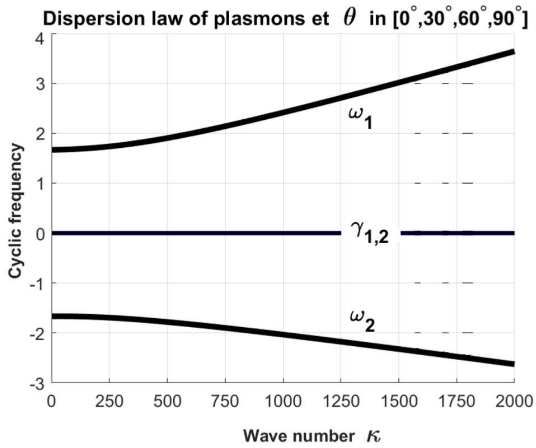

Fig. 1. Dispersion branches of ordinary plasmons in Ge

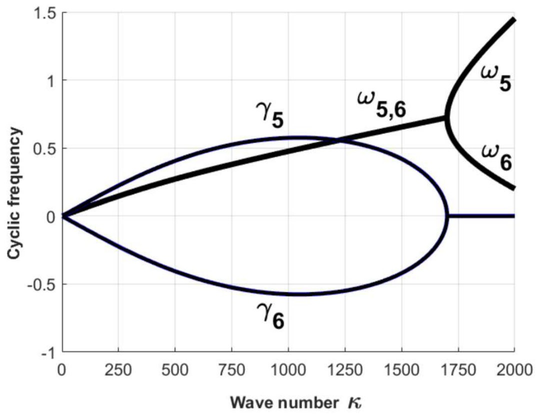

Fig. 2. Dispersion branches of hybrid plasmons at $\theta = 0°$

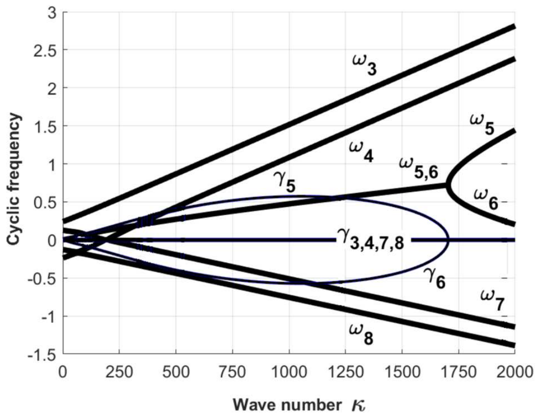

Fig. 3. The same as in Fig. 2, but at $\theta = 30°$

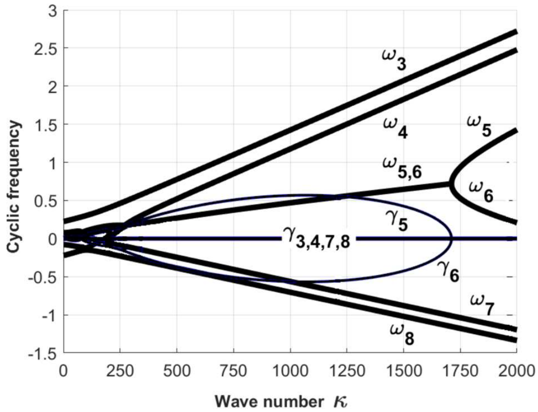

Fig. 4. The same as in Fig. 2, but at $\theta = 60°$

and holes are of the same order of magnitude (see Ref. [13]). Close values of the effective masses of electrons and holes are necessary for optimizing the parameters that determine the physical properties of hybrid quasiparticles (see Ref. [3]). As was shown in Ref. [3], hybrid quasiparticles in germanium correspond to the terahertz frequency interval of electromagnetic waves, $\omega = (10^{11} \div 1.5 \times 10^{13})$ Hz.

Numerical values of germanium parameters required for model calculations can be found in reference book [13]. The magnitude of the magnetostatic field strength $\mathbf{H}_0$ was chosen to be equal to about 10^4 Oe.

It is pertinent to perform numerical calculations in terms of dimensionless variables and choose the electron plasma frequency ω_{pe} and the velocity $\mathbf{v}_{0e}$ of the electron motion that arises due to the action of the electromotive force in the semiconductor as the measurement units.

Based on the data from reference book [13], the following set of quantities was selected and used in dimensionless numerical calculations of dispersion branches $\omega = \omega(\mathbf{k})$ for both plasmons and plasmon-polaritons:

$$\overline{\omega} = \frac{\mathrm{Re}(\omega)}{\omega_{pe}}, \quad \overline{\gamma} = -\frac{\mathrm{Im}(\omega)}{\omega_{pe}}, \quad \boldsymbol{\kappa} = \frac{\mathbf{k}c}{\omega_{pe}},$$
$$a = \frac{\omega_{ph}}{\omega_{pe}} = 0.866, \quad b = \frac{|\mathbf{v}_{0h}|}{|\mathbf{v}_{0e}|} = 0.487, \tag{26}$$
$$u = \frac{|\mathbf{v}_{0e}|}{c} = 1.3 \times 10^{-4}, \quad \mu = \frac{\omega_{0(h)}}{\omega_{0(e)}} = 0.75.$$

The real part $\overline{\omega} = \overline{\omega}(\boldsymbol{\kappa})$ of each solution of the dispersion equations describes the cyclic frequency of quasiparticles, and the imaginary part $\overline{\gamma} = \overline{\gamma}(\kappa)$ characterizes the degree of their instability. The results of calculations of the dispersion branches for plasmons are presented in Figs. 1–5.

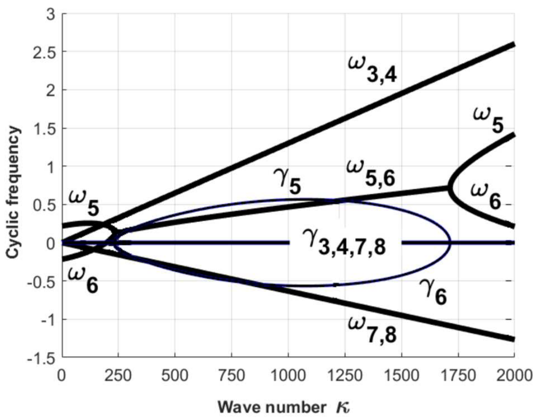

Fig. 5. The same as in Fig. 2, but at $\theta = 90°$

In Fig. 1, the dispersion branches of ordinary plasmons are shown, which were calculated for the following values of the angle $\theta = 0°, 30°, 60°$, and $90°$. The cyclic frequencies $\overline{\omega}_1$ correspond to electron plasmons, and the frequencies $\overline{\omega}_2$ to hole plasmons. In absolute value, the cyclic frequencies $\omega_{1,2}$ are larger than the corresponding plasma cyclic frequencies, i.e., they are in the optical spectral interval. The parameter values $\overline{\gamma}_{1,2} = 0$ indicate the stability of ordinary plasmons.

The dependence of the cyclic frequencies $\overline{\omega}_{1,2}$ on the parameter θ is negligibly weak, which testifies to a weak influence of the magnetostatic field $\mathbf{H}_0$ on the dynamics of ordinary plasmons. The absence of specular symmetry between the dispersion branches $\overline{\omega}_{1,2}$ follows from the difference between the effective masses of electrons and holes, $m_e^* < m_h^*$. In the case of hybrid plasmons, which are "genetically" related to both electrons and holes, and whose frequency band is in the terahertz interval (see Ref. [3]), the influence of the magnetostatic field $\mathbf{H}_0$ will be substantial.

The solutions obtained for dispersion equation (25) show that the relative orientation of the vectors $\mathbf{k}$ and $\boldsymbol{\omega}_0$ affects the number of solutions (see also Figs. 2–5). For instance, there are two branches of hybrid plasmons at $\theta = 0°$ (see Fig. 2) and six branches at $\theta \neq 0°$ (see Figs. 3–5). Furthermore, the dispersion branches of unstable plasmons at $\theta = 0°$ are similar to the corresponding dispersion branches obtained in Ref. [3]. This circumstance can be explained by the fact that in the case of collinear vectors $\mathbf{k}$ and $\boldsymbol{\omega}_0$, the Lorentz force has actually no influence on the longitudinal plasmon oscillations of the electric charge density.

If $\theta \neq 0°$ or $180°$, the influence of the Lorentz force on the plasmon dynamics becomes substantial. As a result, the number of dispersion branches of "hybrid" plasmons increases (see Figs. 3–5). In this case, the additional dispersion branches $\overline{\omega}_{3,4,7,8}$ correspond to stable plasmons.

By comparing the behavior of the dispersion branches in Figs. 3–5, one can see that the additional dispersion branches $\overline{\omega}_{3,4}$ and $\overline{\omega}_{7,8}$ approach each other as the angle θ increases and merge at $\theta = 90°$. Additionally, the region of plasmon instability $\overline{\omega}_{5,6}$ shifts towards the short-wavelength region in the plasmon spectrum. This process can be seen especially clearly in Fig. 5.

We also draw attention to the fact that the dispersion branches at $\theta = 90°$ satisfy the condition $\overline{\omega}_{3,4,7,8}(\mathbf{k} = 0) = 0$. This is in contrast to the dispersion currents at $\theta = 30°$ and $60°$, where $\overline{\omega}_{3,4,7,8}(\mathbf{k} = 0) \neq 0$.

5. Dispersion Law for Plasmon-Polaritons in a Semiconductor with a Direct Electric Current and Located in a Magnetostatic Field

To analyze the influence of the magnetostatic field $\mathbf{H}_0$ on the dynamics of plasmon-polaritons, we use dispersion equation (20). As the analysis has shown, the number of solutions to this equation and the dispersion behavior of the plasmon-polariton branches substantially depend on the relative orientation of the vectors $\mathbf{k}$ and $\boldsymbol{\omega}_0 \sim \mathbf{H}_0$. The maximum number of solutions to dispersion equation (20) equals 24. Some of them are real-valued, and the others are complex conjugate. The complex conjugate solutions describe unstable "hydride" plasmon-polaritons.

Six solutions of Eq. (20) correspond to ordinary plasmon-polaritons. The corresponding dispersion branches are shown in Figs. 6–8. In Fig. 6, the dispersion branches $\omega_{1,2}$ correspond to ordinary longitudinal plasmons, and the branches $\omega_{e(1,2)}$ and $\omega_{h(1,2)}$ to ordinary transverse plasmon-polaritons. The electron, $\omega_{e(1,2)}$, and hole, $\omega_{h(1,2)}$, dispersion branches differ from each other by an infinitesimal value due to magnetostatic splitting.

In the short-wavelength interval of the plasmon-polariton spectrum, ordinary transverse plasmon-

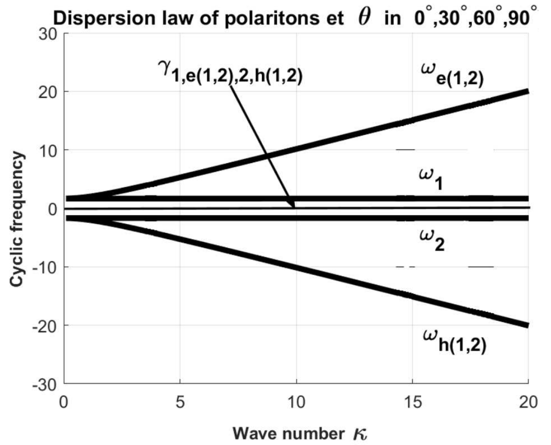

Fig. 6. Dispersion branches of ordinary plasmons and plasmon-polaritons at $\theta = 0°$, $30°$, $60°$, and $90°$

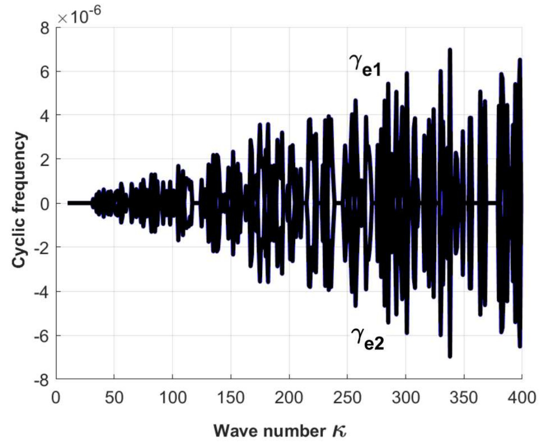

Fig. 7. The parameter γ_{e1} is the increment of growth, and γ_{e2} is the decrement of damping of the amplitude of ordinary electron plasmon-polaritons due to their pseudo-chaotic instability at $\theta = 0°$

polaritons exhibit a pseudo-chaotic instability (see Figs. 7 and 8). Analogous dependences of the instability parameters $\gamma_{e(1,2)}$, $\gamma_{h(1,2)}$ of ordinary plasmon-polaritons on the vector $\mathbf{k}$ are also observed if $\theta \neq 0°$. As concerning hybrid plasmon–polaritons, the dispersion branches $\omega_{5,6}(\boldsymbol{\kappa})$ obtained for $\theta = 0°$ and shown in Fig. 2 are supplemented by 8 stable dispersion branches $\omega_{3,4,7,8,11,12,17,18}(\boldsymbol{\kappa})$ (see Fig. 9). That is, in general, due to the influence of the magnetostatic field $\mathbf{H}_0$, there are 16 quasiparticle dispersion branches at $\theta = 0°$.

If $\theta \neq 0°$ and $90°$, the number of dispersion branches of hybrid plasmon-polaritons increases and becomes equal to 18 (see Figs. 10–13). In total, due

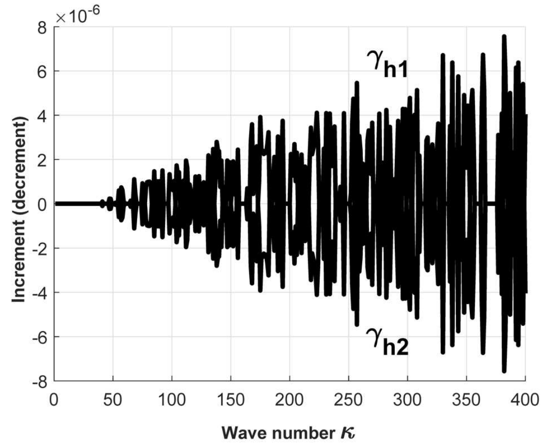

Fig. 8. The parameter γ_{h1} is the increment of growth, and γ_{h2} is the decrement of damping of the amplitude of ordinary hole plasmon-polaritons due to their pseudo-chaotic instability at $\theta = 0°$

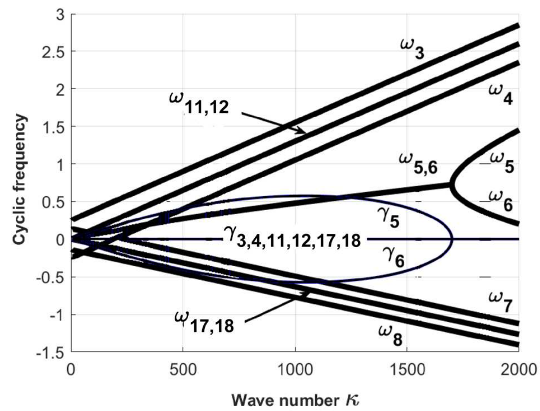

Fig. 9. Dispersion branches of hybrid plasmon-polaritons at $\theta = 0°$

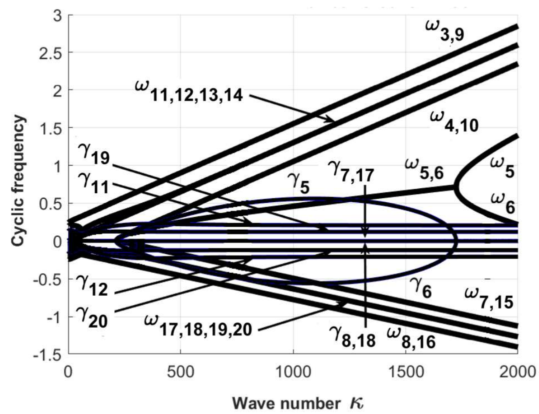

Fig. 10. The same as in Fig. 9, but at $\theta = 30°$

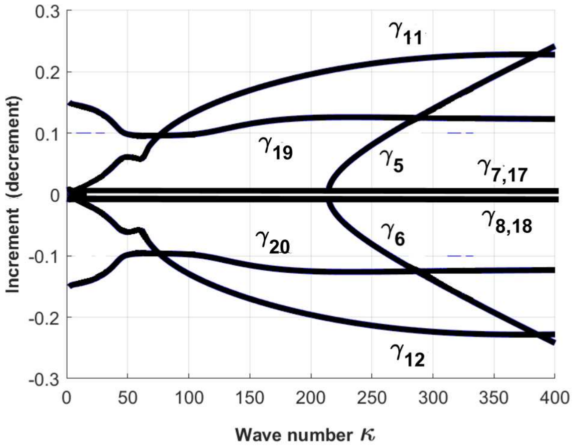

Fig. 11. The instability parameters of hybrid plasmon-polaritons at $\theta = 30°$ enlarged view

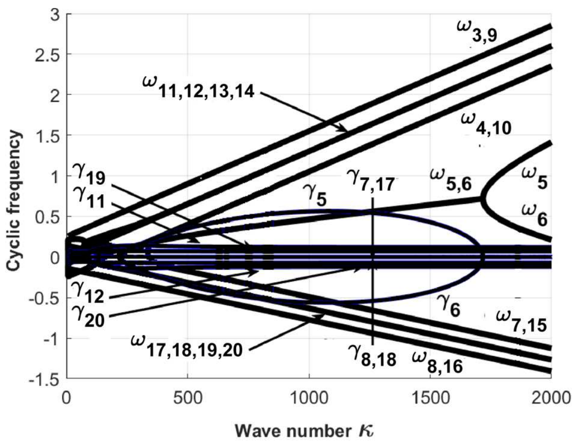

Fig. 12. The same as in Fig. 9, but at $\theta = 60°$

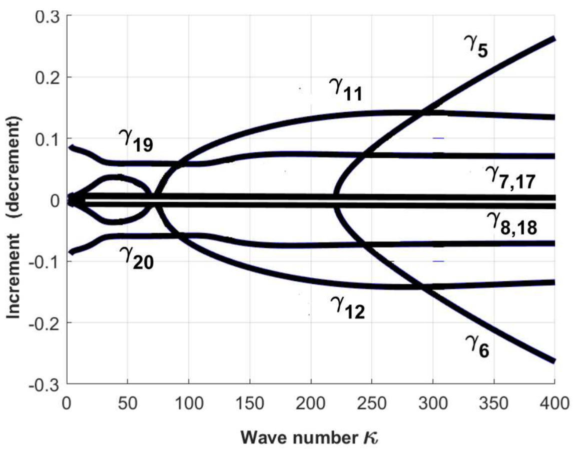

Fig. 13. The same as in Fig. 11, but at $\theta = 60°$

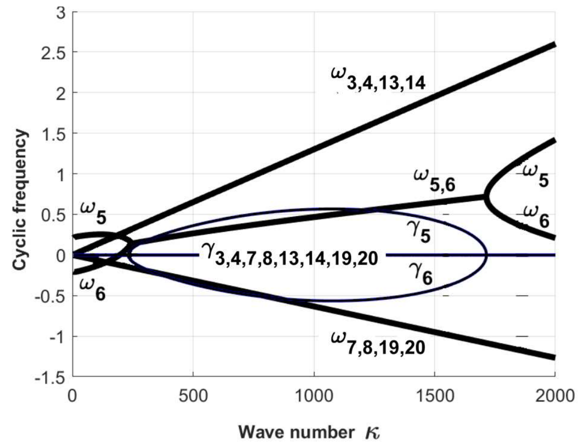

Fig. 14. The same as in Fig. 9, but at $\theta = 90°$

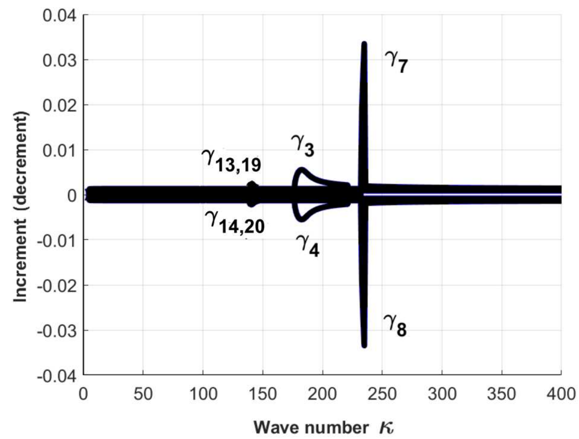

Fig. 15. The same as in Fig. 11, but at $\theta = 90°$

to the influence of the magnetostatic field $\mathbf{H}_0$, there are 24 dispersion branches of quasiparticles if $\theta \neq 0°$ and 90°.

As the value of the angle θ increases, the dispersion branches $\omega_{3,9,4,10}$ and $\omega_{7,15,8,16}$ approach each other (see Figs. 10–13), and the plasmon instability region $\omega_{5,6}$ shifts toward short wavelengths. As a result, the reconstruction of instability regions takes place (see Figs. 11 and 13). In addition, there appear new instability regions, in which the absolute values of instability parameters are orders of magnitude smaller than in the $\gamma_{5,6}$ region.

At $\theta = 90°$, the dispersion branches $\omega_{3,4,12,14}$ and $\omega_{7,8,19,20}$ merge, and Fig. 14 becomes visually similar to Fig. 5. In this case, additional plasmon-polaritons have small values of instability parameters (see Fig. 15).

6. Summary and Conclusions

In this paper, we have analyzed the effect of a magnetostatic field $\mathbf{H}_0$ on the dynamics of quasiparticles (plasmons and plasmon-polaritons) in semiconductors through which a direct electric current flows. It has been shown that, actually, the magnetostatic field $\mathbf{H}_0$ has a weak influence on the dispersion law of ordinary quasiparticles whose frequency band lies in the optical interval. In this case, a "pseudo-chaotization" of plasmon-polariton increments (decrements) in the short-wave side of their spectrum occurs due to the action of the magnetostatic field $\mathbf{H}_0$.

As concerning hybrid quasiparticles arising as a result of the counter-motion of continua of electrons and holes with a frequency band in the terahertz interval $\omega = (10^{11} \div 1.5 \times 10^{13})$ Hz, the influence of the magnetostatic field $\mathbf{H}_0$ on them is substantial. This can be explained by the fact that the Larmor frequencies of electrons and holes turn out close to the frequency bands of hybrid quasiparticles. It has been shown that the dispersion law and the increment (decrement) of the amplitude growth (decrease) for the dynamic variables of hybrid quasiparticles substantially depend on the stationary velocity of the charged particles' motion caused by a constant electric current, the relative orientation of the quasiparticle wave vector $\mathbf{k}$ and the magnetostatic field vector $\mathbf{H}_0$. In particular, the constant electric current is the physical cause of the appearance of unstable quasiparticles in a semiconductor in the terahertz frequency interval $\omega = (10^{11} \div 1.5 \times 10^{13})$Hz of electromagnetic waves. The magnetostatic field $\mathbf{H}_0$ induces a reconstruction of the dispersion branches of hybrid quasiparticles and excites additional frequency bands of hybrid quasiparticles, the number of which depends on the relative orientation of the quasiparticle wave vector $\mathbf{k}$ and the magnetostatic field vector $\mathbf{H}_0$.

The effect of quasiparticle hybridization in a semiconductor is considerably affected by the difference between the values of the electron and hole plasmon frequencies. As this difference increases, the degree of quasiparticle hybridization decreases. Note that the instability phenomenon for hybrid quasiparticles occurs in limited regions of the vector $\mathbf{k}$. Unstable quasiparticles have a certain propagation diagram, which affects their amplitude-frequency characteristics. The latter, in turn, are determined by factors such as the product $(\mathbf{k} \cdot \mathbf{v}_{0(e,h)})$ in formula (14) for the dielectric permittivity of a semiconductor through which a direct electric current flows.

The minimum number of dispersion branches for hybrid quasiparticles equals 4, and this value is realized under the condition of relative collinearity of the quasiparticle wave vector $\mathbf{k}$ and the magnetostatic field vector $\mathbf{H}_0$. This can be explained by the fact that in this case the influence of the magnetostatic field $\mathbf{H}_0$ is absent due to the well-known properties of Lorentz force [11, 12]. The maximum number of dispersion branches for hybrid quasiparticles equals 24, and this value is achieved under the condition that the angle between the vectors $\mathbf{k}$ and $\mathbf{H}_0$ is $\theta = 0°$, 90°, 180°, or 270°.

To analyze the excitation mechanisms of hybrid quasiparticles in a semiconductor as a result of a complex influence of the counter-motion of electrons and holes and the magnetostatic field $\mathbf{H}_0$ on the state of *eh*-plasma, a phenomenological (i.e., model-free) approach was applied. Therefore, the interpretation of the obtained results at the level of microprocesses occurring in semiconductors is senseless.

The availability of two factors influencing the dynamics of hybrid quasiparticles in semiconductors opens up the possibility of flexibly controlling the dynamics of unstable hybrid quasiparticles when this phenomenon is used to solve application problems in terahertz radiophysics.

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ВПЛИВ МАГНІТОСТАТИЧНОГО ПОЛЯ НА МЕХАНІЗМ ЗБУДЖЕННЯ ГІБРИДНИХ ПЛАЗМОН-ПОЛЯРИТОНІВ У НАПІВПРОВІДНИКАХ

Проаналізовано вплив магнітостатичного поля на динаміку квазічастинок (плазмонів та плазмон-поляритонів) у напівпровідниках, через які протікає постійний електричний струм. Зустрічний рух континуумів електронів і дірок призводить до появи нестійких гібридних квазічастинок, які "генетично" зв'язані як з електронами, так і з дірками. Показано, що закон дисперсії та інкремент (декремент) зростання (спадання) амплітуд динамічних змінних гібридних квазічастинок суттєво залежать від стаціонарної швидкості руху заряджених частинок, зумовленої постійним електричнним струмом, та взаємної орієнтації хвильового вектора квазічастинок та вектора магнітостатичного поля. Так, постійний електричний струм є фізичною причиною появи у напівпровіднику нестійких квазічастинок терагерцового частотного діапазону, а магнітостатичне поле викликає збудження додаткових частотних зон гібридних квазічастинок, кількість яких залежить від взаємної орієнтації хвильового вектора квазічастинок та вектора магнітостатичного поля. Наявність двох факторів впливу на динаміку гібридних квазічастинок у напівпровідниках відкриває можливість гнучкого керування динамікою нестійких гібридних квазічастинок у випадку використання цього явища при вирішенні прикладних задач у терагерцовій радіофізиці.

Ключові слова: плазма, електрони, дірки, електричне поле, магнітостатичне поле, густина електричного заряду, густина електричного струму, плазмонна частота, ефективна маса, поляризація, плазмони, поляритони, циклічна частота, хвильовий вектор, діелектрична проникність, дисперсійне рівняння, частотна зона, просторова дисперсія, нестійкість, інкремент зростання, декремент спадання.